THE DERIVE - NEWSLETTER #102

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THE BULLETIN OF THE

DERIVE

USER GROUP

+ CAS-TI

Contents:

July 2016

Statistics and Probability in High School

Carmen Batanero, Universidad de Granada, Spain, and Manfred Borovcnik, University of Klagenfurt, Austria.

In this book, the authors demonstrate how investigations and experiments provide promising teaching strategies to help high-school students acquire statistical and probabilistic literacy. The educational principles relevant for teaching statistics and probability form the backbone of the book and are established in an introductory chapter. In the other chapters (exploratory data analysis, modelling information by probabilities, exploring and modelling association, and sampling and inference), the development of a cluster of fundamental ideas is centred around a statistical study or a real-world problem that leads to statistical questions requiring data in order to be answered. The concepts developed are designed to lead to meaningful solutions rather than remain abstract entities. For each cluster of ideas, the authors review the relevant research on misconceptions and synthesise the results of research in order to support teaching of statistics and probability in high school.

What makes this book unique is its rich source of worked-through tasks and its focus on the interrelations between teaching and empirical research on understanding statistics and probability.

See the free preview of the first chapter on the educational principles in the background of the book:

https://www.sensepublishers.com/catalogs/bookseries/other-books/statistics-and-probability-in-high-school/

Eberhard Lehmann †

I received the sad message that our friend Eberhard Lehmann passed away.
Eberhard was not only an excellent mathematician and a devoted propagator of using modern technologies in mathematics education. He was an enthusiastic teacher in secondary school and a teachers' trainer as well. Eberhard was an excellent lecturer at pre- and in-service courses and author of numerous books and papers. You can find some of his "artistic" results produced with his program "animato" on pages 35 and 41.
But moreover we loose a wonderful person and very close friend. The picture shows Eberhard together with Manuela when we spent some great days in our mountains.
Our deep sympathy is with Manuela and Eberhard's family.

Josef & Noor
on behalf of the DUG Community

Dear DUG Members,

Our newsletter is approximately one month late this time. I was not able to finish it by end of June because there were a lot of preparations to do in advance of TIME 2016.

Then I spent a short week in Mexico attending this conference. We enjoyed a lot of great talks. Unfortunately TIME 2016 was not as well attended as Krems 2014 was. We don't know the reason, maybe that the date of the conference was too early in summer and many schools and universities had not finished their terms.

We had expected more delegates from South and Central America to participate, but they didn't. Hopefully next TIME better. The big question is, if there is any organisation willing to organize TIME 2018. We should relaunch TIME widening the scope (all CAS, all Dynamic Geometry and all Spreadsheet tools are welcome). Although this was valid for all earlier TIME Conferences, too, we should propagate that we don't are a community of "old fashioned DERIVIANs" sticking on a program which is off the market for years. Let's see what future will bring.

We don't have so many contributions in this issue. The main part is devoted to LAPLACE Transforms and Differential Equations. I didn't want to split this paper based on a talk of our Canadian friends from ETS Montreal. It was the first time for me to deal with this objective. It is a shame but I never had to cope with LT during my study and later as teacher. I had a very exciting time and many interesting hours producing the final version of this paper. Many thanks to Michel Beaudin for his never ending patience.

Duncan McDougall's contribution is from the opposite side – lower secondary school level and Wolfgang Alvermann offers an example from an end examination. I left this example in its German origin form but I promise to give a translation in the next DNL.

Please notice the info page containing the sad message about Eberhard Lehmann. He was a great advocate of technology supported math education.

Best regards until next time

Josef

Download all *DNL*-DERIVE- and TI-files from

http://www.austromath.at/dug/

All four parts of "CAS for Physics Examples" are now ready for download from

http://www.acdca.ac.at/

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a content of 40 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE*, *TI*-CAS and other CAS as well to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

Editor: Mag. Josef Böhm
D´Lust 1, A-3042 Würmla, Austria
Phone: ++43-(0)660 3136365
e-mail: nojo.boehm@pgv.at

Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: September 2016

Preview: Contributions waiting to be published

Some simulations of Random Experiments, J. Böhm, AUT, Lorenz Kopp, GER
Wonderful World of Pedal Curves, J. Böhm, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
Graphics World, Currency Change, P. Charland, CAN
Cubics, Quartics – Interesting features, T. Koller & J. Böhm, AUT
Logos of Companies as an Inspiration for Math Teaching
Exciting Surfaces in the FAZ / Pierre Charland´s Graphics Gallery
BooleanPlots.mth, P. Schofield, UK
Old traditional examples for a CAS – What´s new? J. Böhm, AUT
Where oh Where is It? (GPS with CAS), C. & P. Leinbach, USA
Mandelbrot and Newton with *DERIVE*, Roman Hašek, CZK
Tutorials for the NSpireCAS, G. Herweyers, BEL
Some Projects with Students, R. Schröder, GER
Dirac Algebra, Clifford Algebra, D. R. Lunsford, USA
A New Approach to Taylor Series, D. Oertel, GER
Henon & Co; Find your very own Strange Attractor, J. Böhm, AUT
Rational Hooks, J. Lechner, AUT
Statistics of Shuffling Cards, H. Ludwig, GER
Charge in a Magnetic Field, H. Ludwig, GER
Factoring Trinomials, D. McDougall, CAN

and others

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Robert Setif, France

Hello !

Here is a file "sudan.dfw" where my definition of this function answers, but that is wrong. Wikipedia[1] gives the definition of sudan and some values. The answers of DERIVE with my definition are wrong.

Best regards.

[1] https://en.wikipedia.org/wiki/Sudan_function

```
     sudan(n, x, y) :=
       If n = 0
          RETURN x + y
#1:       If y = 0
             RETURN x
             RETURN sudan(n - 1, sudan(n, x, y - 1), sudan(n, x, n - 1) + y)

#2:  sudan(0, 0, 194) = 194

#3:  sudan(0, 12, 46) = 58

#4:  sudan(1, 0, 3) = 6

#5:  sudan(1, 1, 3) = 10

#6:  sudan(1, 2, 3) = 14

#7:  sudan(2, 2, 2)

#8:  Memory exhausted!
```

Fred J. Tydeman [tydeman@TYBOR.COM]

I want matrix q[0..5, 0..3]
I want to set elements of q[] from high index down to low index, like:

```
for a = amax downto 0 by -1
  for b = bmax downto 0 by -1
    q[a,b] = f(a,b)
  end b
end a
```

where f(a,b) is a function. How do that in Derive Windows 6.1 or Derive Dos 4.11?

Also, when I ask for Help in Derive Windows, I get a popup saying Help is not supported. Is there a way to get that to work in 64-bit Windows 10?

Without the Help, I cannot look up Prog().

Ignacio Larrosa Cañestro [ilarrosa@MUNDO-R.COM]

In one line to copy in the input line:

```
VECTOR(VECTOR(f(a,b),b,bmax,0,-1),a,amax,0,-1)
```

amax, bmax and f(a,b) must be defined.

Or in a prog, to copy in the input line (f(a,b) must be defined):

```
fq(amax, bmax):=PROG(q:=VECTOR(VECTOR(f(a,b),b,bmax,0,-1),a,amax,
                0,-1),RETURN q)
```

The statement `RETURN q` is optional, the function gives the last assignation if no there is no explicit `RETURN` statement,.

Fred Tydeman

Sorry, but that will not work (I already tried it).
It has two problems:

That matrix has bounds of 1...bmax+1 and 1...amax+1 (there may be no way around that limitation as I believe all vectors have a lower index of 1 in Derive)

`q sub 1 sub 1` is f(amax,bmax).

(I want q sub 1 sub 1 to be f(1,1) or f(0,0))

Is there a way to set (assign to) just one element of a 2D MxN matrix? Something like:

```
q[a,b] := f(a,b)
```

Ignacio Larrosa Cañestro

Yes,

```
q sub a sub b := f(a,b)
```

The matrix must be defined with adecuated dimensions.

Saludos,

A Coruña (España)
ilarrosa@mundo-r.com
http://www.xente.mundo-r.com/ilarrosa/GeoGebra/

Fred Tydeman

Interesting. That works in Derive for Windows 6.10 but gets a syntax error in Derive for DOS 4.11
Anyone know of a list of the features added in the various versions?

DNL: What concerns the Online Help under in WINDOWS 10.

There is a patch for WINDOWS 10. DUG Member Günter Schoedl gave the advice:

The archive winhlp32-windows-10.zip is contained in the package MTH101.zip, which can bed downloaded from the DUG website (see below).

Unzip the archive and then run install.cmd as administrator.

Fred Tydeman

That helped a lot. Derive's Help mostly works now.
However the Search "button" does nothing. One can search the Index.

Before I ran into this lack of Help working in Windows 10, I had a problem installing Derive 6 into Windows 10. I have an external DVD reader attached to an ultrabook laptop. I ran Setup four times in Windows. The first three times, it hung on the same file; but, ran to completion on the fourth try.

Heinz Rainer Geyer

That´s really a great help with the help by Günter Schoedl. Thanks a lot for that contribution.

Laplace Transforms, Differential Equations and Systems of Differential Equations

Chantal Trottier, Lars Fredericksen, Philippe Fortin,
Michel Beaudin & Josef Böhm

Preparing my presentation for TIME 2016 "Computer Algebra Systems for Physics" together with working on "CAS for Physics Examples" (see Letter of the Editor) I became more and more involved in differential equations describing physical procedures.

In order to accomplish my lecture I asked Michel Beaudin from ETS Montreal for one or the other example using his Nspire tools (solving circuits and other problems). I wanted to compare how to obtain the solutions using various CAS (DERIVE, TI-*n*spireCAS, Maxima, …).

Michel sent one of his "*devoirs*" (home work for the students) – fortunately together with the solutions. Two of the tasks were solved by differential equations containing the unit step function and the Dirac Delta function respectively. This needs – *Laplace Transform*.

So I had to deal with Laplace transform and inverse Laplace transform …

Fortunately I found a paper written by Chantal Trottier in July 2011 describing functions and programs collected in a library ETS_specfunc.tns for TI-NspireCAS. The functions and programs are based on a package originally produced for the Voyage 200 by Lars Fredericksen (from Denmark) adapted for TI-NspireCAS by Philippe Fortin (Lycée Loius Barthou, Pau, France).

Respective websites are:

Chantal's paper: http://seg-apps.etsmtl.ca/nspire/documents/transf%20Laplace%20prog.pdf

Download ETS_specfunc.tns: http://cours.etsmtl.ca/seg/mbeaudin/ETS_specfunc.tns

Original version for V200: http://www.seg.etsmtl.ca/ti/laplace.html

These are the links to the original sources (in French):

http://www.univers-ti-nspire.com/

http://www.univers-ti-nspire.fr/files/pdf/TI-Nspire_chap17_capes.pdf

http://www.univers-ti-nspire.fr/activites.php?lang=&ress_id=82

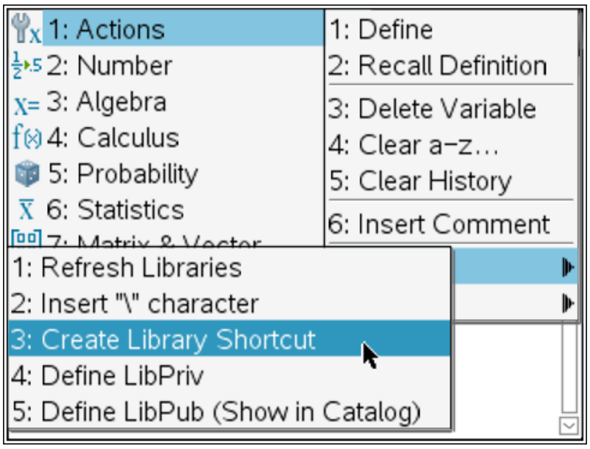
http://www.univers-ti-nspire.com/reussir-en-prepa/prepa-bibliotheques-de-progammes

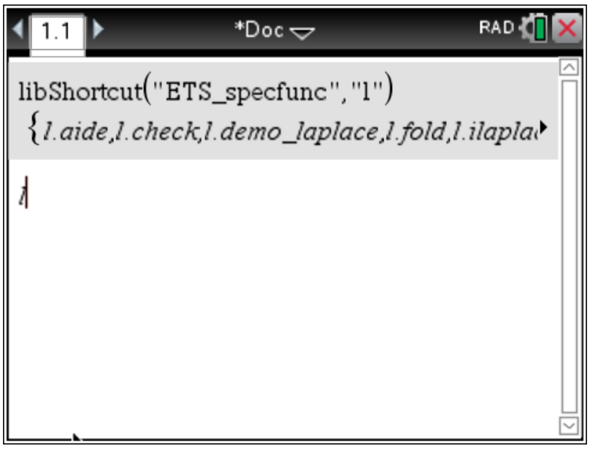
In the following I will try to present this library following Chantal's paper (originally written in French) solving some problems and using other tools, too.

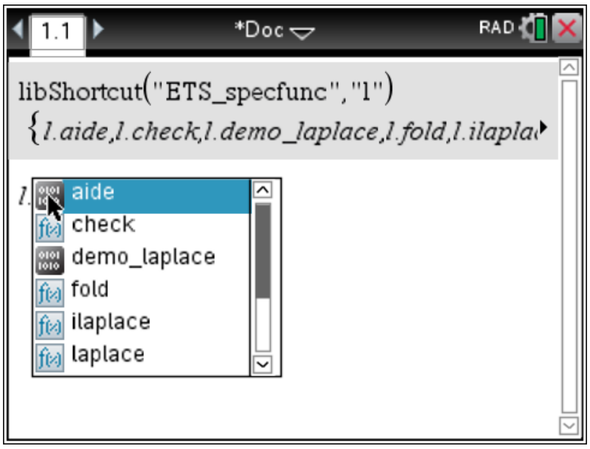
You can find more links on page 24.

Shortcut

It is recommended to store ETS_specfunc.tns in the MyLib folder and create a shortcut for calling the functions and programs provided in the library.





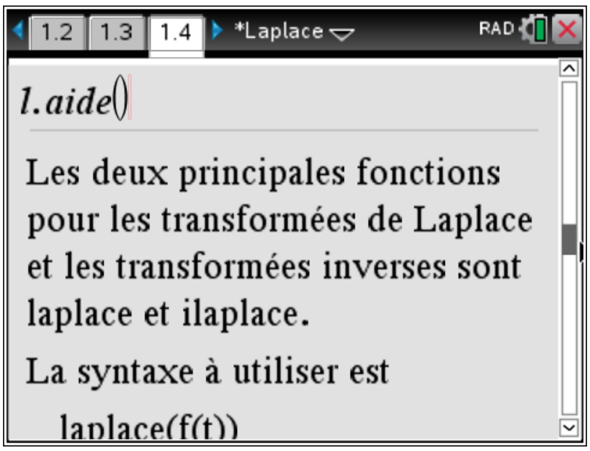


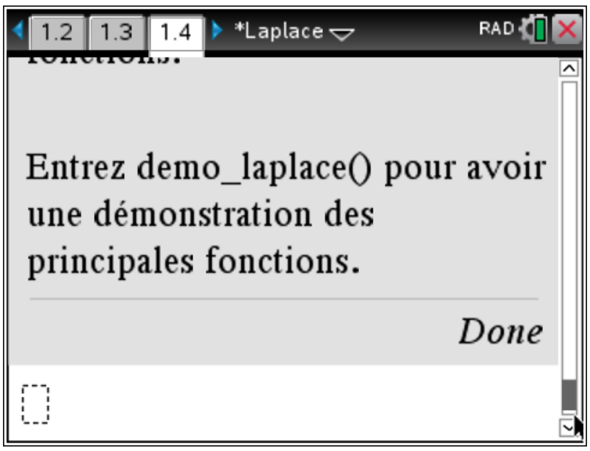
Calling one of the functions contained in this library is easy: just enter l. and a menu is offering all tools provided:

aide, check, demo_laplace, fold, iLaplace, laplace, solved,simultd and fold.

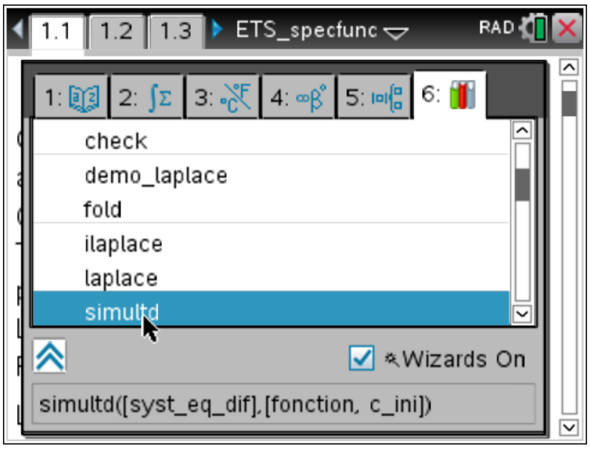
aide() and **demo_laplace()**

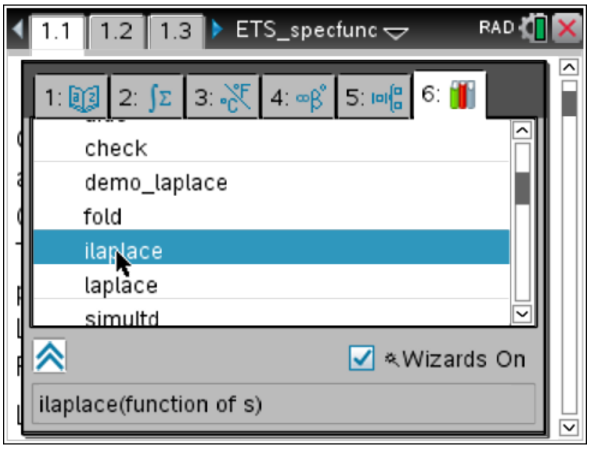
aide() gives a short introduction and the invitation to run demo_laplace(), which demontrates some features of the library.





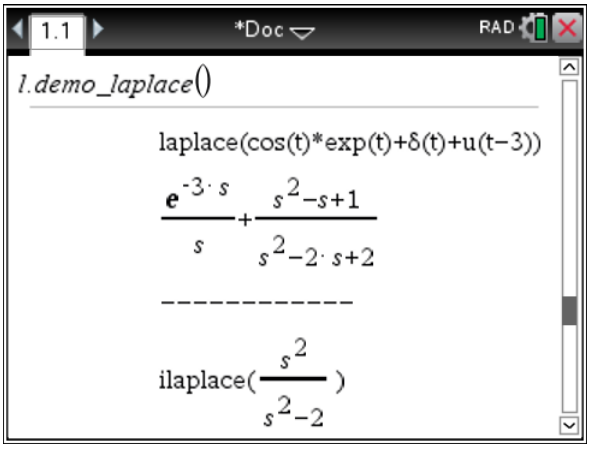
If you install ETS_specfunc_e.tns instead of ETS_specfunc.tns then you will receive English syntax hints when calling the tools from the library.

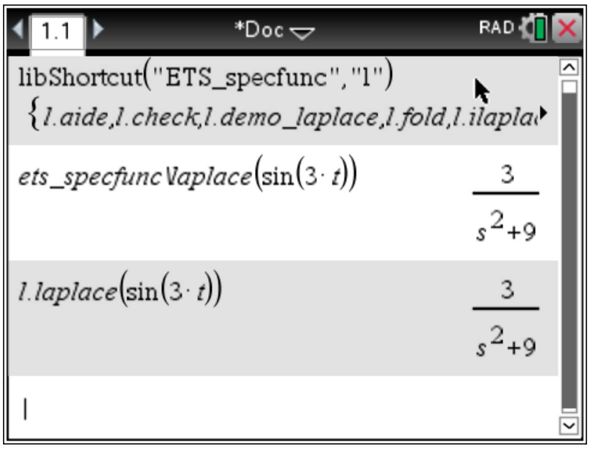




laplace(function of *t*)

For obtaining the Laplace transform of a function it is necessary to enter this function as a function of *t*. The answer is given as a function of *s*. See the following examples:





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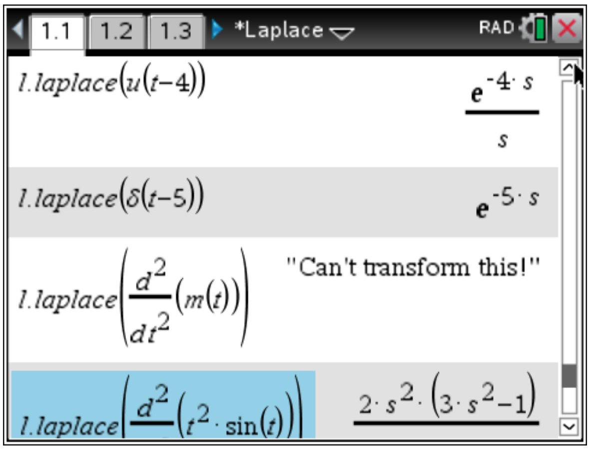
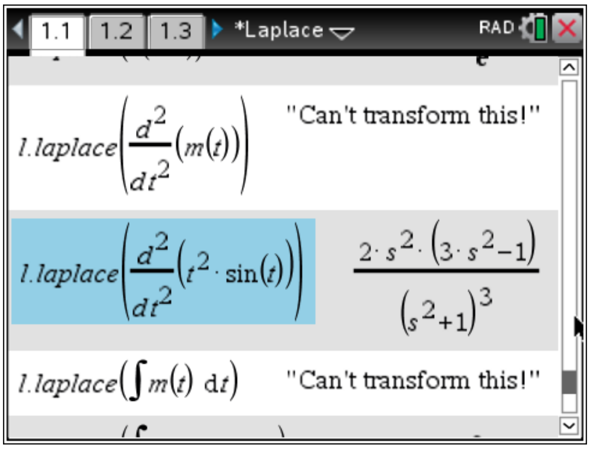
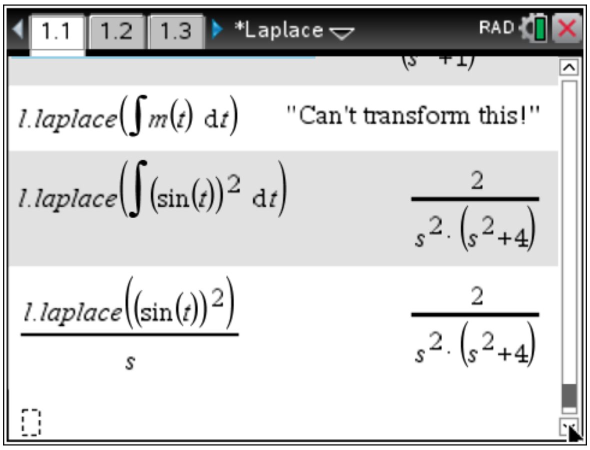




Unlike *DERIVE* the tools provided enable transforming the unit-step function (Heaviside) and the Dirac-Delta function as well. We cannot transform the derivatives and integral of a generic function (but it is possible in connection with solving differential equation, which will be presented later).



The rules for transforming the special functions are well known:

$$laplace(u(t-a)) = \frac{e^{-as}}{s}$$
$$laplace(\delta(t-a)) = e^{-as}$$

Transformation of the integral of a given function is possible giving the correct result

$$L\left(\int f(t)dt\right) = \frac{L(f(t))}{s} = \frac{F(s)}{s}.$$

There is a file `LaplaceTransforms.dfw` contained in the Derive6/Users/Transforms/ folder which was written by Terence Etchells in 2002/2003 which performs the transformation for "regular" functions:

$$\text{Laplace}(\text{SIN}(3\cdot t),\ t,\ s) = \frac{3}{s^2 + 9}$$

$$\text{Laplace}\left(\int_0^t \text{SIN}(a)\ da,\ t,\ s\right) = \frac{1}{s\cdot(s^2 + 1)}$$

$$\text{Laplace}(\text{STEP}(t - 4),\ t,\ s) = \frac{\int_0^\infty e^{-s\cdot t}\cdot\text{SIGN}(t - 4)\ dt}{2} + \frac{1}{2\cdot s}$$

Let's compare the first steps with LAPLACE using *Maxima* (see link on page 32):

(%i2) laplace(sin(3*t),t,s);

(%o2) $\frac{3}{s^2+9}$

(%i24) laplace(unit_step(t-4),t,s);

(%o24) $\frac{\%e^{-4\,s}}{s}$

(%i4) laplace(delta(x-5),x,y);

(%o4) $\%e^{-5\,y}$

(%i5) laplace(diff(f(t),t,2),t,s);

(%o5) $-\left.\frac{d}{d\,t}\,f(t)\right|_{t=0} + s^2\,\text{laplace}(f(t),t,s) - f(0)\,s$

As we can see *Maxima* is very powerful what concerns Laplace transforms (although it needs a little trick for the integral transform).

```
(%i6)  laplace(integrate(f(t),t),t,s);
```

(%o6) $\mathrm{laplace}\left(\int f(t)\,dt\;, t, s\right)$

```
(%i7)  laplace(integrate(f(τ),τ,0,t),t,s);
```

(%o7) $\dfrac{\mathrm{laplace}\left(f(t), t, s\right)}{s}$

```
(%i23)  ratsimp(laplace(integrate(sin(t)^2,t),t,s));
```

(%o23) $\dfrac{2}{s^4+4\,s^2}$

```
(%i17)  laplace((sin(t)^2),t,s)/s;
```

(%o17) $\dfrac{2}{s\left(s^3+4\,s\right)}$

I let perform *DERIVE* the Laplace transform of the second derivative of a generic function $x(t)$:

#1: x(t) :=

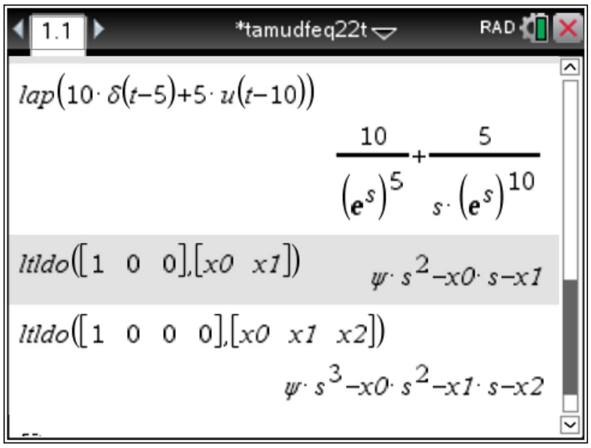
#2: $\mathrm{LAPLACE}\left(\left(\frac{d}{dt}\right)^2 x(t),\ t,\ s\right)$

#3: $-\ \mathrm{SUBST}(x'(t),\ t,\ 0) + s^2\cdot\int_0^{\infty} e^{-\ s\cdot t}\cdot x(t)\ dt + \left(\lim_{t\to\infty} e^{-\ s\cdot t}\cdot(x'(t) + s\cdot x(t))\right) - s\cdot x(0)$

Considering that the limit usually tends to zero, we receive the correct result. Compare with the *Maxima* expression %o5 from above.

Look at this:

Preparing this article I did some research in the web and came across the website of Art Belmonte from the Texas A & M University.



Replace ψ by Y and you will receive the common form of the Laplace transforms of x''(t) and x'''(t):

Y s^2 – s x(0) – x'(0) and

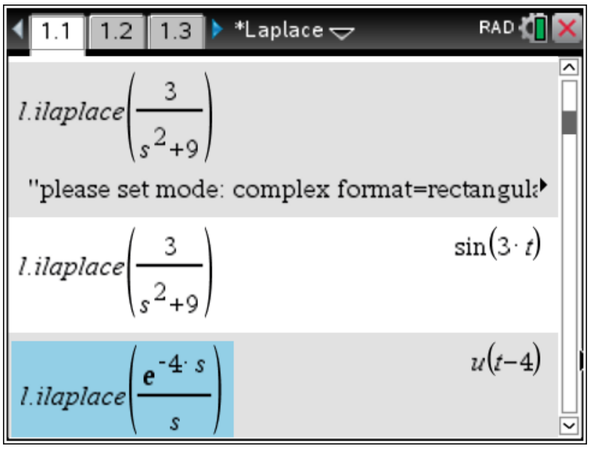
$Y s^3 - s^2 x(0) - s x'(0) - x''(0)$.

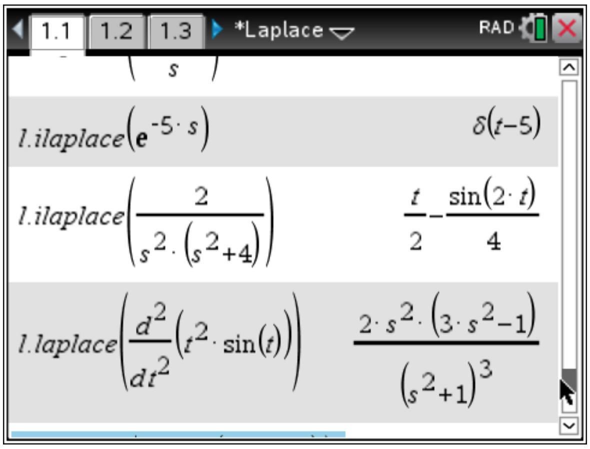
You can download another couple of TI-NspireCAS libraries for treating differential equations with and without applying Laplace transforms from Belmonte's website (see page 32):

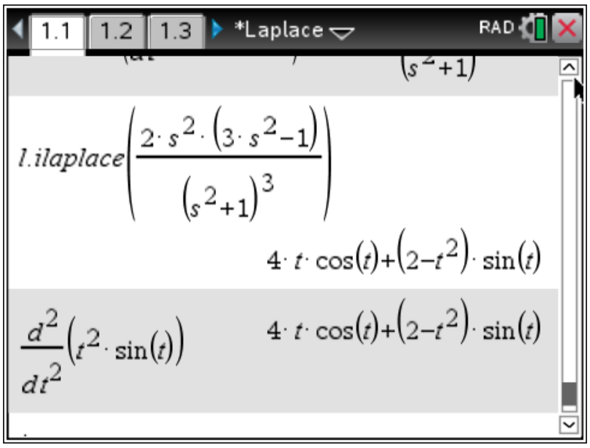
Let's go back to TI-NspireCAS and explore the next function, which is

ilaplace(function of *s*)

Next tool – not surprisingly – is the reverse Laplace transform:







It is interesting that *Maxima* is able to transform unit step and delta function as well but is not able to perform the reverse transformation leading to a step function.

(%i1) `ilt(3/(s^2+9),s,t);`

(%o1) $\sin(3\,t)$

(%i2) `ilt(%e^(-4*s)/s,s,t);`

(%o2) $\mathrm{ilt}\left(\frac{\%e^{-4\,s}}{s}, s, t\right)$

(%i3) `ilt(%e^(-5*s),s,t);`

(%o3) $\mathrm{ilt}\left(\%e^{-5\,s}, s, t\right)$

(%i4) `ilt(2/(s*(s^3+4*s)),s,t);`

(%o4) $\frac{t}{2}-\frac{\sin(2\,t)}{4}$

(%i5) `ilt(2*s^2*(3*s^2-1)/(s^2+1)^3,s,t);`

(%o5) $-t^2\sin(t)+2\sin(t)+4\,t\cos(t)$

Please compare with the results achieved applying `ilaplace` above.

This is the answer of *DERIVE* applying Terence's function:

$$\text{INVLaplace}\left(\frac{2\cdot s^2\cdot(3\cdot s^2-1)}{(s^2+1)^3},\ s,\ t\right) = 4\cdot t\cdot \text{COS}(t) + (2 - t^2)\cdot \text{SIN}(t)$$

In the centre of all efforts lies the aim to solve differential equations. All CAS provide tools supporting this goal. ETS_specfunc.tns offers

solved(diff_eq, {function, ini_values})

This program enables solving differential equations and integro-differential equations as well. The program performs a Laplace transform of the given equation, solves the resulting linear equation and finally performs the inverse Laplace transformation to obtain the solution of the given DE. The order of the DE is of no importance, it is only limited by the memory of the calculator (computer). For a too complex equation the message "Out of memory" is displayed.

The derivatives and the integrals must be entered by the templates provided. The function must be entered together with its argument like *f*(*x*), *f*(*t*), etc.

The initial conditions must be given in the list {…} following the function name in the order *f*(0), *f* '(0), *f* "(0), …

The best will be to demonstrate the program solving some problems:

Example 1

Solve the differential equation $\dfrac{d^2x}{dt^2}+2\dfrac{dx}{dt}+5x=\sin(2t),\ \ x(0)=1,\ \ x'(0)=3.$

$$\mathit{l.solved}\left(\frac{d^2}{dt^2}(x(t))+2\cdot\frac{d}{dt}(x(t))+5\cdot x(t)=\sin(2\cdot t),\{x(t),x0,x1\}\right)$$

$$x(t)=\left(\frac{x0+\frac{4}{17}}{e^t}-\frac{4}{17}\right)\cdot\cos(2\cdot t)+\left(\frac{\frac{x0}{2}+\frac{x1}{2}+\frac{1}{17}}{e^t}+\frac{1}{17}\right)\cdot\sin(2\cdot t)$$

$$\left(\frac{x0+\frac{4}{17}}{e^t}-\frac{4}{17}\right)\cdot\cos(2\cdot t)+\left(\frac{\frac{x0}{2}+\frac{x1}{2}+\frac{1}{17}}{e^t}+\frac{1}{17}\right)\cdot\sin(2\cdot t)|x0=1\text{ and }x1=3$$

$$\left(\frac{21\cdot e^{-t}}{17}-\frac{4}{17}\right)\cdot\cos(2\cdot t)+\left(\frac{35\cdot e^{-t}}{17}+\frac{1}{17}\right)\cdot\sin(2\cdot t)$$

$$\mathit{l.solved}\left(\frac{d^2}{dt^2}(x(t))+2\cdot\frac{d}{dt}(x(t))+5\cdot x(t)=\sin(2\cdot t),\{x(t),1,3\}\right)$$

$$x(t)=\left(\frac{21}{17\cdot e^t}-\frac{4}{17}\right)\cdot\cos(2\cdot t)+\left(\frac{35}{17\cdot e^t}+\frac{1}{17}\right)\cdot\sin(2\cdot t)$$

$$\text{deSolve}(x''+2\cdot x'+5\cdot x=\sin(2\cdot t),t,x)$$

$$x=\boldsymbol{c1}\cdot e^{-t}\cdot\cos(2\cdot t)+\boldsymbol{c2}\cdot e^{-t}\cdot\sin(2\cdot t)-\frac{8\cdot(\cos(t))^2}{17}+\frac{2\cdot\sin(t)\cdot\cos(t)}{17}+\frac{4}{17}$$

$$\text{deSolve}(x''+2\cdot x'+5\cdot x=\sin(2\cdot t)\text{ and }x(0)=1\text{ and }x'(0)=3,t,x)$$

$$x=\frac{21\cdot e^{-t}\cdot\cos(2\cdot t)}{17}+\frac{35\cdot e^{-t}\cdot\sin(2\cdot t)}{17}-\frac{8\cdot(\cos(t))^2}{17}+\frac{2\cdot\sin(t)\cdot\cos(t)}{17}+\frac{4}{17}$$

$$\text{tExpand}\left(\left(\frac{21}{17\cdot e^t}-\frac{4}{17}\right)\cdot\cos(2\cdot t)+\left(\frac{35}{17\cdot e^t}+\frac{1}{17}\right)\cdot\sin(2\cdot t)\right)$$

$$\frac{\left(-8\cdot e^t\cdot(\cos(t))^2+42\cdot(\cos(t))^2+2\cdot e^t\cdot\sin(t)\cdot\cos(t)+70\cdot\sin(t)\cdot\cos(t)+4\cdot e^t-21\right)\cdot e^{-t}}{17}$$

I did it in two ways and using two programs solved() and deSolve():

Solving first the general case followed by calculating the integration constants by substituting the initial conditions. (I skipped presenting calculating ***c1*** and ***c2*** in the fourth row.) Then I entered the ini-conds as additional parameters in the programs.

tExpand of both results shows the identity of both solutions.

You might ask, why working with solved() when deSolve() is a built-in function of TI-NspireCAS? (Presentation how TAMUDEQ is working will be shown later.) The next example will demonstrate the difference between the two tools.

Example 2

Solve the differential equation $\dfrac{d^2x}{dt^2}+5\dfrac{dx}{dt}+6=\begin{cases}3 \text{ for } 0\le t<6\\ 0 \text{ for } \quad t\ge 6\end{cases}$, $x(0)=0,\ x'(0)=2.$

First of all we have to rewrite the right hand side of the equation using the unit step (Heaviside) function as a sum of step functions: 3(step(t) – step(t – 6)). For the syntax of our programs we need to enter this as 3(u(t) – u(t – 6)). Built-in deSolve can neither handle the step function nor the delta function, that makes the difference.

$$l.solved\left(\frac{d^2}{dt^2}(x(t))+5\cdot\frac{d}{dt}(x(t))+6=3\cdot(u(t)-u(t-6)),\{x(t),0,2\}\right)$$

$$x(t)=\frac{\frac{-3\cdot e^{30}\cdot u(t-6)}{25}-\frac{13\cdot u(t)}{25}}{(e^t)^5}+u(t-6)\cdot\left(\frac{93}{25}-\frac{3\cdot t}{5}\right)+u(t)\cdot\left(\frac{13}{25}-\frac{3\cdot t}{5}\right)$$

$$x(t)=\frac{\frac{-3\cdot e^{30}\cdot u(t-6)}{25}-\frac{13\cdot u(t)}{25}}{(e^t)^5}+u(t-6)\cdot\left(\frac{93}{25}-\frac{3\cdot t}{5}\right)+u(t)\cdot\left(\frac{13}{25}-\frac{3\cdot t}{5}\right)$$

$$x(t)=e^{-5\cdot t}\cdot\left(\frac{-3\cdot e^{30}\cdot u(t-6)}{25}-\frac{13\cdot u(t)}{25}\right)+u(t-6)\cdot\left(\frac{93}{25}-\frac{3\cdot t}{5}\right)+u(t)\cdot\left(\frac{13}{25}-\frac{3\cdot t}{5}\right)$$

© solved partially "by hand"

$$s^2\cdot x-2+5\cdot s\cdot x+\frac{6}{s}=3\cdot\left(\frac{1}{s}-\frac{e^{-6\cdot s}}{s}\right) \qquad s^2\cdot x+5\cdot s\cdot x+\frac{6}{s}-2=\frac{3\cdot e^{-6\cdot s}\cdot\left(e^{6\cdot s}-1\right)}{s}$$

$$\text{solve}\left(s^2\cdot x+5\cdot s\cdot x+\frac{6}{s}-2=\frac{3\cdot e^{-6\cdot s}\cdot\left(e^{6\cdot s}-1\right)}{s},x\right) \qquad x=\frac{e^{-6\cdot s}\cdot\left((2\cdot s-3)\cdot e^{6\cdot s}-3\right)}{s^2\cdot(s+5)}$$

$$l.ilaplace\left(\frac{e^{-6\cdot s}\cdot\left((2\cdot s-3)\cdot e^{6\cdot s}-3\right)}{s^2\cdot(s+5)}\right) \qquad e^{-5\cdot t}\cdot\left(\frac{-3\cdot e^{30}\cdot u(t-6)}{25}-\frac{13\cdot u(t)}{25}\right)+u(t-6)\cdot\left(\frac{93}{25}-\frac{3\cdot t}{5}\right)+u(t)\cdot\left(\frac{13}{25}-\frac{3\cdot t}{5}\right)$$

We want to plot the solution function. So we factor out the u-functions:

$$x(t)=\left(\frac{13}{25}-\frac{3t}{5}-\frac{13e^{-5t}}{25}\right)\cdot u(t)+\left(\frac{93}{25}-\frac{3t}{5}-\frac{3e^{30-5t}}{25}\right)\cdot u(t-6).$$

It's clear that the result is a piecewise defined function. There are three ways to make the function term ready to be plotted

- rewrite it manually,
- use Michel Beaudin's step() – function (works like in *DERIVE*)
- use u_to_piece() which converts into a piecewise defined function.

step and u_to_piece are both contained in the library kit_ets_mb.tns.

I replace the *u*-functions by *uu*-functions and apply u_to_piece on this expression which gives *f1*(*x*).

Then I replace the *u*-functions by step functions giving *f2*(*x*).

$$s^2\cdot x-2+5\cdot s\cdot x+\frac{6}{s}=3\cdot\left(\frac{1}{s}-\frac{e^{-6\cdot s}}{s}\right) \qquad s^2\cdot x+5\cdot s\cdot x+\frac{6}{s}-2=\frac{3\cdot e^{-6\cdot s}\cdot\left(e^{6\cdot s}-1\right)}{s}$$

$$\text{solve}\left(s^2\cdot x+5\cdot s\cdot x+\frac{6}{s}-2=\frac{3\cdot e^{-6\cdot s}\cdot\left(e^{6\cdot s}-1\right)}{s},x\right) \qquad x=\frac{e^{-6\cdot s}\cdot\left((2\cdot s-3)\cdot e^{6\cdot s}-3\right)}{s^2\cdot(s+5)}$$

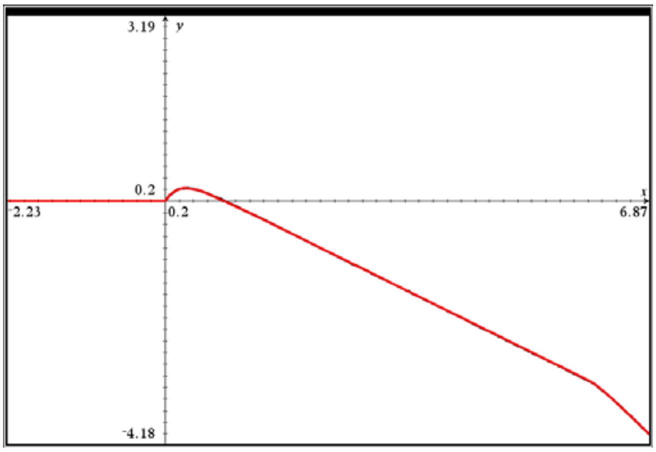
$$l.ilaplace\left(\frac{e^{-6\cdot s}\cdot\left((2\cdot s-3)\cdot e^{6\cdot s}-3\right)}{s^2\cdot(s+5)}\right) \qquad e^{-5\cdot t}\cdot\left(\frac{-3\cdot e^{30}\cdot u(t-6)}{25}-\frac{13\cdot u(t)}{25}\right)+u(t-6)\cdot\left(\frac{93}{25}-\frac{3\cdot t}{5}\right)+u(t)\cdot\left(\frac{13}{25}-\frac{3\cdot t}{5}\right)$$

$$f1(x):=m.u\_to\_piece\left(e^{-5\cdot x}\cdot\left(\frac{-3\cdot e^{30}\cdot m.uu(x-6)}{25}-\frac{13\cdot m.uu(x)}{25}\right)+m.uu(x-6)\cdot\left(\frac{93}{25}-\frac{3\cdot x}{5}\right)+m.uu(x)\cdot\left(\frac{13}{25}-\frac{3\cdot x}{5}\right),x\right) \qquad \textit{Done}$$

$$f1(x) \qquad \begin{cases}\dfrac{-e^{-5\cdot x}\cdot\left(2\cdot(15\cdot x-53)\cdot e^{5\cdot x}+3\cdot e^{30}+13\right)}{25}, & 6<x<\infty\\[2ex] \dfrac{-e^{-5\cdot x}\cdot\left((15\cdot x-13)\cdot e^{5\cdot x}+13\right)}{25}, & 0<x\le 6\\[2ex] 0, & -\infty<x\le 0\end{cases}$$

$$f2(x):=e^{-5\cdot x}\cdot\left(\frac{-3\cdot e^{30}\cdot m.step(x-6)}{25}-\frac{13\cdot m.step(x)}{25}\right)+m.step(x-6)\cdot\left(\frac{93}{25}-\frac{3\cdot x}{5}\right)+m.step(x)\cdot\left(\frac{13}{25}-\frac{3\cdot x}{5}\right) \qquad \textit{Done}$$

$$\mathbf{f2}(x):=e^{-5\cdot x}\cdot\left(\frac{-3\cdot e^{30}\cdot \mathbf{m.step}(x-6)}{25}-\frac{13\cdot \mathbf{m.step}(x)}{25}\right)+\mathbf{m.step}(x-6)\cdot\left(\frac{93}{25}-\frac{3\cdot x}{5}\right)+\mathbf{m.step}(x)\cdot\left(\frac{13}{25}-\frac{3\cdot x}{5}\right)$$



Both functions are plotted superimposed.

The plot is not very exciting.

Let's give the "competitors", *Maxima* and TAMUDFEQ a try.

As it is not able to find the ILT resulting in a step function, I invented my own invl-function according to the rule: $L(u(t-a)\,f(t-a)) = e^{-as}\,F(s)$.

```
(%i1)   invl(a_,f,s,t):=trigsimp(unit_step(t-a_)*ilt(f,s,t-a_))$
(%i12)  de:'diff(x(t),t,2)+5*'diff(x(t),t)+6=
             3*unit_step(t)-3*unit_step(t-6);
```

(de) $$\frac{d^2}{d\,t^2}\,x(t)+5\left(\frac{d}{d\,t}\,x(t)\right)+6=3\,\mathrm{unit\_step}(t)-3\,\mathrm{unit\_step}(t-6)$$

```
(%i3)   laplace(de,t,s);
```

(%o3) $$-\left.\frac{d}{d\,t}\,x(t)\right|_{t=0}+5\left(s\,\mathrm{laplace}(x(t),t,s)-x(0)\right)+s^2\,\mathrm{laplace}(x(t),t,s)-x(0)\,s+\frac{6}{s}=\frac{3}{s}-\frac{3\,\%e^{-6\,s}}{s}$$

```
(%i4)   eq_L:subst([laplace(x(t),t,s)=X,x(0)=0,
                  at('diff(x(t),t,1),t=0)=2],%);
```

(eq_L) $$X\,s^2+5\,X\,s+\frac{6}{s}-2=\frac{3}{s}-\frac{3\,\%e^{-6\,s}}{s}$$

```
(%i5)   solve(eq_L,X);
```

(%o5) $$[X=\frac{\%e^{-6\,s}\left((2\,s-3)\,\%e^{6\,s}-3\right)}{s^3+5\,s^2}]$$

```
(%i6)   expand(%e^(-6*s)*((2*s-3)*%e^(6*s)-3));
```

(%o6) $$-3\,\%e^{-6\,s}+2\,s-3$$

```
(%i7)   part1:ilt((2*s-3)/(s^3+5*s^2),s,t);
```

(part1) $$-\frac{13\,\%e^{-5\,t}}{25}-\frac{3\,t}{5}+\frac{13}{25}$$

```
(%i8)   ilt((-3*%e^(-6*s))/(s^3+5*s^2),s,t);
```

(%o8) $$\mathrm{ilt}\left(-\frac{3\,\%e^{-6\,s}}{s^3+5\,s^2},s,t\right)$$

```
(%i9)   part2:invl(6,-3/(s^3+5*s^2),s,t);
```

(part2) $$-\frac{\%e^{-5\,t}\left(\left(15\,\mathrm{unit\_step}(t-6)\,t-93\,\mathrm{unit\_step}(t-6)\right)\%e^{5\,t}+3\,\%e^{30}\,\mathrm{unit\_step}(t-6)\right)}{25}$$

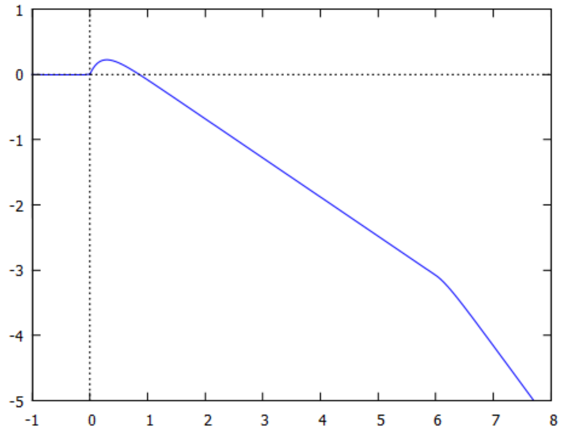
```
(%i10)  part2:expand(part2);
```

(part2) $$-\frac{3\,\mathrm{unit\_step}(t-6)\,\%e^{30-5\,t}}{25}-\frac{3\,\mathrm{unit\_step}(t-6)\,t}{5}+\frac{93\,\mathrm{unit\_step}(t-6)}{25}$$

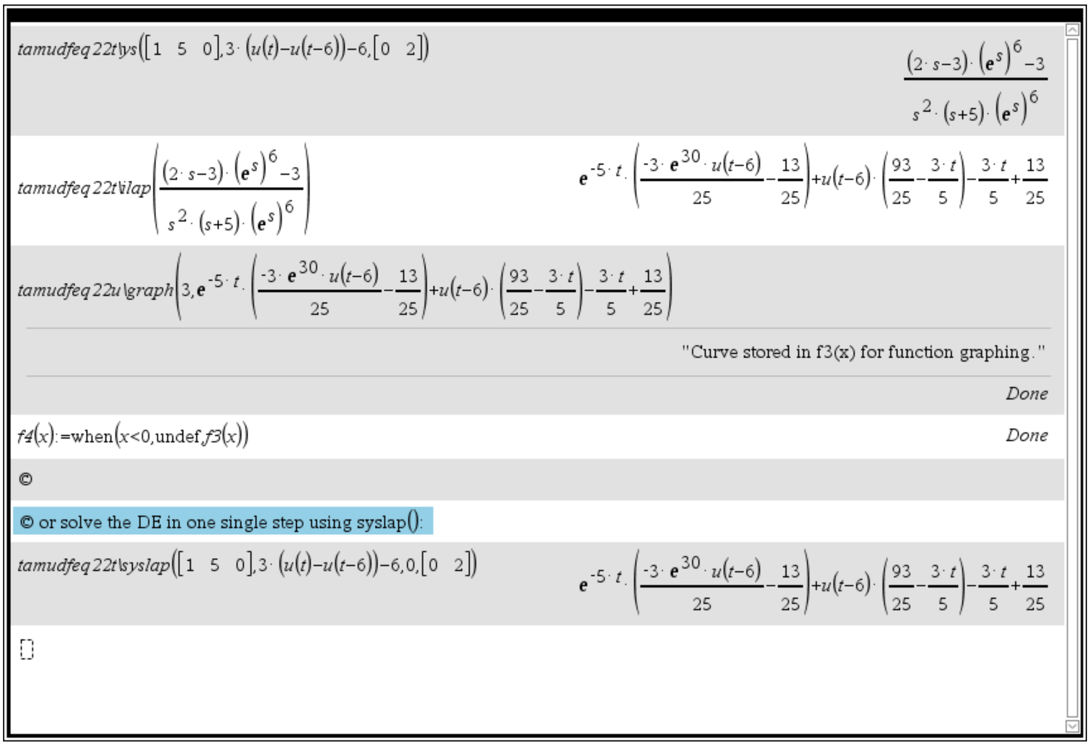
```
(%i11)  plot2d(if t<0 then 0 else part1+part2,[t,-1,8],[y,-5,1]);
plot2d: some values were clipped.
(%o11)  [C:/Users/Josef/maxout5608.gnuplot]
```

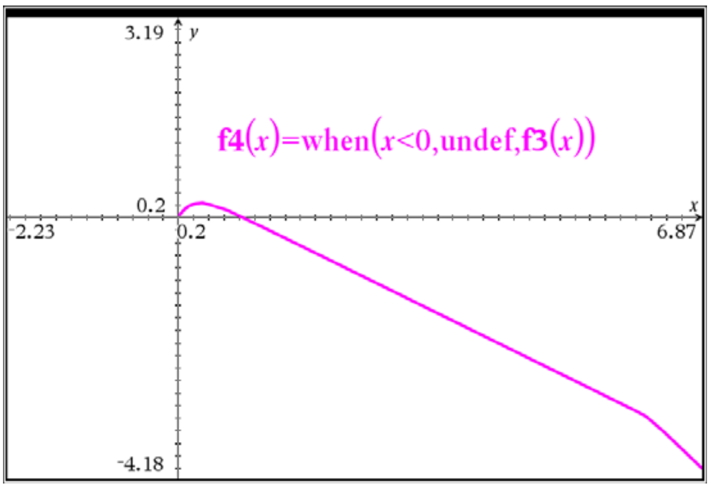
ilaplace automatically includes that the solution is defined for $t \ge 0$. Here we have to consider this.

The maxima plot:



TheTAMUDFEQ treatment is following. It does not consider the domain for $t \geq 0$. Its graph-function converts the result in t and u-function to x and $\hat{u}(t)$ which is nothing else than when(t≥0,1,0).





Next example will contain the Dirac Delta function (one of Michel Beaudin's homework problems).

Example 3

The ODE for a mess-spring system is

$$m\,x'' + b\,x' + k\,x = f(t),\;\; x(0) = x_0,\;\; x'(0) = v_0$$

where $y(t)$ denotes the position of the object at time t, m is the mass of the object, b is the damping constant, k is the spring constant and $f(t)$ is the external force (could be 0) and where the initial position and initial velocity are y_0 and v_0, respectively.

Consider the (undamped) mass-spring problem with two impulses acting as external force:

$$x'' + 4x = 50\,\delta(t-\pi) - 100\,\delta(t-2\pi),\;\; x(0) = 10,\;\; x'(0) = 5.$$

Solve the ODE and plot the graph of the position for $0 \le t \le 10\pi$.

$$\mathit{1.solved}\left(\frac{d^2}{dt^2}(y(t))+4\cdot y(t)=50\cdot \delta(t-\pi)-100\cdot \delta(t-2\cdot \pi),\{y(t),10,5\}\right)$$

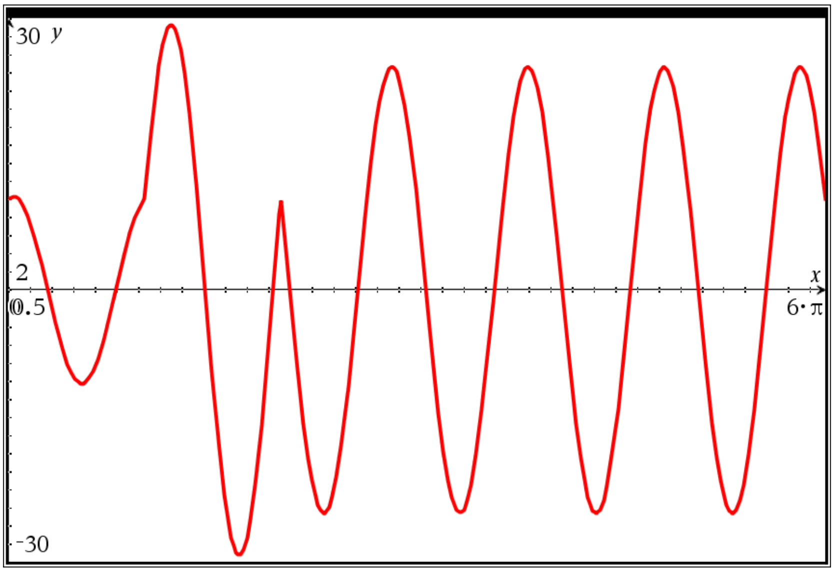
$$y(t)=10\cdot \cos(2\cdot t)\cdot u(t)+\sin(2\cdot t)\cdot \left(25\cdot u(t-\pi)-50\cdot u(t-2\cdot \pi)+\frac{5\cdot u(t)}{2}\right)$$

$$\mathit{1.solved}\left(\frac{d^2}{dt^2}(y(t))+4\cdot y(t)=50\cdot \delta(t-\pi)-100\cdot \delta(t-2\cdot \pi),\{y(t),10,5\}\right)$$

$$y(t)=10\cdot \cos(2\cdot t)\cdot u(t)+\sin(2\cdot t)\cdot \left(25\cdot u(t-\pi)-50\cdot u(t-2\cdot \pi)+\frac{5\cdot u(t)}{2}\right)$$

$$\text{◂}):=10\cdot \cos(2\cdot x)\cdot \mathit{1.step}(x)+\sin(2\cdot x)\cdot \left(25\cdot \mathit{1.step}(x-\pi)-50\cdot \mathit{1.step}(x-2\cdot \pi)+\frac{5\cdot \mathit{1.step}(x)}{2}\right)$$

Done



The impact of the external force (Dirac Delta) is very clear to observe.

Next page shows the *Maxima* procedure.

`desolve` does not work (see expression %o2), so we apply Laplace transform:

```
--> de:'diff(y(t),t,2)+4*y(t)=50*delta(t-%pi)-100*delta(t-2*%pi);
```
(de) $$\frac{d^2}{d\,t^2}\,y(t)+4\,y(t)=50\,\delta(t-\pi)-100\,\delta(t-2\,\pi)$$

```
--> desolve(de,y(t));
```
(%o2) $$y(t)=\mathrm{ilt}\left(\left(\%e^{-2\,\pi\,g20104}\left(\%e^{2\,\pi\,g20104}\left(\left.\frac{d}{d\,t}\,y(t)\right|_{t=0}\right)+y(0)\,g20104\,\%e^{2\,\pi\,g20104}+50\,\%e^{\pi\,g20104}-100\right)\right)/\left(g20104^2+4\right),g20104,t\right)$$

```
--> eq_L:laplace(de,t,s);
```
(eq_L) $$-\left.\frac{d}{d\,t}\,y(t)\right|_{t=0}+s^2\,\mathrm{laplace}(y(t),t,s)+4\,\mathrm{laplace}(y(t),t,s)-y(0)\,s=50\,\%e^{-\pi\,s}-100\,\%e^{-2\,\pi\,s}$$

```
--> eq_L:subst([-at('diff(y(t),t,1),t=0)=5,y(0)=10,
           laplace(y(t),t,s)=Y],eq_L);
```
(eq_L) $$Y\,s^2-10\,s+4\,Y+5=50\,\%e^{-\pi\,s}-100\,\%e^{-2\,\pi\,s}$$

```
--> solve(eq_L,Y);
```
(%o5) $$[Y=\frac{\%e^{-2\,\pi\,s}\left((10\,s-5)\,\%e^{2\,\pi\,s}+50\,\%e^{\pi\,s}-100\right)}{s^2+4}]$$

```
--> expand(%e^(-2*%pi*s)*((10*s-5)*%e^(2*%pi*s)+50*%e^(%pi*s)-100));
```
(%o6) $$50\,\%e^{-\pi\,s}-100\,\%e^{-2\,\pi\,s}+10\,s-5$$

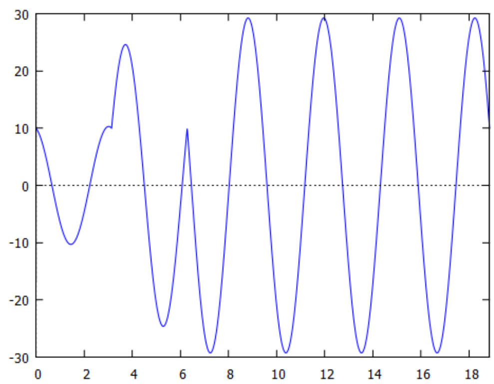
```
--> part1:ilt((10*s-5)/(s^2+4),s,t);
```
(part1) $$10\cos(2\,t)-\frac{5\sin(2\,t)}{2}$$

```
--> invl(a_,f,s,t):=trigsimp(unit_step(t-a_)*ilt(f,s,t-a_))$
```

```
--> part2:invl(%pi,50/(s^2+4),s,t)+invl(2*%pi,-100/(s^2+4),s,t);
```
(part2) $$25\sin(2\,t)\,\mathrm{unit\_step}(t-\pi)-50\sin(2\,t)\,\mathrm{unit\_step}(t-2\,\pi)$$

```
--> m(t):=part1+part2$
```

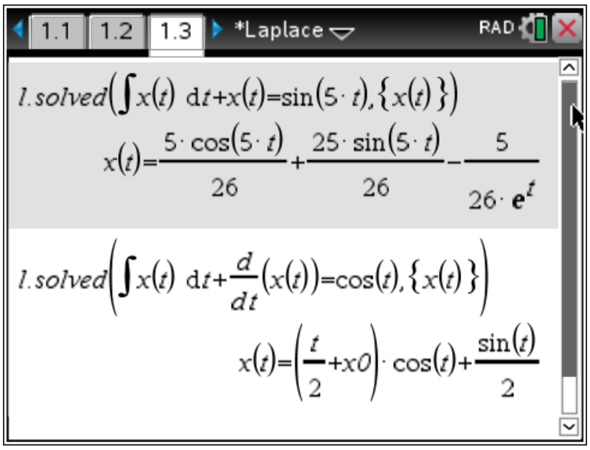
```
--> plot2d(m(t),[t,0,6*%pi]);
```
(%o11) [C:/Users/Josef/maxout3668.gnuplot]



Examples 4 and 5

Solve the integral equation $\int x\,dt + x = \sin(5t)$.

Solve the integro-differential equation $\int x\,dt + \dfrac{dx}{dt} = \cos(t)$ without initial condition.



We need some knowledge about Laplace transforms, then we can solve the first equation (and the second one, too) supported by *DERIVE* and Terence's utility file:

$$\#5:\quad \frac{x}{s} + x = \mathrm{Laplace(SIN(5\cdot t),\ t,\ s)}$$

$$\#6:\quad \frac{x}{s} + x = \frac{5}{s^2 + 25}$$

$$\#7:\quad \mathrm{SOLVE}\left(\frac{x}{s} + x = \frac{5}{s^2 + 25},\ x\right)$$

$$\#8:\quad x = \frac{5\cdot s}{(s + 1)\cdot(s^2 + 25)}$$

$$\#9:\quad x(t) := \mathrm{INVLaplace}\left(\frac{5\cdot s}{(s + 1)\cdot(s^2 + 25)}\right)$$

$$\#10:\quad x(t) := -\frac{5\cdot e^{-t}}{26} + \frac{5\cdot \mathrm{COS}(5\cdot t)}{26} + \frac{25\cdot \mathrm{SIN}(5\cdot t)}{26}$$

I will solve the second equation supported by *Maxima*, which offers more potential

```
(%i1) de:laplace(integrate(f(t_),t_,0,t)+diff(x(t),t)=cos(t),t,s);
```

(de) $s\,\mathrm{laplace}\left(x(t),t,s\right)+\dfrac{\mathrm{laplace}\left(f(t),t,s\right)}{s}-x(0)=\dfrac{s}{s^2+1}$

```
(%i2) de_1:s*X+X/s-x(0)=s/(s^2+1);
```

(de_1) $X\,s+\dfrac{X}{s}-x(0)=\dfrac{s}{s^2+1}$

```
(%i3) solve(de_1,X);
```

(%o3) $\left[X=\dfrac{x(0)\,s^3+s^2+x(0)\,s}{s^4+2\,s^2+1}\right]$

```
(%i4) ilt((x(0)*s^3+s^2+x(0)*s)/(s^4+2*s^2+1),s,t);
```

(%o4) $\dfrac{\sin(t)}{2}+\dfrac{t\cos(t)}{2}+x(0)\cos(t)$

Next program to explore is a tool for solving systems of differential and integro-differential equations

solved([eq1;eq2;…], [f1(var), f1(0), f1'(0), …; f2(0, f2'(0), …; …])

Examples 6 and 7

Solve the system of differential equations $\begin{array}{c}\dfrac{dx}{dt}+\dfrac{dy}{dt}+5x+3y=e^{-t}\\ 2\dfrac{dx}{dt}+\dfrac{dy}{dt}+x+y=3\end{array}$ with $x(0)=2,\ y(0)=1.$

© Example for simultd

$mat:=\begin{bmatrix}\frac{d}{dt}(x(t))+\frac{d}{dt}(y(t))+5\cdot x(t)+3\cdot y(t)=\mathbf{e}^{-t}\\ 2\cdot\frac{d}{dt}(x(t))+\frac{d}{dt}(y(t))+x(t)+y(t)=3\end{bmatrix}$ $\begin{bmatrix}\frac{d}{dt}(x(t))+\frac{d}{dt}(y(t))+5\cdot x(t)+3\cdot y(t)=\mathbf{e}^{-t}\\ 2\cdot\frac{d}{dt}(x(t))+\frac{d}{dt}(y(t))+x(t)+y(t)=3\end{bmatrix}$

$l.simultd\left(mat,\begin{bmatrix}x(t)&2\\y(t)&1\end{bmatrix}\right)$ $\begin{bmatrix}x(t)=\frac{25\cdot\mathbf{e}^t}{3}-\frac{11}{6\cdot(\mathbf{e}^t)^2}-\frac{9}{2}\\ y(t)=\frac{-25\cdot\mathbf{e}^t}{2}+\frac{1}{2\cdot\mathbf{e}^t}+\frac{11}{2\cdot(\mathbf{e}^t)^2}+\frac{15}{2}\end{bmatrix}$

$l.check\left(mat,\begin{bmatrix}x(t)=\frac{25\cdot\mathbf{e}^t}{3}-\frac{11}{6\cdot(\mathbf{e}^t)^2}-\frac{9}{2}\\ y(t)=\frac{-25\cdot\mathbf{e}^t}{2}+\frac{1}{2\cdot\mathbf{e}^t}+\frac{11}{2\cdot(\mathbf{e}^t)^2}+\frac{15}{2}\end{bmatrix}\right)$ $\begin{bmatrix}0\\0\end{bmatrix}$

$$\boxed{x(t)=\frac{25}{3}e^t-\frac{11}{6}e^{-2t}-\frac{9}{2},\quad y(t)=-\frac{25}{2}e^t+\frac{1}{2}e^{-t}+\frac{11}{2}e^{-2t}+\frac{15}{2}}$$

Let's proceed with a second example consisting of three DEs:

$$mat2:=\begin{bmatrix}\frac{d}{dt}(x(t))=3\cdot x(t)-2\cdot y(t)-9\cdot t+13\\ \frac{d}{dt}(y(t))=-x(t)+3\cdot y(t)-2\cdot z(t)+7\cdot t-15\\ \frac{d}{dt}(z(t))=-y(t)+3\cdot z(t)-6\cdot t+7\end{bmatrix}\quad \begin{bmatrix}\frac{d}{dt}(x(t))=3\cdot x(t)-2\cdot y(t)-9\cdot t+13\\ \frac{d}{dt}(y(t))=-x(t)+3\cdot y(t)-2\cdot z(t)+7\cdot t-15\\ \frac{d}{dt}(z(t))=-y(t)+3\cdot z(t)-6\cdot t+7\end{bmatrix}$$

$$inic:=\begin{bmatrix}x(t) & x0\\ y(t) & y0\\ z(t) & z0\end{bmatrix}\quad \begin{bmatrix}x(t) & x0\\ y(t) & y0\\ z(t) & z0\end{bmatrix}$$

$$l.simultd(mat2,inic)$$

$$\begin{bmatrix}x(t)=\left(\frac{x0}{4}-\frac{y0}{2}+\frac{z0}{2}+\frac{5}{2}\right)\cdot\left(\mathbf{e}^{t}\right)^{5}+\left(\frac{x0}{2}-z0\right)\cdot\left(\mathbf{e}^{t}\right)^{3}+\left(\frac{x0}{4}+\frac{y0}{2}+\frac{z0}{2}-\frac{5}{2}\right)\cdot\mathbf{e}^{t}+3\cdot t\\ y(t)=\left(\frac{-x0}{4}+\frac{y0}{2}-\frac{z0}{2}-\frac{5}{2}\right)\cdot\left(\mathbf{e}^{t}\right)^{5}+\left(\frac{x0}{4}+\frac{y0}{2}+\frac{z0}{2}-\frac{5}{2}\right)\cdot\mathbf{e}^{t}+5\\ z(t)=\left(\frac{x0}{8}-\frac{y0}{4}+\frac{z0}{4}+\frac{5}{4}\right)\cdot\left(\mathbf{e}^{t}\right)^{5}+\left(\frac{z0}{2}-\frac{x0}{4}\right)\cdot\left(\mathbf{e}^{t}\right)^{3}+\left(\frac{x0}{8}+\frac{y0}{4}+\frac{z0}{4}-\frac{5}{4}\right)\cdot\mathbf{e}^{t}+2\cdot t\end{bmatrix}$$

$$inic2:=\begin{bmatrix}x(t) & 0\\ y(t) & 0\\ z(t) & 0\end{bmatrix}\quad \begin{bmatrix}x(t) & 0\\ y(t) & 0\\ z(t) & 0\end{bmatrix}$$

$$l.simultd(mat2,inic2)$$

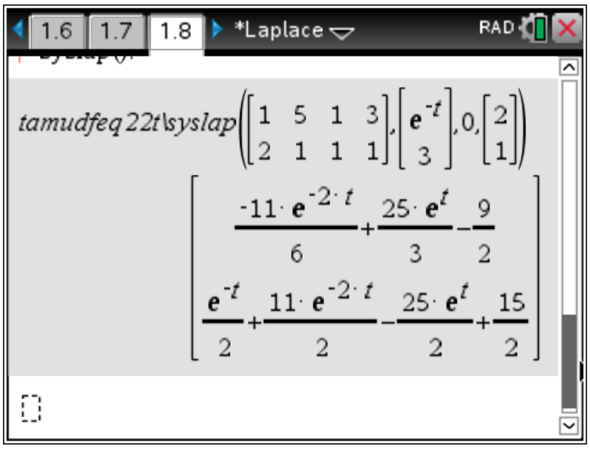
$$\begin{bmatrix}x(t)=\frac{5\cdot\left(\mathbf{e}^{t}\right)^{5}}{2}-\frac{5\cdot\mathbf{e}^{t}}{2}+3\cdot t\\ y(t)=\frac{-5\cdot\left(\mathbf{e}^{t}\right)^{5}}{2}-\frac{5\cdot\mathbf{e}^{t}}{2}+5\\ z(t)=\frac{5\cdot\left(\mathbf{e}^{t}\right)^{5}}{4}-\frac{5\cdot\mathbf{e}^{t}}{4}+2\cdot t\end{bmatrix}$$

$$l.check\left(mat2,\begin{bmatrix}x(t)=\frac{5\cdot\left(\mathbf{e}^{t}\right)^{5}}{2}-\frac{5\cdot\mathbf{e}^{t}}{2}+3\cdot t\\ y(t)=\frac{-5\cdot\left(\mathbf{e}^{t}\right)^{5}}{2}-\frac{5\cdot\mathbf{e}^{t}}{2}+5\\ z(t)=\frac{5\cdot\left(\mathbf{e}^{t}\right)^{5}}{4}-\frac{5\cdot\mathbf{e}^{t}}{4}+2\cdot t\end{bmatrix}\right)\quad\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$l.check\left(mat2,\begin{bmatrix}x(t)=\left(\frac{x0}{4}-\frac{y0}{2}+\frac{z0}{2}+\frac{5}{2}\right)\cdot\left(\mathbf{e}^{t}\right)^{5}+\left(\frac{x0}{2}-z0\right)\cdot\left(\mathbf{e}^{t}\right)^{3}+\left(\frac{x0}{4}+\frac{y0}{2}+\frac{z0}{2}-\frac{5}{2}\right)\cdot\mathbf{e}^{t}+3\cdot t\\ y(t)=\left(\frac{-x0}{4}+\frac{y0}{2}-\frac{z0}{2}-\frac{5}{2}\right)\cdot\left(\mathbf{e}^{t}\right)^{5}+\left(\frac{x0}{4}+\frac{y0}{2}+\frac{z0}{2}-\frac{5}{2}\right)\cdot\mathbf{e}^{t}+5\\ z(t)=\left(\frac{x0}{8}-\frac{y0}{4}+\frac{z0}{4}+\frac{5}{4}\right)\cdot\left(\mathbf{e}^{t}\right)^{5}+\left(\frac{z0}{2}-\frac{x0}{4}\right)\cdot\left(\mathbf{e}^{t}\right)^{3}+\left(\frac{x0}{8}+\frac{y0}{4}+\frac{z0}{4}-\frac{5}{4}\right)\cdot\mathbf{e}^{t}+2\cdot t\end{bmatrix}\right)\quad\begin{bmatrix}0\\0\\0\end{bmatrix}$$

We will discuss check() later.

TAMUDFEQ requires its own syntax but has no problems to solve the problem:



Maxima's `desolve` function is very powerful. We can find the general solution first followed by the special solution.

(%i1) `eq1:'diff(x(t),t)+'diff(y(t),t)+5*x(t)+3*y(t)=%e^(-t);`

(eq1) $\frac{d}{d\,t}\,y(t)+\frac{d}{d\,t}\,x(t)+3\,y(t)+5\,x(t)=\%e^{-t}$

(%i2) `eq2:2*'diff(x(t),t)+'diff(y(t),t)+x(t)+y(t)=3;`

(eq2) $\frac{d}{d\,t}\,y(t)+2\left(\frac{d}{d\,t}\,x(t)\right)+y(t)+x(t)=3$

(%i3) `gensol:desolve([eq1,eq2],[x(t),y(t)]);`

(gensol) $[x(t)=\frac{(2\,y(0)+6\,x(0)+11)\,\%e^{t}}{3}-\frac{(4\,y(0)+6\,x(0)-5)\,\%e^{-2\,t}}{6}-\frac{9}{2},\ y(t)=-\frac{(2\,y(0)+6\,x(0)+11)\,\%e^{t}}{2}+\frac{\%e^{-t}}{2}+\frac{(4\,y(0)+6\,x(0)-5)\,\%e^{-2\,t}}{2}+\frac{15}{2}]$

(%i4) `ev(subst([x(0)=2,y(0)=1],gensol));`

(%o4) $[x(t)=\frac{25\,\%e^{t}}{3}-\frac{11\,\%e^{-2\,t}}{6}-\frac{9}{2},\ y(t)=-\frac{25\,\%e^{t}}{2}+\frac{\%e^{-t}}{2}+\frac{11\,\%e^{-2\,t}}{2}+\frac{15}{2}]$

(%i6) `atvalue(x(t),t=0,2)$ atvalue(y(t),t=0,1)$`

(%i7) `desolve([eq1,eq2],[x(t),y(t)]);`

(%o7) $[x(t)=\frac{25\,\%e^{t}}{3}-\frac{11\,\%e^{-2\,t}}{6}-\frac{9}{2},\ y(t)=-\frac{25\,\%e^{t}}{2}+\frac{\%e^{-t}}{2}+\frac{11\,\%e^{-2\,t}}{2}+\frac{15}{2}]$

I must admit that it is very exciting for me solving the DEs using Laplace transforms because I cannot remember having this done during my study time. I didn't teach at a technical vocational school so I didn't have the necessity to inform about this objective.

And I didn't solve systems of DEs – not in my study time and not later. But I do it now in my old days. I wanted to solve the system given above in the classical way – but CAS supported. I work with *DERIVE* and follow the instructions given in the respective textbooks and a paper "*Betriebsanleitung für gewöhnliche lineare Differentialgleichungen*" (*Prof. Dr. Dirk Ferus*).

dx and dy are my notations für x' and y'. I rewrite the equations solving them for x' and y' (dx, dy).

#1: InputMode := Word

#2: $eq1 := dx + dy + 5\cdot x + 3\cdot y = e^{-t}$

#3: $eq2 := 2\cdot dx + dy + x + y = 3$

#4: SOLVE([eq1, eq2], [dx, dy])

#5: $\left[dx = - e^{-t} + 4\cdot x + 2\cdot y + 3 \wedge dy = 2\cdot e^{-t} - 9\cdot x - 5\cdot y - 3\right]$

#6: $eq1 := dx = - e^{-t} + 4\cdot x + 2\cdot y + 3$

#7: $eq2 := dy = 2\cdot e^{-t} - 9\cdot x - 5\cdot y - 3$

#8: $m := \begin{bmatrix} 4 & 2 \\ -9 & -5 \end{bmatrix}$

#9: EIGENVALUES(m) = [1, -2]

#10: ev1 := EXACT_EIGENVECTOR(m, 1)

#11: ev2 := EXACT_EIGENVECTOR(m, -2)

#12: $a\cdot ev1\cdot e^{t} + b\cdot ev2\cdot e^{-2\cdot t} = \begin{bmatrix} \dfrac{2\cdot a\cdot e^{t}}{3} + \dfrac{b\cdot e^{-2\cdot t}}{3} \\ - a\cdot e^{t} - b\cdot e^{-2\cdot t} \end{bmatrix}$

First of all we find a solution for the respective homogeneous system. We have two distinct real eigenvalues which give two eigenvectors. The solution functions for the homogeneous system are given by the two components of the vector in expression #12 with *a* and *b* arbitrary constants.

Next step is finding a particular solution. I apply the method of variation of constants:

#13: $m1 := \begin{bmatrix} \dfrac{2\cdot a\cdot e^{t}}{3} & \dfrac{b\cdot e^{-2\cdot t}}{3} \\ - a\cdot e^{t} & - b\cdot e^{-2\cdot t} \end{bmatrix}$

#14: $SOLVE\left(m1\cdot\begin{bmatrix} cc1 \\ cc2 \end{bmatrix} = \begin{bmatrix} 3 - e^{-t} \\ -3 + 2\cdot e^{-t} \end{bmatrix}, [cc1, cc2]\right)$

#15: $cc1 = \dfrac{6\cdot e^{-t}}{a} - \dfrac{e^{-2\cdot t}}{a} \wedge cc2 = - \dfrac{3\cdot e^{2\cdot t}}{b} - \dfrac{e^{t}}{b}$

parts (expression #18) is the desired particular solution.

$$\#16:\quad \left(c1 := \int \left(\frac{6\cdot e^{-t}}{a} - \frac{e^{-2\cdot t}}{a}\right) dt\right) = c1 := \frac{e^{-2\cdot t}}{2\cdot a} - \frac{6\cdot e^{-t}}{a}$$

$$\#17:\quad \left(c2 := \int \left(-\frac{3\cdot e^{2\cdot t}}{b} - \frac{e^{t}}{b}\right) dt\right) = c2 := -\frac{3\cdot e^{2\cdot t}}{2\cdot b} - \frac{e^{t}}{b}$$

#18: parts := c1·m1↓↓1 + c2·m1↓↓2

$$\#19:\quad parts := \left[-\frac{9}{2}, \frac{e^{-t}}{2} + \frac{15}{2}\right]$$

Now we have the general solution and it remains to solve the initial value problem, i.e. finding the constants *a* and *b*:

$$\#21:\quad gensol := \left[\frac{2\cdot a\cdot e^{t}}{3} + \frac{b\cdot e^{-2\cdot t}}{3} - \frac{9}{2}, -a\cdot e^{t} + \frac{e^{-t}}{2} - b\cdot e^{-2\cdot t} + \frac{15}{2}\right]$$

#22: SOLUTIONS(SUBST(gensol, t, 0) = [2, 1], [a, b])

$$\#23:\quad \left[\left[\frac{25}{2}, -\frac{11}{2}\right]\right]$$

$$\#24:\quad SUBST\left(gensol, [a, b], \left[\frac{25}{2}, -\frac{11}{2}\right]\right)$$

$$\#25:\quad \left[\frac{25\cdot e^{t}}{3} - \frac{11\cdot e^{-2\cdot t}}{6} - \frac{9}{2}, -\frac{25\cdot e^{t}}{2} + \frac{e^{-t}}{2} + \frac{11\cdot e^{-2\cdot t}}{2} + \frac{15}{2}\right]$$

Fortunately we can see the same result as above. We have already checked its correctness using the next tool on page 19:

check(equation, solution)

$l.check\left(\frac{d^2}{dt^2}(x(t))+2\cdot\frac{d}{dt}(x(t))+5\cdot x(t)=\sin(2\cdot t),x(t)=\left(\frac{21}{17\cdot e^t}-\frac{4}{17}\right)\cdot\cos(2\cdot t)+\left(\frac{35}{17\cdot e^t}+\frac{1}{17}\right)\cdot\sin(2\cdot t)\right)$	0
$l.check\left(\int x(t)\ dt+x(t)=\sin(5\cdot t),x(t)=\frac{5\cdot\cos(5\cdot t)}{26}+\frac{25\cdot\sin(5\cdot t)}{26}-\frac{5}{26\cdot e^t}\right)$	0
$l.check\left(\int x(t)\ dt+\frac{d}{dt}(x(t))=\cos(t),x(t)=\left(\frac{t}{2}+x0\right)\cdot\cos(t)+\frac{\sin(t)}{2}\right)$	0

The given DE (first argument) must be entered without the initial condition(s).

Checking DEs with the unit step function or the δ-function is not so easy.

I wrote to Michel:

Dear Michel,
short question:
Do I read the French text correctly understanding that the tool check() (from your library ETS_specfunc.tns) cannot be applied for DEs containing the unit-step function or the delta function?

This is his answer:

Dear Josef, take a look at the following example. When we are using Laplace transforms with students, we tell them that the « derivative of u(t) is delta(t) » (in the sense of generalized functions or « distributions »). But we don't tell them how to deal with products of such functions. We will have a beer together in Mexico and we will discuss this. Best regards,

Michel

$$ets\_specfunc\backslash solved\left(\frac{d}{dt}(x(t))+3\cdot x(t)=\delta(t-2),\{x(t),1\}\right) \qquad x(t)=\frac{e^{6}\cdot u(t-2)+u(t)}{(e^{t})^{3}}$$

$$ets\_specfunc\backslash check\left(\frac{d}{dt}(x(t))+3\cdot x(t)=\delta(t-2),x(t)=\frac{e^{6}\cdot u(t-2)+u(t)}{(e^{t})^{3}}\right) \qquad \frac{e^{6}\cdot\frac{d}{dt}(u(t-2))}{(e^{t})^{3}}+\frac{\frac{d}{dt}(u(t))}{(e^{t})^{3}}-\delta(t-2)$$

©This last answer is equivalent to the following:

$$\frac{e^{6}\cdot\delta(t-2)}{(e^{t})^{3}}+\frac{\delta(t)}{(e^{t})^{3}}-\delta(t-2) \qquad (\delta(t-2)\cdot e^{6}+\delta(t))\cdot e^{-3\cdot t}-\delta(t-2)$$

©Let's write the last answer in this form: $\delta(t-2)\cdot e^{-3(t-2)}+\delta(t)\cdot e^{-3\cdot t}-\delta(t-2)$

©Now, for a function $f(t)$ continuous at $t = a$, we have the following: $f(t)\cdot\delta(t-a)= f(a)\cdot\delta(t-a)$

©So the answer provided by "check" is $\delta(t-2)\cdot e^{(0)}+\delta(t)\cdot e^{0}-\delta(t-2)= \delta(t)$. This is 0 when $t > 0$.
So be careful when using "check" with impulse or unit step functions contained in the DE!

Preparing the functions for plotting and the plots are following.

$uu(t):=\frac{1+\text{sign}(t)}{2}$ — *Done*

$e^{-3\cdot t}\cdot\left(e^{6}\cdot uu(t-2)+uu(t)\right)$ — $\frac{e^{-3\cdot t}\cdot\left(e^{6}\cdot\text{sign}(t-2)+\text{sign}(t)+e^{6}+1\right)}{2}$

$kit\_ets\_mb\backslash u\_to\_piece\left(\frac{e^{-3\cdot t}\cdot\left(e^{6}\cdot\text{sign}(t-2)+\text{sign}(t)+e^{6}+1\right)}{2},t\right)$ — $\begin{cases}\left(e^{6}+1\right)\cdot e^{-3\cdot t}, & 2<t<\infty\\ e^{-3\cdot t}, & 0<t\le 2\\ 0, & -\infty<t\le 0\end{cases}$

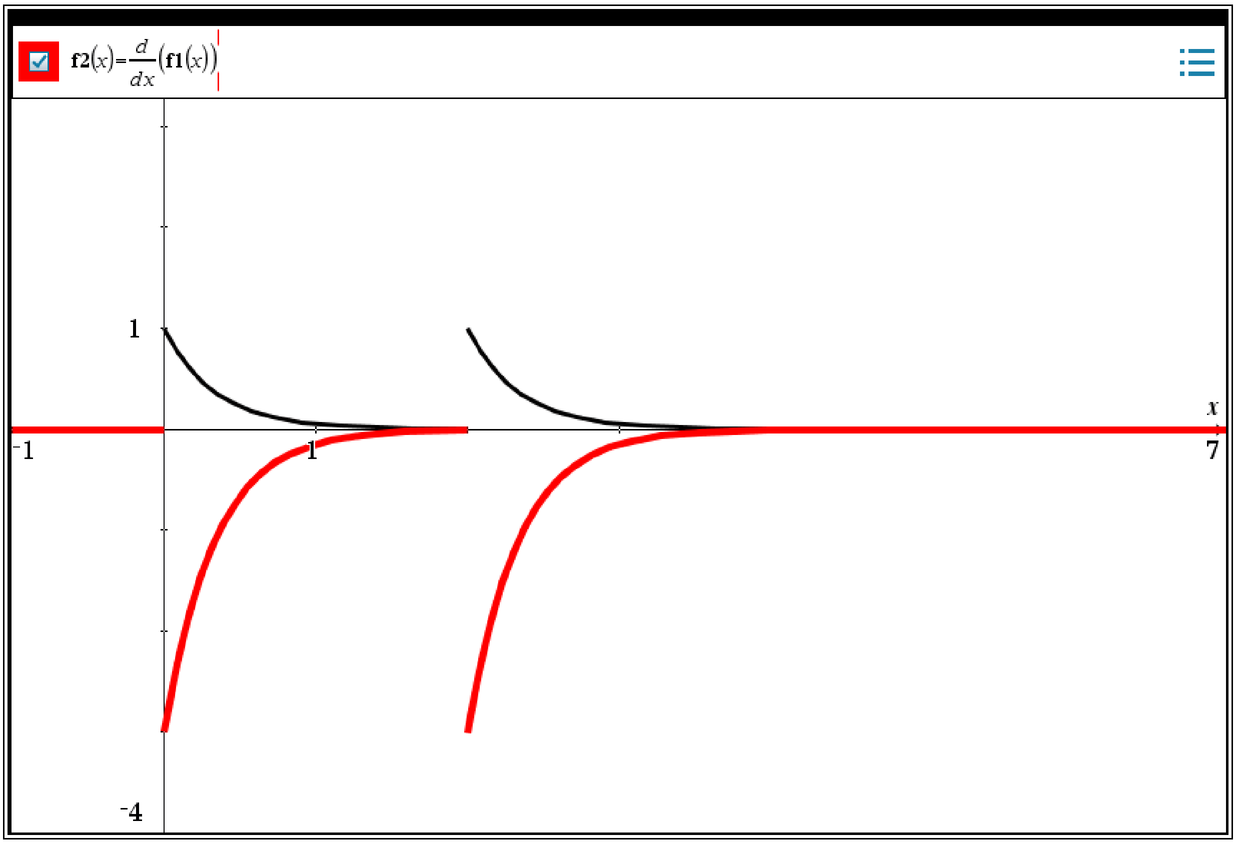
$\begin{cases}\left(e^{6}+1\right)\cdot e^{-3\cdot t}, & 2<t<\infty\\ e^{-3\cdot t}, & 0<t\le 2\\ 0, & -\infty<t\le 0\end{cases}\Bigg|t=x$ — $\begin{cases}\left(e^{6}+1\right)\cdot e^{-3\cdot x}, & 2<x<\infty\\ e^{-3\cdot x}, & 0<x\le 2\\ 0, & -\infty<x\le 0\end{cases}$

$f1(x):=\begin{cases}\left(e^{6}+1\right)\cdot e^{-3\cdot x}, & 2<x<\infty\\ e^{-3\cdot x}, & 0<x\le 2\\ 0, & -\infty<x\le 0\end{cases}$ — *Done*

$f2(x):=\frac{d}{dx}\left(f1(x)\right)$ — *Done*

$kit\_ets\_fh\backslash grouper\_fct\left(f2(x)+3\cdot f1(x),x\right)$ — $\begin{cases}0, & 2<x<\infty\\ 0, & 0<x<2\\ 0, & -\infty<x<0\end{cases}$

©Adding the red curve and 3· the black curve yields 0. See page 2. But there are discontinuities at 0 and 2. This is also confirmed by doing the computations by hand. We will do this with a beer!



As I wrote above, checking the solution using check() is not easy. See example 3 from above followed by Michel's explication (given in TI-Nspire-Notes pages):

$1.\ solved\left(\frac{d^2}{dt^2}(x(t))+4\cdot x(t)=50\cdot \delta(t-\pi)-100\cdot \delta(t-2\cdot\pi),\{x(t),10,5\}\right)$ $\quad x(t)=10\cdot\cos(2\cdot t)\cdot u(t)+\sin(2\cdot t)\cdot\left(25\cdot u(t-\pi)-50\cdot u(t-2\cdot\pi)+\frac{5\cdot u(t)}{2}\right)$

$1.\ check\left(\frac{d^2}{dt^2}(x(t))+4\cdot x(t)=50\cdot \delta(t-\pi)-100\cdot \delta(t-2\cdot\pi),x(t)=10\cdot\cos(2\cdot t)\cdot u(t)+\sin(2\cdot t)\cdot\left(25\cdot u(t-\pi)-50\cdot u(t-2\cdot\pi)+\frac{5\cdot u(t)}{2}\right)\right)$

$25\cdot\frac{d^2}{dt^2}(u(t-\pi))\cdot\sin(2\cdot t)+100\cdot\frac{d}{dt}(u(t-\pi))\cdot\cos(2\cdot t)-50\cdot\frac{d^2}{dt^2}(u(t-2\cdot\pi))\cdot\sin(2\cdot t)-200\cdot\frac{d}{dt}(u(t-2\cdot\pi))\cdot\cos(2\cdot t)+\frac{d^2}{dt^2}(u(t))\cdot\left(10\cdot\cos(2\cdot t)+\frac{5\cdot \text{si}}{}\right.$ ▸

Checking answers in Laplace transforms problems involving impulse functions

Suppose we have the following mass–spring problem:
$x''(t)+4x(t)=50\delta(t-\pi)-100\delta(t-2\pi),\ x(0)=10,\ x'(0)=5$.

Applying the Laplace transform yields the following solution (I take a shortcut with my "ressort" function. This could also be obtained using "solveD" from ETS_specfunc library):

kit_ets_mb\ressort$(1,0,4,50\cdot\delta(t-\pi)-100\cdot\delta(t-2\cdot\pi),10,5)$

▸ $10\cdot\cos(2\cdot t)\cdot u(t)+\sin(2\cdot t)\cdot\left(25\cdot u(t-\pi)-50\cdot u(t-2\cdot\pi)+\frac{5\cdot u(t)}{2}\right)$

Let's rewrite the last answer this way:
$(10\cos(2t)+5/2\sin(2t))\cdot u(t)+25\sin(2t)\cdot u(t-\pi)-50\sin(2t)\cdot u(t-2\pi)$

Now, let's recall the following rules regarding Heaviside $(u(t))$ and Dirac delta function $(\delta(t))$:

$\frac{d}{dt}(u(t))=\delta(t)$, $\ f(t)\cdot\delta(t-a)=f(a)\cdot\delta(t-a)$ for any continuous function f at $t=a$.

Moreover, $t\cdot\delta'(t)=-\delta(t)$.

Applying the product rule for derivatives and these last remarks, we find this:
$x''+4x=5\delta(t)+10\delta'(t)+50\delta(t-\pi)-100\delta(t-2\pi)$ Because we want a solution defined for $t>0$, the first two terms on the right are 0, so our solution satisfies the ODE!

I strongly prefer to write the answer using piecewise functions and, then, substitute into the DE: students don't know how to deal with Dirac delta function when comes time to compute derivatives.

For achieving this, I will use my function "u_to_piece": $\mathbf{uu}(t):=\frac{1+\text{sign}(t)}{2}$ ▸ *Done*

kit_ets_mb\u_to_piece$\left(10\cdot\cos(2\cdot t)\cdot\mathbf{uu}(t)+\sin(2\cdot t)\cdot\left(25\cdot\mathbf{uu}(t-\pi)-50\cdot\mathbf{uu}(t-2\cdot\pi)+\frac{5\cdot\mathbf{uu}(t)}{2}\right),t\right)$

$$\blacktriangleright\begin{cases}10\cdot\cos(2\cdot t)-\frac{45\cdot\sin(2\cdot t)}{2}, & 2\cdot\pi<t<\infty\\ 10\cdot\cos(2\cdot t)+\frac{55\cdot\sin(2\cdot t)}{2}, & \pi<t\le 2\cdot\pi\\ 10\cdot\cos(2\cdot t)+\frac{5\cdot\sin(2\cdot t)}{2}, & 0<t\le\pi\\ 0, & -\infty<t\le 0\end{cases}$$

$$\mathbf{sol}(t):=\begin{cases}10\cdot\cos(2\cdot t)-\frac{45\cdot\sin(2\cdot t)}{2}, & 2\cdot\pi<t<\infty\\ 10\cdot\cos(2\cdot t)+\frac{55\cdot\sin(2\cdot t)}{2}, & \pi<t\le 2\cdot\pi\\ 10\cdot\cos(2\cdot t)+\frac{5\cdot\sin(2\cdot t)}{2}, & 0<t\le\pi\\ 0, & -\infty<t\le 0\end{cases} \blacktriangleright \textit{Done}$$

Now Nspire won't group the piecewise functions together but Frédérick' s function "grouper_fct" will: I use it:

$$\mathbf{kit\_ets\_fh\backslash grouper\_fct}\left(\frac{d^2}{dt^2}(\mathbf{sol}(t))+4\cdot\mathbf{sol}(t),t\right) \blacktriangleright \begin{cases}0, & 2\cdot\pi<t<\infty\\ 0, & \pi<t<2\cdot\pi\\ 0, & 0<t<\pi\\ 0, & -\infty<t<0\end{cases}$$

We "have lost" the 2 Dirac functions! This is the price to pay when we don't want to deal with the calculus involving Dirac.

On page 2, $\mathbf{f1}(x)$ is $\mathbf{sol}(x)$ in blue color. Because Nspire CAS unlike Derive does not have a "Simplify before plotting" command, it is better to compute the second derivative in order to be able to plot the graph rapidly!

$$\frac{d^2}{dt^2}(\mathbf{sol}(t)) \blacktriangleright \begin{cases}90\cdot\sin(2\cdot t)-40\cdot\cos(2\cdot t), & 2\cdot\pi<t<\infty\\ -40\cdot\cos(2\cdot t)-110\cdot\sin(2\cdot t), & \pi<t<2\cdot\pi\\ -40\cdot\cos(2\cdot t)-10\cdot\sin(2\cdot t), & 0<t<\pi\\ 0, & -\infty<t<0\end{cases}$$

$$\begin{cases}90\cdot\sin(2\cdot t)-40\cdot\cos(2\cdot t), & 2\cdot\pi<t<\infty\\ -40\cdot\cos(2\cdot t)-110\cdot\sin(2\cdot t), & \pi<t<2\cdot\pi\\ -40\cdot\cos(2\cdot t)-10\cdot\sin(2\cdot t), & 0<t<\pi\\ 0, & -\infty<t<0\end{cases}\Bigg|t=x \blacktriangleright \begin{cases}90\cdot\sin(2\cdot x)-40\cdot\cos(2\cdot x), & 2\cdot\pi<x<\infty\\ -40\cdot\cos(2\cdot x)-110\cdot\sin(2\cdot x), & \pi<x<2\cdot\pi\\ -40\cdot\cos(2\cdot x)-10\cdot\sin(2\cdot x), & 0<x<\pi\\ 0, & -\infty<x<0\end{cases}$$

$$\mathbf{f2}(x):=\begin{cases}90\cdot\sin(2\cdot x)-40\cdot\cos(2\cdot x), & 2\cdot\pi<x<\infty\\ -40\cdot\cos(2\cdot x)-110\cdot\sin(2\cdot x), & \pi<x<2\cdot\pi\\ -40\cdot\cos(2\cdot x)-10\cdot\sin(2\cdot x), & 0<x<\pi\\ 0, & -\infty<x<0\end{cases} \blacktriangleright \textit{Done}$$

$\mathbf{f2}(x)$ is in red color..

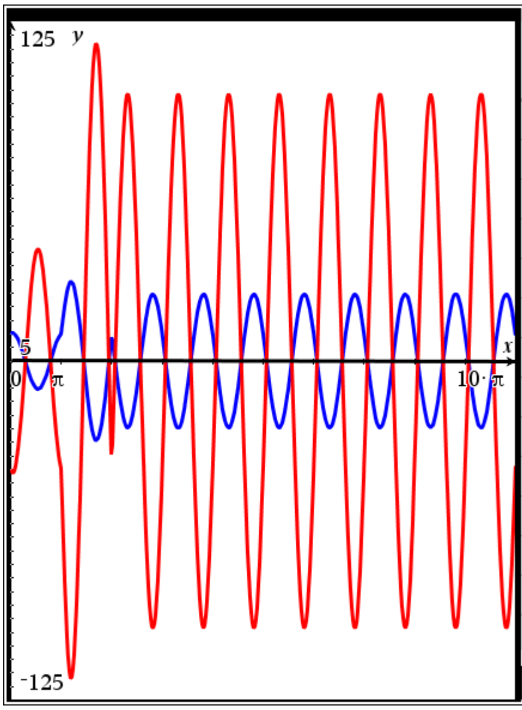
On page 2, $\mathbf{f3}(x)$ is $\mathbf{f2}(x)+4\mathbf{f1}(x)$ and has black color. Take a look and observe that at $x=\pi$ and $x=2\pi$, the function $\mathbf{f3}(x)$ seems to be also defined (and being identically 0) but:

$\mathbf{f3}(\pi) \blacktriangleright$ undef

$\mathbf{f3}(2\cdot\pi) \blacktriangleright$ undef

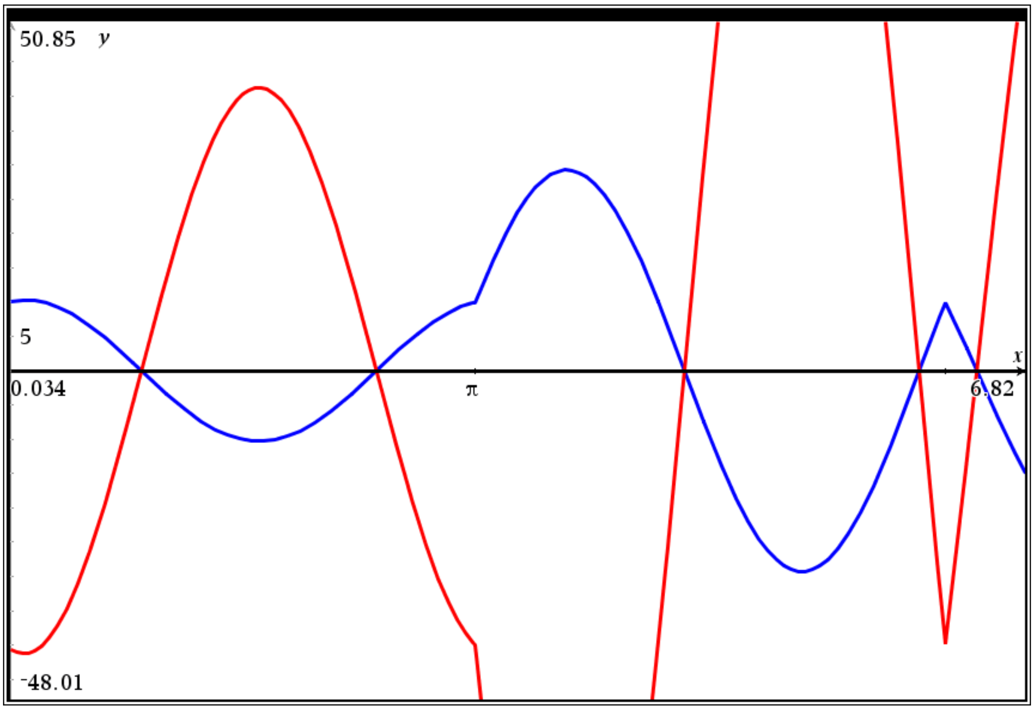
In fact, $\mathbf{f2}(x)$ is not defined at $x=\pi$ and $x=2\pi$.

The plots are following:



x	f1(x):= sol(x)	f2(x):= piecewise.	f3(x):= f2(x)+4*f...
0.3141...	9.55963	-38.2385	0.
0.6283...	5.46781	-21.8712	-1.E-12
0.9424...	-0.7125...	2.85011	4.E-13
1.25664	-6.62071	26.4828	0.
1.5708	-10.	40.	0.
1.88496	-9.55963	38.2385	0.
2.19911	-5.46781	21.8712	0.
2.51327	0.712529	-2.85011	4.E-13
2.82743	6.62071	-26.4828	-1.E-12
3.14159	10.	-40.	2.E-12
3.45575	24.2543	-97.0171	2.E-12
3.76991	29.2442	-116.977	-1.E-11
4.08407	23.0639	-92.2555	-2.E-12
4.39823	8.07392	-32.2957	0.

5.4678112344876



Additional note from Michel:

See http://mathworld.wolfram.com/DeltaFunction.html (equation (14) for the derivative of Dirac).

A few days before leaving for TIME 2016 in Mexico I had a very intense email exchange with Michel. He was very patient answering my questions and moreover gave a lot of input with regard to this article.

It was not sufficient for him to give all the explications above. As true DERIVIAN he gave advice how to treat DEs involving the δ-function (impulse-function) with *DERIVE*.

Dear Josef, I have to tell you Derive is "responsible" of my education on the Dirac delta function. Here is why:

There is no delta Dirac delta function d(t-a) in Derive but you can "replace" it by 1/b*CHI(a,t,a+b) where CHI is the built-in indicator function. Then you use the "dsolve2_iv" command and solve your ODE. This can be done because Derive knows how to integrate functions involving CHI, STEP and products of these with other functions. Then, you take a limit as b goes to 0 from right. You will exactly obtained the solution the Laplace method would have given!!! Moreover, before taking the limit, you can use a slider for b and "animate" the effect of the Dirac delta function. I have been using this year after year in the classroom with my students. Now, with Nspire CAS, I do the same, replacing the "dsolve2" command by the "ressort" or "solveD" commands and the CHI function by u functions because CHI(a, t, b) = u(t-a) -u(t-b) where "u" is what Nspire wants for STEP inside the ETS_specfunc library.
The animation with the slider helps students for the understanding that Dirac delta function is a limit --that does not exist in the classical sense-- of indicator functions.

If you want to insert this into the next DNL, we can do again the example of
x" + 4*x = 5*delta(x-pi) -100*delta(x-2*pi), x(0)=5, x'(0)=10 with good old Derive when we will meet in Mexico.

See you soon,
Michel

I didn't want to wait for our meeting in Mexico and tried to follow Michel's detailed instructions. I had the idea that it might be nicer to have a beer – or two – instead of solving DEs in the summer heat of Mexico City.

$$\#1: \quad \delta(a) := \frac{1}{b}\cdot\chi(a,\ t,\ a + b)$$

$$\#2: \quad \mathrm{DSOLVE2\_IV}(0,\ 4,\ 50\cdot\delta(\pi) - 100\cdot\delta(2\cdot\pi),\ t,\ 0,\ 10,\ 5)$$

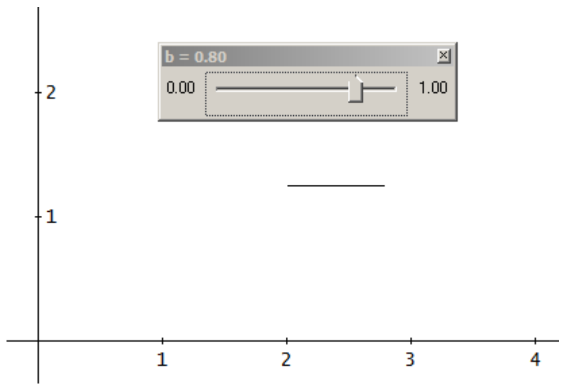
$$\#3: \quad \lim_{b\to+0} \mathrm{DSOLVE2\_IV}(0,\ 4,\ 50\cdot\delta(\pi) - 100\cdot\delta(2\cdot\pi),\ t,\ 0,\ 10,\ 5)$$

$$\#4: \quad - 25\cdot\mathrm{SIGN}(t - 2\cdot\pi)\cdot\mathrm{SIN}(2\cdot t) + \frac{25\cdot\mathrm{SIGN}(t - \pi)\cdot\mathrm{SIN}(2\cdot t)}{2} + 10\cdot\mathrm{COS}(2\cdot t) - 10\cdot\mathrm{SIN}(2\cdot t)$$

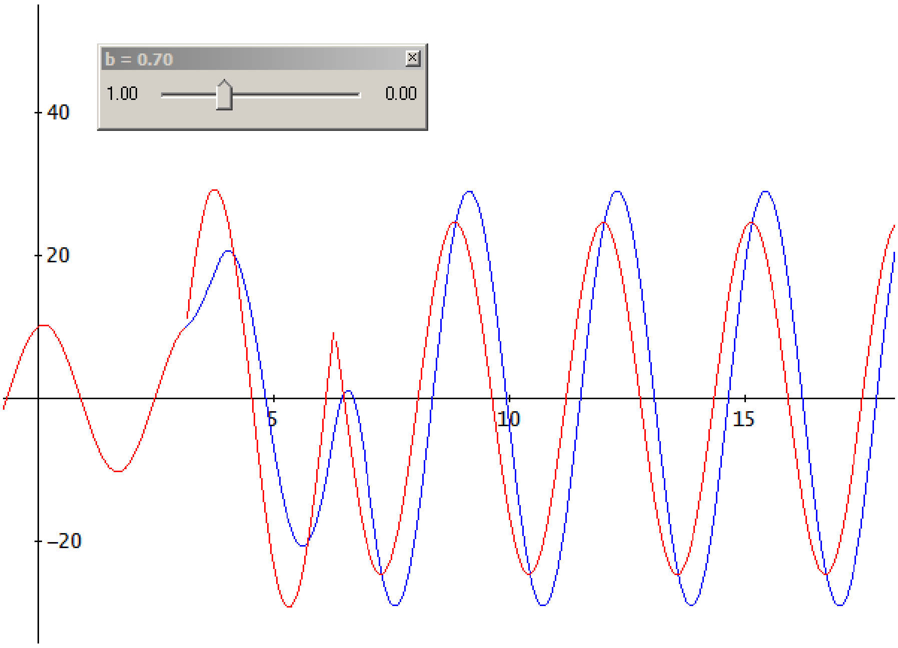
According to Michel's mail I "invented" the delta-function (expression #1).

This is the plot of δ(2) with a slider for b.

Moving the slider is very informative for understanding the Dirac Delta function.



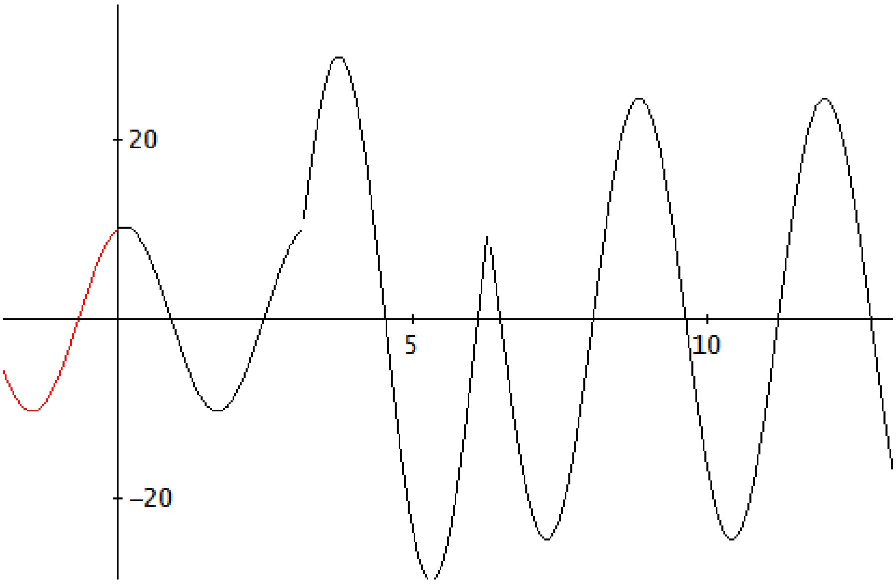
This is the plot of the solution function (expression #2) including variable *b* which tends to zero. The solution is given in expression #4 (red graph).



The solution obtained via Laplace transform in DERIVE syntax is given below:

$$10\cdot\mathrm{COS}(2\cdot t)\cdot\mathrm{STEP}(t) + \mathrm{SIN}(2\cdot t)\cdot\left(25\cdot\mathrm{STEP}(t-\pi) - 50\cdot\mathrm{STEP}(t-2\cdot\pi) + \frac{5}{2}\cdot\mathrm{STEP}(t)\right)$$

Its plot is superimposed (in black) and we notice identity with one exception: the black solution graph is defined for $t > 0$ only.



I'd like to add that you can use the STEP function, too. So it is no problem to solve Example 2 applying `DSOLVE2_IV()`. Give it a try!

I sent my results to Michel and very soon after his next – very welcome – lecture concerning Dirac Delta came in.

Good! It shows –again—how Derive was special! Looking inside the « dsolve2_iv » command made me learn so many mathematics. I miss so much these 2 guys – Stoutemyer and Rich.

Because you are involved into physics with your book and Leon, I think the following explanation about Dirac delta function can be good for the readers. But maybe you already know this, so ignore it if it is the case.

Suppose a force f(t) acts on an object of mass m during an interval of time a < t < b. Then the impulse due to this force is defined as int(f(t), t, a, b). But according to Newton's second law, we also have m*a =f(t) where a is the acceleration: a = dv/dt. So, integrating on both sides, we find m*(v(b)- v(a)) = int(f(t), t, a , b): That is the impulse equals the change of momentum.

Thinking of mass-spring problem, let Fo > 0 be any constant, let to >= 0, and consider the ODE m*x'' + b*x' + k*x = Fo*delta(t- to), x(0) = xo and x'(0) = vo. The (infinite) external force is Fo*delta(t-to) and can be compared to a hammer striking the object with total impulse of Fo N.s. Students have a better understanding of this « external force » if you replace it by the (real) indicator force Fo/b*CHI(to, t, to+b), then solve the ODE, then take a limit as b goes to 0 from the right. Exactly what you have done in your Derive file!

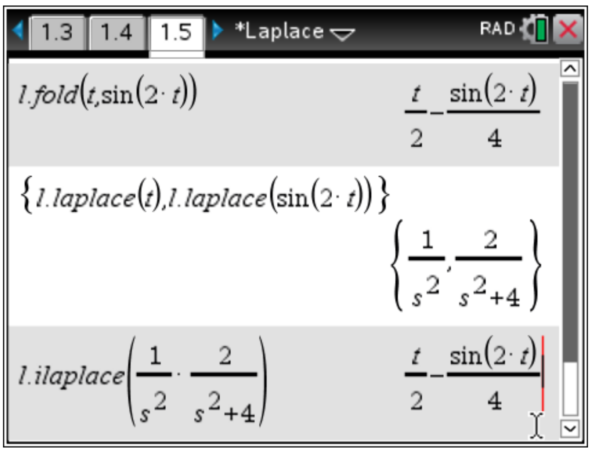
The last tool provided in ETS_specfunc.tns performs the convolution of two functions:

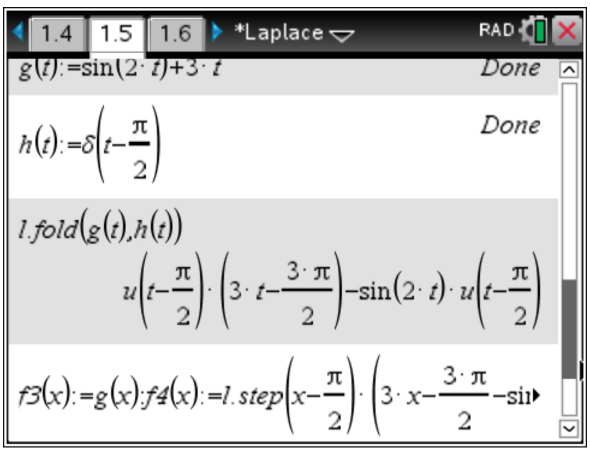
Convolution of $f(t)$ and $g(t)$ is here defined as follows: $f(t) * g(t) = \int_0^t f(\tau)\cdot g(t-\tau)\,d\tau$. See also earlier DNLs: DNL#8 and DNL#57.

fold(*f*(*t*), *g*(*t*))

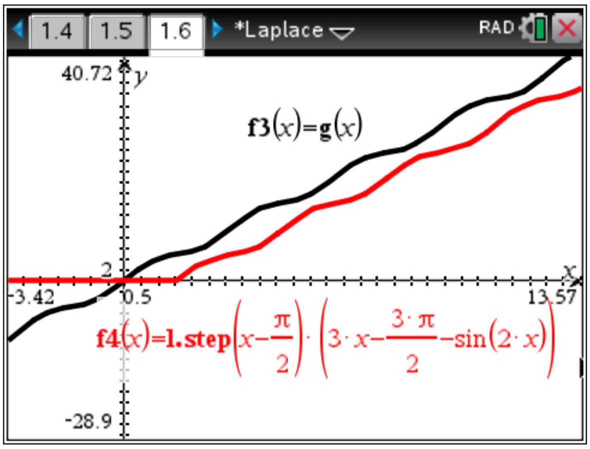
Let $F(s)$ and $G(s)$ the Laplace transforms of $f(t)$ and $g(t)$, then $f(t)*g(t) = L^{-1}(F(s)\cdot G(s))$.

Examples 8 and 9





Convolution of signal $x(t)$ with $\delta(t-a)$ results in a translation: $x(t) * \delta(t-a) = x(t-a)$.



DERIVE does not provide a function for performing convolution of two functions. But it is easy work to define one's own convolution tool.

$$\#5:\quad \text{convol}(u, v, t) := \int_0^t \left(\lim_{t \to \tau} u\right) \cdot \lim_{t \to t - \tau} v \; d\tau$$

$$\#6:\quad \text{convol}(t, \text{SIN}(2 \cdot t)) = \frac{t}{2} - \frac{\text{SIN}(2 \cdot t)}{4}$$

$$\#7:\quad [\text{Laplace}(\text{EXP}(5 \cdot t), t, s), \text{Laplace}(\text{COS}(t), t, s)]$$

$$\#8:\quad \left[\frac{1}{s - 5}, \frac{s}{s^2 + 1}\right]$$

$$\#9:\quad \text{convol}(\text{EXP}(5 \cdot t), \text{COS}(t)) = \frac{5 \cdot e^{5 \cdot t}}{26} - \frac{5 \cdot \text{COS}(t)}{26} + \frac{\text{SIN}(t)}{26}$$

$$\#10:\quad \text{INVLaplace}\left(\frac{1}{s - 5} \cdot \frac{s}{s^2 + 1}, s, t\right) = \frac{5 \cdot e^{5 \cdot t}}{26} - \frac{5 \cdot \text{COS}(t)}{26} + \frac{\text{SIN}(t)}{26}$$

Recommended links:

http://calclab.math.tamu.edu/~belmonte/m308/D/d6/d6.html

http://calclab.math.tamu.edu/~belmonte/TAMUDFEQ/TAMUDFEQ.html

http://calclab.math.tamu.edu/~belmonte/TAMUCALC/TAMUCALC.html

http://www.math.tu-berlin.de/fileadmin/i26_matheservice/Module/Skripten/anleitungDG.pdf

https://sourceforge.net/projects/wxmaxima/

http://www.t3europe.eu/resources/engineering-mathematics/laplace-transforms/

Reducing Fractions and its Application to Rational Expressions

Duncan E. McDougall

(I)

There is a variety of reasons today why a given student doesn’t learn or master a presented method or technique. As teachers , we are aware of diverse learning styles and conditions in the classroom. And, despite hard work and willingness to learn on the part of the student, the set objective is not met. Logically then, what do we do ? Alternative approaches to a problem are often sought and this is where “ thinking outside the box “ may come in handy; especially when a novel idea works and appeals to others.

Imagine the plight of the Math 10 or Math 11 student whose factoring skills are less than adequate. That person may find factoring a chore due to a lack of success with a conventional method.

In order to increase their comprehension, a student needs another approach to make the task more suitable. Consider reducing the fraction 39/65 to lowest terms. If we know that 13 is a factor of both 39 and 65, then we can write it as $\frac{39}{65}=\frac{13\times 3}{13\times 5}=\frac{3}{5}$. The educator knows that 13 is the greatest common factor but the student may not. Similarly, how would it be apparent to a student in an expression like $\frac{x^2-2x-3}{x^2-7x+12}$?

(II)

For reference and clarity I would now like to refer to my publication in the article, “Reducing Fractions”, published in the Scientific Journal of Junior Math and Science, London, England 1990, which demonstrates the premise that the only possible factors available to reduce a fraction to lowest terms, come from the difference between the numerator and the denominator. Numerically, it looks like this:

Reduce $\frac{39}{65}$ Steps:

(1) 65 – 39 = 26 demonstrates the difference between numerator and denominator.

(2) Factors of 26 are: 1, 2, 13, 26

(3) Disregard 1, 2, and 26 because they are even.

(4) Try 13. If 13 doesn't work, then nothing else will.

(5) $\frac{39}{56}\div\frac{13}{13}=\frac{3}{5}$

(III)

For reference and clarity I would now like to refer to my publication in the article, “Reducing Fractions”, published in the Scientific Journal of Junior Math and Science, London, England 1990, which demonstrates the premise that the only possible factors available to reduce a fraction to lowest terms, come from the difference between the numerator and the denominator. Numerically, it looks like this:

eg 1) Reduce $\frac{x^2-2x-3}{x^2-7x+12}$

Steps:

(1) $(x^2-2x-3)-(x^2-7x+12)$

$= -5x - 15$

$= 5\,(x - 3)$

(2) Disregard 5 and consider (x– 3) because 5 doesn't divide evenly into the numerator or the denominator but (x– 3) might.

At this stage, we can factor easily because we know that (x – 3) is one of the desired factors; failing that, use long division

(3) $x^2-2x-3=(x-3)(x+1)$ and
$x^2-7x+12=(x-3)(x-4)$

(4) $\frac{x^2-2x-3}{x^2-7x+12} \div \frac{x-3}{x-3} = \frac{x+1}{x-4}$

(IV)

eg 2) Reduce $\frac{x^3-1}{x-1}$

Steps:

(1) $(x^3-1)-(x-1)$

$= x^3 - 1 - x + 1$
$= x^3 - 1 = x\,(x^2 - 1)$
$= x\,(x - 1)\,(x + 1)$

(2) Disregard x and (x + 1) because neither x nor (x + 1) divide evenly into neither the numerator nor the denominator; the only remaining factor to consider is (x – 1).

(3) $x^3 - 1 = (x - 1)(x^2 + x + 1)$

(4) $\frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1} = x^2+x+1$

What I also like about this method is that we can discover which factors will not work in a given situation:

Consider $\frac{x^2 - x - 12}{x^2 - x - 6}$

Steps: (1) $(x2 - x - 12) - (x^2 - x - 6)$

$= -6$

Immediately we can see that there cannot be a common factor of the form ($x + a$). (Assuming that the algebra is done correctly of course)

(V)

As we can see, there is no point in factoring and looking far a common term if indeed none exists to begin with.

Now it's no secret that a method for finding the g.c.f. of two polynomials does exist, but it does involve long division, and therefore, it would look something like this:

Find the g.c.f. for $x^2 - 2x - 3$ and $x^2 - 7x + 12$, or g.c.f. ($x^2 - 2x - 3$, $x^2 - 7x + 12$).

Divide one into the other, and keep track of the remainder. Now divide the remainder into the previous divisor, and again keep track of the remainder. Continue this last step until the remainder is zero. The divisor, which gives zero as a remainder, is our g.c.f.

This means we would have:

$(x^2 - 7x + 12) : (x^2 - 2x - 3) = 1$

$\underline{-x^2 + 2x + 3}$

$-5x + 15 = -5(x - 3)$

Take only (x– 3) as –5 is not a factor of the form ($x + a$) and because –5 does not divide evenly into either the numerator or the denominator.

$(x^2 - 2x - 3) : (x - 3) = x + 1$

$\underline{-x^2 + 3x}$

$x - 3$

$\underline{-x + 3}$

0

The g.c.f. is ($x - 3$).

(VI)

Actually, finding the g.c.f. in this manner is part of the reason why the above method of subtraction works. The teacher now has more than one way of presenting this material to various types of learners, and can provide alternatives for the reluctant student.

A welcome application of this approach is the calculation of limits for the calculus student. In general, we have:

$$\lim_{x \to -a} \frac{x^2 + x(a+b) + ab}{x+a}$$

Instead of evaluating directly, and giving the indeterminate form $\frac{0}{0}$, we can subtract the two polynomials, factor this difference, and then try to reduce it to its lowest terms.

This would create the following:

$$\begin{aligned}(x^2 + x(a+b) + ab) - (x+a) &= x^2 + ax + bx + ab - x - a \\ &= x(x+a) + b(x+a) - 1(x+a) \\ &= (x+a)(x+b-1)\end{aligned}$$

(VII)

This reveals that (1) the expression can be reduced, and that (2) $(x + a)$ is the common factor. A numerical example would look like:

$$\lim_{x \to -2} \frac{x^2 + 3x + 2}{x+2} =$$

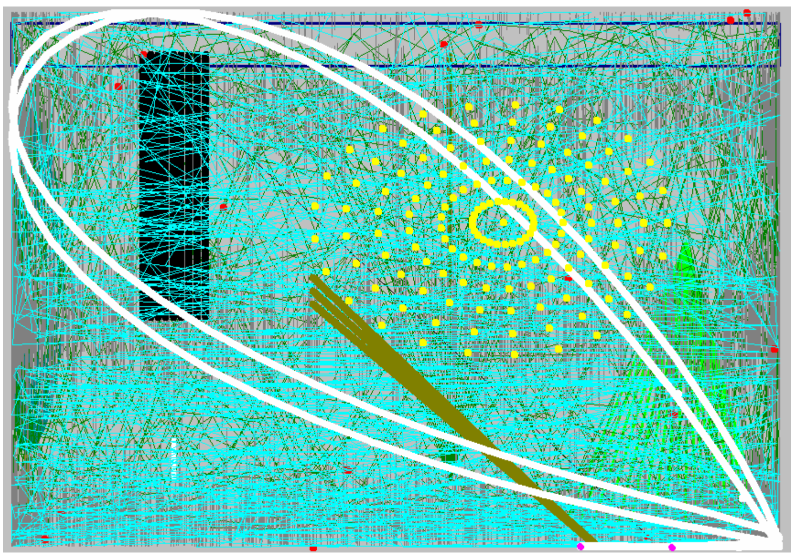
$$\begin{aligned}(x^2 + 3x + 2) - (x + 2) &= x^2 + 2x \\ &= x(x+ 2)\end{aligned}$$

$$= \lim_{x \to -2} \frac{(x+1)(x+2)}{x+2}$$

$$= \lim_{x \to -2} (x+1) = -1$$

disregard x and consider $(x + 2)$ because x doesn't divide evenly into the numerator or the denominator.

In summary, some students see math as a necessary evil. <however, now they can "get into it" a bit more because someone has found a method which makes sense to them.

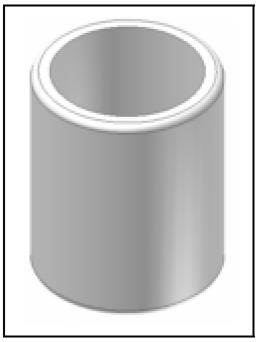


Eberhard Lehmann: Downhill in Hinterstoder

Wolfgang Alvermann: Eine Abituraufgabe

Newtonsches Abkühlungsgesetz → Regression / DGL

Bei der Herstellung von Werkstücken in der Metallbearbeitung finden Stähle Verwendung, die über bestimmte Werkstoffeigenschaften verfügen müssen. Diese Eigenschaften können unter anderem durch Wärmebehandlung gezielt beeinflusst werden. Folgende Wärmebehandlungen sollen zum Einsatz kommen:



1. Erwärmung auf eine bestimmte Anfangstemperatur.
2. Abschreckung (schnelle Abkühlung in einem Ölbad zur Steigerung der Härte).

Untersucht werden soll die Wärmebehandlung einer Hülse, die vor Beginn der Wärmebehandlung Umgebungstemperatur aufweist.

a) Während der Erwärmung der Hülse in einem Ofen, der eine Temperatur von $\vartheta = 1000$ Grad Celsius (°C) aufweist, werden zu bestimmten Zeiten die folgenden Stahltemperaturen gemessen:

t in Sekunden [s]	100	200	400	800	1000	1500
ϑ in [°C]	237	406	640	867	920	977

Zur Veranschaulichung soll der Erwärmungsprozess visualisiert werden.

Skizzieren Sie den Temperaturverlauf mit Hilfe der Messwerte in ein geeignetes Koordinatensystem.

Beschreiben Sie die Art des Temperaturanstiegs.

Bestimmen Sie eine Funktionsgleichung mittels geeigneter exponentieller Regression, mit der sich der Temperaturanstieg beschreiben lässt, und dokumentieren Sie dabei die Lösungsschritte.

b) Für die Beschreibung der Erwärmung gilt nun folgender funktionaler Zusammenhang

$$\vartheta(t) = 1000 - 980 \cdot e^{-0.0025t}$$

Berechnen Sie die Umgebungstemperatur, ab der der Erwärmungsprozess beginnt.

Für den anschließenden Abschreckvorgang der Hülse ist eine Anfangstemperatur von $\vartheta_A = 800°C$ erforderlich.

Untersuchen Sie, wie viele Hülsen im Ofen pro Stunde erwärmt werden können.

Für die vollständige Durchwärmung der Hülse darf die Temperaturänderung am Ende des Erwärmungsprozesses nicht mehr als $v_\vartheta = 0.6\frac{°C}{s}$ betragen.

Prüfen Sie, ob diese Vorgabe bei $\vartheta_A = 800°C$ eingehalten wird.

In den folgenden Aufgabenteilen wird von einer Anfangstemperatur von $\delta_A = 800°C$ und einer Ölbadtemperatur von $\vartheta_U = 20°C$ ausgegangen.

c) Die Hülse wird nach der Erwärmung in einem Ölbad von ϑ_A auf ϑ_U abgeschreckt. Der Abkühlungsprozess kann beschrieben werden mit Hilfe einer Funktionsschar ϑ_k mit $\vartheta_k(t) = \vartheta_U + (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t}$ (Newtonsches Abkühlungsgesetz), wobei der Parameter *k* vom verwendeten Öl abhängt.

Es gilt:
- t : Zeit in s
- $\vartheta(t)$: Temperatur in °C zum Zeitpunkt t
- k : Abkühlungskoeffizient in $\frac{1}{s}$

Beschreiben Sie den Einfluss der Parameter ϑ_U, ϑ_A und k auf den Verlauf des Graphen von ϑ_k.

Ermitteln Sie die Gleichung der Funktionsschar, die den Abschreckvorgang beschreibt.

Der Abschreckvorgang bis zu einer Temperatur von 50 °C soll genau 80s dauern.

Berechnen Sie zur Auswahl eines geeigneten Öls den Abkühlungskoeffizienten k .

d) Das Newtonsche Abkühlungsgesetz ist die Lösungsfunktion der Differenzialgleichung (DGL) $\vartheta_k{}'(t) = -k \cdot (\vartheta(t) - \vartheta_U)$.

Weisen Sie nach, dass die Funktionenschar $\vartheta_k(t) = \vartheta_U + (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t}$ eine Lösung der DGL ist.

Die in der Hülse während der Erwärmung gespeicherte Wärmeenergie berechnet sich mit der Gleichung $Q = m \cdot C \cdot \Delta\delta$

Es gilt:
- Q : Wärmemenge in KJ (kJ)
- m : Masse des Werkzeugs in Kilogramm (kg) — $m_{St} = 0.5$ kg
- $\Delta\delta$: Temperaturänderung in Kelvin (K)
- C : Spezifische Wärmekapazität in $\left(\frac{kJ}{kg \cdot K}\right)$ — $C_{St} = 0.47 \frac{kJ}{kg \cdot K}$

Während der Abschreckung auf Umgebungstemperatur wird die durch die Erwärmung in der Hülse gespeicherte Wärmeenergie Q auf das Ölbad übertragen. Das Öl muss gekühlt werden, wenn die abgegebene Wärmeenergie pro Hülse mehr als 200 kJ beträgt. Entscheiden Sie, ob das Öl gekühlt werden muss.

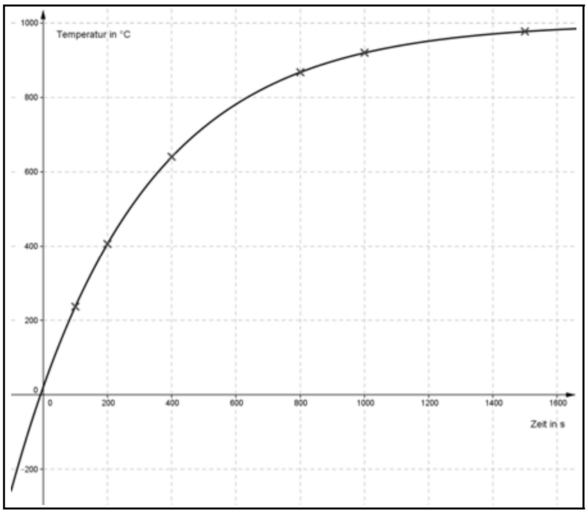
Die Wärmeenergie lässt sich andererseits auch als Fläche unter der Abkühlungskurve interpretieren.

Dann gilt: $Q(t) = A \cdot \int_0^{t_{ab}} (\vartheta_k(t) - \vartheta_U(t))\, dt$, mit $\vartheta_{0.0407}(t) = 20 + 780 \cdot e^{-0.0407 \cdot t}$

Weisen Sie nach, dass für die Hülse der Koeffizient $A = 0.00956\ \frac{kJ}{kg \cdot s}$ gilt und dokumentieren Sie einen Rechenweg, der auch ohne den Einsatz eines CAS nachvollziehbar ist. Berechnen Sie die Wärmeenergie, die in den ersten 50 Sekunden übertragen wird.

Lösung zu a)

Skizze des Temperaturverlaufs / Beschreibung



Die Erwärmung zeigt den Verlauf begrenzten Wachstums, da die Temperatur zunächst stark ansteigt, der Anstieg dann abnimmt um gegen den Sättigungswert 1000°C zu streben.

Berechnung der Funktionsgleichung mittels Regression

Erweiterung der Tabelle um die Zeile *1000-δ,* um eine Regressionsgleichung der Form $\vartheta(t) = a \cdot b^t$ bestimmen zu können.

t in Sekunden [s]	100	200	400	800	1000	1500
ϑ in [°C]	237	406	640	867	920	977
1000 - ϑ	763	594	360	133	80	23

Das CAS liefert aus den Zeilen *t und 1000-*ϑ die Werte a = 979.88 und b = 0.9975.
Daraus folgt:
$\vartheta(t) = 1000 - 979.88 \cdot 0.9975^t$ bzw. nach Umrechnung auf die Basis e

$$\boxed{\vartheta(t) \approx 1000 - 980 \cdot e^{-0.0025 \cdot t}}$$

Lösung zu b)

Berechnung der Umgebungstemperatur und der Hülsenzahl

Die Umgebungstemperatur beträgt ca. 20 °C $[\vartheta(0) = 20]$

Ansatz für die Hülsenzahl:

$800 = 1000 - 980 \cdot e^{-0.0025 \cdot t} \Rightarrow t = 635.69\ s$

$\frac{3600\ s}{635.69\ s} \approx 5.66$

Pro Stunde können ca. 5 Hülsen erwärmt werden.

Ermittlung der Temperaturänderung

$\vartheta'(t) = 2.45 \cdot e^{-0.0025 \cdot t} \Rightarrow \vartheta'(635.69) = 0.5 \Rightarrow v_f = 0.5 \frac{°C}{s}$

Die Vorgabe wird eingehalten.

Lösung zu c)

Einfluss der Parameter

ϑ_U: Eine Erhöhung von ϑ_U verschiebt den Grenzwert des Graphen von ϑ_k nach oben, eine Verringerung nach unten.

ϑ_A: Eine Erhöhung von ϑ_A verschiebt den Schnittpunkt des mit der Temperaturachse nach oben, eine Verringerung nach unten.

k: Eine Erhöhung von k staucht den Graphen von ϑ_k in t-Richtung, eine Verringerung streckt ihn.

Gleichung der Funktionenschar / Berechnung des Abkühlkoeffizienten

$\vartheta_U = 20°C,\ \vartheta_A = 800°C$ eingesetzt in $\vartheta_k(t) = \vartheta_U + (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t}$ ergibt

$\vartheta_k(t) = 20 + 780 \cdot e^{-k \cdot t}$

$\vartheta_k(80) = 50 \Rightarrow 50 = 20 + 780 \cdot e^{-80 \cdot k}$

$k = 0.0407$

Der Abkühlungskoeffizient hat einen Wert von $k = 0.0407 \frac{1}{s}$

Lösung zu d)

Nachweis der DGL – Möglichkeit 1

Einsetzen der Funktionsgleichung und ihrer Ableitung in die DGL

$$\left.\begin{aligned} \vartheta_k(t) &= \vartheta_U + (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t} \\ \vartheta_k{}'(t) &= -k \cdot (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t} \end{aligned}\right\} \text{Einsetzen ergibt } -k \cdot (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t} = -k \cdot \left(\vartheta_U + (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t} - \vartheta_U\right)$$

Nachweis der DGL – Möglichkeit 2

Lösung der DGL durch Trennung der Variablen

$$\left.\begin{aligned} &\vartheta_k{}'(t) = \frac{d\vartheta_k}{dt} = -k \cdot (\vartheta_k(t) - \vartheta_U) \\ &\int \frac{d\vartheta_k}{\vartheta_k(t) - \vartheta_U} = -\int k \cdot dt \\ &\ln(\vartheta_k(t) - \vartheta_U) = -k \cdot t + const \\ &\vartheta_k(t) - \vartheta_U = e^{-k \cdot t + C} = e^{-k \cdot t} \cdot \underbrace{e^{Const}}_{=C} \\ &\vartheta_k(t) = \vartheta_U + C \cdot e^{-k \cdot t} \end{aligned}\right\} \text{Für den Abkühlungsprozess gilt die Anfangsbedingung } \vartheta_k(0) = \vartheta_A \Rightarrow C = \vartheta_A - \vartheta_U$$

Also: $\boxed{\vartheta_k(t) = \vartheta_U + (\vartheta_A - \vartheta_U) \cdot e^{-k \cdot t}}$ q.e.d.

Die Möglichkeit 1 ist zu bevorzugen.

Entscheidung über Ölkühlung

$$Q = m \cdot C \cdot \Delta\vartheta = 0.5kg \cdot 0.47 \frac{kJ}{kg \cdot °C} \cdot (800 - 20)°C = 183.3kJ$$

In der Hülse ist die Wärmeenergie von 183.3 *kJ* < 200 *kJ* gespeichert; daher muss das Öl nicht gekühlt werden.

Nachweis von A ohne CAS

$$Q(t) = A \cdot \int_0^{t_{ab}} (\vartheta_{0.0407}(t) - 20)\,dt \Rightarrow \begin{aligned} 183.3 &= A \cdot \lim_{t_{ab}\to\infty} \int_0^{t_{ab}} 780 \cdot e^{-0.0407 \cdot t} dt \\ 183.3 &= A \cdot \lim_{t_{ab}\to\infty} \left[-19165 \cdot e^{-0.0407 \cdot t}\right]_0^{t_{ab}} \\ &= A \cdot \lim_{t_{ab}\to\infty} \left[-19165 \cdot e^{-0.0407 \cdot t_{ab}} - (-19165 \cdot e^{-0.0407 \cdot 0})\right] \\ &= 19165 \cdot A \end{aligned}$$

$$A = 0.00956 \frac{kJ}{K \cdot s}$$

Die abgegebene Wärmemenge in den ersten 50 s

$$Q(50) = 0.00956 \cdot \int_0^{50} (\vartheta_{0.0407}(t) - 20)dt = 159.27$$

In den ersten 50s wird eine Wärmemenge von 159.27 *kJ* abgegeben.

Anmerkungen

Vorgabe in der gymnasialen Oberstufe und beim Zentralabitur an den niedersächsischen Beruflichen Gymnasien sind berufsbezogene Aufgaben aus den Bereichen Technik oder Wirtschaft unter Einbeziehung eines GTR bzw. CAS. Wirkliche Anwendungsprobleme sind in den Schulbüchern nur ansatzweise zu finden, eigene Aufgaben zu entwickeln bleibt daher Aufgabe des Lehrers bzw. von Teams zur Erstellung von Beispielen, die annähernd praxisrelevant sind und den Schülern (und Lehrern) auch vermittelt werden können.

Bei dieser Aufgabe wurde eine Anleihe gemacht bei der Wärmebehandlung von Eisenwerkstoffen; dabei unterscheidet man u. a. das

- ***Glühen*** → langsames Erwärmen, Halten auf Glühtemperatur und langsames Abkühlen, um u. a. entstandene Spannungen im Werkstoff zu verringern
- ***Härten*** → Erwärmen und Halten auf Härtetemperatur, Abschrecken in Öl, Wasser oder Luft (der Stahl wird sehr hart und spröde), Anlassen (Erwärmung auf eine bestimmte Temperatur, um die Sprödigkeit und Bruchempfindlichkeit zu mindern)
- Weitere Verfahren wie ***Vergüten, Nitrierhärten …***

Die Aufgabe stammt aus dem niedersächsischen Zentralabitur 2012 für Berufliche Gymnasien; ich habe die Aufgabenstellung übernommen, der Lösungsteil in dieser Form stammt von mir!