THE DERIVE - NEWSLETTER #53

THE BULLETIN OF THE DERIVE USER GROUP + CAS-TI

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March 2004

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Liebe DUG-Freunde,

Da ist er nun, der erste Newsletter, der nicht in gedruckter Form erscheint. Ich wünsche ihm eine freundliche Aufnahme. Die Reaktionen unserer Mitglieder auf die Änderung war sehr unterschiedlich (Seite 28). Ich danke für das Verständnis, das Sie der Umstellung entgegengebracht haben. Ich hoffe, dass der Inhalt dieses DNL Sie daon überzeuget, dass wir unseren Standard halten. Die freie Mitgliedschaft bietet auch die Chance, unseren Kreis zu erweitern. Bitte weisen Sie CAS-Interessenten auf unsere Existenz hin. Alle Unterlagen können auf der homepage – geänderte Adresse im Editorial – nachgelesen werden.

Falls es Probleme mit den pdf-Dateien geben sollte (Zeichensätze, ...), dann schreiben Sie bitte. Gleichzeitig mit dem DNL#53 werden aktualisierte Ausgaben der DNLs #1 und #2 aus 1991 zum Download angeboten. Auch die zugehörigen mth-Dateien wurden überarbeitet. Die Endungen v5, bzw. v6 weisen auf Derive Versionen 5 und 6 hin.

Es war reizvoll, aber auch sehr mühsam, die alten Dateien mit der modernen Textverarbeitung und Derive-Version wieder zum Leben zu erwecken. Ich habe mich bemüht, die originale Erscheinungsform so weit wie möglich unverändert zu belassen und dabei gleichzeitig die Inhalte zu aktualisieren. Machen Sie einen Blick hinein und freuen Sie sich über den Fortschritt des CAS.

Ich wurde mehrfach ermahnt, dass ich in meinem Rückblick auf die ersten 13 Jahre DUG vergessen habe, mich auch bei meiner Frau Noor für ihren Einsatz und ihre Einsicht für die vielen von mir hinter dem PC verbrachten Stunden zu bedanken. Das sei hiermit "offiziell" nachgeholt. Ich möchte die Gelegenheit auch nützen, meinen Dank an Bernhard Kutzler nachzutragen, er erwies sich immer als ein sehr hilfreicher Freund und Ratgeber.

Beachten Sie bitte das reichhaltige Programm der Derive & TI-CAS, bzw. ACDCA-Konferenz TIME-2004 in Montreal. Im Rahmen dieser Konferenz wird ein Derive User Group Treffen stattfinden. Es wäre schön, dort viele Freunde zu treffen.

Ich möchte Sie auch auf den Wunsch unseres Freundes David Halprin hinweisen, der mit einer Gruppe von Interessierten dem Lebenswerk von Ernesto Cesáro die gebührende Aufmerksamkeit widmen möchte (Seite 44).

Mit herzlichen Grüßen

Dear DUG-Community,

Here it is, the first newsletter which does not appear in printed form. I hope that it will be welcomed by you. The reactions of our members on the change of publishing were very different (see page 28). I am very grateful for your understanding. The free membership gives the chance to find new members and contributors. Please inform CAS-interested people about the existence of the DERIVE & CAS-TI –Usergroup. All information can be downloaded from the homepages (one address has changed – see in Editorial).

If there are problems with the pdf-files (type fonts, ...) then please contact me. Together with DNL#53 I offer adapted issues of DNL#1 and #2 from 1991 for downloading. The respective MTH-files were also revised and can now be used with Derive 5 and Derive 6. I hope that the contents of DNL#53 will convince you that we tried to keep our standard.

It was a rewarding task, but also a tough one to revive the old files using modern text processors and recent Derive versions. I tried to leave the original appearance as unchanged as possible and simultaneously to revise and updroupate the contents for compatibility with Derive of today. Have a look and enjoy the evolution.

Many of you reminded me that in my view back to the past I forgot to thank my wife Noor for her work and for her understanding for my many hours spent in front of the PC. I´d like to make this good. And I´d like to take the occasion to express my gratefulness to Bernhard Kutzler, he always was a very helpful friend and adviser.

Please notice the rich program of TIME-2004 in Montreal (Derive & CAS-TI and ACDCA-Conference). We will have a DERIVE Usergroup meeting in the frame of this conference. It would be great to meet there many of you.

Finally I´d like to call your attention to our friend David Halprin´s idea to form a group of interested people to spread the work of the mathematician Ernesto Cesáro (page 44).

With my best regards as ever

DUG-Meeting 2004
TIME-2004, Montreal
Saturday, 17 July 2004, Lunchtime (tentative schedule)

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a contents of 44 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE* and the *TI*-89/92/*Voyage* 200 as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

As many of the *DERIVE* Users are also using the CAS-*TIs* the *DNL* tries to combine the applications of these modern technologies.

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Contributions:

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: June 2004
Deadline 15 May 2004

Preview: Contributions waiting to be published

Finite continued fractions St. Welke, GER
Some simulations of Random Experiments, J. Böhm, AUT
Wonderful World of Pedal Curves, J. Böhm
Another Task for End Examination, J. Lechner, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
ANOVA with *DERIVE* & *TI*, M. R. Phillips, USA
Hill-Encription, J. Böhm
CAD-Design with *DERIVE* and the TI, J. Böhm
Avoiding Convolution and Transforming Methods, M. Lesmes-Acosta, COL
Farey Sequences on the TI, M. Lesmes-Acosta, COL
Fuzzy Logic, G. Hagen, AUT
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
Modelling real date: enthalpie values
A TVM-Calculator for *DERIVE*, M. R. Phillips, USA
Shares, Put and Call, J. Böhm, AUT

and
Setif, FRA; Vermeylen, BEL; Leinbach, USA; Koller, AUT,
Keunecke, GER, ...and others

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Vladimir Bondarenko algebraistxxi@yahoo.com

Hello All,

I use Derive 6. Could I somehow suppress running the startup banner?

Any word of help is appreciated.

tbfowler100 tfowler@mitretek.org

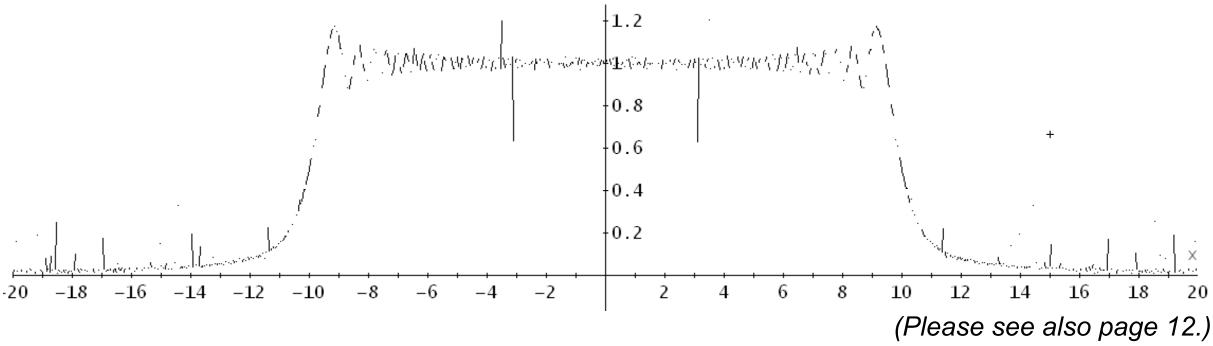
I just upgraded to Derive 6, despite the paucity of information on the TI website about new capabilities it has. I am pleased to report that while changes are not substantial, and the "look and feel" is about the same, it does have some new and important capabilities. Specifically, I was trying to do a two-dimensional Fourier integral over a rectangle (int(int(#e^(-2*pi*#i*(f*x+g*y)),x,-a/2,a/2),y,-b/2,b/2), but could not get Derive 5 to simplify its answer (which was correct) to a product of sines (which Mathematica does). This simplification requires putting two terms over a common denominator (Simplify-factor-trivial content -- available in Derive 5), but then a new setting: Options-mode setting-trigonometry-expand. Also, in Derive 5 I could not directly plot Bessel functions (Bessel_J(0,z))--program kept telling me it could not plot that function.
Derive 6 does so nicely. The manual by Kutzler and Kokol-Voljc also has some nice new chapters. So I recommend the upgrade!

$$\#1:\quad \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} e^{-2\cdot\pi\cdot i\cdot(f\cdot x+g\cdot y)}\,dx\,dy$$

$$\#2:\quad \frac{\cos(\pi\cdot a\cdot f-\pi\cdot b\cdot g)}{2\cdot\pi^2\cdot f\cdot g}-\frac{\cos(\pi\cdot a\cdot f+\pi\cdot b\cdot g)}{2\cdot\pi^2\cdot f\cdot g}$$

tbfowler100

Though the plotting routines of Derive work well, I have encounted a problem with a relatively simple plot. Perhaps someone can get this to work. Try plotting the chirp function, abs(int(#e^(#i*pi*x^2)*#e^(#i*2*pi*f*x),x,-10,10)) between (-20,20). This should give a rapidly oscillating function of about value 1 between (-10,10), and then rapidly fall to zero outside of that region. Derive gives a more or less rapidly oscillating function, though the points don't always connect, in the (-10,10) region, but instead of falling to zero outside this region, it just oscillates wildly. Looks like some convergence problem, but there doesn't seem to be any way to control any of the plot parameters. This function can be shown to be equivalent to another involving fresnel_cos and fresnel_sin functions: abs(#e^(-#i*pi*f^2)*(fresnel_cos(2^(1/2)*(10-f))-fresnel_cos(2^(1/2)*(-10-f))+#i*fresnel_sin(2^(1/2)*(10-f))-#i*fresnel_sin(2^(1/2)*(-10-f)))). When I tried that, it just went into some infinite loop.



(Please see also page 12.)

Manuel Garcia Mateos garcia_mateos@WEB.DE

Limit of a Sequence

Dear Derivians,

I need to know how to calculate a limit on a recursive sequence! For example I defined the following sequence `mittelwert(n):=iterates([b,(a+b)/2],[a,b],[0,1],n)`
I received a list and I know that the sequence `mittelwert(n)` converges to 2/3.
I think there is no problem to calculate the limit of a explicit sequence, but what about a recursive?

Thanks a lot for your help!
Manuel

Stefan Welke Spwelke@AOL.COM

Dear Manuel,

you can obtain your sequence of vectors by matrix-multiplication:

`f_0_([a,b]) = [[ 0 , 1 ],[ 1/2 , 1/2 ]]*[a,b]` and generally:

`f_n_([a,b]) = [[ 0 , 1 ],[ 1/2 , 1/2 ]]^n*[a,b]`

You can compute the eigenvalues l_1 , l_2 and eigenvectors e_1 , e_2 of this matrix and similar matrices, which describe recursive sequences. Then you can deompose [a,b] as a linear combination `[a,b] = alpha*e_1 + beta*e_2` and by linear algebra you obtain:
`f_n([a,b]) = alpha * l_1^n * e_1 + beta * l_2^n * e_2 ,`
which is the sum of two geometric sequences.

I hope this helps.
Stefan Welke

Ignacio Larrosa Cañestro ilarrosa@MUNDO-R.COM

`a(n+1) = (a(n)+ a(n-1)/2  ===>  2a(n+1) = a(n) + a(n-1)`

`2a(n+1) + a(n) = 2a(n) + a(n-1)`, for each n >= 1

Then,

`2a(n+1) + a(n) = 2a(2) + a(1)`

As a(n+1) is between a(n) and a(n-1), and |a(n+1) - a(n)| = |(a(n) - a(n-1)|/2, a(n) has limit as n ---> inf.

Let it a. Then

`2a + a = 3a = 2a(2) + a(1)  ====>  a = (2a(2) + a(1))/3`

Saludos,

Ignacio Larrosa Cañestro

Wim De Jong WimdeJongTwo@AOL.COM

Hi Manuel,

You are dealing with the sequence of vectors (a(n),b(n)) defined by

(a(n+1),b(n+1))=(b(n),0.5(a(n)+b(n))) with (a(0),b(0))=(01).

So you have a(n+1)=b(n) and b(n+1)=0.5(a(n)+b(n)), i.e. 2b(n+1)-b(n)-b(n-1)=0. This homogeneous linear recurrence relation (with constant coefficients) of degree 2 is treated in the same way as a homogeneous 2nd order linear differential equation (w.c.c.). Try b(n)=t^n. This yields the auxiliary equation 2t^2-t-1=0, which has the solutions -0.5 and 1. So the general solution of the recurrence relation is b(n)=A(-0.5)^n+B, where A and B are arbitrary constants. Hence (a(n),b(n)) = (A(-0.5)^(n-1)+B, A(-0.5)^n+B). Now (a(0),b(0))=(0,1) yields the special solution ((1/3)(-0.5)^(n-1)+2/3,(1/3)(-0.5)^n+2/3). And as n tends to infinity (a(n),b(n)) tends to (2/3,2/3).

Another interesting technique for solving 2b(n+1)-b(n)-b(n-1)=0 with b(0)=1 and b(1)=0.5 is the generating function method. Consider the so-called generating function G(x)=b(0)+b(1)x+b(2)x^2+ b(3)x^3+ ... (this is not really a function; it is an expression for b(n) which allows convenient algebraic operations).

G(x) = 1+0.5x+0.5(b(1)+b(0))x^2+0.5(b(2)+b(1))x^3+ ...=
= 1+0.5x{1+b(1)x+b(2)x^2+...} + 0.5x^2{b(0)+b(1)x+b(2)x^2+...} = 1+0.5xG(x)+0.5x^2G(x),

which yields

G(x)=1/(1-0.5x-0.5x^2)=1/{(1+0.5x)(1-x)} = (use partial fractions) (1/3)/(1+0.5x)+(2/3)/(1-x) = (use the binomial expansion) (1/3){1-0.5x+(0.5^2)x^2-(0.5^3)x^3+...}+(2/3){1+x+x^2+...}. In this formal power series the coefficient of x^n (n=0,1, 2, ...) is (1/3)(-0.5)^n +2/3.

Thus b(n)=(1/3)(-0.5)^n+2/3 for n=0,1,2,... .

Cheers, Wim

Patrick West patrick@fpwest.com, www.fpwest.com

Folks,

I just recently moved to Pocket PC from the HP 200LX palmtop.

I am happy to report that my old DOS version of Derive works nicely under PocketDOS on my Ipaq 3845.

I also obtained the ti-89 emulator shell for Pocket PC and loaded the rom from my 89 and it works nicely.

James F. Gordon jfgordon@FRONTIERNET.NET

Is Derive 6 able to perform implicit differentiation? For example, how can I find dy/dx for the following implicit function of x:

y^3*sin(x*y) = x^4.5

I was not able to find implicit differentiation in the introductory text that accompanied the CD.
Thank you for any help you can give me on this.

James

Louis F. Lowell

Does anyone know how to do implicit differentiation with Derive? Specially something like

x + sqrt(x*y) – y = 4 or **y^2 + e^(x*y) = sin(x^2*y^2)+2**

Thanks for any help anyone can provide.

Aleksey D. Tetyorko chib@MEGASTYLE.COM

I cannot do best then you can see in `implicit.mth`.
Aleksey

```
==============implicit.mth====================================
y^3*SIN(x*y)=x^(4.5)

DIF(y^3*SIN(x*y)=x^(4.5),x)*t+DIF(y^3*SIN(x*y)=x^(4.5),y)*u

;Simp(#2)
(u*x*y^3+t*y^4)*COS(x*y)+3*u*y^2*SIN(x*y)=9*t*x^(7/2)/2

;Solve(#3,u)
SOLVE((u*x*y^3+t*y^4)*COS(x*y)+3*u*y^2*SIN(x*y)=9*t*x^(7/2)/2,u)

;Simp(Solve(#3,u))
u=t*(9*x^(7/2)-2*y^4*COS(x*y))/(2*y^2*(x*y*COS(x*y)+3*SIN(x*y)))

u/t

SUBST(u/t,u,RHS(u=t*(9*x^(7/2)-
2*y^4*COS(x*y))/(2*y^2*(x*y*COS(x*y)+3*SIN(x*y)))))

;Simp(#7)
(9*x^(7/2)-2*y^4*COS(x*y))/(2*y^2*(x*y*COS(x*y)+3*SIN(x*y)))
==============implicit.mth====================================
```

MacDonald Phillips phillipsm@GAO.GOV

There is a command in Derive 6 for implicit differentiation. It is in one of the utility files and will be automatically called up when you type it in. First, bring the x^4.5 to the left hand side so you have an expression that is implicitly equal to zero. The command is:

`IMP_DIF(u,x,y,n)`

This gives the nth implicit derivative of y wrt to x of expression u. Since n defaults to 1 all you have to enter for your example is:

`IMP_DIF(y^3*sin(x*y)-x^4.5,x,y)`

Hope this helps.

Terence Etchells

`IMP_DIF(u, x, y, n)` *can be used also in earlier Derive versions (function of DIFF_APPS.mth utility file)*

$$\mathrm{IMP\_DIF}(x + \sqrt{(x\cdot y)} - y = 4,\ x,\ y) = ?$$

$$\mathrm{IMP\_DIF}(y^2 + \hat{e}^{x\cdot y} = \mathrm{SIN}(x^2 + y^2) + 2,\ x,\ y) = \left[- \frac{y\cdot \hat{e}^{x\cdot y}}{x\cdot \hat{e}^{x\cdot y} + 2\cdot y} = - \frac{x}{y}\right]$$

It´s obvious, that it doesn´t make any sense to apply imp_dif on equations. Students must know this and bring all expressions on one side of the equation (according to Don´s mail) and are not allowed to believe the result without any doubts. In case if there is a constant on one side of the equation Derive 5 returns a ? showing that something must be wrong. Derive 6 does not and offers a "result".

$$\mathrm{IMP\_DIF}(x + \sqrt{(x\cdot y)} - y = 4) = \left(\frac{y}{x} = -1\right)$$

*It is very interesting to follow **Stepwise Simplification** of Implicit Differentiation of DERIVE 6. See the first step:*

$$\#1: \quad \mathrm{IMP\_DIF}(y^3 \cdot \mathrm{SIN}(x \cdot y) - x^{4.5})$$

$$\frac{d}{dx}(F(x) + G(x)) \Rightarrow \frac{d}{dx}F(x) + \frac{d}{dx}G(x)$$

$$\#2: \quad \mathrm{ITERATE}\left(\frac{d}{dx}dk - \frac{\left(\frac{d}{dy}dk\right)\cdot\left(\frac{d}{dx}(-x^{9/2}) + \frac{d}{dx}(y^3 \cdot \mathrm{SIN}(x \cdot y))\right)}{\frac{d}{dy}(y^3 \cdot \mathrm{SIN}(x \cdot y) - x^{9/2})}, dk, -\frac{\frac{d}{dx}(-x^{9/2}) + \frac{d}{dx}(y^3 \cdot \mathrm{SIN}(x \cdot y))}{\frac{d}{dy}(y^3 \cdot \mathrm{SIN}(x \cdot y) - x^{9/2})}, 0\right)$$

and the final ones:

$$\#17: \quad \mathrm{ITERATE}\left(0, d \cdot k, \frac{\frac{9 \cdot x^{7/2}}{2} - y^4 \cdot \mathrm{COS}(x \cdot y)}{y^3 \cdot \mathrm{COS}(x \cdot y) \cdot x + \left(\frac{d}{dy}y^3\right) \cdot \mathrm{SIN}(x \cdot y)}, 0\right)$$

$$\frac{d}{dx}x^n \Rightarrow n \cdot x^{n-1}$$

$$\#18: \quad \mathrm{ITERATE}\left(0, d \cdot k, \frac{\frac{9 \cdot x^{7/2}}{2} - y^4 \cdot \mathrm{COS}(x \cdot y)}{y^3 \cdot \mathrm{COS}(x \cdot y) \cdot x + 3 \cdot y^2 \cdot \mathrm{SIN}(x \cdot y)}, 0\right)$$

$$\#19: \quad \mathrm{ITERATE}\left(0, d \cdot k, \frac{9 \cdot x^{7/2} - 2 \cdot y^4 \cdot \mathrm{COS}(x \cdot y)}{2 \cdot y^2 \cdot (x \cdot y \cdot \mathrm{COS}(x \cdot y) + 3 \cdot \mathrm{SIN}(x \cdot y))}, 0\right)$$

I tried to extend imp_dif() to imp_diff(), which works also for implicitly given expressions, without collecting both sides on on side of the equation: Josef

```
        imp_diff(u, x, y, n := 1) :=
          If RHS(u) = 0
#1:          IMP_DIF(u, x, y, n)
             IMP_DIF(LHS(u) - RHS(u), x, y, n)
             IMP_DIF(LHS(u) - RHS(u), x, y, n)
```

$$\#2: \quad \mathrm{imp\_diff}(x + \sqrt{x \cdot y} - y = 4) = \frac{y \cdot (\sqrt{x \cdot y} + 2 \cdot x)}{x \cdot (2 \cdot y - \sqrt{x \cdot y})}$$

$$\#3: \quad \mathrm{imp\_diff}(4 - x - \sqrt{x \cdot y} + y) = \frac{y \cdot (\sqrt{x \cdot y} + 2 \cdot x)}{x \cdot (2 \cdot y - \sqrt{x \cdot y})}$$

The following is a letter from Kathrin who needed support in programming:

Hello all!

For my skilled work I have to program a function (in Derive) which computes the error of Taylor series. This function which I called "`fehlerfunktion`" works:

```
fehlerfunktion(funktion, n, epunkt, F) :=
  Prog
    F := funktion - TAYLOR(funktion, x, epunkt, n)
    RETURN F
```

(`funktion` is e.g. sin(x) and `epunkt` is the the position at which the polynomial shall approximate)

Then I tried to compute the error at the point `bpunkt` because I am not allowed to set `x:=bpunkt` and to use the function solve in a second step. I had to write a new function which I called `fehler` and this one does not work and I don't know why:

```
FEHLER(funktion, n, epunkt, bpunkt, M) :=
  Prog
    M := funktion - TAYLOR(funktion, x, epunkt, n)
    x := bpunkt
    RETURN M
```

My third job was to find a function which computes the n which approximates at `bpunkt` as good as the user wants (the acceptable error I called `fehlerwert`):

```
napproximation(funktion, n, epunkt, bpunkt, fehlerwert) :=
  Prog
    Loop
      If FEHLER(funktion, n, epunkt, bpunkt) > fehlerwert
         RETURN n
      n :+ 1
```

Because my second function does not work I don't know if the third one is correct (using this function Derive just computes and computes without displaying a result) .

This is my first time using Derive and maybe my questions sound very banal. Nevertheless I would be very happy if you could help me! Thanks for any kind of help anybody can provide!

with kind regards

Kathrin Becker

DNL:

Dear Kathrin,
I wanted to follow your ideas and tried to correct your programming efforts:

```
FEHLER(funktion,n,epunkt,bpunkt):=
                        SUBST(funktion-TAYLOR(funktion,x,epunkt,n),x,bpunkt)
```

and then using this auxiliary function use the LOOP as follows:

```
napproximation(funktion, epunkt, bpunkt, fehlerwert, n := 0) :=
  Loop
    If ABS(FEHLER(funktion, n, epunkt, bpunkt)) < fehlerwert
       RETURN n
    n :+ 1
```

which must be typed in as:

```
napproximation(funktion,epunkt,bpunkt,fehlerwert,n:=0):=
LOOP(IF(ABS(FEHLER(funktion, n, epunkt, bpunkt)) < fehlerwert, RETURN n),
     n :+ 1)
```

Then it works as you can see below. Johann Wiesenbauer provided a very compact function, which is more general because it allows also variables ≠ x.

Josef

Johann Wiesenbauer

Hi Kathrin,

For a start, try the following routine

```
napprox(u, x, x0, x1, epsilon, n := 0) :=
 LOOP(
  IF(ABS(SUBST(u - TAYLOR(u, x, x0, n), x, x1)) < epsilon, RETURN n),
  n :+ 1)
```

and tell me, if it does what you wanted it to do.

Cheers, Johann

$$\text{napprox}\left(\text{SIN}(x),\ x,\ 1,\ 2,\ \frac{1}{1000}\right) = 6$$

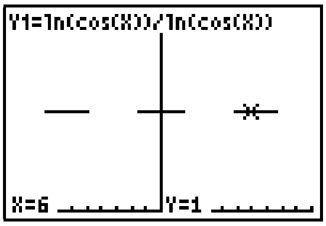
$$\text{napproximation}(\text{SIN}(x),\ 1,\ 2,\ 0.0001) = 6$$

Some weeks ago I received an e-mail from Kim Hendrickx, TI-Support Centre, who forwarded a problem, which was sent to him recently: This was the problem:

Prof. Dr. P.

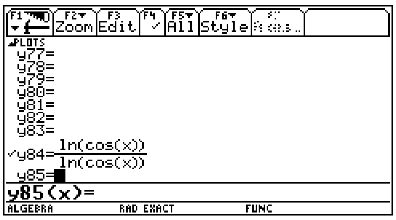
Prof P. wants to plot the graph of
$y = \ln(\cos(x)) / \ln(\cos(x))$ over the domain $Db(f) = \{x \mid -pi/2 + 2k*pi < x < pi/2 + 2k*pi,\ k$ integer).
The function value is 1 where the function is defined.

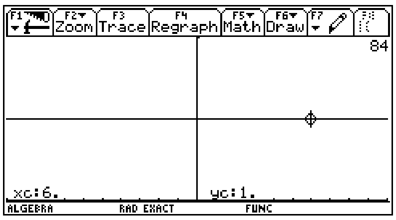
On the TI-83Plus:



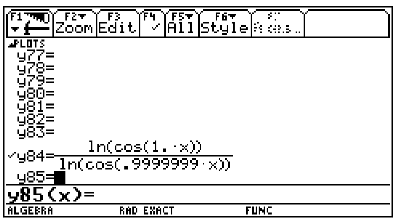
The graph is correct! But ....

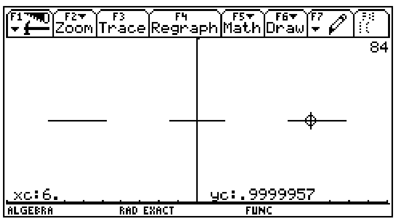
On the TI-89, TI-92Plus, Voyage 200:





The CAS-Calculator plots the line y = 1 and represents incorrectly function values outside of the domain. But if I change the function definition slightly then:





This was my answer and I would like to ask all of you about your opinion, because I am not sure if I am right with my conclusions. Josef

Dear Professor P., Dear Kim,

Many thanks for the interesting graphics problem, which seems to be more a mathematical than a graphical problem.
First of all, DERIVE and MuPad are both plotting $y = 1$ as graph of your function.

I believe that there are two reasons which are responsible for the graphic representation of this quotient-function of two logs. (By the way, ln(x)/ln(x) shows also $y = 1$, which should have a graph only for $x > 0$ following your intentions??)

(1) The Computeralgebra System seems to recognize numerator and denominator as identical and cancels **before** evaluating numerator and denominator ....

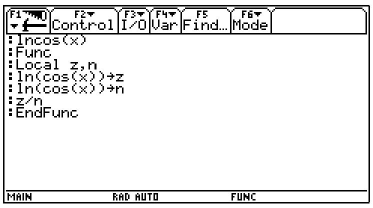
(2) More likely the CASs are calculating with complex numbers.

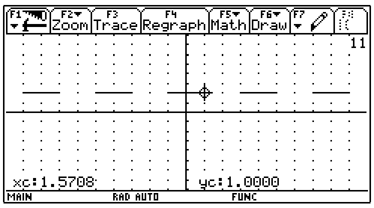
Before demonstrating a possible solution of your problem, i.e. to reproduce the TI-83 plot on the TI-CAS-Calculators I´d like to mention some remarkable results, which could inspire discussion in classroom.

Investigating the "wrong" graph using TRACE you will find that there is no function value given for all places x with cos(x) = 1 (log = 0). This happens for $x = 0, 2\pi, 4\pi$, ... but for $x = \pi, 3\pi$, ... we find non-real result. If we change Complex Format (MODE) from REAL to RECTANGULAR, we obtain again the quotient 1, because of $\ln(-1) = \pi * i$.

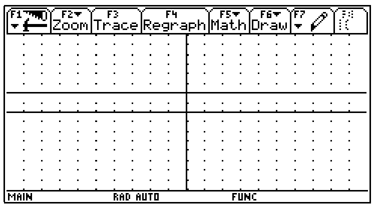
The TI-83 cannot perform all these calculations, because of its lack of computeralgebra. I believe that this calculator tries to evaluate numerator and denominator numerically and being unable to do this in the complex region leaves the function value undefined – and consequently doesn´t plot any pixel.

I tried to produce the desired plot and wrote a function which prevents cancellation and and simulates the TI-83+ more or less. Function lncos(x) is written into the Y=-Editor and then plotted.





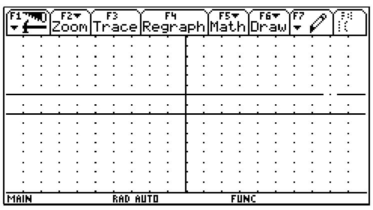
Complex Format REAL



Complex Format: RECTANGULAR

Playing around with this function I came across another interesting fact. Increasing the xres-value in WINDOW, the resulting plot shows a gap (see the picture below). But this happens only for xres = 5 with the ZoomTrig-setting. Exchanging cos by sin, the gap appears in the left halfplane.

Any explanation??



I hope that my attempt to interpret this interesting phenomenon could help and remain ....

Unfortunately I only received an very friendly answer from Kim, but non from Prof P. So I don´t know if I could satisfy him.

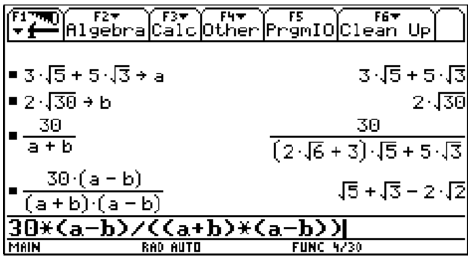
Gerhard Pachler, Austria

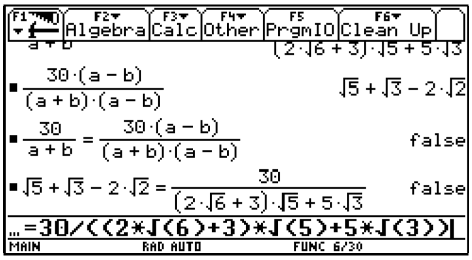
The students solved the problem to rewrite the following fraction without roots in the denominator:

$$\frac{30}{3\sqrt{5}+5\sqrt{3}+2\sqrt{30}}$$

Calculation by hands reults in: $\dfrac{30}{3\sqrt{5}+5\sqrt{3}+2\sqrt{30}} = \sqrt{5}+\sqrt{3}-2\sqrt{2}$

Then we repeat the calculation using the TI-92:

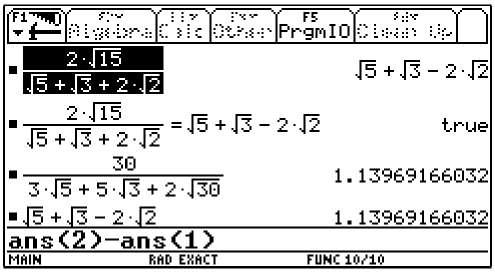


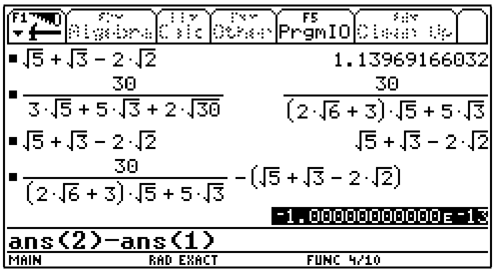


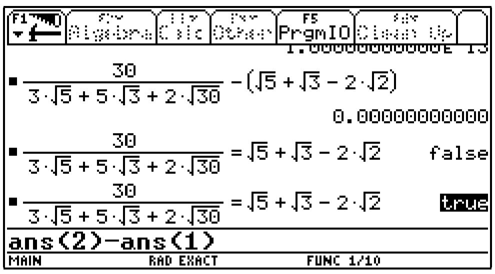
Why does the TI not recognize the equivalence of the expressions? How can I convince the machine?

DNL:

This is my treatment of the roots (fortunately not at the dentist!)







After some tries which were answered with **false** *I factored out* sqrt(15) *in numerator and denominator and cancelled the root – and now the equivalence became* **true***!!*

Then I proceed:

Copying the two expressions as ans(2) *and* ans(1) *into the edit line and then approximating* ans(2) - ans(1) *I receive a result which differs from 0 – which might be an internal numerical affair??*

But if I copy the expressions into the edit line or if I type them in the Exact Mode refuses to perform any calculation, approximating the difference I obtain 0.00000…. for all decimal places.

Evaluating the equation exactly I receive **false***, but approximating it I get* **true** *again.*

So I am not surprised that approximating ans(2)=ans(1) *is* **false***.*

It seems that we could write a PhD on the behaviour of the CAS-TIs in treating expressions containing roots.

Best regards
Josef

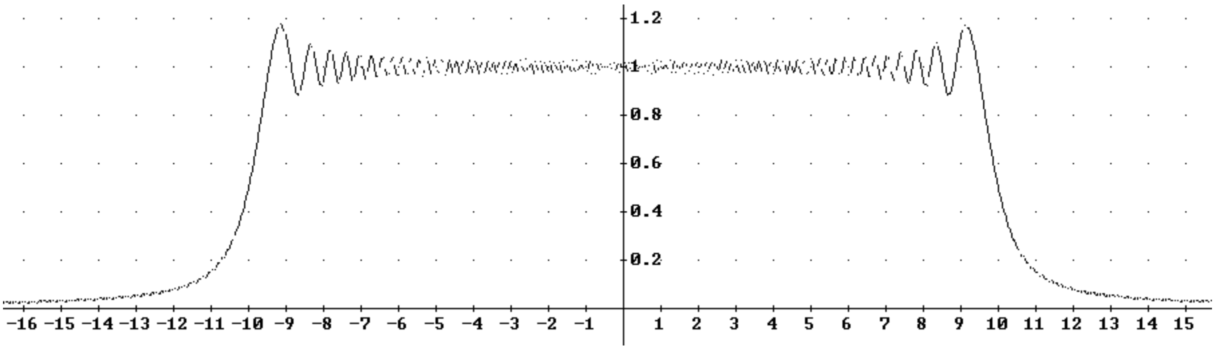
Are there any other findings??

Mate Matica

The expression (see page 3) *abs(int(#e^(#i*pi*x^2)*#e^(#i*2*pi*f*x),x,-10,10))* equals

$$\frac{\left|\mathrm{ERF}\left(\frac{\sqrt{2}\cdot\sqrt{\pi}\cdot(1-\hat{\imath})\cdot(f-10)}{2}\right)-\mathrm{ERF}\left(\frac{\sqrt{2}\cdot\sqrt{\pi}\cdot(1-\hat{\imath})\cdot(f+10)}{2}\right)\right|}{2}$$

(it can be obtained using Derive with some tricks). Now the numerical errors (generated by INT) are eliminated and the plot is ok.



Volker Loose

Hello all,
is there a function in Derive which returns the length of the periodicalpart of a decimal fraction,
is there a function in Derive which returns the length of the preperiodical part of a decimal fraction?

Example: 1/28=0.03571428571428...

preperiodical part 03, length 2
periodical part 571428, length 6

Thanks
Volker

Johann Wiesenbauer

Actually this is the same question Peter Schofield asked me some time ago.
I promised to write a utility function then, but sadly enough forgot about it. Sorry, Peter! Well, at last in the attached file there is a first version which might live up to the expectations of both of you.

Cheers,
Johann

Albert Rich

Hello Johann,
In your Titbits 26 in DNL#52 you challenged the reader to determine the number of digits in the recently discoved largest known Mersenne prime: 2^20996011-1 using Derive. That's easy, simply approximate it giving

```
1.259768954·10^(6.320429·10^6)
```

The number of digits is this power of 10 plus 1.
Probably the easiest challenge you have ever issued. :=)
Aloha, Albert

On Josephus' Permutations

Stefan Welke, Bonn, Germany, spwelke@aol.com

1 Faster computation

The following code presents two functions, which compute Josephus' permutaions. These permutations were described in DNL #52 by Rüdeger Baumann as a programming challenge. Rüdeger's function is named `josephus1(n, k)`. You can find the code in DNL #52.

```
        josephus2(n, k) :=
          Prog
            [v := [1, ..., n], w := [], q := n, p := 1, j := 1]
            Loop
#1:           If q = 0
                 RETURN w
              If p = k
                 [w := APPEND(w, [v↓j]), p := 1, v := DELETE(v, j), IF(j = q, j := 1), q :- 1]
                 [p :+ 1, IF(j < q, j :+ 1, j := 1)]
```

`josephus2(n,k)` is slightly faster than Rüdeger's function, but it shows the same time behaviour with respect to the arguments n and k: The computation time depends nearly linearly on n and on k.

```
josephus3(n, k, v, w, q, p, j, m, z) :=
  Prog
    [v := [1, ..., n], w := [], q := n, p := 1, j := 0, m, z, v]
    Loop
      If k ≥ q exit
      m := FLOOR((q - j)/k)
      z := VECTOR(j + i·k, i, 1, m)
      w := APPEND(w, v↓z)
      v := DELETE(v, z)
      j := m·k + j - q
      q := q - m
    j := 1 + MOD(j + k - 1, q)
    Loop
      If q = 0
         RETURN w
      w := APPEND(w, [v↓j])
      v := DELETE(v, j)
      q :- 1
      j := 1 + MOD(j - 2 + k, q)
```

Derive 6 provides a new `DELETE`-command: `DELETE([a,b,c,d,e],[1,3,5])` removes elements a, c and e from the first vector.

The second version is reasonably faster than the preceeding one. The time behaviour depends mainly on n and not on k. The following table shows some computation times in seconds on my 3.6 Ghz Pentium.

n=300	k=20	k=40	k=150	k=200	k=280	k=400	k=600
josephus1	1.77	3.55	13.3	17.4	24.5	34.9	52.4
josephus2	0.250	0.453	1.58	2.09	2.91	4.14	6.28
josephus3	0.015	0.031	0.047	0.047	0.047	0.047	0.047

2 The set of special Josepus permutations

The Josephus functions generate lists of numbers, which represent permutations of the set $\{1, 2, ..., n\}$ of the first n natural numbers. Now the question is, if every permutation is a Josephus permutation. The number of different pemutations of n different objects is $n!$, so the sequence $\{josephus(n,1), josephus(n,2), ..., josephus(n,1+n!)\}$ must contain at least two equal permutaions, and the next example demonstrates this fact:

```
#7:   josephus3(20, 1 + 20!)

#8:   [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
```

But the following theorem shows, that Josephus permutations are only a subset of the group of all permutations for $n \geq 4$, because $\text{lcm}(1,2,...,n) \leq n!$ for $n \geq 4$.

Theorem

The equation $josephus(n,k) = josephus(n, k + \text{lcm}(1,2,...,n))$ holds for all $n, k \in \mathbb{N}^*$.

```
#9:  LCM([1, ..., 200])

#10: 337293588832926264639465766794841407432394382785157234228847021917234018060677390066992000

#11: josephus3(200, 1 + LCM([1, ..., 200]))

#12: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28,
. . . . . .
      163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182,

      183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
```

(Computation time: 0.031 sec)

3 Encryption

A slight modification of `josephus3(n,k)` allows us to permute the order of characters in a given string by a Josephus permutation, where n is the number of characters in that string.

```
       josephus4(list, k, v, w, j, q, m, z) :=
         Prog
           [v := list, w := [], j := 0, q := DIM(list)]
           Loop
             If k ≥ q exit
             m := FLOOR((q - j)/k)
             z := VECTOR(j + i·k, i, 1, m)
             w := APPEND(w, v↓z)
             v := DELETE(v, z)
#13:         j := m·k + j - q
             q := q - m
           j := 1 + MOD(j + k - 1, q)
           Loop
             If q = 0
                RETURN w
             w := APPEND(w, [v↓j])
             v := DELETE(v, j)
             q :- 1
             j := 1 + MOD(j - 2 + k, q)

#14:  josephus4([a, b, c, d, e], 5)

#15:                                              [e, a, c, d, b]
```

We now use this function to define encoding and decoding functions for strings:

```
#16:  encode(string, k) := CODES_TO_NAME(josephus4(NAME_TO_CODES(string), k))

#17:  encode(Hallo, fellow derivians, 11)

#18:                                          lnlswlrfavdoe iH o,aeil

      decode(string, k, w, v, txt) :=
        Prog
#19:      w := NAME_TO_CODES(string)
          v := josephus3(DIM(w), k)
          CODES_TO_NAME(SORT([v, w]')'↓2)

#20:  decode(lnlswlrfavdoe iH o,aeil, 11)

#21:                                          Hallo, fellow derivians
```

Permutations are no good means for encryption. But here the whole text is permuted, and not only the alphabet. Thus the longer the text, the more possible permutations are at hand. For example: A text of 200 characters admits `LCM([1,…,200])`, which is about $3.372935888 \cdot 10^{89}$, possible Josephus permutations.

The given decoding function works painfully slow, due to the two matrix tranpositions in the code. Function `decode2(string,k)` works considerably faster:

```
      decode2(string, k, v_, n_, q_, z_, w_) :=
        Prog
          v_ := NAME_TO_CODES(string)
          n_ := DIM(string)
          q_ := 1
          z_ := [1, ..., n_]
#22:      w_ := josephus3(n_, k)
          Loop
            If q_ = n_ + 1
               RETURN CODES_TO_NAME(z_)
            z_ := REPLACE(v_↓q_, z_, w_↓q_)
            q_ :+ 1

#23:  decode2(lnlswlrfavdoe iH o,aeil, 11)

#24:                                          Hallo, fellow derivians
```

Decoding a string is easy, if we know the number of the permutation given by the Josephus function. Here is my challenge for the readers of the *Derive News Letter*:

> Decode the following string, which was encoded with `josephus4` and the resulting encoding function.

```
"ruimunlasee,dvretoI -uo noqudne  avmdaaW eowmckadwao h,ot  ni e ei aamo
ntphnm sobayoc, o,p,or eeoi nn  l stpe pogut rf tauot edstyiplwrdnI,n gpi
dhnselt or as iib cnrnamln  a .onlae f naha  a rydi ra,ierg gent .naemnidgo
yppg rmp m eongdrhoy yrpetOa pshun  enseogadthorti ceo te h miryeryynrvI
ii dtc  leT lan f ddge iade-r.ermm p "
```

There is no space character at the end of the first four lines.

Der Artikel von Franz SCHLÖGLHOFER im DNL52 hat mir ausgezeichnet gefallen, ebenso Ihre Ergänzungen. Es ist mir gelungen, die "threads" für vier Punkte, also für die Bézier-Kurve 3. Grades in der Ebene hinzubekommen. Wenn Sie Zeit und Lust haben, werfen Sie einen Blick darauf.
Allerdings beschleicht mich der Verdacht, dass das einfacher auch gehen könnte/müsste. Das "riecht" förmlich nach einer Rekursion, oder?
Gerhard Hagen

Bézier Curve of degree 3 – Threads for four points

Gerhard Hagen, Althofen

We start with an example: A(-5|0), B(0|0), C(8|0), D(9|9) and develop threads for groups of three points:

```
[A := [-5, 9], B := [0, 0], C := [8, 0], D := [9, 9]]
```

$$\text{threads2}(p1\_, p2\_, p3\_, n) := \text{VECTOR}\left(\left[p1\_ + \frac{i}{n}\cdot(p2\_ - p1\_),\ p2\_ + \frac{i}{n}\cdot(p3\_ - p2\_)\right], i, 0, n\right)$$

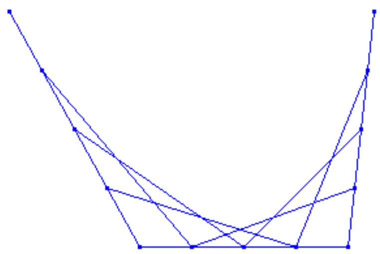
Two `threads2` are composed using 4 points and resulting in `help2`.

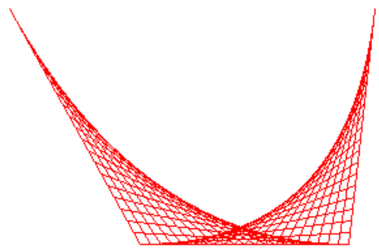
```
help2(p1_, p2_, p3_, p4_, n) := APPEND(threads2(p1_, p2_, p3_, n), threads2(p2_, p3_, p4_, n))
```

Simplfying `help2` we note that some threads appear twice. `help3` resolves this "problem".

```
help3(p1_, p2_, p3_, p4_, n) := DELETE(help2(p1_, p2_, p3_, p4_, n), n + 1)
```

```
help3(A, B, C, D, 4)

help3(A, B, C, D, 20)
```

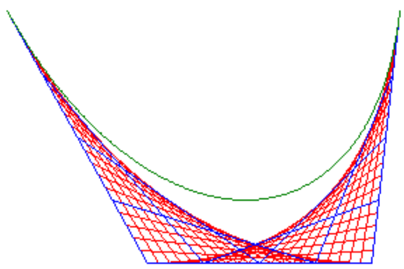




This auxiliary construction does not osculate the curve except in the end points.

$$\text{bez3}(a\_, b\_, c\_, d\_, t) := (1 - t)^3\cdot a\_ + 3\cdot(1 - t)^2\cdot t\cdot b\_ + 3\cdot(1 - t)\cdot t^2\cdot c\_ + t^3\cdot d\_$$

```
bez3(A, B, C, D, t)
```



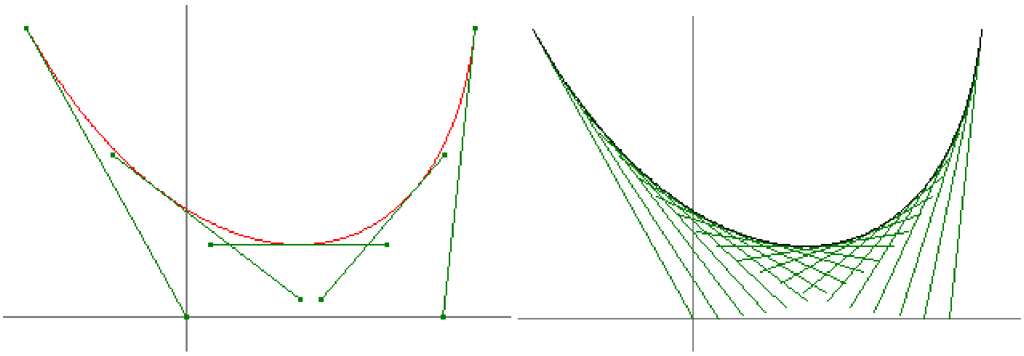
Final composition of `threads3` from `threads2` and `help3` requires some "Try and Error Testing and Playing":

The groups must fit together and groups of three points must be chosen correctly. In the respective `thread2` only each i-th segment is tangent of the curve.

$$\text{threads3}(p1\_, p2\_, p3\_, p4\_, n) := \text{VECTOR}\left[\left(\text{threads2}\left((\text{help3}(p1\_, p2\_, p3\_, p4\_, n))_{i,1}, (\text{help3}(p1\_, p2\_, p3\_, p4\_, n))_{i,2}, (\text{help3}(p1\_, p2\_, p3\_, p4\_, n))_{i+n,2}, n\right)\right)_i, i, 1, n+1\right]$$

threads3(A, B, C, D, 4)

threads3(A, B, C, D, 20)



An open question: What is the locus of the endpoints of the threads?

Albert Rich & Johann Wiesenbauer: ***Facing the Challenge***

Hello Johann,
In your Titbits 26 in DNL#52 you challenged the reader to determine the number of digits in the recently discovered largest known Mersenne prime: 2^20996011-1 using Derive. That's easy, simply approximate it giving

```
1.259768954·10^(6.320429·10^6)
```

The number of digits is this power of 10 plus 1.
Probably the easiest challenge you have ever issued. :=)
Aloha, Albert

Hello Albert,
It's certainly very impressive that Derive can cope in approximate mode with the problem of determining the number of digits of this huge number, in particular in the light of the recent discussion about APPROX on the newsgroup. I have a slight preference for my solution though, which I gave at the end of the my "Titbis(26)" for arbitrary Mersenne numbers 2^p-1, namely

```
digits(p):=floor(p*log(2,10))+1
```

for two reasons:

- This method will still work for even higher exponents without adjusting the precision (in other words, FLOOR will take care of that adjustment itself!)

- It can be asily adapted to other bases b>2 (you only have to replace 10 by b in the formula above)

Nevertheless, I agree that this challenge wasn't too hard for a change, in particular, if you can resort to such a powerful tool as Derive :))

Happy Easter,
Johann

Jan Vermeylen, Belgium

Hello Josef and Albert,

Can I bother you with a problem I got from an engineer?

He wants to estimate the value of the parameters K and k in an expression where he does experiments measuring x and t, while changing constants Ca, Cb, Cc, Cd for every experiment.

The expression is:

$$a = Ca(K-1)$$
$$b = -(K\cdot(Ca+Cb)+Cc+Cd)$$
$$\mu = K\cdot Cb - \frac{Cc\cdot Cd}{Ca}$$
$$q^2 = b^2 - 4\cdot a\cdot \mu$$

with

$$t = \frac{K}{k\cdot q}\cdot \ln\left(\frac{1+\frac{2ax}{b-q}}{1+\frac{2ax}{b+q}}\right)$$

I think this is a straightforward application of the FIT function, but to my surprise, I get no good answer.

What do I do wrong?

Jan successfully produced a expression for *t* depending on *K* and *k* using the Computeralgebra by substituting step by step. As you can imagine the resulting expression is a very bulky one. Finally Jan tried regression using given data from below. It didn´t work – `Derive Online Help: The dependence on the parametric variables should be linear.`

$$\text{data} := \begin{bmatrix} x & ca & cb & cc & cd & expr \\ 1 & 0.3 & 0.5 & -1.8 & 2.3 & 7.6 \\ 2 & -1.9 & 3.4 & 2.4 & -5.7 & -0.21 \\ 3 & -5.4 & 7.4 & -0.3 & -0.45 & 2.56 \end{bmatrix}$$

Any idea, how to solve the problem??

A Macro for Derive 6

Johann Wiesenbauer wrote a Macro for earlier Derive versions which happened to fail for upgraded Derive versions, because Derive became “more clever”. Johann used all his tricks to keep Derive not too clever for this special purpose. And again was a call from German teachers who wanted to have Johann´s valueable tool adapted for using it with Derive 6. (See also DNL#31 and DNL#42.)

Albert Rich´s wrote

In Derive 4 and 5, when an equation is simplified (as distinct from being solved), the left and right sides of the equation are simplified independently, and the results returned as an equation. So basically, the equality operator is just a place holder.

In Derive 6, when an equation is simplified, it is replaced by equivalent, but simpler, expression having the same Boolean truth-value. This is the way virtually all other expressions are simplified in Derive. In addition to being more consistent for the user, throughout the Derive 6 equation and inequality solver the assumption is made that equations will be fully simplified.

Therefore, I wonder if it would be possible for you to modify your routine for step-by-step equation solving so that equations are represented as 2-element vectors. Stored in this way, the left and right sides of equations can be independently simplified and manipulated.

And it was possible for Johann:

Johann Wiesenbauer

Hello Derivers,

On request of many users I have updated my macro for solving equations to DfW6 by taking into account that version 6 handles equations a bit differently. In particular, I had to deal with each side of the equation separately. The output of every DO-command is now a 2-row matrix in order to suppress commas. I hope you can live with this. On the other hand, I have added a new command sqr(@) to compute square roots. In a similar way, other useful functions (e.g. for expanding and factoring polynomials) could be added, if there is a need for it.

I wish you much fun with this macro and look forward to any comments or suggestions you might have as to further improvements!

Cheers
Johann

Here is the file (it is very enjoyable to follow Johann´s programming skills. Josef

```
#1:  RECORD(u) :=
       Prog
         o := []
         @1 := LHS(u)
         @2 := RHS(u)
         l := [@1]
         r := [@2]
         u

#2:  sqr(u, u_) :=
       Prog
         If NUMBER?(u)
            RETURN ± √u
         u_ := POLY_GCD(u, DIF(u, x))
         If NUMBER?(u/u_^2)
            √(u/u_^2)·u_

#3:  DO(u) :=
       Prog
         If u = sqr(@)
            Prog
              @1 := sqr(@1)
              @2 := sqr(@2)
              o := ADJOIN([± √@], o)
            false
            Prog
              @1 := SUBST(u, @, @1)
              @2 := SUBST(u, @, @2)
              o := ADJOIN([u], o)
         l := ADJOIN(@1, l)
         r := ADJOIN(@2, r)
         ["", "↓", ""; @1, "=", @2]

#4:  UNDO(u) :=
       Prog
         o := REST(o)
         l := REST(l)
         r := REST(r)
         @1 := FIRST(l)
         @2 := FIRST(r)
         ["", "↓", ""; @1, "=", @2]

#5:  play := [REVERSE(l), VECTOR(=, k_, DIM(l)), REVERSE(r), APPEND(REVERSE(o), [])]`
```

A first example:

#6: $\mathrm{RECORD}('(3^{2\cdot x+1} - 7 = 20))$

#7: $3^{2\cdot x+1} - 7 = 20$

#8: $\mathrm{DO}(@ + 7) = \begin{bmatrix} & & \downarrow & \\ 3^{2\cdot x+1} & & = & 27 \end{bmatrix}$

#9: $\mathrm{DO}(\mathrm{LOG}(@, 3)) = \begin{bmatrix} & \downarrow & \\ 2\cdot x + 1 & = & 3 \end{bmatrix}$

#10: $\mathrm{DO}\left(\dfrac{@}{2}\right) = \begin{bmatrix} & \downarrow & \\ \dfrac{2\cdot x + 1}{2} & = & \dfrac{3}{2} \end{bmatrix}$

#11: $\mathrm{UNDO}() = \begin{bmatrix} & \downarrow & \\ 2\cdot x + 1 & = & 3 \end{bmatrix}$

#12: $\mathrm{DO}(@ - 1) = \begin{bmatrix} & \downarrow & \\ 2\cdot x & = & 2 \end{bmatrix}$

#13: $\mathrm{DO}\left(\dfrac{@}{2}\right) = \begin{bmatrix} & \downarrow & \\ x & = & 1 \end{bmatrix}$

#14: $\mathrm{play} = \begin{bmatrix} 3^{2\cdot x+1} - 7 & = & 20 & [@ + 7] \\ 3^{2\cdot x+1} & = & 27 & \left[\dfrac{\mathrm{LN}(@)}{\mathrm{LN}(3)}\right] \\ 2\cdot x + 1 & = & 3 & [@ - 1] \\ 2\cdot x & = & 2 & \left[\dfrac{@}{2}\right] \\ x & = & 1 & \end{bmatrix}$

Now let´s see solving a quadratic equation step by step. (Don´t forget the quote operator ‘ which is of essential importance. Josef)

$$\#15:\quad \mathrm{RECORD}(\,'(x^2 - 4\cdot x + 7 = 10))$$

$$\#16:\quad x^2 - 4\cdot x + 7 = 10$$

$$\#17:\quad \mathrm{DO}(@ - 3) = \begin{bmatrix} & \downarrow & \\ x^2 - 4\cdot x + 4 & = & 7 \end{bmatrix}$$

$$\#18:\quad \mathrm{DO}(\sqrt{@}) = \begin{bmatrix} & \downarrow & \\ |x - 2| & = & \sqrt{7} \end{bmatrix}$$

That was the "standard square root", let´s try **sqr**

$$\#19:\quad \mathrm{UNDO}() = \begin{bmatrix} & \downarrow & \\ x^2 - 4\cdot x + 4 & = & 7 \end{bmatrix}$$

$$\#20:\quad \mathrm{DO}(\mathrm{sqr}(@)) = \begin{bmatrix} & \downarrow & \\ x - 2 & = & \pm\sqrt{7} \end{bmatrix}$$

$$\#21:\quad \mathrm{DO}(@ + 2) = \begin{bmatrix} & \downarrow & \\ x & = & \pm\sqrt{7} + 2 \end{bmatrix}$$

$$\#22:\quad \mathrm{play} = \begin{bmatrix} x^2 - 4\cdot x + 7 & = & 10 & [@ - 3] \\ x^2 - 4\cdot x + 4 & = & 7 & [\pm\sqrt{@}] \\ x - 2 & = & \pm\sqrt{7} & [@ + 2] \\ x & = & \pm\sqrt{7} + 2 & \end{bmatrix}$$

I tried to add the features `EXPAND` *and* `FACTOR`*. Doing so I had to change Johann´s syntax:*
As my function is called `doo`*, I use* `doo(@,"e")`, `doo(@,"f")` *and* `doo(@,"r")` *for expanding, factoring and drawing the square root. You are friendly invited to test your programming skills. I could not "forbid" Derive simplifying the given equation to present unchanged in the first row of the* `play`*-table. Josef*

$$\#46:\quad \mathrm{RECORD}\left(\,'\left(\frac{b+x}{b-x}-\frac{4\cdot b\cdot(3\cdot b-x)}{b^2-2\cdot b\cdot x+x^2}=\frac{b-x}{b+x}\right)\right)$$

$$\#47:\quad \frac{b+x}{b-x}-\frac{4\cdot b\cdot(3\cdot b-x)}{b^2-2\cdot b\cdot x+x^2}=\frac{b-x}{b+x}$$

$$\#48:\quad \mathrm{doo}(@\cdot(b-x)^2\cdot(b+x))=\begin{bmatrix} & \downarrow & \\ -(x+b)\cdot(x^2-4\cdot b\cdot x+11\cdot b^2) & = & (b-x)^3 \end{bmatrix}$$

$$\#49:\quad \mathrm{doo}(@,e)=\begin{bmatrix} & \downarrow & \\ -x^3+3\cdot b\cdot x^2-7\cdot b^2\cdot x-11\cdot b^3 & = & -x^3+3\cdot b\cdot x^2-3\cdot b^2\cdot x+b^3 \end{bmatrix}$$

$$\#50:\quad \mathrm{doo}(@+x^3-3\cdot b\cdot x^2+7\cdot b^2\cdot x)=\begin{bmatrix} & \downarrow & \\ -11\cdot b^3 & = & 4\cdot b^2\cdot x+b^3 \end{bmatrix}$$

$$\#51:\quad \mathrm{doo}(@-b^3)=\begin{bmatrix} & \downarrow & \\ -12\cdot b^3 & = & 4\cdot b^2\cdot x \end{bmatrix}$$

$$\#52:\quad \mathrm{doo}\left(\frac{@}{4\cdot b^2}\right)=\begin{bmatrix} & \downarrow & \\ -3\cdot b & = & x \end{bmatrix}$$

#53: play

$$\#54:\quad \begin{bmatrix} -\dfrac{x^2-4\cdot b\cdot x+11\cdot b^2}{(x-b)^2} & = & \dfrac{b-x}{x+b} & [@\cdot(x+b)\cdot(x-b)^2] \\ -(x+b)\cdot(x^2-4\cdot b\cdot x+11\cdot b^2) & = & (b-x)^3 & [\mathrm{expand}(@)] \\ -x^3+3\cdot b\cdot x^2-7\cdot b^2\cdot x-11\cdot b^3 & = & -x^3+3\cdot b\cdot x^2-3\cdot b^2\cdot x+b^3 & [x^3-3\cdot b\cdot x^2+7\cdot b^2\cdot x+@] \\ -11\cdot b^3 & = & 4\cdot b^2\cdot x+b^3 & [@-b^3] \\ -12\cdot b^3 & = & 4\cdot b^2\cdot x & \left[\dfrac{@}{4\cdot b^2}\right] \\ -3\cdot b & = & x & \end{bmatrix}$$

Derive 6 acts as a teacher!

Guiseppe Ornaghi, Italy

Using Derive 5.06 is it possible to simplify the following integral:
INT(x^3/(exp(x)-1),x,0,infinity) to (pi)^4/15.

But the function x^3/(exp(x)-1) does not have closed form anti-derivative.

I'd be interested to see how Derive works to find the exact form of the integral.

Best regards

Giuseppe

Mike Law

Maybe a couple of ideas (I am reaching ... these days I use tokens to ride the bus, I make many mistakes counting change, tokens minimize my embarrassment)

division by zero ...

First guess is some sort of expansion ...

I seem to recall Euler or Bernoulli numbers ...... (exp(x)-1)^(-1) expansion of this

if x/(exp(x)-1) mult. by x, then divide by x so that 1=(B0+B1x/1!+B2x^2/2!+...)(1+x/2!+x^2/3!+...)

if x^3/() try mult. x^3 then divide by x^3 etc

Not sure this is useful, however, ... a little more down memory lane I recall a topic in contour integration

Cauchy's theorem

In the complex plane ... f(z) = 1/2*pi*i Integral on close path f(x)dx/(x-z)

integral on closed path f(z) dz =2*pi*i SUM residues

from:

* Mathematical Methods of Physics by Mathews/Walker 1970
They had some examples, none exactly like your problem, it is late at night and this got me out of bed when I felt something rising from the muddy bottom of the river of my past ... maybe I was dreaming.

Regards,
mike

Albert Rich,

Hello Giuseppe,

The new Display Steps feature in the recently released Derive 6 makes it easy to answer your question. The definite integration rule applied to your integral is:

If a>0 and p>-1, INT(x^p/(c*#e^(a*x)-c),x,0,inf) ==> p!*ZETA(p+1)/a^(p+1)/c

Information about Derive 6 is available at www.derive.com and www.derive-europe.com.

Aloha,
Albert D. Rich
Co-author of Derive

DNL:

Follow Derive 6 stepwise to enhance your mathematical knowledge. You will find some of Mike Law´s ideas realized. So Derive 6 acts as a teacher in mathematics. Johann Wiesenbauer provides some more information on this subject.

#1: $\int_0^{\infty} \frac{x^3}{e^x - 1}\, dx$

#2: $\frac{\pi^4}{15}$

Now I repeat the calculation -
- starting stepwise simplification!

$e^0 \Rightarrow 1$

If a>0 and p>-1,

$$\int_0^{\infty} \frac{x^p}{c \cdot e^{a \cdot x} - c}\, dx \Rightarrow \frac{\frac{p! \cdot \zeta(p+1)}{a^{p+1}}}{c}$$

#3: $3! \cdot \zeta(4)$

If n>=0 is an integer,

$n! \Rightarrow 1*2*3*...*n$

#4: $6 \cdot \zeta(4)$

If n>0 is even,

$$\zeta(n) \Rightarrow \frac{2^{n-1} \cdot \pi^n}{n!} \cdot |\mathrm{BERNOULLI}(n)|$$

#5: $\frac{4 \cdot 6 \cdot \pi^4}{15 \cdot 4!}$

If n>=0 is an integer,

$n! \Rightarrow 1*2*3*...*n$

#6: $\frac{\pi^4}{15}$

Johann Wiesenbauer

Well, as you can see from Albert's answer, Derive doesn't actually "compute" this integral, but essentially looks up the underlying formula in its memory. Just in case, you wonder where this formula comes from, here are some more lines, which might help to this end. (Note that all equations below are the result of a simplification apart from the second one.)

I hope these lines are fairly self-explanatory, but if this is not the case, please just tell me so and I will fill in the details.

Cheers,
Johann

n :∈ Integer [0, ∞)

k :∈ Integer [0, ∞)

x :∈ Real (0, ∞)

$$x^n \cdot \sum_{k=0}^{\infty} \hat{e}^{-(k+1)\cdot x} = \frac{x^n}{\hat{e}^x - 1}$$

$$\int_0^{\infty} \frac{x^n}{\hat{e}^x - 1}\, dx = x^n \cdot \sum_{k=0}^{\infty} \int_0^{\infty} \hat{e}^{-(k+1)\cdot x}\, dx$$

$$\int_0^{\infty} x^n \cdot \hat{e}^{-(k+1)\cdot x}\, dx = (k+1)^{-n-1} \cdot n!$$

$$\sum_{k=0}^{\infty} (k+1)^{-n-1} \cdot n! = n! \cdot \zeta(n+1)$$

Guiseppe

I want to thank everyone who replied to my query.

Mike Law

It was a fun question, as usual I got more than I gave as your question took me back from a side trip to a wonderful book by Julian Havil called: "Gamma ... Exploring Euler's Constant" ... a very good book with plenty of detailed examples.

Regards,
mike law

Floating Point Error?

Valerui Anisiu, Romania anisiu@MATH.UBBCLUJ.RO

Hello Derivers,

I have found an annoying floating point error in Derive at least when working in high precision (it seems to be very old).
E.g. if n:=10^50-1, then APPROX((n+10^23)/n,p) is simplified to 1 for p<=50, so that almost half of the digits are wrong!
Using precision 51 one obtains a correct result, but changing APPROX((n+10^20)/n,51), 1 appears again!
While writing this email I discovered that even for the implicit precision 10, 10000000500/9999999999 and 20000001/20000000 are approximated to 1 instead of 1.000000050.

It is very strange that the bug (feature?) was not discovered earlier!

I found this error trying to compute with high precision an integral. Derive cannot approximate any integral with 30 digits in a reasonable amount of time (the Simpson's method is not suitable for high precision); even APPROX(INT(x^4,x,0,1),30) is without hope [if you need it :-)]. So I have programmed a Gauss-like quadrature which was fast enough up to the precision 150 but less than half of the digits were correct due to the preceding FP error.

Best regards,
Valeriu

Johann Wiesenbauer

Hello Valeriu,

As we all know, Derive has got a lot of strengths for which we love it, but it has also got some weaknesses. I hate to say it, but the bad performance of APPROX() you reported in your posting (and I found myself many more examples) belongs definitely to the latter.

I tried many different remedies, but none of them seems to work in all cases. In my opinion, for the time being the most promising one is to use a "self-made" approx() like

$$\text{approx}(e,\ p := 10) := \text{INSERT}(.,\ \text{STRING}(\text{FLOOR}(e\cdot 10^{p})),\ -p)$$

using the very powerful FLOOR(), which you can really rely on unlike APPROX(). For example, you get

$$n := 10^{50} - 1$$

$$\text{approx}\left(\frac{n + 10^{23}}{n},\ 27\right) = 1.000000000000000000000000001$$

In the very crude form above, it works only for positive expressions e and displays trailing zeros, but I hope you got my point anyway. Maybe the underlying idea could also be used by Albert to resolve the current problems with APPROX() in a more "professional" way.

Cheers,
Johann

Hello Derive users,

Recently there have been several posting on the ***Derive*** **Newsgroup** email list concerning the accuracy of results returned by ***Derive***. I felt the issue was of sufficient concern and interest that I wrote the following explanation:

Rational vs. Floating Point Arithmetic

Although ***Derive*** can display numbers in rational, decimal or scientific notation, internally all numbers are stored as integers or the ratio of two integers. There are numerous advantages, especially for a computer algebra system, to store numbers as rationals and use rational arithmetic, instead of storing them as floating point and using floating point arithmetic. These include:

1. Rational arithmetic is fundamental to mathematics and taught in elementary school. Floating point is a relatively recent invention designed for the convenience of calculators and computers.

2. Floating point is unable to represent all rational numbers exactly (for example, 0.3333333 is not quite 1/3). The ability to represent ratios exactly is critical when symbolically simplifying expressions.

3. The elementary operations (addition, subtract, multiplication, and division) can be performed exactly without roundoff errors using rational arithmetic. For example, the result of inverting a matrix of numbers twice using exact rational arithmetic is the original matrix. Whereas, successive floating point operations often result in the accumulation of roundoff errors. It is difficult to explain a result of 4.99999999 to students.

Of course, there are disadvantages to rational arithmetic. These include:

1. Slower performance since arithmetic operations must be done in software rather than hardware (that is, until chip maker recognize the advantages of rational arithmetic and produce "floating slash processors"...). However, note that floating point processors only provide arithmetic accurate to a fixed precision, but applications such as computer algebra require adjustable precision arithmetic. So the performance advantage of floating point is lost since the adjustable precision floating point arithmetic must be implemented in software.

2. The more serious criticism of rational arithmetic is the phenomenon pointed out by Valeriu Anisiu and others. Unfortunately, the problem is not a bug in ***Derive*** but inherent in the uneven distribution of rational numbers along the real number line.

D. W. Matula and P. Kornerup present an algorithm for approximating rational numbers in their article "Finite Precision Rational Arithmetic: Slash Number Systems" published in the January 1985 issue of the IEEE Transactions of Computers. This algorithm, extended for adjustable precision arithmetic, was implemented in assembly language for ***muLISP*** and inherited by ***Derive*** (***muLISP*** is the LISP language interpreter in which ***Derive*** is written).

Around small integers and simple fractions (for example, 1/2 and 2/3) there are relatively large intervals in which there is no better approximation for rational numbers than the integers or simple fractions themselves. Unfortunately rational numbers near the ends of these intervals may not be within the current digits of precision of the integer or simple fraction. Perhaps Johann Wiesenbauer can better explain the sparsity of rational numbers whose numerators and denominators do not exceed a given size around integers and simple fractions.

In the worse case around the integer 1, rational numbers of only half the current digits of precision can be represented. Therefore, the bottom line is: If you want to ensure that rational numbers of at least n digits are representable, use the Simplification tab of ***Derive***'s Options > Mode Settings command to set the digits of precision to 2*n.

Of course, there is still the problem of catastrophic cancellation that occurs when approximating the difference of two nearly equal numbers (for details, see the description in the on-line help for the Precision Field of the Simplification tab of the Options > Mode Settings command).

I hope this explanation and work-around helps.

Aloha, Albert D. Rich, Applied Logician, Co-author of ***Derive***

Hello Albert,

First of all, many thanks for your taking the trouble to explain the internal workings of Derive, when it is trying to approximate an expression, which was certainly very instructive for many users. The bottomline of you message that you should use double presision in certain situations is a little bit unsatisfactory though.

Hoping not to arouse your anger even more, I have still some more questions. In the first place, you could use Maple inputting

```
n:=10^50;
evalf((n+10^23)/n,28);
```

or Mathematica inputting

```
n:=10^50
N[(n+10^23)/n,28]
```

to get the correct result. I don't know, how they do this. Maybe they use a completely different approach or they use the same approach as Derive, but found a way to circumvent the problem. At any rate, this failure is not inherent to all CAS. (I know you didn't claim this, but the fact remains that Derive seems to be an exception here.)

I remember that some years ago there was a quite similar problem with FLOOR() in Derive and I'm still very proud of the fact that pointing out this problem to you along with some suggestions how to fix it led to an extremely reliable implementation of FLOOR(). I have used it on many occasions since then without the slightest problems and - just to make up for the comparisons with Maple and Mathematica above - its performance is far superior to the performance of those rival programs on that score.

I may be mistaken, but I think the situations with FLOOR and APPROX are quite similar in the way that certain numbers have to be dealt with more carefully. In the case, of FLOOR() these are the numbers very close to an integer. Please be patient with me and maybe this is completely unrealistic for some reason, but what about Derive detecting those cases on its own and taking counter measures like boosting the internal precision rather than leaving this to the user?

There is one more thing concerning APPROX() that is "inconsistent" this time as I would call it. If you call APPROX from outside a program, you will always get a floating point number, which is very meaningful, indeed, and just what you expected. If APPROX() is returned by a program, it will be always a rational number represented as a fraction when using the default settings. I'm almost sure you will not understand me, but this tiny inconsistency is driving me crazy! There is simply no way of getting a floating point result other than adjusting the default settings which is committing sort of a sacrilege to me!

Albert´s reply:

Hello Johann,

Thank you for your comments concerning this difficult problem. Requiring the user to use double precision to prevent this worst case error is certainly not optimal, but unfortunately I do not know how to fix the problem other than to internally use twice as much precision as the user specifies. I feel that would unnecessarily slow down the system.

And yes, I did not claim that the problem is inherent to cas; however, I do claim that it is inherent to approximate rational arithmetic. I believe Maple and Mathematica use infinite precision rational arithmetic for exact calculuations, and adjustable precision floating point arithmetic for approximate ones. Derive uses infinite precision rational arithmetic for exact calculuations, and adjustable precision rational (aka floating slash) arithmetic for approximate ones. In addition to the considerable difficulty of implementing infinite precision floating point arithmetic in Pentium assembly language, there are innumerable problems (suffered by Derive's competitors) caused by the need to convert from numbers from rational to floating point, and back.

The `APPROX` problem is much more fundamental than the `FLOOR` problem. It is not just a matter of getting the right rational approximation, there is NO rational number that approximates the exact answer to within the current digits of precision whose numerator and denominator is small enough. Your idea of increasing the number of digits for the problem cases is intriguing, but knowing when and how much to increase the precision is beyond me. Perhaps if the result of approximation is an integer or simple fraction, and the approximation error exceeds the normal amount for the current precision setting, then the precision could be increased. Sounds like a research project for a numerical analysis course.

As I said in my previous email, internally all numbers in Derive are represented as rationals. Therefore, `APPROX` always returns a rational number. However, when an expression is simplified, a special check is made to see if `APPROX` is being called from the top-level. If so, the rational result it returns is displayed in decimal or scientific notation. For example, try simplifying `[APPROX(pi)]` versus `APPROX(pi)`.

As far as programming is concerned, the `APPROX` function is only able to set the precision to approximate mode and the notation to scientific <u>within</u> the scope of its call. Therefore, if you want to approximate something and then analyze its decimal representation, include the analysis within the scope of the APPROX function. For example, `APPROX(STRING(pi))` simplifies to "3.141592653"; whereas, `STRING(APPROX(pi))` simplifies to "1146408/364913".

Josef,

I noted with interest the problem of "Who makes himself a present?" on page 21 of issue #46. I recently encounter the same type of problem and also did a simulation to find the answer. But there is an exact solution to the matching problem as presented in *Introduction to Probability Theory*, by Hoel, Port, and Stone.

As a reminder (from DNL#46):

> ***Who makes himself a present?***
>
> *At the occasion of a celebration 22 people bring a gift. All the gifts are distributed randomly among the 22 guests. Each of the participants receives one gift, so it can happen that one or the other will get back his own package. What is the average value of people getting back their own gift?*

The probability of no matches for n objects is:

$$\#1: \quad P(n) := \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

And the probability of exactly r matches among n randomly permuted objects is:

$$\#2: \quad \text{Matches}(r,\ n) := \frac{P(n - r)}{r!}$$

The probabilities of 0 to 22 matches among the 22 people bringing gifts is therefore:

#3: t := TABLE(Matches(r, 22), r, 0, 22)`

#4:

0	1	2	3	4	5
0.3678794	0.3678794	0.1839397	0.06131324	0.01532831	0.003065662
6	7	8	9	10	
0.0005109436	$7.299195 \cdot 10^{-5}$	$9.123994 \cdot 10^{-6}$	$1.013777 \cdot 10^{-6}$	$1.013777 \cdot 10^{-7}$	
11	12	13	14	15	
$9.216155 \cdot 10^{-9}$	$7.680130 \cdot 10^{-10}$	$5.907788 \cdot 10^{-11}$	$4.219880 \cdot 10^{-12}$	$2.813063 \cdot 10^{-13}$	
16	17	18	19	20	
$1.759113 \cdot 10^{-14}$	$1.030867 \cdot 10^{-15}$	$5.857202 \cdot 10^{-17}$	$2.740211 \cdot 10^{-18}$	$2.055158 \cdot 10^{-19}$	
21	22				
0	$8.896791 \cdot 10^{-22}$				

Taking the dot product of the rows of the matrix gives the expected number of matches, which is 1.

$$\#8: \quad t_1 \cdot t_2 = 1$$

t is interesting to note that there are 0 matches for 21. This is because if there are 21 matches, the last one is automatically a match also!

But, what kind of probability distribution does this represent? If we take the limit of Matches() as n approaches infinity we get:

$$\#9: \quad \lim_{n\to\infty} \text{Matches}(r,\ n) = \frac{\hat{e}^{-1}}{r!}$$

And this is simply the **Poisson distribution with a mean of 1**.

Sincerely,

Don Phillips
phillipsm@gao.gov
donphillips@starpower.net

Some comments on the change of publication form of the DERIVE Newsletter

Lieber Josef,

zum Ende der Print-Ausgabe der DNL möchte ich Dir und Deiner Gattin recht herzlich für die vielen Stunden der Arbeit danken, die ihr beide in diese Publikationen gesteckt habt. Alle DNL waren stets liebevoll zusammengestellt und immer sehr interessant, so daß ich sämtliche Exemplare aufbewahrt habe.

Sicherlich werden auch die zukünftigen Ausgaben in gleicher Qualität das Internet bereichern, worauf ich mich schon jetzt freue.
Es erleichtert die Arbeit ungemein, wenn man auf die schnelle Verfügbarkeit von Artikeln zurückgreifen und bei Fortbildungsveranstaltung darauf "verlinken" kann.Ich würde es also sehr begrüßen, wenn auch ältere Ausgaben so nach und nach in elektronischer Form erscheinen würden, wenn ich auch die diesbezügliche Arbeit als nicht gering ansehe.

Gregor Noll

Dear Josef,
I was reading the last issue of the DUG bulletin and I was feeling sorry that it will not be printed again. I know there are reasons that probably make this a wise decision, but receiving regularly the bulletin made me remember, from the distance, my very good friends in the first house of a little street in a small village in Austria...

The lyrics of a Spanish song begins saying "Algo se muere en el alma, cuando un amigo se va..." ("Something dies in your soul, when a friend goes..."). The DUG Bulletin was our friend and the connection between many friends for a long time. Now its somehow gone, but its spirit (and virtual .PDF version) will stay with us.

Thank you for your (Noor's and Josef's) hard work along all these years.

Your friend,
Eugenio

Lieber Josef,

nachträglich noch alles Gute zum neuen Jahr. Heute ist dein 'XXL' Newsletter angekommen – immer ein höchsterfreulicher Posteingang, verspricht er doch stets interessante Lektüre, viele Anregungen und neue Erkenntnisse. Ich kann allerdings nicht verhehlen, dass mich die Nachricht, den DNL zukünftig in elektronischer Form zu versenden, mich nicht besonders erfreut hat, wenngleich ich die Argumente natürlich nachvollziehen kann. Ich, für meinen Teil, hätte lieber höhere Mitgliedsbeiträge in Kauf genommen, da ich nach wie vor gerne blättere. Deshalb habe ich die DNL ja auch, zu je drei Jahrgängen zusammengefasst, binden lassen. In diesen Bänden kann man wunderbar herumblättern und entdeckt dabei immer wieder neue oder in Vergessenheit geratene Dinge. Ich werde mir dann wohl den DNL immer selber ausdrucken müssen.

Peter Lüke-Rosendal

Dear Josef,

I just received the latest DNL in the mail today. Thank you so much for the nice words regarding the Derive team in your "Letter of the Editor". We really enjoy working with you over the years! Derive wouldn't be the same product without all your questions, comments and suggestions.

I too look forward to the new electronic era of the DUG, but backward with nostalgia at the printed versions. When you have new electronic information regarding the DUG membership and or a new DUG link, please let me know so I can update TI's Derive site at: http://education.ti.com/us/product/software/derive/features/groups.html .

Additionally, many thanks for the article, "Wonderful World of DERIVE 6". You do a great job of explaining the new features and their application for classroom use.

Aloha,
Theresa

Lieber Josef,

danke für das neue Heft. Die Ankündigung, es künftig ohne Papier zu verschicken schmeckt mir sehr gut. Denn meine Ordner nehmen einfach nix mehr auf. Und die Hoffnung auf ein richtiges Register (und evtl. auch Links auf die *.dfw files etc.?) macht richtig geizig (d.h. geil, weil dieser ja lt. Werbung so ist).

Allerdings muss ich auch eine kleine Rüge mitschicken:
Hast du bei der Danksagung im Editorial nicht die Noor vergessen?
Deshalb besonderen Gruß auch an sie.

Viele Gruesse,
Wolfgang

Dear Josef,

I am glad that the DUG membership is now free. After devaluation of our currency it has become hard to buy goods abroad. Please, take note of my E-mail so that you can send me the publication dates of future DNL's.

Best regards,

Marcelo.

Dear Josef,
Many thanks Josef for your work on the DNL,
Please add me to the list for DNL publication date notification.
I'll miss looking forward to the orange envelope dropping through the door.
On the other hand an electronic copy will lend itself to more rapid dissemination of information.
I look forward to the next issue as always.

All the best,
Ian Pigram

Kaprekar's Sequence and his "Selfnumbers"

Richard Schorn, Kaufbeuren (eMail richard.schorn@t-online.de)

The "special number" 6174 (D-N-L #30, p 21 was discovered by D. R. Kaprekar (born in 1905), B.Sc. , S.T.C. of Devlali (India) in 1946. The introduction to his booklet "The New Constant 6174" reads as follows:

6174 is an invariant. It comes again and again from many four digital numbers by the "Reversed subtraction process"

It is a matter of great pleasure and joy to bring this 6174 from the working of the process. This is a new mathematical constant. (first discovered in 1946 by the author.)
311 Devlali Camp 1-6-1959 D.R. Kaprekar

Later on he writes:

An article on this subject was first published by me in Scripta Mathematica of New York, vol XV, page 244-245. ...I am the inventor of this new constant 6174 and ...

Kaprekar did a lot of "number-crunching" (without computer or DERIVE!):

...Hundreds of papers and note books were filled with several sorts of calculations and at last the secrets fully investigated. ...

I learned about him by Martin Gardner's famous column "*Mathematical Games*" in "*Scientific American*" in 1980 and I therefore bought several booklets from Mr. Kaprekar.

The sequence 1, 2, 4, 8, 16, 23 (yes. 23, not 32!), 28, 38, 49, 62, ... seems to be Kaprekar's invention too:

$$k_0 = 1,\ k_n = k_{n-1} + \text{sum of the digits of } k_{n-1}$$

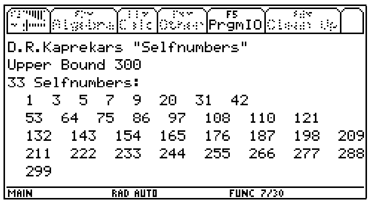
In his booklet "The Mathematics of the New Self Numbers" (1963) Kaprekar studied the problem whether a certain number appears in the sequence and how to find its predecessor. 3 has no generator. He now (as an example) asks:

Is there any generator for 86? The answer is No.……It will be seen by several trials that there is no generator for 86. 86 is the Self-Number. It is really Self-born. It is a <follows a Sanskrit expression> ...A number which has no generator is called a Self-Number.…Thus between one and 100 here are in all 13 Self-Numbers. They are 1, 3, 5, 7, 9, 20, 31,42,53, 64, 75, 86 and 97.…

There are contradictory opinions of the mathematical significance concerning Kaprekar's research, but in any case the results are well known in the world of "recreational mathematics" and give nice examples for practicing computer programming.

The TI-92-Programs use German words for the variables and inputs, but they are easily translated to other languages.

Program KAPREKAR yields the sequence for different starting values. Program SELFNUMB gives the so-called Self-Numbers up to a predetermined value. Both programs need the function QUER (from german *Quersumme*) which calculates the total of the digits of a given number.



Here are some programs and their results to manage Kaprekars sequence. First one has to define a function which calculates the sum of the digits for a given number. Using a “normal” programming language one has to construct a loop using functions MOD and DIV (if available!). With DERIVE a very simple sum-of-the-digits-function is possible, following an idea of Josef Böhm in DNL #44, page 33:

```
Q(n) := Σ(VECTOR(CODES_TO_NAME(k), k, NAME_TO_CODES(n)))
```

A step-by-step demonstration with the number 47110815 will clarify the process:

- `NAME_TO_CODES(47110815) = [52, 55, 49, 49, 48, 56, 49, 53]`
  One gets a vector with the ASCII-Codes of the eight digits.
- `VECTOR(CODES_TO_NAME(k),k, [52,55,49,49,48,56,49,53]) = [4,7,1,1,0,8,1,5]`
  The "ASCII-names" of the eight digits make up a new vector.
- `SUM([4, 7, 1, 1, 0, 8, 1, 5])=27`
  The sum (Σ) of the elements is determined.

```
N(n) := n + Σ(VECTOR(CODES_TO_NAME(k), k, NAME_TO_CODES(n)))
```

Obviously this function determines the next element in Kaprekars sequence. Without the help of computers Kaprekar discovered remarkable numbers with three and four generators:

```
 N(9999999999999999999893) = 10000000000000000000102
 N(9999999999999999999902) = 10000000000000000000102
N(10000000000000000000091) = 10000000000000000000102
N(10000000000000000000100) = 10000000000000000000102
```

In [3] he writes on page 18:

> *This number will have a historical value in future as discovered by D. R. Kaprekar on 7-6-61 by his methods and also discovered by Prof. Gunjikar on 7-9-61 by his different methods.*

With function `N(n)` using `ITERATES` one gets the sequence up to a given index:

```
ITERATES(N(n), n, 1, 25) = [1, 2, 4, 8, 16, 23, 28, 38, 49, 62, 70, 77, 91,
 101, 103, 107, 115, 122, 127, 137, 148, 161, 169, 185, 199, 218]
```

In a similar way we calculate the m^{th} element of the sequence:

```
KAPREKAR(m) := ITERATE(N(n), n, 1, m)
```

Some examples:

```
   KAPREKAR(10) = 70
  KAPREKAR(100) = 1213
KAPREKAR(10000) = 213703
KAPREKAR(99999) = 2609882
KAPREKAR(100000) = 2609917
```

The function `SELF(n)` returns `true`, if the argument n has no predecessor or in Kaprekars words if n is a "self-number".

```
SELF(20) = true
SELF(70) = false
SELF(1000000) = true
SELF(2609917) = false
```

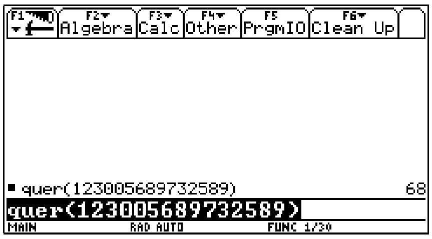
The last program yields a list of selfnumbers up to a given number:

```
SELFNUMBER(1000) = [1, 3, 5, 7, 9, 20, 31, 42, 53, 64, 75, 86, 97, 108,
  110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 211, 222, 233, 244,
  255, 266, 277, 288, 299, 310, 312, 323, 334, 345, 356, 367, 378, 389,
  400, 411, 413, 424, 435, 446, 457, 468, 479, 490, 501, 512, 514, 525,
  536, 547, 558, 569, 580, 591, 602, 613, 615, 626, 637, 648, 659, 670,
  681, 692, 703, 714, 716, 727, 738, 749, 760, 771, 782, 793, 804, 815,
  817, 828, 839, 850, 861, 872, 883, 894, 905, 916, 918, 929, 940, 951,
  962, 973, 984, 995]
```

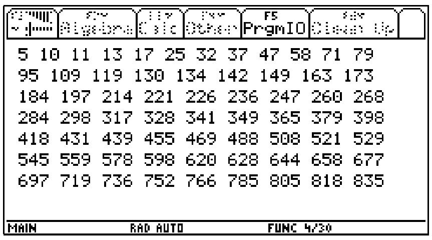
Note the the curious part of this list, starting with 211!

TI-92-Programs KAPREKAR.9XP and SELFNUMB.9XP using Function QUER.9XF

```
quer(x)
Func
Local h,s
Ø→s
x→h
While h≠Ø
  s+remain(h,1Ø)→s
  intDiv(h,1Ø)→h
EndWhile
s
EndFunc
```




```
kaprekar()
Prgm

Local aus,z,g
ClrIO
Ø→z
""→aus
Input "Initial Number",g
ClrIO
While z≤79
  aus&" "&string(g)→aus
     If dim(aus)>34 Then
        Disp aus
        ""→aus
     EndIf
  g+quer(g)→g
  z+1→z
EndWhile
Disp aus
EndPrgm
```




```
selfnumber()
Prgm

local q,flag,i,n
{1}→selfn
{1}→folge
1→selfz
ClrIO
input "Upper Bound?",n
For i,2,n
  1→flag
  for k,1,selfz
    if i>folge[k]
      fol-
ge[k]+quer(folge[k])→folge[k]
    if i=folge[k]
       Ø→flag
  EndFor
  If flag=1 then
     selfz+1→selfz
     augment(selfn,{i})→selfn
     augment(folge,{i})→folge
  endIf
endFor

ClrIO
"D.R.Kaprekar's "&char(34)→tt
tt&"Selfnumbers"&char(34)→tt
Disp tt
disp "Upper Bound "&string(n)
disp string(selfz)&" Selfnumbers:"
""→aus
for i,1,selfz
  aus&"  "&string(selfn[i])→aus
  if mod(i,8)=Ø then
     Disp aus
     ""→aus
   EndIf
EndFor
Disp aus

EndPrgm
```

[1] D.R.Kaprekar, The New Constant 6174, Devlali, 1959

[2] D.R.Kaprekar, The Mathematics of the New Self Numbers", 1963
(This is the first booklet of a series with the same title)

[3] D.R.Kaprekar, Self numbers and numbers with 3 and 4 generators, Devlali, 1962

Hallo Josef,
als Dank für deine schöne Nr. 51 möchte ich dir eine kleine Knobelei schicken:
As a thank you for fine DNL#51, I´d like to send a little brain twister:

SEP – Zahlen *SEP – Numbers*

Rüdeger Baumann, Celle

Nimm eine natürliche Zahl ≥ 10, zerlege sie in ihre Ziffern und bilde deren Summe.
Take any natural number ≥ 10 and calculate its sum of digits

Beispiel: 28 → 2,8: Summe: 10.

Nun hänge die Summe hinten an und bilde wieder eine Summe aus zwei Summanden, indem du die erste Ziffer weglässt.
Attach the sum and calculate again the sum of all numbers after removing the first one.

Im Beispiel: 2, 8, 10,
Auf diese Weise machst du weiter: 2, 8, 10, 18, 28 – oh Wunder: die Startzahl ist wieder erschienen. Dies ist natürlich ein seltener Glücksfall.
Dir zu Ehren soll die 28 und jede solche Zahl SEP-Zahl (sich **se**lbst **p**roduzierende Zahl) heißen.

*So you go on: 2, 8, 10, 18, 28 – oh what a miracle, the initial number appears again. This is a rare lucky chance. To honor you, I´ll 28 and all such numbers a SEP-number (**self p**roducing number). (That needs some explication: Sepp is an Austrian and Bavarian form for Josef, Guiseppe is its Italian origin.. In Vienna – and Eastern Austria we have another form for Josef: Pepi or Peperl.)*

Zweites Beispiel (langsam): **Startzahl 1104**

1+1+0+4 = 6
1+0+4+6 = 11
0+4+6+11 = 21
4 + 6 + 11 + 21 = 42
6 + 11 + 21 + 42 = 80
11 + 21 + 42 + 80 = 154
21 + 42 + 80 + 154 = 297
42 + 80 + 154 + 297 = 573
80 + 154 + 297 + 573 = ???? **1104**

BINGO!! SEP-Zahl/*Number*

Herausforderung: Man schreibe ein DERIVE-Programm zur Ermittlung von SEP-Zahlen. Wie viele davon $\le 10^{10}$ gibt es?
Challenge: Write a DERIVE program to find SEP-numbers. How many of them can be found $\le 10^{10}$*?*

I tried to do my "home work":

```
sep(11436171)
[1, 1, 4, 3, 6, 1, 7, 1, 24, 47, 93, 182, 361, 716, 1431, 2855, 5709, 11394, 22741,
  45389, 90596, 180831, 360946, 720461, 1438067, 2870425, 5729456, 11436171]
```

$$\text{seppp}(10^5 + 1,\ 10^6)$$

$$\left[\begin{array}{c} 10 \\ [120284,\ 129106,\ 147640,\ 156146,\ 174680,\ 183186,\ 298320,\ 355419,\ 694280,\ 925993] \end{array}\right]$$

Sierpinski-Tetraeder: Darstellung in 3D unter *DERIVE* 5
AG CoMa, Gutenbergschule Wiesbaden
2.Hj. 2001/02
www.gutenberg-gym.de/coma/

1. **Berechnung der Eckpunktskoordinaten des Tetraeders; k ist hierbei die Kantenlänge**

   Calculating the coordinates of the vertices of a tetrahedron with edge = k

$$\left[a := [0, 0, 0], b := [k, 0, 0], c := \left[\frac{1}{2}\cdot k, \frac{k}{2}\cdot\sqrt{3}, 0\right], d := \left[\frac{k}{2}, \frac{k}{6}\cdot\sqrt{3}, k\cdot\sqrt{\left(\frac{2}{3}\right)}\right]\right]$$

Die Herleitung, durch Anwendung des Satzes von Pythagoras, zeigen wir mit der Kantenlänge k = 5. Die Grundfläche des Tetraeders ABC sieht so aus:

Firstly we apply the Pythagorean Theorem and work with k = 5. We start with the base triangle ABC.

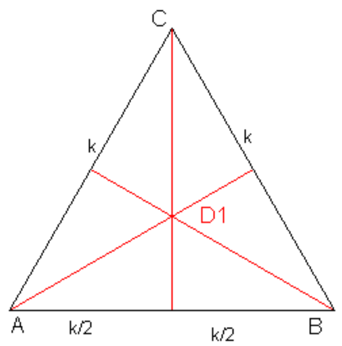
$$\#2: \quad \text{dreieck} := \begin{bmatrix} 0 & 0 \\ 5 & 0 \\ 2.5 & 2.5\cdot\sqrt{3} \\ 0 & 0 \end{bmatrix}$$

Die Spitze des Tetraeders D liegt über dem Schnittpunkt der Seitenhalbierenden:

The top of the tetrahedron D is above the intersection point of the perpendicular bisectors of the sides.

$$\text{seitenhalb} := \begin{bmatrix} [0, 0] & \dfrac{[5, 0] + [2.5, 2.5\cdot\sqrt{3}]}{2} \\ [5, 0] & \dfrac{[2.5, 2.5\cdot\sqrt{3}]}{2} \\ [2.5, 2.5\cdot\sqrt{3}] & \dfrac{[5, 0]}{2} \end{bmatrix}$$

D1 (k/2, k√3/6)



Das Dreieck AD_1D ist rechtwinklig und es gilt: $k^2 = dz^2 + |AD_1|^2 = dz^2 + \left(\frac{k}{2}\right)^2 + 3\left(\frac{k}{6}\right)^2$.

Wir lösen nach dz (der z-Koordinate von D) auf.

Triangle AD_1D is a right triangle and we solve the above expression vor dz = 3rd coordinate of D.

$$\#4: \quad \text{InputMode} := \text{Word}$$

$$\#5: \quad k^2 = dz^2 + \left(\frac{k}{2}\right)^2 + \left(\frac{k}{6}\right)^2\cdot 3$$

$$\#6: \quad \text{SOLVE}\left(k^2 = dz^2 + \left(\frac{k}{2}\right)^2 + \left(\frac{k}{6}\right)^2\cdot 3, dz\right)$$

$$\#7: \quad dz = -\frac{\sqrt{6}\cdot k}{3} \vee dz = \frac{\sqrt{6}\cdot k}{3}$$

Damit sind die Koordinaten von D bekannt:

So we know the coordinates of vertex D:

$$D\left(\frac{k}{2}, \frac{k\sqrt{3}}{6}, \frac{k\sqrt{6}}{3}\right)$$

2. Errechnen des Mittelpunkts des Tetraeders:

Calculating the center point of the tetrahedron

Wir errechnen den Mittelpunkt des Tetraeder M, um das Tetraeder zu zentrieren.
Die x-Koordinate ist schon bekannt, da sie mit der von Punkt D identisch sein muss, d.h. sie ist k/2.

We calculate the center point of the tetrahedron M to center the body.
x-coordinate is known – the same as the x-coordinate of point D, i.e. k/2.

$$\#8: \quad m = \left[\frac{k}{2}, y, z\right]$$

$$\#9: \quad MA := \sqrt{\left(\left(\frac{k}{2}\right)^2 + y^2 + z^2\right)}$$

$$\#10: \quad MB := MA$$

$$\#11: \quad MC := \sqrt{\left(0^2 + \left(y - \frac{k}{2}\cdot\sqrt{3}\right)^2 + z^2\right)}$$

$$\#12: \quad MD := \sqrt{\left(0^2 + \left(y - \frac{k}{6}\cdot\sqrt{3}\right)^2 + \left(z - k\cdot\sqrt{\left(\frac{2}{3}\right)}\right)^2\right)}$$

Um die Koordinaten von M zu erhalten stellen wir gleich und lösen die Gleichungen für y bzw. z.

To find the coordinates of M we set equal and solve for y and z.

$$\#15: \quad \text{SOLVE}(MA = MC, y) = \left(y = \frac{\sqrt{3}\cdot k}{6}\right)$$

$$\#16: \quad \text{SOLVE}(MA = MD, z) = \left(z = \frac{\sqrt{2}\cdot(\sqrt{3}\cdot k - 2\cdot y)}{8}\right)$$

Wir setzen y ein:

We substitute for y:

$$\#17: \quad z = \frac{\sqrt{2}\cdot\left(\sqrt{3}\cdot k - \frac{2\cdot\sqrt{3}\cdot k}{6}\right)}{8}$$

Daraus folgt:

This results in:

$$\#18: \quad m := \left[\frac{k}{2}, \frac{\sqrt{3}\cdot k}{6}, \frac{\sqrt{6}\cdot k}{12}\right]$$

Der Mittelpunkt M des Tetraeders liegt also genau unter dem Punkt D in einem 1/4 der Höhe des Tetraeders.

Center M of the tetrahedron lies exactly under D at the body´s height first quarter.

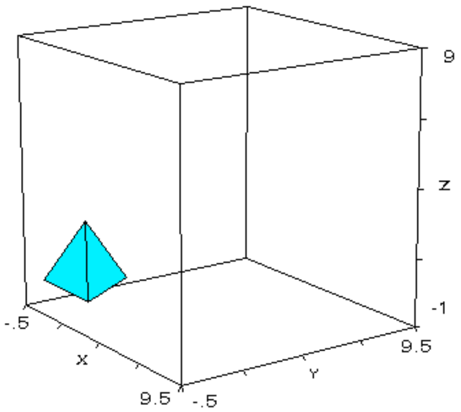
3. Wir zeichnen das Tetraeder in der Flächenansicht:

We plot the tetrahedron and show the faces:

Zuerst definieren wir die Flächen f1 – f4
At first we define the faces f1 – f4

```
#19:  [f1 := [a, b, d, a], f2 := [a, c, d, a], f3 := [b, c, d, b], f4 := [a, b, c, a]]
#20:  tetraeder := [f1, f2, f3, f4]
```

Damit kann die Figur im 3D-Fenster dargestellt werden. Der Zeichenbereich wird angepasst.
We can present the figure in the 3D-Plot Window. We have to adjust the plot region.




```
k := 3
```

Nun haben wir unser Tetraeder schon 3D dargestellt, aber wir wollten es am Ursprung zentrieren.

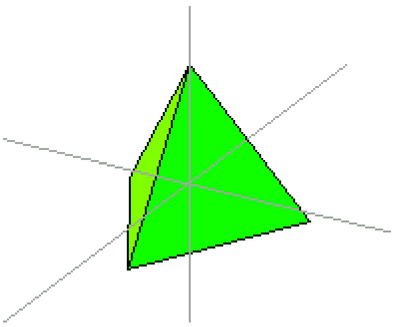
Now we see our tetrahedron in a 3D-view, but we would like to have it centered in the origin.

4. Wir verschieben nun unsere Punkte a,b,c,d, so dass das Tetraeder zentriert liegt.

We translate points a, b, c, d such that the origin will become the center.

Dazu subtrahieren wir den Vektor von M von jedem Punkt.
To do so we perform 4 substractions: point minus vector M

#22: $\text{tetraeder1} := \text{VECTOR}(\text{VECTOR}(\text{tetraeder}_{i,j} - m, j, 1, 4), i, 1, 4)$



Origin O is center of the tetrahedron with edge k = 3.

Wir automatisieren jetzt die Darstellung des Tetraeders.
Zur Vereinfachung nehmen wir hier für die Kantenlänge k=3 an.
Die Definition des Tetraeders:

We automate the presentation of the tetrahedron*, to make it easier we take k = 3 and repeat the definition:*

#23: $k := 3$

#24: $\left[a := [0, 0, 0], b := [k, 0, 0], c := \left[\frac{1}{2}\cdot k, \frac{k}{2}\cdot\sqrt{3}, 0\right], d := \left[\frac{k}{2}, \frac{k}{6}\cdot\sqrt{3}, k\cdot\sqrt{\left(\frac{2}{3}\right)}\right]\right]$

#25: $[f1 := [a, b, d, a], f2 := [a, c, d, a], f3 := [b, c, d, b], f4 := [a, b, c, a]]$

#26: $\text{tetraeder} := [f1, f2, f3, f4]$

Da wir häufiger verschieben wollen definieren wir uns eine Funktion `trans()`, die für die Verschiebung eines Objektes mit Vektor v, wobei das Objekt in der Flächendarstellung vorliegt, zuständig ist.

We intend to apply a translation more often, so we develop a function ***`trans()`****, which is responsible for translating an object given by its faces by a vector v.*

```
#27:  trans(obj, v) := VECTOR(VECTOR(obj     + v, j, 1, 4), i, 1, DIM(obj))
                                        i,j

#28:  trans(tetraeder, b - a)
```

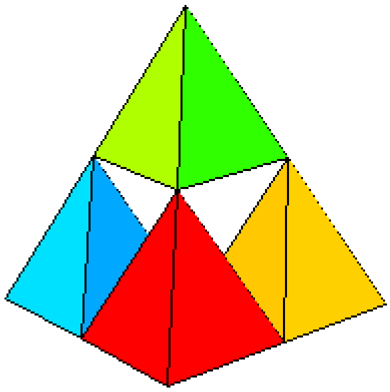
Jetzt wird das Tetraeder in drei Richtungen verschoben:

Now we translate the tetrahedron into three directions:

```
#30:  [tetraeder, trans(tetraeder, b - a), trans(tetraeder, c - a), trans(tetraeder, d -
       a)]
```

$0 \le x,y,z \le 6$

A click with the right mouse key into each single tetrahedron opens the `Edit Window` *and one can change the* `Plot Color` *using* `Scheme Custom`.



Für die Verschiebung des aufzubauenden Sierpinski-Tetraeders braucht man eine modifizierte Verschiebungsfunktion, weil die Struktur des Sierpinski-Tetraeders eine Liste von Listen ist.

For translating the requested Sierpinski-tetrahedron we need a modified translation function, because the structure of the Sierpinski-tetrahedron is a list of lists.

```
#31:  trans1(obj, v) := VECTOR(VECTOR(VECTOR(obj       + v, k, 1, 4), j, 1, 4), i, 1,
                                                i,j,k
       DIM(obj))
```

Nun endlich kann das Sierpinski-Tetraeder rekursiv aufgebaut werden.
Zu Beginn (n=0) steht das Orginal-Tetraeder und seine 3 verschobenen kleinen Brüder.
Dieses Objekt wird nun als Ganzes um die doppelte Kantenlänge in 3 Richtungen verschoben.

Finally we can build the Sierpinski tetrahedron recursively.
We start with n = 0 – the initial tetrahedron – and its three translated little brothers (or sisters).
Then we translate this object as a whole by double side length into three directions

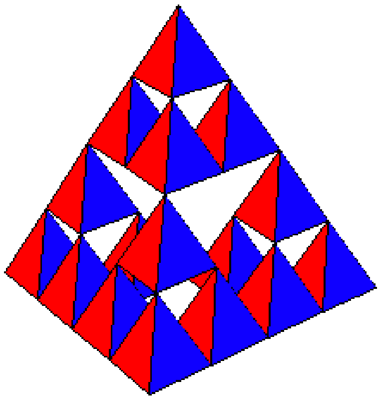
```
      sier(n) :=
        If n = 0
#31:       [tetraeder, trans(tetraeder, b - a),
           APPEND([sier(n - 1), trans1(sier(n -

       trans(tetraeder, c - a), trans(tetraeder, d - a)]
       1), 2^n·(b - a)), trans1(sier(n - 1), 2^n·(c - a)), trans1(sier(n - 1), 2^n·(d -

       a))])
```

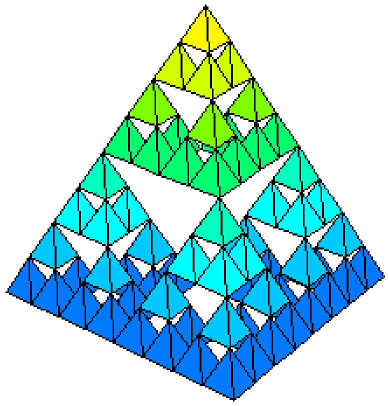
to edit as:

```
sier(n) := IF(n = 0,
          [tetraeder, trans(tetraeder, b - a), trans(tetraeder, c - a), trans(tetraeder, d - a)],
          APPEND([sier(n - 1), trans1(sier(n - 1), 2^n·(b - a)), trans1(sier(n - 1), 2^n·(c - a)),
                    trans1(sier(n - 1), 2^n·(d - a))])
          )
```

Teste die folgenden Beispiele / *Test the following examples*:

```
#32:  sier(0)
#33:  sier(1)
#34:  sier(2)
#35:  sier(3)
#36:  sier(4)
```

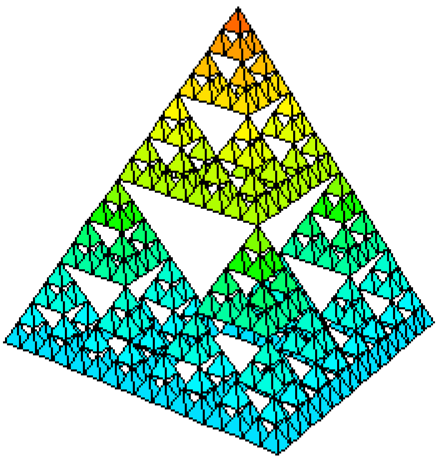




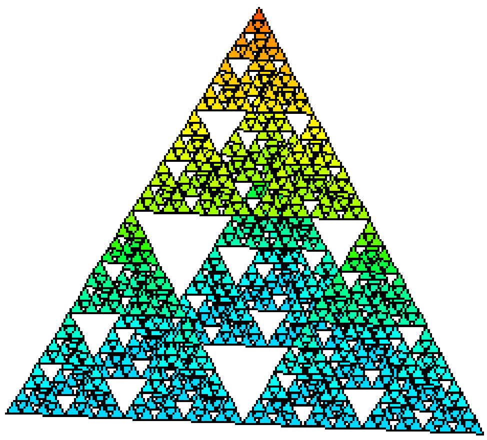
Nun sollte das Tetraeder am besten noch den Mittelpunkt im Ursprung haben!
Alles kein Problem, verschieben wir es also. *Final translation into the origin.*

$$\#37:\quad \mathrm{sier\_c}(n) := \mathrm{trans1}\left(\mathrm{sier}(n),\ -2^{n+1}\cdot k\cdot\left[\frac{1}{2}, \frac{\sqrt{3}}{6}, \frac{\sqrt{6}}{12}\right]\right)$$

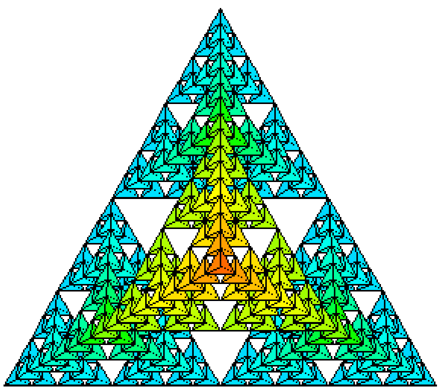
sier_c(3)



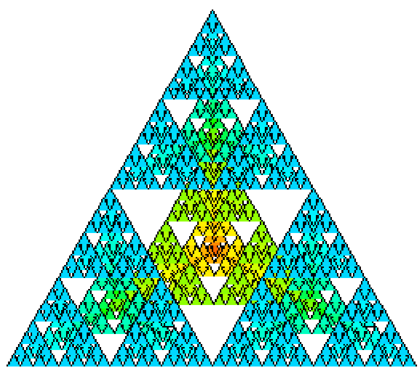
sier_c(4)



view from above



view from the bottom



Titbits from Algebra and Number Theory (27)

by Johann Wiesenbauer, Vienna

In the current issue of the "Titbits" I'll try to keep an old promise I gave Peter Schofield back in December 2002. At that time he sent me a short email in which he was "wondering if DERIVE would be strengthened by including some kind of decimal notation corresponding to rational numbers". It soon became clear by the programs he included that he meant some kind of conversion of a rational number to a form which makes visible the preperiod and the period of it. I wrote back that I found his idea very interesting, but was still thinking about an optimal implementation. I particular, I was aiming at some sort of a "Swiss Army Knife", which could do all sorts of conversions in terms of changing to different bases. In fact, I was trying hard for some time to achieve this goal, but no solution looked really satisfactory to me then. Sadly enough, I forgot about this matter eventually.

Now, as some of you might know, quite recently Volker Loose has brought up exactly the same issue on the Derive-newsgroup again. And once more I looked into this problem, but this time I succeeded in writing a routine that comes close enough to my picture of a "Swiss Army Knife" to be worth being published on the newsgroup. In the meantime, I have even added some more improvements (hopefully you agree that these are improvements, indeed!) and you can see the outcome convert(r,k,b) on the next page along with some examples.

Basically, you can use this utility function to convert a positive rational number to another representation, where you can see both the preperiod and period of r w.r.t. any base b with the usual restriction 2<=b<=36. As a spin-off, you may also use this function to convert positive integers to other bases b than 10, also with a fixed block-length k, as this is often needed in coding theory. The parameters r, b and k have the following meaning:

r is the nonnegative rational number to be converted,

b is the base for all calculations and the output (2<=b<=36; b:=10 by default),

k is sort of a "switch". To be more precise,

- if k and r are both positive integers, then the output consists of the k least significant digits of r
- if k=0, then the output consists of as many digits as necessary, which is the default case
- if k=-1, then the output is only the period length of r
- if k=-2, then the output is only the preperiod length of r
- if k=-3, then the output is a vector containing both the preperiod length and the period length of r

```
convert(r, b ≔ 10, k ≔ 0, a_, d_, n_, r_ ≔ 0, s_) ≔
  Prog
    If r = 0
       RETURN IF(k < 0, 0, REST(STRING(10^k)))
    n_ ≔ NUMERATOR(r)
    d_ ≔ DENOMINATOR(r)
    Loop
      a_ ≔ GCD(d_, b)
      If a_ = 1 exit
      r_ :+ 1
      n_ :* b/a_
      d_ :/ a_
    s_ ≔ b
    Loop
      If MOD(s_, d_) ≤ 1 exit
      s_ :* b
    n_ :* (s_ - 1)/d_
    OutputBase ≔ b
    a_ ≔ STRING(FLOOR(n_, s_ - 1))
    Loop
      If DIM(a_) > r_ exit
      a_ ≔ ADJOIN("0", a_)
    a_ ≔ INSERT(".", a_, -r_)
    s_ ≔ REST(STRING(s_ + MOD(n_, s_ - 1)))
    OutputBase ≔ Decimal
    If k = -1
       RETURN DIM(s_) - 0^(d_ - 1)
    If k = -2
       RETURN r_
    If k = -3
       RETURN [r_, DIM(s_) - 0^(d_ - 1)]
    If d_ = 1
       Prog
         If r_ > 0
            RETURN a_
         a_ ≔ DELETE(a_, -1)
         If k = 0
            RETURN a_
         Loop
           If DIM(a_) = k
              RETURN a_
           If DIM(a_) < k
              a_ ≔ ADJOIN("0", a_)
              a_ ≔ REST(a_)
    n_ ≔ APPEND(a_, "<", s_, ">")
```

$$\mathrm{convert}\left(\frac{1}{28}\right) = 0.03<571428>$$

$$\mathrm{convert}(13,\ 2) = 1101$$

$$\mathrm{convert}\left(\frac{1}{3},\ 2\right) = 0.<01>$$

p= 2231588810593399 is the smallest prime such 1/p has period 2003 (Cf. www.primepuzzles.net and puzzle 208 there.) Indeed, we get:

```
#5:   convert(1/2231588810593399)

#6:   0.<000000000000000448111226966625292420798398833515682905127413360~
        6093400509692366772061856346118381238015662344060034958096567671~
        8465239302279637019360464382481161390973304491412032169179317368~
        4363304603984145005324888834688688141604620765892780956425386302~
        7921045155758188142921476163065425554949408238482149451490093731~
        7548342921070719501851040433529102728286338723009466465331893113~
        8297471379355688192826936599518389164526410843653223163360019280~
        6511374821246283580552313564246559241032247330963586193127370501~
        8194689153289081804961825447591063042310608638405893039308877640~
        5753492360271462100989507408730431522985979959583881619285805717~
        4956985794545949343822990493183545432966827488090387733838229816~
        6874784898242373565302056595598059535989834556216101551511150280~
        1606335958025171245075306417378727690982587588301548934406714991~
        4916403128781830242186951419655355421202318965031001893431061122~
        5192071835842486485977183386982311206045204856823054908205090005~
        1457095252402440185349219597697282968516959889113713793339827398~
        0169042709280151940426635818586478183624967375739407827647070489~
        8731124298744735851336865455313228325285689895982135213079015639~
        3973517005569009365310608190240051620674573829012580756056830914~
        0861091665055521167919693410936946690301528609529890600515052971~
        4051434420936889742716746533408914756039524769207326344980705363~
        1062052234605625785530469775250398735345956785311378483544651453~
        6175180821393428602003951104683777930590963511671871505368152083~
        7999021520569561660741408127099234360647041167779653106470237958~
        5214299697314497545770214930786241140134761337903093696212628931~
        2954218500487228960441587275257453254772998479084037652756473151~
        9208332955045286516205950157354439047655931391572096101419557550~
        7988570568757731946625834990006145506154130277608644778693485112~
        5051414053848753593570942608521413148260947750152547980381398390~
        7679025224708477593900111958464815686165291730713241052811379472~
        9517308487175627422606797290336183884481746804928823750331514216~
        26685329883038153401>

        convert(1/2231588810593399, 10, -1) = 2003

        convert(1/2231588810593399, 10, -2) = 0

        convert(1/2231588810593399, 10, -3) = [0, 2003]
```

As you might have noticed I haven't mention any computation times so far. The reason for this is very simple: It takes usually only fractions of a second to get those results. For example, the last 4 computations took 0.08s each on my 2GHZ-PC. Apart from all those "bells and whistles" in my program, which you might find useful or not, this may well be the biggest difference to Peter's sample programs mentioned above (dealing with the basic case b=10 only), which are programmed in a completely different way and are say " a little bit slower", though giving the correct results.

Let's close this topic with a look at the period lengths of 1/p for all primes p below 100. These are given by the following table:

$$\text{TABLE}\left(\text{convert}\left(\frac{1}{p}, 10, -1\right), p, \text{SELECT}(\text{PRIME}(q\_), q\_, 1, 100)\right)' =$$

$$\left[\begin{array}{ccccccccccccccccccccccccc} 2 & 3 & 5 & 7 & 11 & 13 & 17 & 19 & 23 & 29 & 31 & 37 & 41 & 43 & 47 & 53 & 59 & 61 & 67 & 71 & 73 & 79 & 83 & 89 & 97 \\ 0 & 1 & 0 & 6 & 2 & 6 & 16 & 18 & 22 & 28 & 15 & 3 & 5 & 21 & 46 & 13 & 58 & 60 & 33 & 35 & 8 & 13 & 41 & 44 & 96 \end{array}\right]$$

You will immediately see the interesting property, that the period length of 1/p is always a divisor of p-1, if p doesn't divide 10. Only for a few primes p, whose values you can conclude from the table, the maximum p-1 is actually reached, though. What do those primes have in common? Well, it's not my intention to put you on tenterhooks: These are exactly the primes for which 10 is a primitive root, i.e. a generator in the cyclic prime residue class group mod p.

```
SELECT(PRIMITIVE_ROOT(p, 10) = 10, p, SELECT(PRIME(q_), q_, 1,100))
                  [7, 17, 19, 23, 29, 47, 59, 61, 97]
```

Hope to see you all in Montréal. And as always, if you have any comments or suggestions, please let me know! (j.wiesenbauer@tuwien.ac.at)

http://time-2004.etsmtl.ca/start.html

ACDCA - Section

Lectures

Using a programmable calculator to allow high school learners …
Significance of use of technology in mathematics in vocational educ …
A learning system for mathematical modelling with dynamic geometry …
Combining the possibilities of Derive and Excel while studying bases …
Utilisation de Maple en calcul numérique
Representations and graphic calculator in mathematical teaching …
A practical example of mathematics teaching using the internet
Using CAS to Explore Precalculus and Calculus Concepts Through
Experiences with the obligatery use of graphic calculators like TI-83 ...
Preparing for the "Full Monty"
Rule dialogue in problem solving environment T-Algebra
Designing instructional tools by Flash MX ActionScript-some examples ...
The application of CAS in teaching calculus
Developing Control Over the Use of a CAS : the Teacher's Perspective
DGS and CAS as tools supplementing each other in an inquiry task
Assessing Geometry Concepts of Post-Bac Teacher Candidates with

Computer-Based Mathematics Assessment of Engineering Students
A mathematics and science domain e-learning platform IWT based
Calculatrices symboliques dans l'enseignement des mathématiques …
Integrating writing and technology into mathematical learning
Mathematical Reasoning and its formalization within a Dynamic World
Classical and computer methods in elementary geometry
A Course of ODE with a CAS
Using CAS to Develop Precalculus Concepts in a US Curriculum
Using a Computer Screen as a Whiteboard while Recording the Lecture …
New Models in Assessment in Computer integrated Mathematical …
Modeling some dynamic phenomena with Maple6 in a CAS-based math …
Creating Visualizations using Maple
Promenades, bombardements aléatoires et distribution ….
Modelling with Sketchpad in the teaching of mathematics
La grande révélation
Assessment using Technology: A Case Study in Computer Aided Drafting
Preservice Mathematics Teachers' Beliefs about Teaching with Technology
Refocusing Mathematics Education Because of Technology
Adding an Interactive Component to Computer Algebra in Differential Equ.
The assessing of mathematics skills in a secondary school CAS environm.
Mathematics and the Web: Lessons Learned
Cognitive Tools for Exploring Linear and Exponential Growth
Using CAS in Traditional and Alternative Assessment Models
Reflexions sur les potentialités des logiciels et des calculatrices …
The Correctness, Completeness and Compactness Standards of Computer
Area Estimation and the TI-83: An Application for Economics
Learning College Calculus in a CAS Environment: Theory and Practice
Can CAS improve the Mathematical Abilities of Pupils?
On the CAS and the coordination of semiotic representations
On Reforms of calculus teaching by means of Mathematica
Three-fold activities for discovering conceptual connections within …
Assessment Issues in the introduction of a CAS pilot in the
Problèmes corrigés par ordinateur et disponibles sur le réseau
Why DO we teach theorems in calculus?
Artificial Intelligence vs. trained life bacteria
Un échange de bons procédés entre maths et info

Panel Discussion:

Boon or Bust: What are the implications of calculator

Workshops

Interconnectivity - Data Exchange Between Derive 6 and TI CAS Calcs
Make New from Old
Teaching Mathematics by Math-XP

Hi fellow Derivers,

I am hoping to set up an international group of mathematicians, who are now, or will be, interested in promoting the many mathematical achievements of Ernesto Cesàro.

He seems to be known mainly for the Cesàro summation formula for certain divergent series. This single achievement, is only a minuscule `drop in the bucket' of his vast host of accomplishments.

The centenary of his death will be on 12 September 2006. As a group we could publish papers, that bring his work before the mathematical diaspora, not merely for the purpose of reviving his name, but also because of its innate worth. There is also a great deal of untapped potential in his writings, and we all can be beneficiaries. I am sure that many college curricula, at least, would benefit with some of his findings.

Personally, I have found so much advantage from Intrinsic Geometry, that it seems to have no end. What I have in mind is that there could be occasional talks/lectures on campus to `sound out' the locals and we could enlist all interested parties in a Cesàro subgroup on a list server, either Jiscmail or Yahoo Groups. Eventually, two years down the track, we can have a centenary celebration, the nature of which is yet to be determined.

I have included a short biography below. Please let me have feedback as to your own ideas and also please tell anyone that you think may be interested, even if they are not fellow Derivers. I welcome all suggestions.

Many thanks, David Halprin (convenor of eDUG)

http://groups.yahoo.com/group/eDug

CESÀRO, Ernesto

Mathematician, born in Naples on 12th. March 1859. At the age of 14, due to unforseen circumstances, he was obliged to go and live with a brother at Lieges, where he attended the "School of Mines", and, because of his unusual approach to mathematical research, very soon attracted the attention of Catalan, who then became his close friend and protector. Not very drawn to studying engineering, he succeeded after many trials to begin the study of mathematics at the University of Rome, but because of poor health and financial difficulties was not able to attend lectures regularly, nor to graduate. However, even in this difficult period, he succeeded in publishing about 100 works, which helped him to gain, in 1886, the Chair of Analytical Algebra at the University of Palermo. In 1891 he went to Naples to teach Infinitesimal Calculus, and there, while on the point of taking up the Chair of Rational Mechanics at the University of Bologna, in an attempt to save his son from drowning, died with him, (12th. September 1906).

An original and most prolific mind, which soared into most widely dispersed fields:-

From elementary geometry via Intrinsic Coordinates, [including two conditions for immobility in a plane, (of a point and of a line)] to the theory of numbers, etc.

From analysis, both algebraic and infinitesimal, to the calculation of probability, u.s.w.

From differential geometry to mathematical-physics, etc..

I have a set of three volumes of most of his papers "Opere Scelte", (Edizioni Cremonese, Roma 1968), but unfortunately many of his best papers on Intrinsic Geometry are not therein, but are dispersed throughout European Universities. (I have most of these too.)

The partly self-taught nature of his scientific development, far from diverting his research activity, led him into every field; to ask and answer new questions, and cast new light upon problems, already very old. He was a definitive polymath (polyMATH), in every sense of the word.

The work of Cesàro, because of its great volume, and the fragmentary nature found in every part, makes it difficult to draw a conclusion. It is enough to remember his contributions to asymptotic arithmetic, particularly "Excursions in Arithmetic of the Infinite" in Annals of Mathematics (Milan 1885), the "Course of Algebraic Analysis" (Turin 1884), which reflects the arithmetical view of algebra, characteristic of Cesàro, and which was then translated into German; the "Introduction to the Mathematical Theory of Elasticity (Turin 1894); the "Lessons in Intrinsic Geometry" (Naples 1896), a distinctly personal work of Cesàro, and also this was translated into German, and finally the "Elements of Infinitesimal Calculus" (Naples 1899, 2nd. edition in 1905).