THE DERIVE - NEWSLETTER #54

THE BULLETIN OF THE

DERIVE®

USER GROUP

+ CAS-TI

Contents:

June 2004

Interesting websites:

The following are two excellent websites from German teachers using Derive in Math education. On Mr. Grude´s homepage you can find a very fine contribution on Cellular Automata.

http://home.arcor.de/gallinat.richard/arcor/index.htm

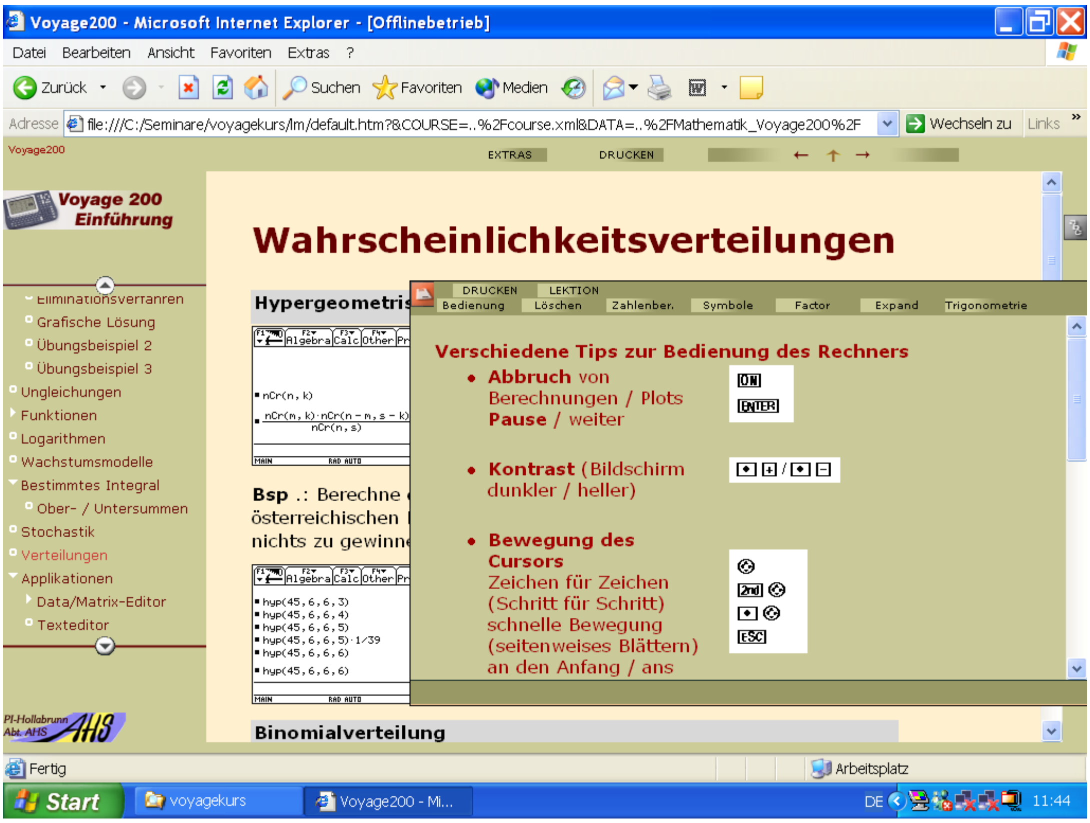
http://www.dietrichgrude.de/kranich/indexkranich.htm

Our French friends produced an online course for TI-calculators:

http://www.t3ww.org/france/t3-online.htm

Gerhard Hainscho produced a very extended Voyage200 Course for Beginners. Walter Wegscheider put this course as an online course on the web (together with many additional resources and links). Many thanks Gerhard and Walter from T^3-Austria and ACDCA.

http://www.austromath.at/daten/voyage200



Dear DUG Members,

this DNL does not contain so many but more extended contributions. Revising DNL#3 and preparing its pdf-publication I got the idea to treat the Differential Equations from 1991 with the TI-92+/Voyage200. I gave a workshop on DEs based on course materials provided by Günter Redl, so I added this paper too (together with some English explanations). I hope that the DERIVE users are not too disappointed, but we didn´t have so many CAS-Calculator contributions in the last DNLs.
Many tasks in the WS are provided for students and WS-participants. The 2nd part (page 20) refers to DNLs#2/#3. You can download both in revised and updated form as pdf-files and there you will find more explanations. It was fascinating to apply the DE-Mode of the TIs , to compare the results and in case if the TI´s DE-mode fails to program the respective functions according to the auxiliary functions provided in Derive´s ODE1.MTH.

The second extended article is the result of a Swedish-Austrian cooperation between David Sjöstrand an me. Last summer David presented his Excel/Derive cooperation on Cellular Automata in Gothenburg. This was a challenge for me to wrap the procedures in a program. Finally we decided to give a joint presentation at the Matematik-Bienallen in Malmö, last January. We had much fun in the audience and we would like to share our product with you. I´d like to call your attention to D Grude´s website (see Information).

On page 43 a tool is offered which might overcome compatibility problems between *DERIVE*6- and *DERIVE*5 files. I am very grateful for many valueable comments and support from Theresa Shelby and Albert Rich.

Finally after a long time we have again an ACDC contribution (Amazing Corner of the Deriver´s Curiosity, founded by Alfonso Población in DNL#22, 1996), which was sent by Milton Lesmes from far Colombia. Muchas Gracias, Milton.

Many new members have joint the DUG during the first months of 2004. A warm welcome to you all. Torbjörn Alm sent a contribution about Pollard-Factorization, which will be published soon. Among others I received an article from H.R. Geyer, who presents a roboter in 3D using the slider bars to move his "little man", and an extended article dealing with "Actuarial Mathematic" from MacDonald Phillips.

In the meanwhile an interesting discussion was raised about the function value of sign(0), which is not finished. I´ll report about it next time.

I´ll close repeating the invitation to join the DUG meeting in Montreal and wishing you all a wonderful summer. I am looking forward to meeting you with DNL#55 in September.

DUG-Meeting 2004
TIME-2004, Montreal
Saturday, 17 July 2004, Lunchtime

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a contents of 44 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE* and the *TI*-89/92/*Voyage* 200 as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

As many of the *DERIVE* Users are also using the CAS-*TIs* the *DNL* tries to combine the applications of these modern technologies.

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Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: September 2004
Deadline 15 August 2004

Preview: Contributions waiting to be published

Finite continued fractions St. Welke, GER
Some simulations of Random Experiments, J. Böhm, AUT
Wonderful World of Pedal Curves, J. Böhm
Another Task for End Examination, J. Lechner, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
ANOVA with *DERIVE* & *TI*, M. R. Phillips, USA
Hill-Encription, J. Böhm
CAD-Design with *DERIVE* and the TI, J. Böhm
Avoiding Convolution and Transforming Methods, M. Lesmes-Acosta, COL
Farey Sequences on the TI, M. Lesmes-Acosta, COL
Fuzzy Logic, G. Hagen, AUT
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
Modelling real date: Enthalpie Values
A TVM-Calculator for *DERIVE*, M. R. Phillips, USA
Shares, Put and Call, J. Böhm, AUT
Actuarial Mathematics, M. R. Phillips, USA
A Roboter with Derive, H.R. Geyer, GER

.........and others

Impressum:
Medieninhaber: *DERIVE* User Group, A-3042 Würmla, D´Lust 1, AUSTRIA
Richtung: Fachzeitschrift
Herausgeber: Mag.Josef Böhm
Herstellung: Selbstverlag
URLs: http://www.austromath.at/dug/index.htm
http://shop.bk-teachware.com/k.asp?session=2579115&kat=11

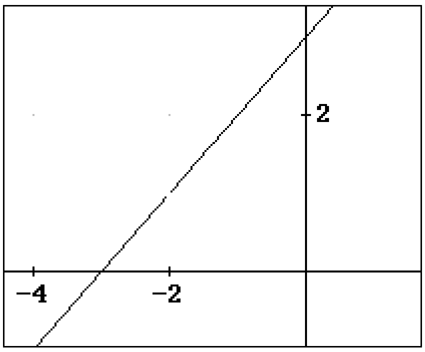
Adam Marlewski algebraistxxi@yahoo.com

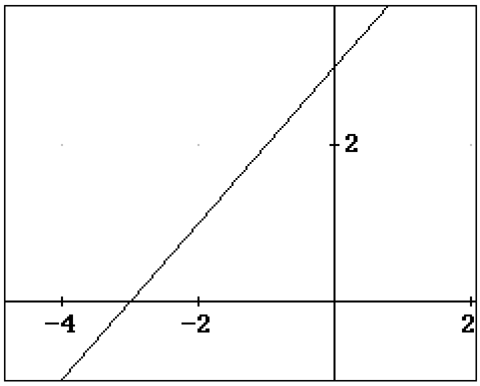
Hello All,

Lou Lowell wrote:
I recently examined the graph of the expression (x^2+5x+6)/(x+2) in both Dfw 4.11 and Dfw 5.06. In ver 4.11 the discontinuity at x= -2 is observable on the graph, however the same function in ver.5 is never observable. Moreover, in ver.4 when tracing along the curve, derive gives a little click when passing the discontinuity; in ver.5 there is no indication of a discontinuity at -2.
Can this be changed to be visible in ver.5?

I answer: In DfW 5.04 the discontinuity at x = -2 is visible if the scale is not too big. See atached file (_discont.dfw) please.
Adam

DNL: I´d like to add another comment: It makes a difference if you have set Option Simplify/Approximate before plotting in the 2D-Plot Window or not. If yes, Derive automatically simplifies the expression $\frac{x^2+5x+6}{x+2}$ to $x + 3$ and subsequently plots the line without any gap.





MacDonald Phillips phillipsm@GAO.GOV

The problem submitted by Jan Vermeylen on page 18 of DNL#53 presents quite a problem in equation fitting. While the resulting, very bulky, equation is linear in k, it is definitely not linear in K. Since there are only two parameters to estimate, it is possible to use any two rows of data and set up two equations in two unknows to obtain initial guesses for the parmeters. However, in solving the equations for the initial guesses, k and K turn out to be complex values. This presented a problem since my nonliear regression routines in the User directory of Derive 6 did not handle datasets with complex numbers or complex parameters. So I modified them so they do! I'm sure complex answers were not expected, but given that there are only three data points, that's the best I can come up with at this time.

So, I get the following: k = 0.041708+0.555127*i, K = -43.2998+45.649*i where i is the square root of -1.

The adjusted R^2 is only .75 and the t and F statistics indicate that the fit is not statistically significant. But that's to be expected with only 3 data points.

Any other thoughts on solving this problem would be greatly appreciated.

If anyone wants a copy of my latest regression routines (they also handle weighted regressions now), please email me at phillipsm@gao.gov.

Don Phillips

Degree versus Radian Mode in Derive 6

Hello All,

In Derive 6, the effect on the system of setting the angle mode to Degree was changed for the following reasons:

1. In Derive 5, a user can not tell just by looking at an expression in the algebra window whether the argument of a trig function is expressed in radians or degrees. For example,

SIN(5)

could mean the sine of 5 radians or the sine of 5 degrees, depending on the current angle mode setting.

2. Changing the angle mode setting during a Derive 5 session resulted in inexplicable behavior, as pointed out on page 148 of the English language manual distributed with Derive 5.

3. I do not know how to resolve the numerous bugs in Derive 5 that result when attempting to implement this feature.

In Derive 6 these problems have been resolved by making the angle mode setting effect only the display of expressions, and by requiring that angles in degrees be entered explicitly using the deg operator. See the Derive 6 on-line help for details.

I hope this explanation helps, and I apologize for the inconvenience it has caused.

Aloha,
Albert D. Rich
Co-author of Derive

Assignments in programs

Presenting some programs produced with Derive 5 I found another – undocumented? - change in Derive´s behaviour.

See the following demo example. I´d like to intriduce "`res`" as a global variable. In Derive 5 I get the output `test(5) = 150`, but in version 6 I get `test(5) = res:=150`. To avoid this I have to add an additional line `RETURN res` or simply `res`. So I have to check all my programs if they are working as expected in Derive 6.

```
#1:   test(n, x, y) :=
        Prog
          x := n^2
          y := n^3
          res := x + y

#2:   test(5) = 150
```

```
#1:   test(n, x, y) :=
        Prog
          x := n^2
          y := n^3
          res := x + y

#2:   test(5) = res := 150
```

```
#3:   test2(n, x, y) :=
        Prog
          x := n^2
          y := n^3
          res := x + y
          res

#4:   test2(5) = 150
```

In case if including `res` into the list of program arguments (and then being a local variable) it works properly.
Josef

This is Albert Rich´s answer:

Albert Rich

Hello Josef,

Theresa ask me to respond to the question raised in your email that follows. In Derive 5, when assignments are simplified, the result is just the simplified right side of the assignment. In Derive 6, when assignments are simplified, the result is the assignment with the right side simplified (this is stated in the Derive 6 on-line help for the Author > Variable Value command). For example, when the assignment

x := 2+3

is simplified in Derive 5, 5 is returned; whereas in Derive 6, the assignment x := 5 is returned. This change in the behavior of Derive was made for two reasons:

1. When an expression is simplified in Derive, the result should be equivalent to the original expression. In the example above, 5 is not equivalent to the assignment x := 2+3; whereas, x := 5 is equivalent to it.

2. In Derive 5 if an assignment is entered and simplified on the Expression Entry line, only the simplified right side of the assignment appears as an expression in the algebra window. In Derive 6, the entire assignment appears as an expression in the algebra window. Thus if the algebra window is saved as a dfw or mth file, it is clear that an assignment was made when the file is subsequently loaded.

However, as you discovered, for efficiency reasons, when assignments to local variables made within function definitions are simplified, only the simplified right side is returned.

Hope this explanation helps and I apologize for the inconvenience. In the next release of Derive 6, I will clearly state this change in behavior of Derive 6 in the on-line help.

Aloha,
Albert

Tom Miles temiles@cox.net

Sharing some interesting observations

Hello Josef,

I thought I would pass along some interesting observations I have made regarding the solutions to my equations.
As I mentioned in a previous communication, I was frustrated that my programs were failing when I was trying to solve for polygons with sides greater than 40. I had been using Derive 6 in the decimal mode. To reach a solution, I often had to increase the decimal precision to over 100. I made two changes that have greatly improved the situation:

First, I converted all the equations to the radians and changed the system to Angle := Radian.

Second, I simply changed NSOLVE((y – y1) = m(x – x1)) to NSOLVE((y – y1) - m(x – x1) = 0). Surprisingly, setting the relationship to "0" made a big difference.

Hope this information may be of interest.

Sincerely,
Tom Miles

Valeriu Anisiu

Hello Albert,

I have some Derive problems for you. Some of them were not present in version 6.01 Beta.

1 $$\int_0^1 \frac{x}{\sqrt{(2 - x^2)} + x + 1}\,dx = -\frac{\sqrt{3}\cdot LN(4\cdot\sqrt{6} - 5\cdot\sqrt{3} - 6\cdot\sqrt{2} + 10)}{12} - \frac{\pi}{8} + \frac{\sqrt{2}}{2} - \frac{\sqrt{3}\cdot\pi\cdot i}{6}$$

For the antideriavative a strange branch is chosen. Same problem with the next integral:

2 $$\int_0^1 \frac{1}{\sqrt{(1 - x)} + \sqrt{(x + 1)} + 1}\,dx = -\frac{\sqrt{3}\cdot LN(4\cdot\sqrt{6} - 5\cdot\sqrt{3} - 6\cdot\sqrt{2} + 10)}{6} - \frac{\pi}{4} + \sqrt{2} - \frac{\sqrt{3}\cdot\pi\cdot i}{3}$$

3 After a long time a complicated multiple SUBST expression is returned; of course the integral can be easily computed (e.g with INTSUBST). Derive should either quit (as in other cases) or give a normal result.

$$\int \frac{1}{x\cdot\sqrt{(x^2 + x + 1)} + 2\cdot x + 2}\,dx$$

4 Memory exhaustion in either exact/approx mode. Also for other sums related to ZETA().

5 $$\text{SUBST}\left(\text{ATAN}(t) - \frac{\sqrt{3}\cdot LN(t\cdot(\sqrt{2} - 1) - \sqrt{3} + \sqrt{2})}{6} + \frac{\sqrt{3}\cdot LN(t\cdot(\sqrt{2} - 1) + \sqrt{3} + \sqrt{2})}{6},\ t,\ \frac{\sqrt{(2 - x^2)} - \sqrt{2}}{x}\right)$$

exhaust memory!

6. (Interface problem)
In Derive 6.0 (and 6.1 beta) the mouse wheel did not work; now it works but in the upward direction the worksheet jumps very fast to the top (being almost useless).

I finish with a question related to the approx mode in Derive.
From your very clear explanation I understood that Derive does not use a floating point arithmetic.
A real number seems to be approximated with PrecisionDigits=n as A/B*10^C with abs(A)<K1(n), 0<B<K2(n) and no limit for the integer C. [possibly 2^C is used instead of 10^C].
I am curious to know:
- which are the values for K1(n) and K2(n)? Are not they too "optimistic"?
- why a limit (maybe optional) for C is absent (for overflow/underflow in approx mode)?.

Because of this, in numerical computations "infinitesimals" may propagate. Derive behaves sometimes strangely when abs(C) is big. For example ITERATE(f(x),...) may appear to diverge even if the iterates of f converge to 0 or to a vector having a 0 component.

I wish you a happy Easter [this year the Western and Eastern Easter coincide as a Derive program posted a couple of years ago shows].

Valeriu

Albert Rich

Hello Valeriu,

The following is in response to your numbered questions:

1. Unfortunately, as plotting both the real and imaginary parts shows, the antiderivative Derive finds for x/(SQRT(2-x^2)+x+1) is discontinuous at x=SQRT(3)/2-1/2. The correct answer to the definite integral is

SQRT(3)*LN(-4*SQRT(6)+5*SQRT(3)-6*SQRT(2)+10)/12-pi/8+SQRT(2)/2

This result can be obtained by adding the integral from x=0 to the discontinuity to the integral from the discontinuity to 1. I will add this problem to my list of discontinuous antiderivatives to resolve.

2. One application of Display Step shows that the antiderivative of 1/(SQRT(1-x)+SQRT(x+1)+1) reduces to the problem described in #1 above.

3. Unfortunately, as the Display Step feature shows, Derive rationalizes the denominator and then applies partial fraction expansion resulting in explosive growth of the problem. Can you tell me what substitution is required to find the antiderivative?

4. In the next release of Derive this summation problem will almost instantly simplify to pi^2/6 - ZETA(2, 50001) and approximate to 1.644914067.

5. This memory exhaustion is due to a stack overflow problem that occurs when trivial factoring some algebraic expressions. For example, try trivially factoring x*(2*SQRT(6)+2*SQRT(2)-8)+4. I hope to have this resolved for the next release of Derive.

6. On my computer running Derive 6.01, scrolling the algebra window using the mouse wheel works normally. I will ask Theresa if she can reproduce and resolve the problem you are having.

Concerning your questions concerning rational approximation in Derive:

Derive uses the approximate rational arithemetic provided by muLISP (the LISP language pseudo-code interpreter in which Derive is written). muLISP approximates a number by finding the closest rational approximation for which the sum of the bit lengths of the numerator and denominator does not exceed a given number of bits. For n digits of precision set by the user, Derive uses an internal precision of 4*CEILING(5*n+6,6) bits. Thus for the default 10 digits of precision, the sum of the bit lengths of the numerator and denominator is limited to about 40 bits. However, there is no limit (other than memory) on the bit lengths allowed for integers and pure reciprocals, since this is the only way to preserve their magnitude.

As you noted, there is no underflow or overflow in Derive other than memory exhaustion. Overflowing to infinity is a bad idea for a computer algebra system, since even big numbers are much closer to 1 than they are infinity. A golden rule (or at least ideal) in Derive is that 1/(1/n)) should equal n. If for n>>0, 1/n approximated to 0, 1/(1/n) would approximate to infinity. However, as I say above even if n>>0, n should not approximate to infinity. Therefore, it is also a bad idea to underflow to zero.

Hope this explanation helps. I have copied this email to several others since the discussion of approximate rational arithmetic may be of interest to them. I hope you do not object.

Aloha,
Albert

Valeriu Anisiu

Hello Albert,
here are some remarks related to your last email.

INT

For an integral of a function R(x,sqrt(ax^2+bx+c)) where R is a rational function, it is not a good idea to rationalize the denominator; the Euler's substitutions are recommended. They are implemented (as a bonus) in INTSUBST (because I noticed that they are not implemented in Derive).
e.g.

INTSUBST(1/(x*SQRT(x^2+x+1)+2*x+2),x, x^2+x+1="euler1")
-->
-SQRT(3*SQRT(17)/17+5/17)*ATAN((4*SQRT(x^2+x+1)-4*x-SQRT(17)+3)*SQRT(11*SQRT(~
17)/64+45/64))+SQRT(3*SQRT(17)/68-5/68)*LN(4*SQRT(x^2+x+1)-4*x-SQRT(22*SQRT(1~
7)+90)+SQRT(17)+3)-SQRT(3*SQRT(17)/68-5/68)*LN(4*SQRT(x^2+x+1)-4*x+SQRT(22*SQ~
RT(17)+90)+SQRT(17)+3)

(the change of variable is sqrt(x^2+x+1)=x+t)

APPROX

I don't think that for numerical computations, over/underflowing is such a bad idea (of course I do not suggest to change Derive for this but it is anyway largely used). Your statement that a huge number is closer to 1 than to inf is also arguable because in the interval [-inf,inf] usually other metrics are used (for example via a bijection to [-1,1]).
The fact that in approx mode 1/(1/n) = n is of course fine but why should it be true only when n is integer; in approx mode this is not essential for a user who wants seriously to approximate and not to "test"
superficially Derive.

I have recently seen a proposal for approximating real numbers in rational arithmetic where x is approximated to the first convergent p/q (of the continuous fraction of x) for which abs(x-p/q)<=eps*abs(x).
With this approach PrecisionDigits would have a full meaning; the problem of the uneven distribution of Q is resolved with a very moderate cost!

The kernel functions should return the result within the requested precision. In Derive (unlike any other CAS I know) this is not the case with trigonometric functions. E.g.:
APPROX(sin(10^11),10) --> 0.7071067811   (which seems to be sqrt(2)/2 ??) instead of 0.9286936604
In this case the argument of the sine must be reduced internally using PrecisionDigits+11
A very rough user solution could be to define:

```
sin_(x):=IF(IDENTICAL?(Precision,Exact),
        SIN(x),
        SIN(APPROX(MOD(x,2*pi),PrecisionDigits+MAX(LOG(ABS(x),10),0))))
```

Salut,
Valeriu

You can find the end of this discussion on page 45.

Differentialgleichungen mit dem TI-89/TI-92+/Voyage 200
Differential Equations with TI-89/TI-92+ and V200

A Workshop

Hier soll nicht die Theorie der DGL und ihre Lösungsmechanismen besprochen, sondern an einigen ausgewählten Aufgaben die Vorgangsweise auf den TI-CAS-Rechnern beschrieben werden. Der TI-89/TI-92PLUS und der Voyage 200 verfügen über ein gehöriges Potential an graphischen und analytischen Werkzeugen zur Behandlung der Differentialgleichungen.

We will not deal with theory of DEs, but show the solving procedure using the TI-CAS calculators.

Beispiel 1/Example 1

$y' = y(x+1)$

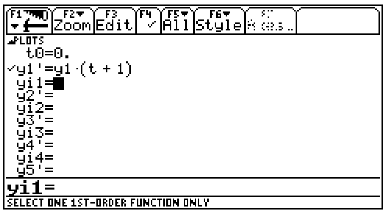
a) Zeichne das Richtungsfeld. Plot the direction field.

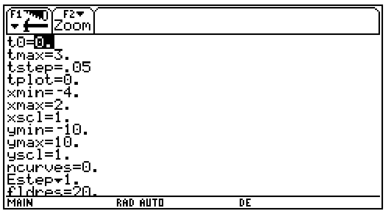
b) Zeichne spezielle Lösungen der DGL für y(0) = 3, y(0) = -3 und y(-1) = 4

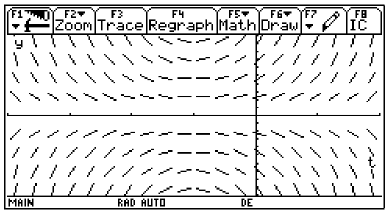
c) Bestimme die allgemeine Lösung und die oben angeführten speziellen Lösungen mit Hilfe des Rechners indem du die händische Vorgangsweise nachvollziehst.
Find the general solution reproducing the manual solving procedure.

d) Bestimme diese Lösung mit allen vom CAS zur Verfügung gestellten Hilfsmitteln. Die Lösung aus b) könnte mit der entsprechenden nun gefundenen Lösung überlagert werden. Use all means of CAS to find the solutions.

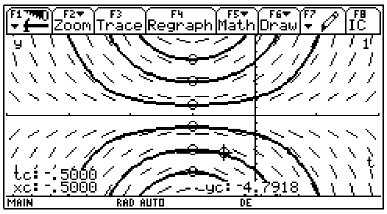
a) Der Rechner wird im MODE auf Graph DIFF EQUATIONS eingestellt. Die Differentialgleichung wird in den [Y=]-Editor geschrieben. Aber Achtung: die Rolle des *x* wird vom Parameter *t* übernommen! (*x* must be replaced by *t*!)

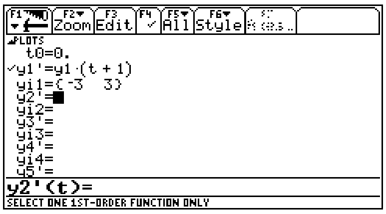


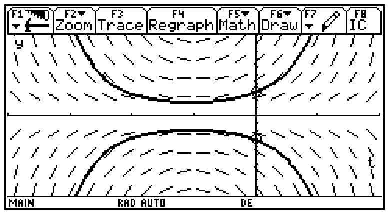




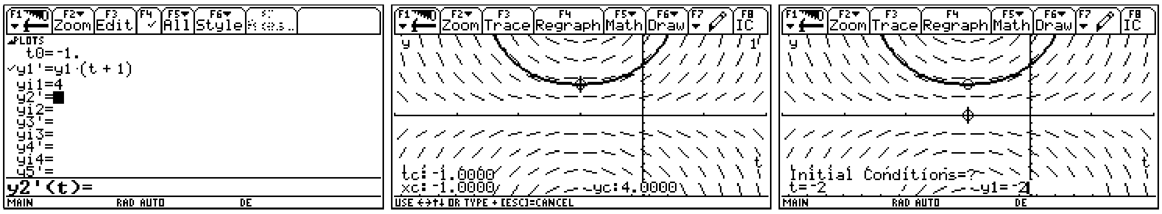
Wenn Du in den [WINDOW]-Werten für ncurves etwa den Wert 6 einsetzt, dann werden gleichmäßig über den *y*-Bereich 6 Lösungskurven eingezeichnet. Über F3 Trace können die Kurven abgetastet werden.



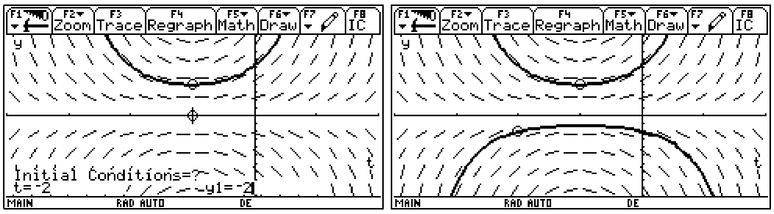




b) Um die beiden Lösungskurven zu den gegebenen Anfangsbedingungen zu sehen, gibt man die beiden Funktionswerte zu $x = t = 0$ in einer Liste zu yi1 an. Für die dritte gesuchte Lösungskurve muss der Wert für t0 auf –1 und yi1 auf 4 gesetzt werden.

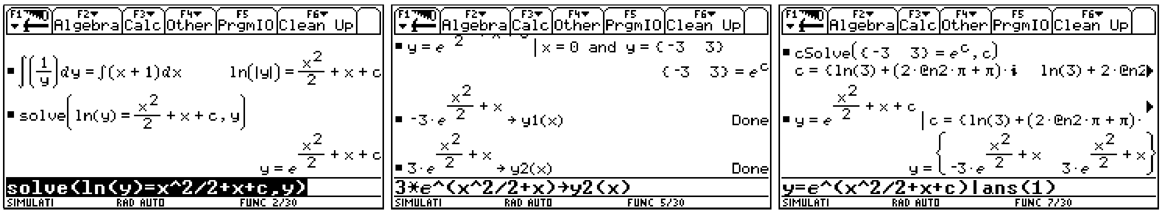


Es gibt noch eine weitere – interaktive – Möglichkeit, Lösungskurven durch bestimmte Punkte zu legen. Über F8 IC können Anfangsbedingungen (Initial Conditions) direkt eingegeben werden. So wollen wir zB die Kurve durch P(-2|-2) legen.

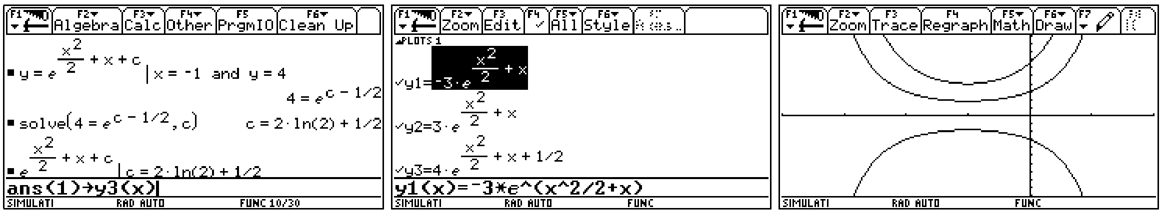


c) Als Lösungsmethode kommt hier die *Methode der Trennung der Variablen* in Frage. We apply Separation of Variables.

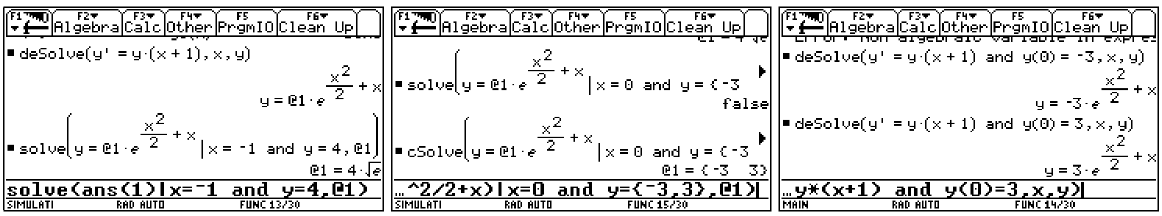
$$\frac{dy}{dx} = y(x+1) \to \frac{dy}{y} = (x+1)\,dx$$



Alle drei speziellen Lösungen werden gefunden und über den Funktioneneditor gezeichnet. Beachte den Einsatz der cSolve-Funktion (wenn man nicht direkt für e^c substituiert).
We find all special solutions and plot them. Notice application of cSolve if not substituting directly for e^c.



d) Mit der implementierten **deSolve**-Funktion lässt sich die DGL direkt lösen. Auch hier führt cSolve zum gewünschten Ergebnis. Die @- Zeichen stehen für freie Parameter.
The @- characters are free parameters.



Auch die Randbedingungen (oder andere gegebene Punkte der Lösungskurve) können sofort mit angegeben werden. We can include points of the integral curve into the **deSolve**-function.

Beispiel 2/Example 2

$$y' = \frac{2y}{x}\ln\left(\frac{3y}{x}\right)$$

a) Bestimme die Lösung mit Hilfe des Rechners, ohne vorerst desolve() einzusetzen.

Tipp: Substituiere $z = \frac{y}{x}$. Solve without using deSolve() – substitution!!.

b) Wie lautet die spezielle Lösung für $y(1) = \sqrt[3]{e}$? Special solution for $y(1) = \sqrt[3]{e}$

c) Zeichne das Richtungsfeld und lege die gefundene Lösungskurve drüber. Direction field.

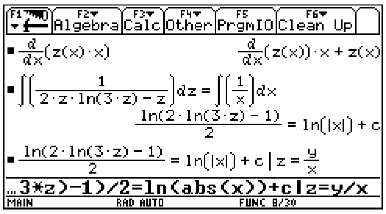
d) Bestimme diese Lösung mit allen vom CAS zur Verfügung gestellten Hilfsmitteln.

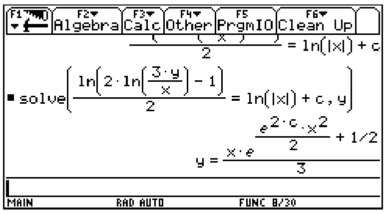
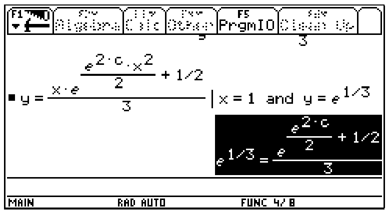
e) Fixiere die Lösungen von a), b) und d) in einem Script (Textfile). Produce a script.

a) $z = \frac{y}{x} \to y = z \cdot x \to y' = z' \cdot x + z$

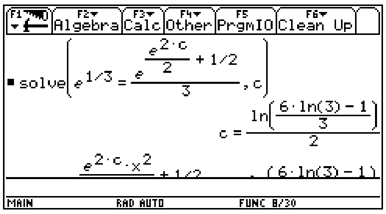
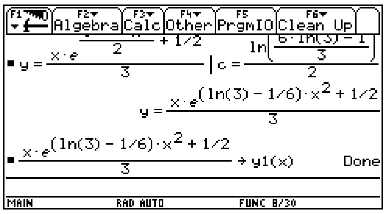
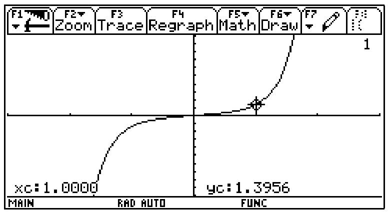
Daher wird aus der DGL: $z' \cdot x + z = 2z \cdot \ln(3z) \to \frac{dz}{dx} \cdot x = 2z \cdot \ln(3z) - z \to \frac{dz}{2z \cdot \ln(3z) - z} = \frac{dx}{x}$.

Die Trennung der Variablen ist geglückt. Es fragt sich nur, ob die Integrale geschlossen ausgewertet werden können? Separation was possible, is integration possible, too?

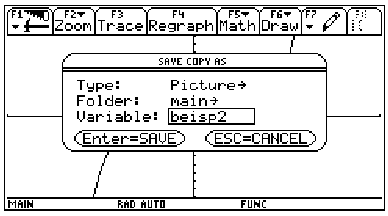


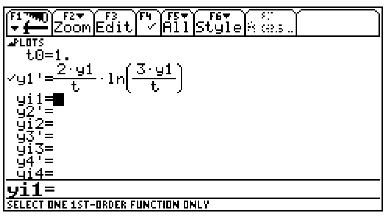
Das Integral lässt sich auswerten. Welche Integrationsregel(n) führen zum Ziel? [1].
Which integration rules are leading us to success? [page 12]

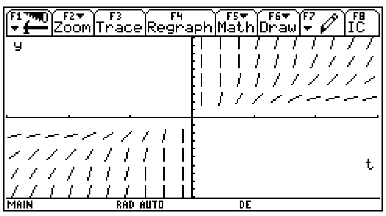
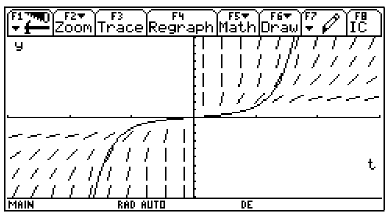




b) Es gibt eine allgemeine und die gesuchte spezielle Lösung. Der Graph der Lösungskurve wird als Bild F1 2:Save Copy As unter dem Namen beisp2 abgespeichert – um ihn später über das Richtungsfeld zu legen. We save the graph for later combining it with the direction field.



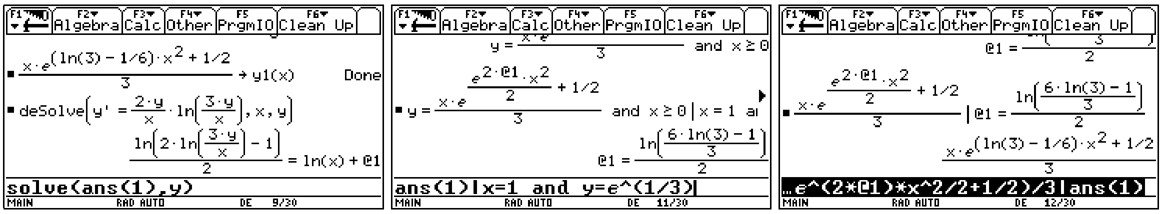
c) Über MODE wird in den DIFF EQUATIONS Modus gewechselt. Die [WINDOW]-Werte sollen mit denen von vorhin identisch sein.



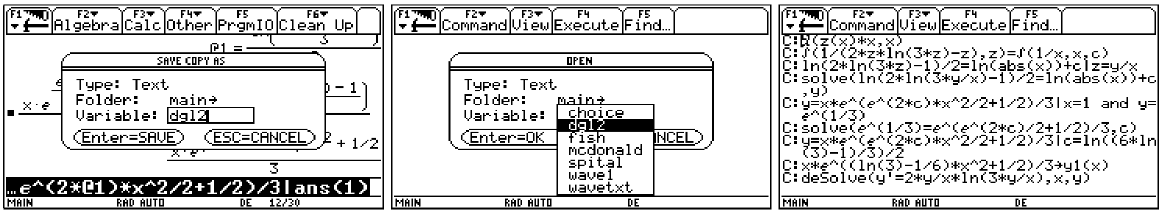



Über F1 1:Open wird beisp2 über das Richtungsfeld gelegt – und alles passt.

d) Wenn es "nur" um die Lösung geht, versucht man gleich desolve().

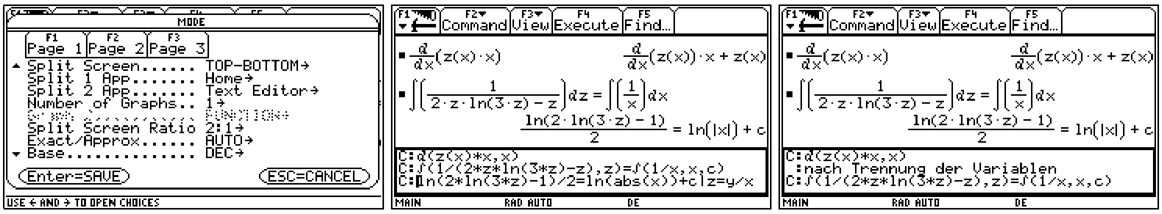


e) Der komplette Prozess wird in einer Textdatei gespeichert (F1 2:Save Copy As).



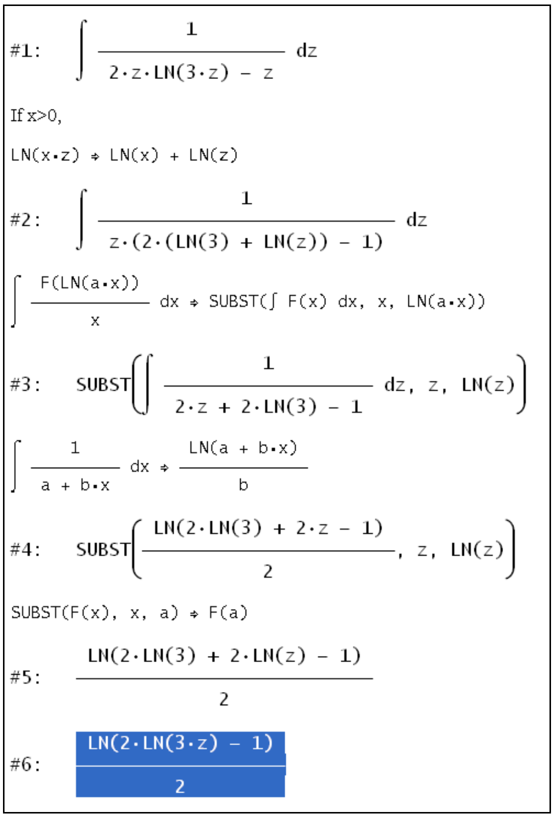
Im Texteditor wird diese Datei wieder geöffnet. Jede mit **C:** eingeleitete Zeile ist ein ausführbares Kommando, das mit **F4** "exekutiert" werden kann. Es empfiehlt sich, den Schirm laut Muster zu teilen.

We open the file in the text editor. Each line showing a **C:** can be "executed" in the Home Screen by pressing **F4**. I recommend to split the screen according to the pictures.



Diese Frage stellen wir *DERIVE* 6. Der eingebaute "schrittweise Simplifier" verrät uns die angewendeten – elementaren – Rechenregeln. So können wir von einem CAS nicht nur die Ergebnisse rascher erhalten, sondern gewinnen auch Kenntnisse in den Regeln – oder wir werden wieder an sie erinnert.

DERIVE 6 gives the answer. Stepwise simplification shows us the secrets how to find the closed form of the integral.
So the CAS does not only help us finding the results very quick, but also helps us to gain new knowledge or reminds us on possibly forgotten knowledge.



Beispiel 3/Example 3

$$\left(3x^2y+8xy-y+2\right)dx+\left(x^3+4x^2-x+\frac{y}{2}\right)dy=0$$

a) Zeige, dass hier eine "exakte DGL" vorliegt, d.h., dass die Integrabilitätsbedingung erfüllt ist. $P(x,y)dx+Q(x,y)dy=0$ ist exakt, wenn gilt: $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$. Show that the DE is exact.

b) Löse die DGL auf traditionelle Weise. Erst allgemein, dann für $y(0) = 2$.
Solve in the traditional way, general and special for y(0)=2.

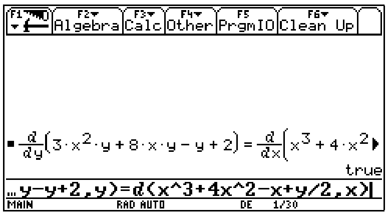
c) Erzeuge für die Lösung dieses Typs von DGL ein Programm.
Produce a program for solving DEs of this kind.

d) Erfinde eine eigene *exakte Differentialgleichung* (die Winkelfunktionen enthält).
Find an own exact DE containing trig-functions.

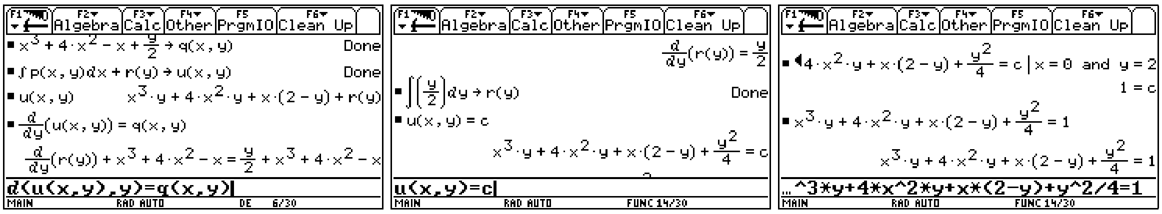
e) Suche die Lösungen von a) und b) auf direktem Weg mit dem CAS.
Find the solutions for a) and b) directly using CAS.

f) Löse auch *deine* Aufgabe mit deSolve und mit dem Programm.
Solve your problem (d) with deSolve and with the program.

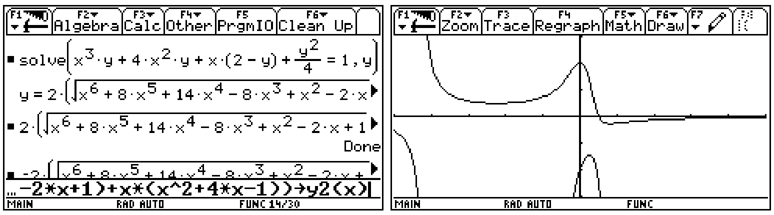
g) Zeichne das Richtungsfeld mit der speziellen Lösung aus b).
Plot the direction field including the special solutions from b).

a) 

b) Der allgemeine Ansatz für die Lösung $u(x,y)$ lautet: $u(x,y)=\int P(x,y)dx+r(y)$. Aus dem so gewonnenen $u(x,y)$ wird nach Ableitung nach y ein Ausdruck für $r'(y)$ bestimmt, der dann nach y integriert zu $r(y)$ führt.

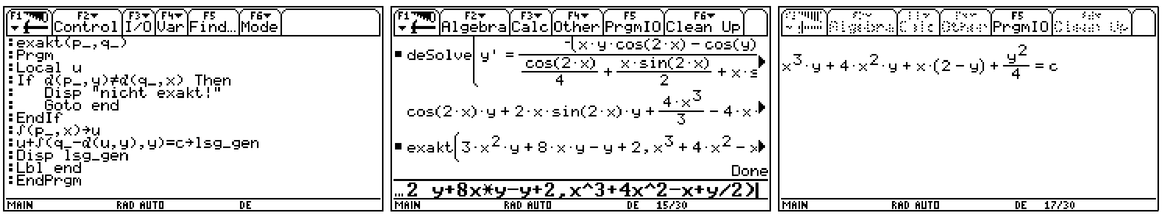


Von den beiden Funktionsgraphen gilt natürlich nur der obere!



Only the upper graph is valid.

c) Ein Programmvorschlag/**Proposal for a program** (following an idea of Günter Redl).



d) Ein Vorschlag ist, mit mit Kern $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = s(x,y)$ zu beginnen. Dieser Term muss sowohl nach x als auch nach y integrierbar sein und ergibt dann die Terme $P(x,y)$ und $Q(x,y)$, die noch mit jeweiligen Integrationskonstanten ergänzt werden dürfen.

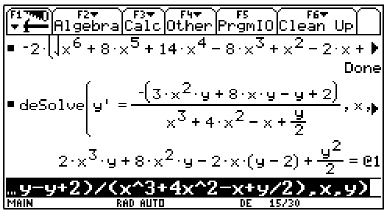
$$s(x,y) = x \cdot \cos(2x) + \sin y$$

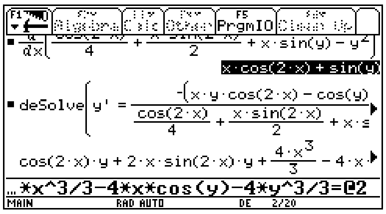
$$P(x,y) = \int s(x,y)dy + konst = x \cdot y \cdot \cos(2x) - \cos y + x^2$$

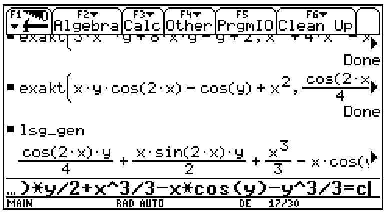
z.B. $$Q(x,y) = \int s(x,y)dx + konst = \frac{\cos(2x)}{4} + \frac{x \cdot \sin(2x)}{2} + x \cdot \sin y - y^2$$

$$\left(x \cdot y \cdot \cos(2x) - \cos y + x^2\right) \cdot dx + \left(\frac{\cos(2x)}{4} + \frac{x \cdot \sin(2x)}{2} + x \cdot \sin y - y^2\right) \cdot dy = 0$$

e) und f) Die CAS-Lösungen:



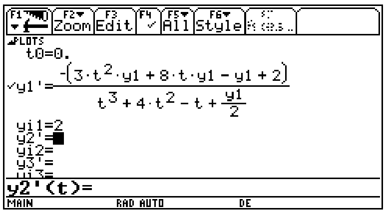


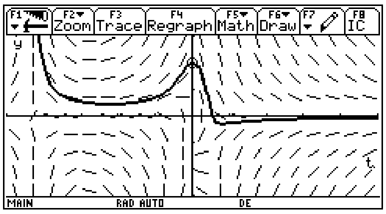


Im dritten Bild wird die Programmlösung der selbst erstellten DGL gezeigt. Sind die Ergebnisse mit **deSolve()** und dem Programm äquivalent?

Are the deSolve()- and the program solutions equivalent?

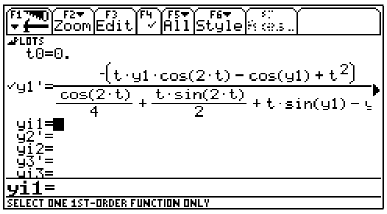
g) Das Richtungsfeld mit der speziellen Lösung.

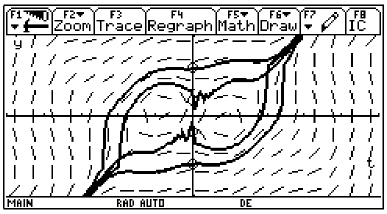
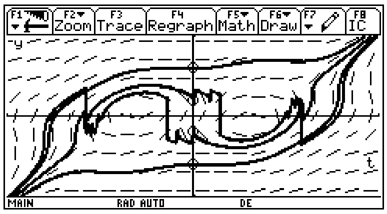


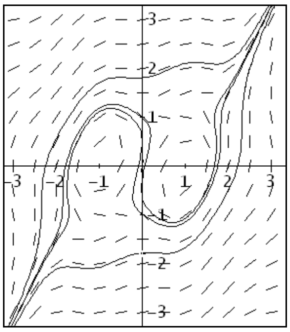


Für „unsere" DGL (d) enstehen recht barocke Lösungskurven (ncurves = 4). Leider kann man am TI keine implizit gegebenen Relationen (Funktionen) zeichnen lassen – mit Umwegen über den R^3 ist dies recht mühsam möglich – siehe später. Über den Funktionseditor wird entweder das EULER- oder das RUNGE-KUTTA-Verfahren für eine numerische Approximation eingesetzt. Ich vergleiche die Ausgabe am TI mit der Ausgabe von *DERIVE*.

"My personal DE" from (d) results in very baroque solution curves. Unfortunately we cannot do implicit plots on the TI in an easy way – we can do it via R^3 representation as shown later. I compare theTI-plot with the respective Derive plot.





Eine Verbesserung der Darstellung lässt sich durch Verkleinerung des **tstep** -Werts bei gleichzeitiger Verkleinerung des Bildausschnitts erreichen. Dass man auch übertreiben kann, zeigt die Abbildung rechts oben. (Zooming in does not necessarily improve the graphic representation.)

Im linken Bild sieht man die *DERIVE*-Darstellung der implizit gegebenen Lösungskurven mit den Anfangsbedingungen $y(0) = -1.8$,

Beispiel 4/Example 4

$$2y'' + 3y' - 27y = 0; \qquad y(0) = -2,\ y'(0) = \frac{1}{2}$$

a) Bestimme die graphische Lösung am TI. Graphic solution on the TI.

b) Löse die DGL auf traditionelle Weise – CAS-unterstützt, aber ohne desolve().

c) Vergleiche mit der reinen CAS-Lösung. Compare the traditional with the CAS-solution.

d) Interpretiere die Werte in der TI-Tabelle ([TABLE]). Discuss the [TABLE]-values.

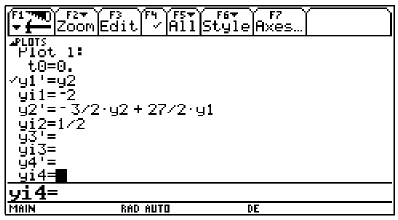
e) Ein Richtungsfeld ergibt sich über F9 Format Fields DIRFLD im [GRAPH]-Fenster. Versuche, auch die Lösungskurve zu zeichnen und vergleiche mit den früher gefundenen Graphen. Plot the direction field together with the solution curve from above.

a) Für die Eintragung im Funktionseditor muss ein kleiner Trick angewendet werden. Die vorliegende DGL muss zuerst durch geeignete Substitution in ein System von zwei DGL umgewandelt werden. The given DE must be transformed into a system of two DEs.

Man setzt: y1 = y, y2 = y', y3 = y'' usw. Durch Differenzieren ergibt sich dann:
y1' = y', y2' = y'', y3' = y''' usw.

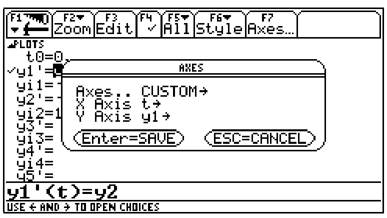
Bei der vorliegenden Aufgabe bedeutet dies:

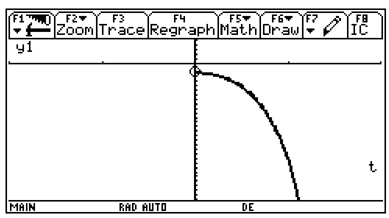
$$y'' = -\frac{3}{2}y' + \frac{27}{2}y$$



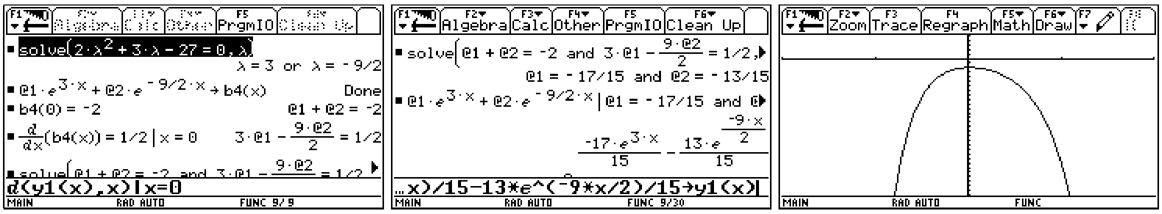
Für die Zeichnung der Lösungskurven müssen über F7 die Axes nach der Abbildung eingestellt werden.

Format Fields FLDOFF



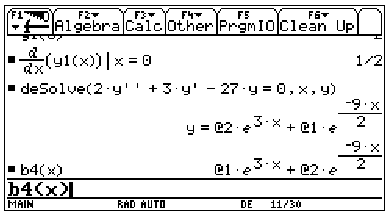


b) Wir lösen die charakteristische Gleichung, die zwei reelle getrennte Lösungen liefert. Die allgemeine Lösung wird als Summe von zwei Exponentialfunktionen mit unbekannten Koeffizienten angesetzt. Diese werden leicht aus einem linearen GLS berechnet, das aus den Anfangsbedingungen gewonnen wird. Mit geeigneten [WINDOW]-Werten lässt sich die Lösungskurve schön darstellen. Solve the characteristical equation and find the general solution as usual.

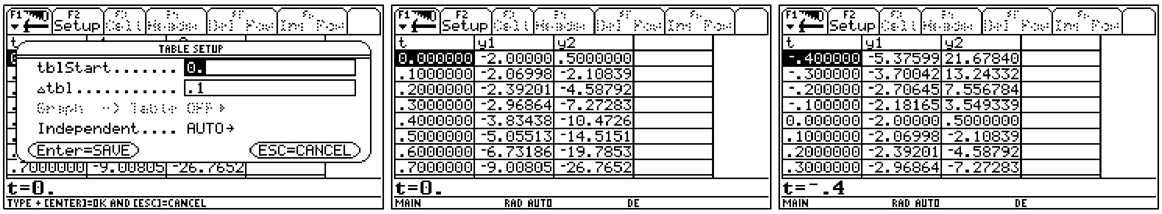


c) Die reine CAS-Lösung ergibt sich sofort in allgemeiner Form. Zur speziellen Lösung muss man vorgehen wie in b) gezeigt.

We obtain the pure CAS-solution in one single step.



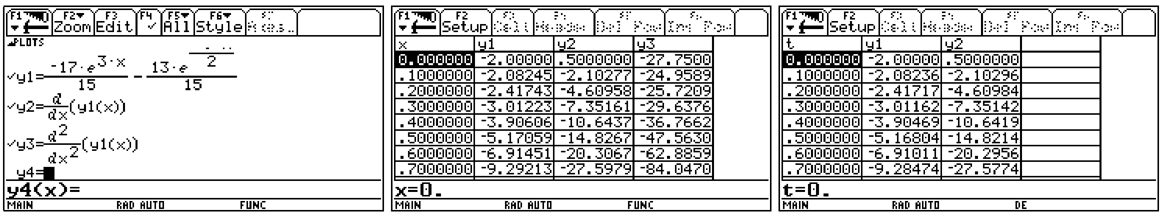
d) Die Tabelle wird zuerst eingerichtet, dann kann sie betrachtet werden. Unter y1 und y2 werden die Werte für $y(x)$ und $y'(x)$ ausgewiesen.



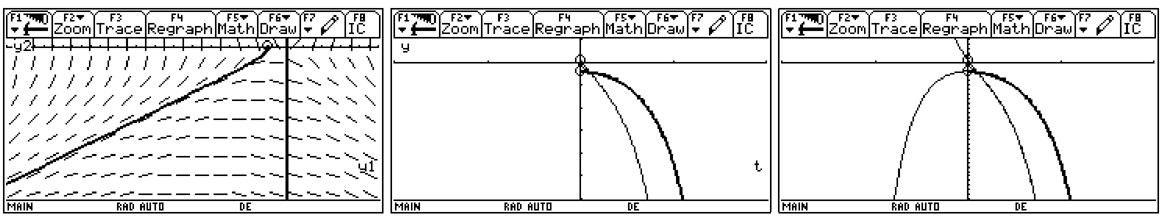
Setup the table and inspect the values of y(x) and y'(x) as y1 and y2.

Wenn man auch die Werte der zweiten Ableitung sehen will, dann muss man wieder in den FUNCTION-Modus zurückwechseln. Der Vergleich zeigt den „Fehler" bei der hier angewendeten EULER-Methode. Wenn man im DE-Modus auf die RUNGE-KUTTA-Methode umstellt, dann fällt der Fehler deutlich geringer aus.

For finding the values of the 2nd derivative one has to switch back to FUNCTION-Mode. Comparison shows the "error" applying EULER-method. Applying RUNGE-KUTTA results in smaller errors.



e) Im Richtungsfeld wird der Zusammenhang zwischen y1 (= Lösungskurve) und y2 (= erste Ableitung) dargestellt. Stellt man die Achsen auf TIME um und aktiviert y1' und y2', dann werden Funktion und erste Ableitung gezeichnet. (Hier wurde die Funktion mit dem Style Thick ausgezeichnet. Anschließend habe ich die Graphen der exakten Lösung und deren erste Ableitung über die numerische Approximation gelegt (RUNGE-KUTTA).



Wird die EULER-Methode eingesetzt, kann man leichte Abweichungen auch optisch erkennen.

Beispiel 5/Example 5

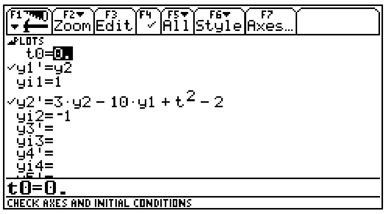
$y'' - 3y' + 10y = x^2 - 2; \qquad y(0) = 1,\ y'(0) = -1$

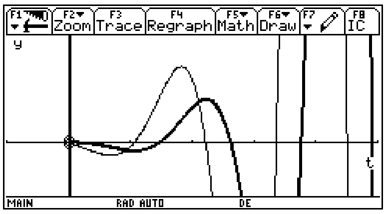
$\qquad\qquad\qquad\qquad\qquad\quad y(1) = 1,\ y'(1) = 2$

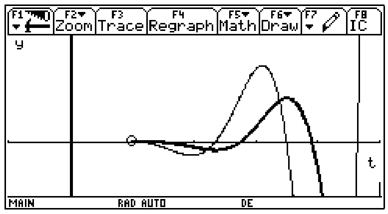
a) Bestimme die graphische Lösung am TI. Graphic solution on the TI.
b) Löse die DGL auf traditionelle Weise – CAS-unterstützt, aber ohne deSolve().
c) Führe die "Probe" durch. First without and then check with deSolve().
d) Vergleiche mit der reinen CAS-Lösung. Compare with the pure CAS solution.

e) Die beiden Lösungskurven werden leicht gefunden. Für die zweite sind die Anfangswerte zu verändern. Diese können auch interaktiv über F8 im Grafikfenster eingestellt werden.

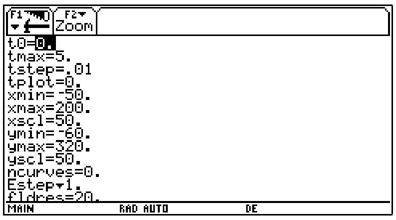
a)

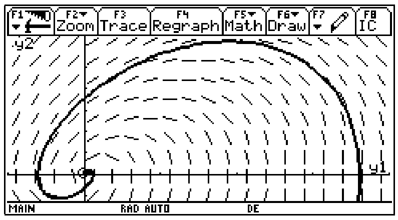




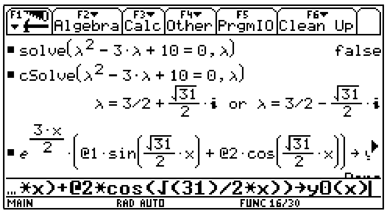


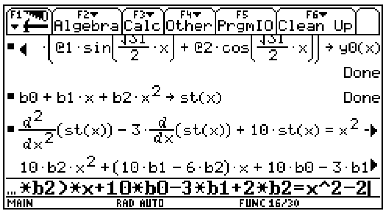
Besonders interessant wird hier das DIRFLD für Funktion und Ableitung. DIRFLD for y1 and y2.

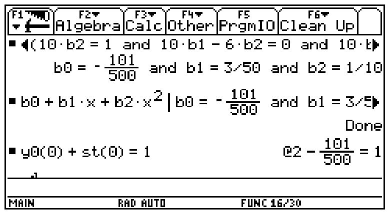




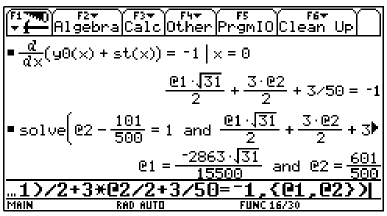
b) Die traditionelle Lösungsmethode beginnt wieder mit der charakteristischen Gleichung, die hier zwei konjugiert komplexe Lösungen ergibt. Der Realteil wird für die Exponentialfunktion und der Imaginärteil für den trigonometrischen Lösungsansatz verwendet.

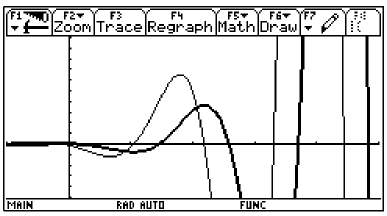






Dabei bleiben zwei Parameter. Für das Störglied $x^2 - 2$ wird ein unbestimmtes quadratischen Polynom st(x) angesetzt, dessen Koeffizienten nach Einsetzen von st(x) in die DGL durch Koeffizientenvergleich gefunden werden (b0, b1 und b2). Durch Einsetzen der Anfangsbedingungen erhält man schließlich die noch offenen Parameter. The traditional way supported by the TI.



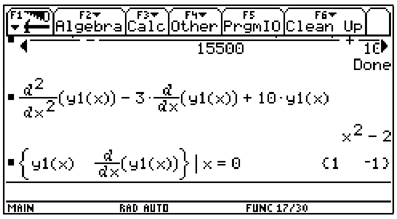


Das ist die Lösung für das erste Paar von Anfangsbedingungen – in „Schönschrift“.

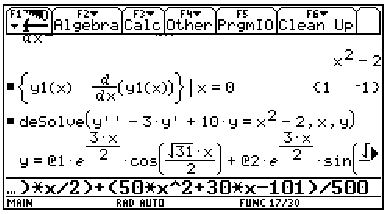
$$y(x)=\frac{601e^{\frac{3x}{2}}\cos\frac{\sqrt{31}x}{2}}{500}-\frac{2863\sqrt{31}\,e^{\frac{3x}{2}}\sin\frac{\sqrt{31}x}{2}}{15500}+\frac{x^2}{10}+\frac{3x}{50}-\frac{101}{500}$$

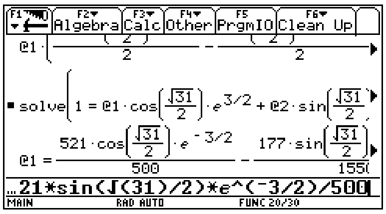
The solution for the first pair of initial conditions. In c) we check the result.

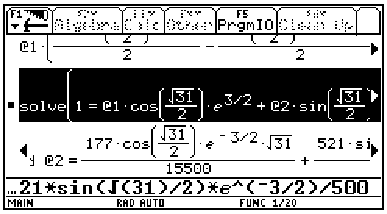
c) Die Probe bestätigt unser Ergebnis.



d) Wir lösen die Aufgabe mit desolve() und verwenden die allgemeine Lösung zur Berechnung der speziellen Lösung für das zweite Paar von Bedingungen.







Die komplette zweite spezielle Lösung kann mit einem mit *DERIVE* produzierten Ausdruck verglichen werden: Compare the 2nd special solution with the respective Derive result.

$$\mathrm{DSOLVE2\_IV}(-3,\ 10,\ x^2-2,\ x,\ 1,\ 1,\ 2)$$

$$e^{3\cdot x/2}\cdot\left(\mathrm{COS}\left(\frac{\sqrt{31}\cdot x}{2}\right)\cdot\left(\frac{521\cdot e^{-3/2}\cdot\mathrm{COS}\left(\frac{\sqrt{31}}{2}\right)}{500}-\frac{177\cdot\sqrt{31}\cdot e^{-3/2}\cdot\mathrm{SIN}\left(\frac{\sqrt{31}}{2}\right)}{15500}\right)+\mathrm{SIN}\left(\frac{\sqrt{31}\cdot x}{2}\right)\cdot\left(\frac{177\cdot\sqrt{31}\cdot e^{-3/2}\cdot\mathrm{COS}\left(\frac{\sqrt{31}}{2}\right)}{15500}+\frac{521\cdot e^{-3/2}\cdot\mathrm{SIN}\left(\frac{\sqrt{31}}{2}\right)}{500}\right)\right)+\frac{50\cdot x^2+30\cdot x-101}{500}$$

Beispiel 6/Example 6

Eine Anwendung: Schwingungsgleichung Vibration Equation

Die Feder eines PKW wird durch das Gewicht des Fahrzeugs (ca. 3500 N pro Rad) verformt. Die Feder wird um weitere 5 cm verformt und dann sich selbst überlassen. Die Schwingungsgleichung $s(t)$ ist aufzustellen und der Graph (Weg-Zeit-Diagramm) zu erzeugen.

Dabei ist m die Masse (m = 350 kg), k die Federkonstante (z.B. 50000 kg/s^2) und l der Dämpfungsfaktor. Verschiedene Werte für l führen zu verschiedenen Fällen. Untersuche die folgenden Fälle und interpretiere die entsprechenden Graphen:

a) Dämpfungsfaktor $l = 0$ dvarious damping factors

b) Dämpfungsfaktor l = 1000 kg/s

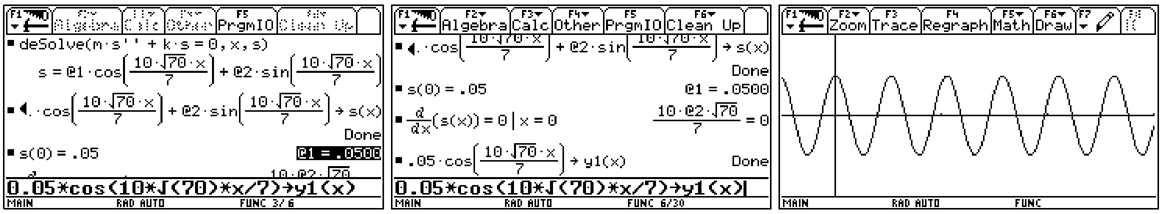
c) Dämpfungsfaktor l = 10000 kg/s

d) Welcher Dämpfungsfaktor ergibt eine gerade ausreichende Dämpfung?
Which damping factor is necessary for sufficient damping?

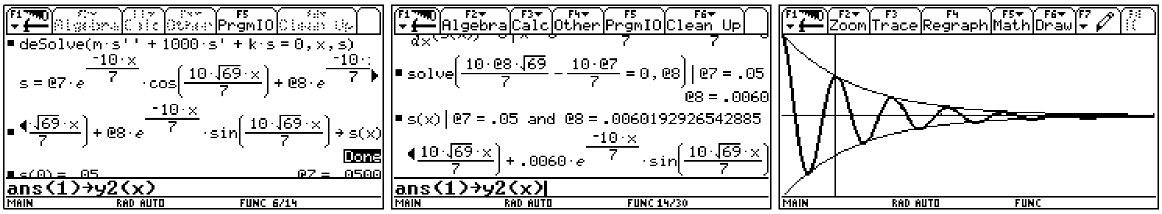
Alle Aufgaben werden mit größtmöglicher CAS-Unterstützung gelöst.

This example was presented by Günter Redl in a T3-ACDCA-Seminar. Many thanks, Günter.

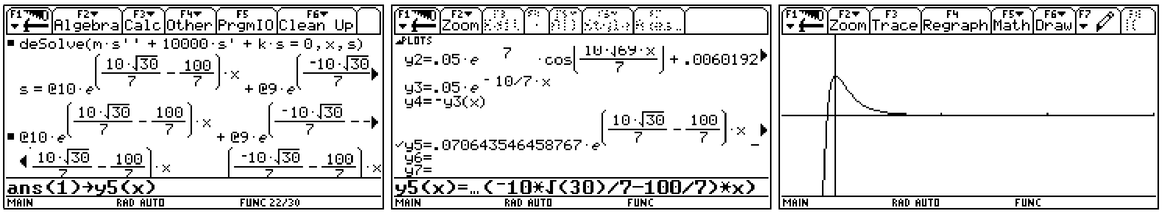
a) Es ergibt sich das Bild einer ungedämpften Schwingung. undamped oscillation



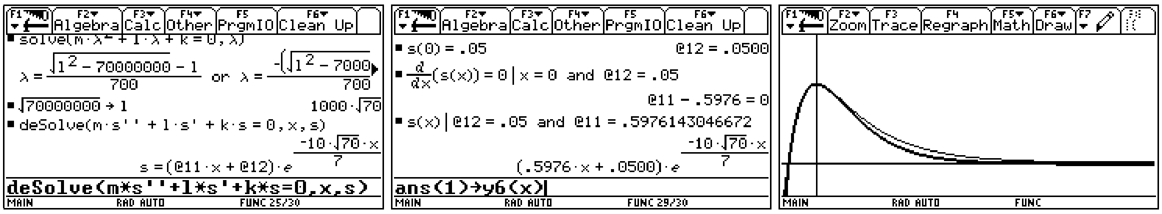
b) Hier sehen wir eine zu geringe Dämpfung – eine gedämpfte Schwingung. Die Funktion $s(t)$ gemeinsam mit $\pm\,0{,}05e^{-(10/7)x}$. too less damping



c) Mehr als ausreichende Dämpfung – Kriechfall, aperiodische Bewegung. too much damping



d) Eine gerade ausreichende Dämpfung ergibt sich, wenn der Dämpfungsfaktor l so gewählt wird, dass die charakteristische Gleichung der DGL eine reelle Doppellösung hat.



Choose damping factor l so that the characteristical equation must have a real double solution.

On the next pages you can find problems from DERIVE Newsletter #2 and #3 (1991). Then we demonstrated the use of the MATH-file ODE1.MTH. Some of the then useful auxiliary functions are missed in recent ODE1.MTH, because programming made it possible to integrate them into one procedure.

I recommend to compare with the updated contributions given in DNL#2 and DNL#3. Josef

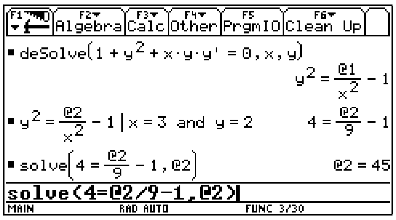
Examples from DNL#3 worked out on the TI-92+/V 200. Compare with the DERIVE procedures!

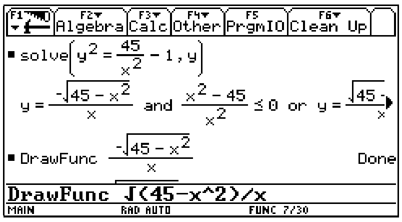
Ex 1: $(1+y^2)\,dx + x\,y\,dy = 0$

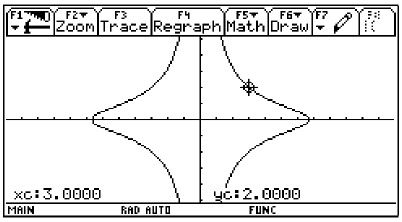
Give the general solution and give the special solution containing P(3|2).

Sketch the direction field, some integral curves and the graph of the special solution from above.

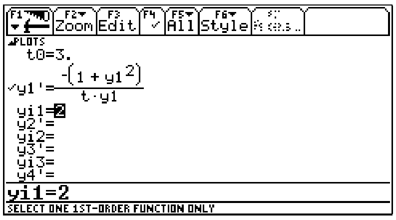
$$(1+y^2)\,dx + x\,y\,dy = 0 \;\rightarrow\; 1 + y^2 + x\;y\;y' = 0$$

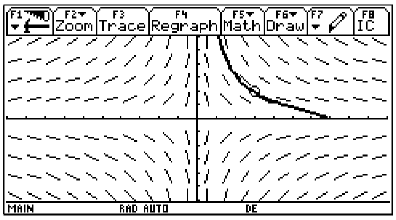




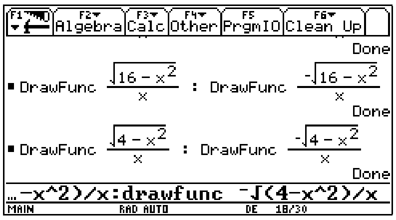


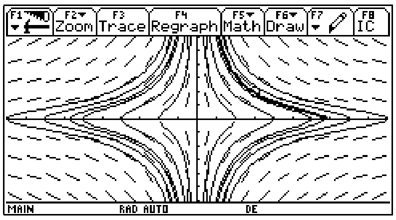
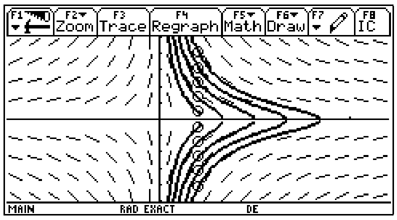
General solution is $y^2 = \dfrac{C - x^2}{x^2}$. We plot the direction field together with the integral curve passing the given point.





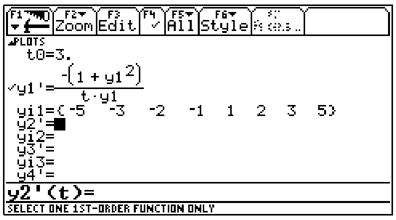
ncurves from the [WINDOW]-settings does not work – (t0 is temporarily set at the middle of the screen and initial conditions are distributed evenly along the y-axis). As there are no defined values on the y-axis ncurves must fail!! We can add solution curves via DrawFunc in the [GRAPH]-window or we change xmax and xmin to shift the y-axis out of the center (ncurves = 10)

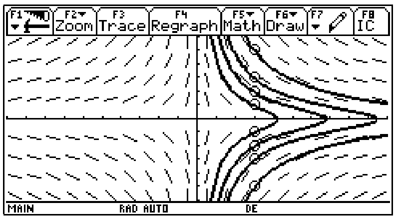


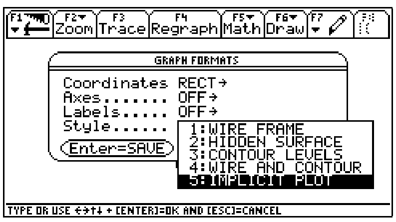



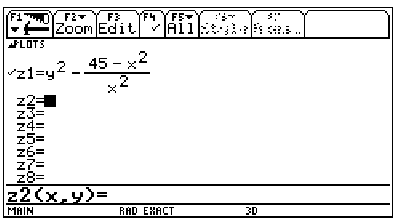
The [Y=]-settings allow plotting several solution curves.

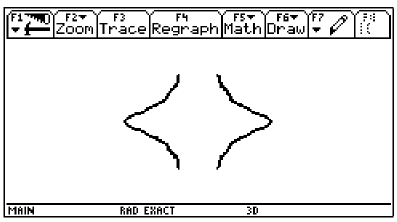
Find below the special solution produced as an implicit plot.



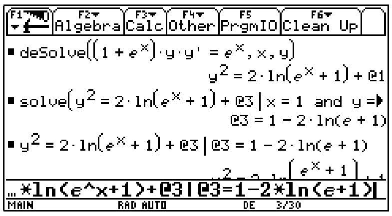


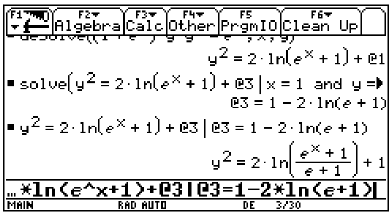


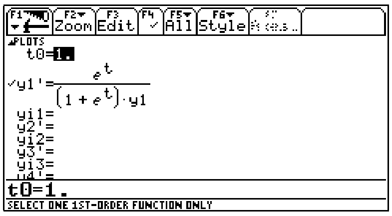


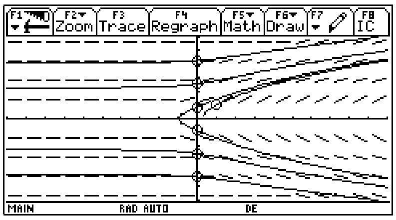


Ex 2: $\quad (1+e^x)\,y\,y' = e^x; \quad P(1|1)$







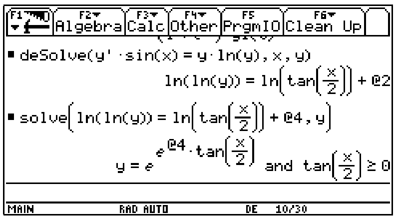


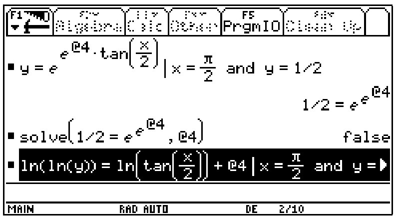
Solutions:

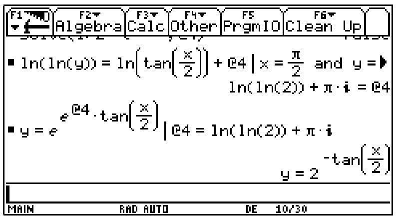
$$y^2 = \ln(e^x+1)+C$$

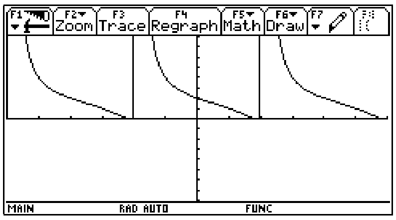
$$y^2 = 2\ln\left(\frac{e^x+1}{e+1}\right)+1$$

Ex 3: $\quad y'\sin x = y\ln y; \quad y\left(\frac{\pi}{2}\right)=\frac{1}{2}$







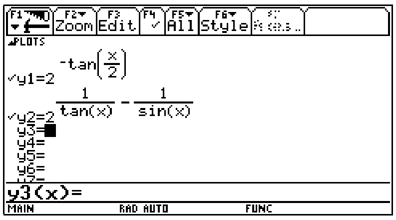


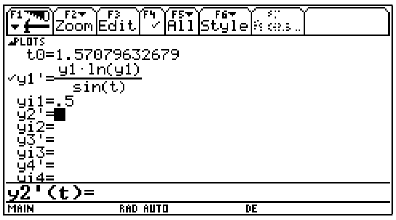
Solutions:

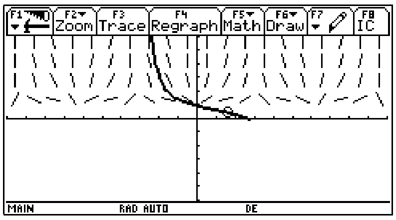
$$y = e^{e^{C}\tan\left(\frac{x}{2}\right)}$$

$$y = 2^{-\tan\left(\frac{x}{2}\right)}$$

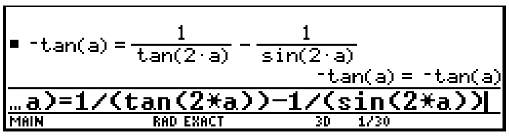
Derive version 2 showed the solution: `y = 2`$^{\texttt{COT(x)-1/SIN(x)}}$





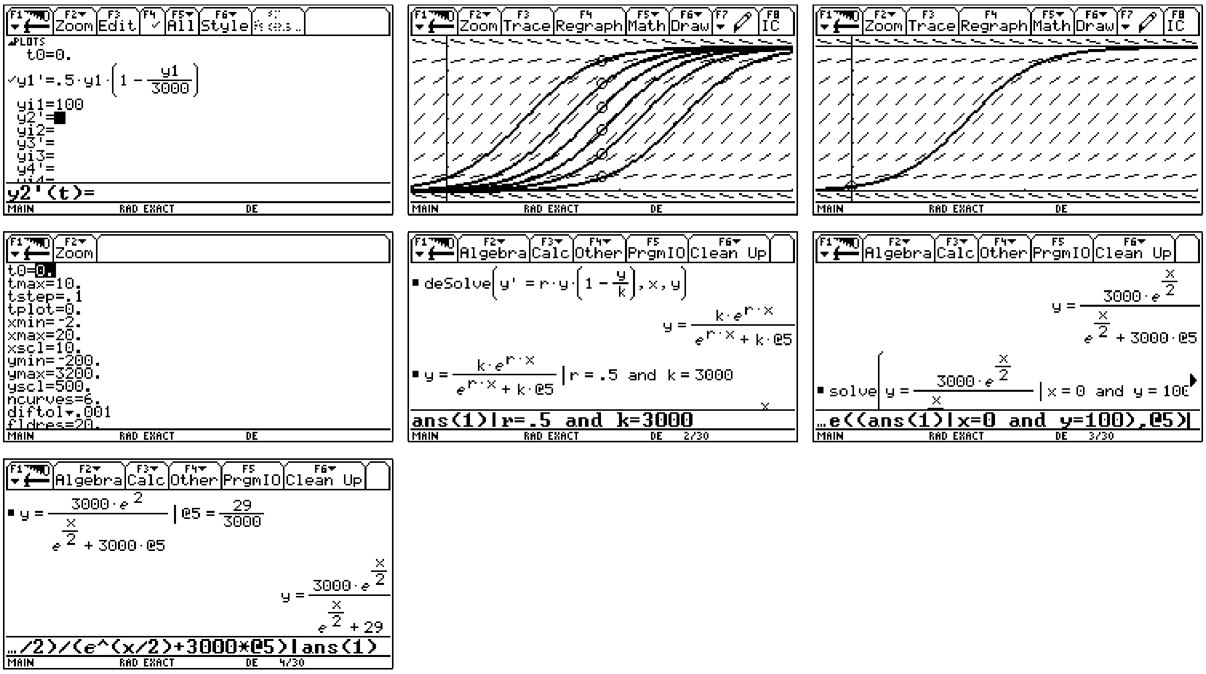


Plotting both functions y1 and y2 on the same axes shows the identity of the solutions. Then we see the direction field together with one branch of the special solution.



Ex 4: The logistic growth curve

$$dy = dx \cdot r \cdot y \cdot \left(1 - \frac{y}{k}\right); \quad r = 0.5;\ k = 3000;\ y(0) = 100$$



Ex 5: The general equation expressing the relation between electromotive force U and current I in a circuit containing the resistance R and the inductance L is:

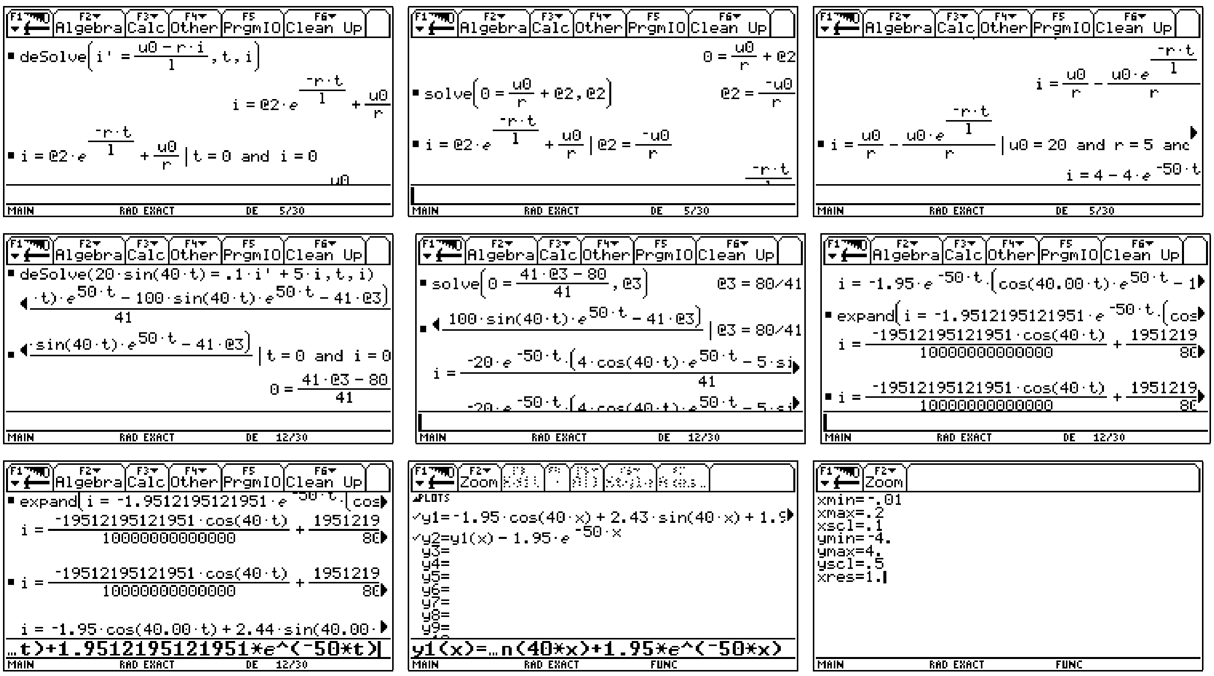
$$U = L \cdot I' + R \cdot I$$

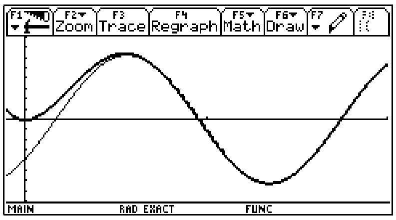
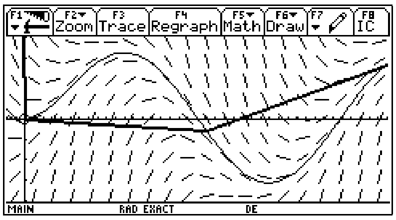
Solve this equation first for a constant $U = U_0$.

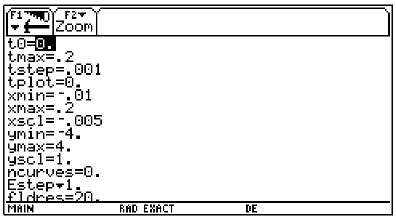
$I(t=0) = 0$

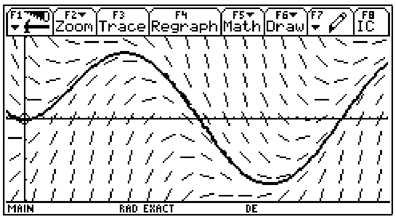
Then for $U_0 = 20V$, $R = 5\Omega$, $L = 0.1$ Henry.

Solve this equation, when $U = U_0 \sin(\omega t)$! ($\omega = 40$)

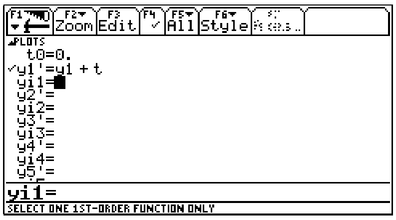


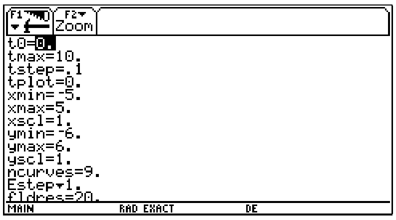
 

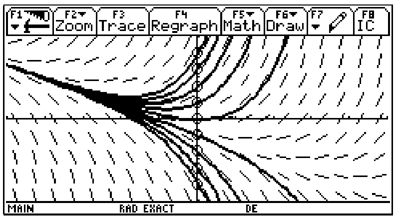




Ex 6: The DE $y' = x + y$ defines a nice direction field. Let the Voyage 200 plot this field together with a family of integral curves.





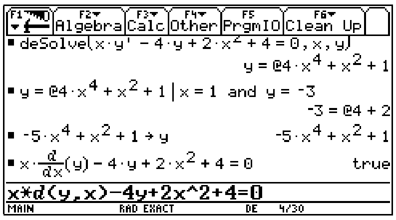


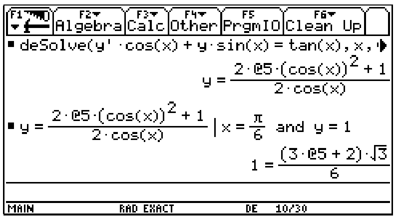
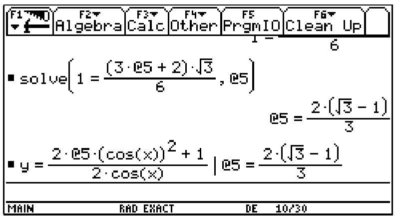
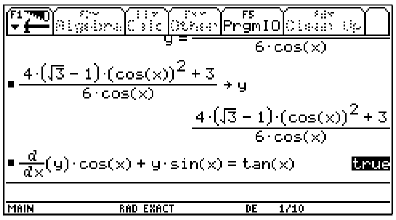
Ex 7: Find the general solutions of the given DEs and verify them. Find also the special solutions containing the given points.

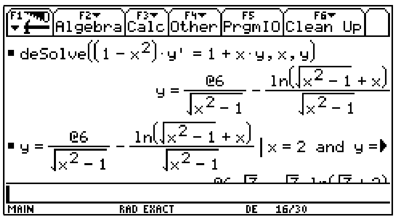
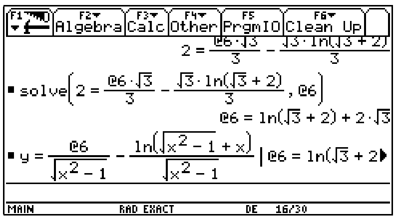
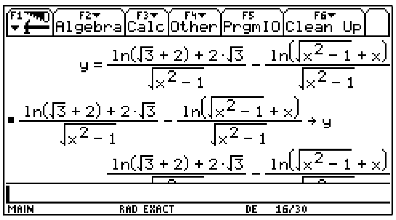
a) $x\,y' - 4y + 2x^2 + 4 = 0$; $P(1|-3)$

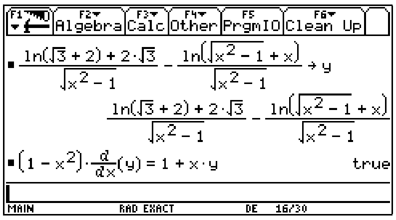
b) $y'\cos x + y\sin x = \tan x$; $P\left(\frac{\pi}{6}|1\right)$

c) $(1-x^2)\,y' = 1 + x\,y$; $P(2|2)$



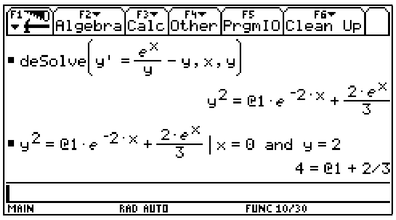


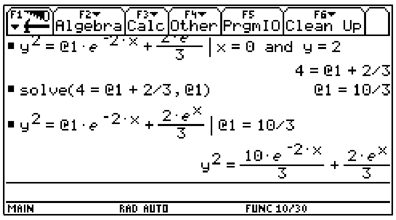
Ex 8: Both of the following equations are of the Bernoulli-type:

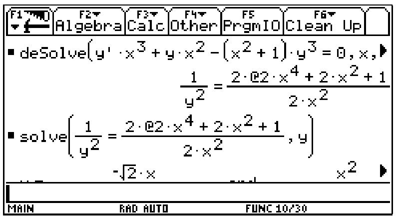
$y' + p(x)\cdot y = q(x)\cdot y^k \quad (k = \text{const.})$

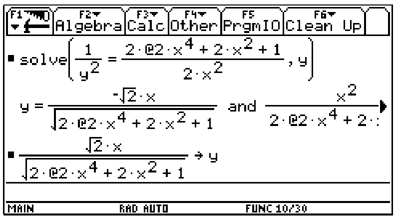
a) $\quad y' = \dfrac{e^x}{y} - y; \quad P(0\,|\,2)$

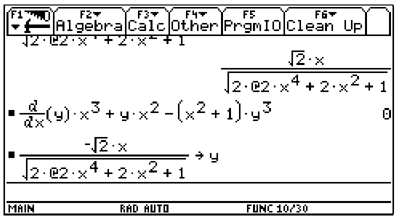
b) $\quad y'x^3 + y\,x^2 - (x^2+1)\,y^3 = 0;$ Verify!

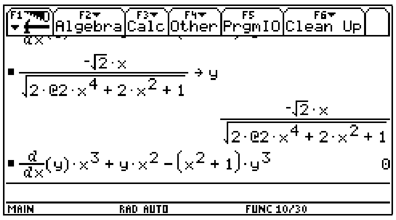










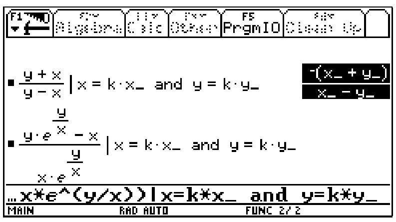


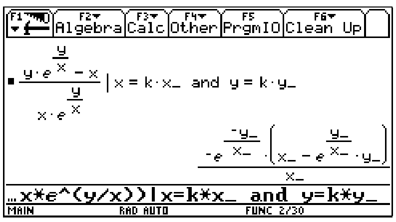
Ex 9: Test the following DEs on homogeneity. Then solve the equations.

a) $\quad y' = \dfrac{y+x}{y-x}; \quad P(1\,|\,2)$

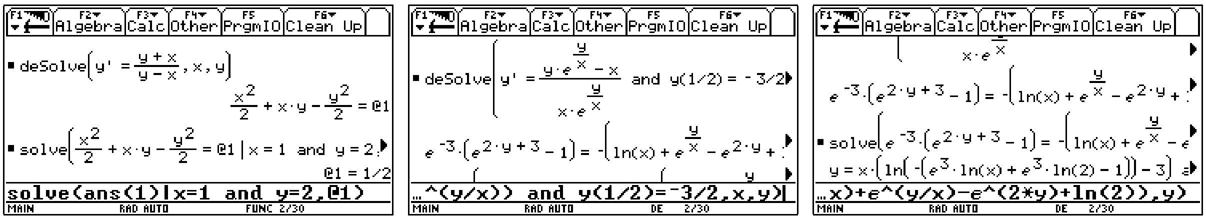
b) $\quad y' = \dfrac{y\,e^{\frac{y}{x}} - x}{x\,e^{\frac{y}{x}}}; \quad P\left(\dfrac{1}{2}\middle|-\dfrac{3}{2}\right)$, verify the solution!

The test:





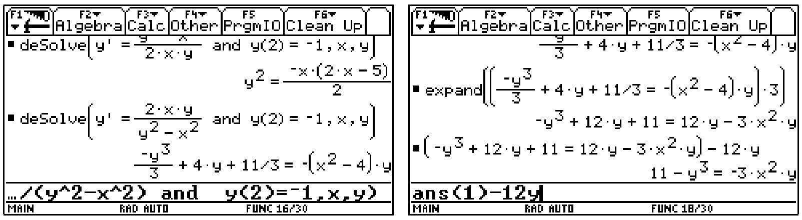
and the solutions:



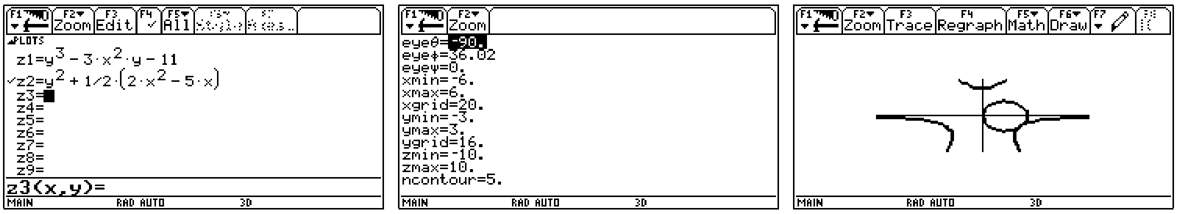
Ex 10: Compare the integral curves of

$$y' = \frac{y^2 - x^2}{2xy} \quad \text{and} \quad y' = \frac{2xy}{y^2 - x^2} \quad \text{intersecting in } P(2|-1).$$

Plot both curves together with the tangents in P.



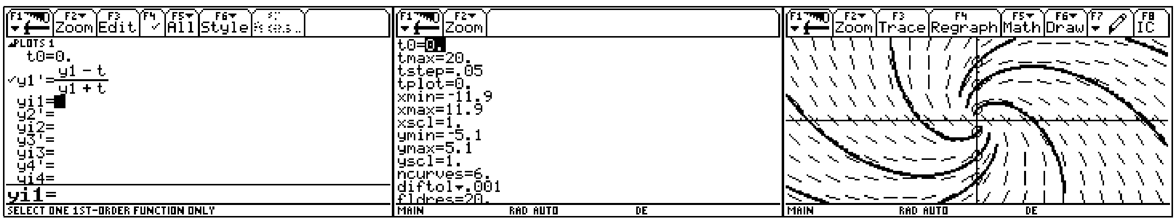
I tried to superimpose two implicit plots using the 3D mode. On the TI I cannot plot the tangents.

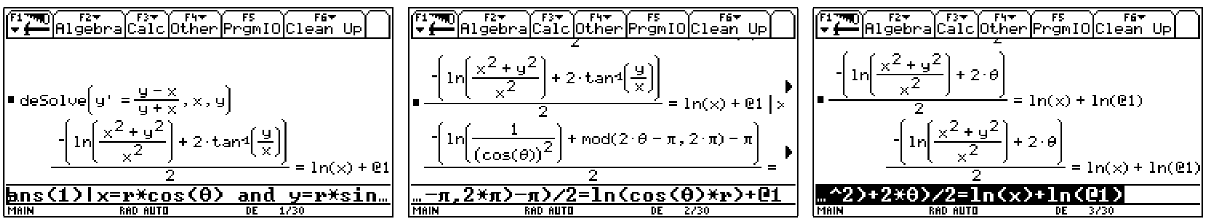


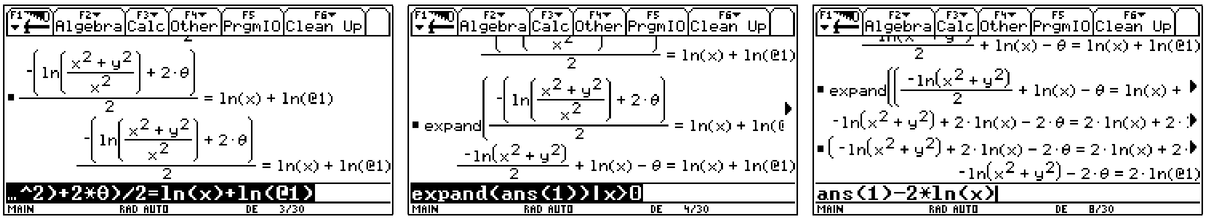
Ex 11: Find the curves which intersect the circles $x^2 + y^2 = r^2$ at angles of $\pi/4$.

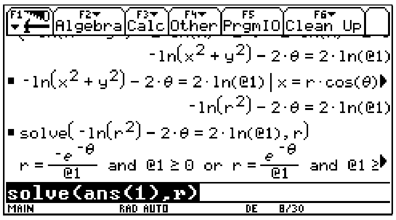
Plot some circles together with a family of curves representing the general solution of the underlying differential equation.

According to [2] the problem leads to the homogeneous DE $y' = \dfrac{y-x}{y+x}$.

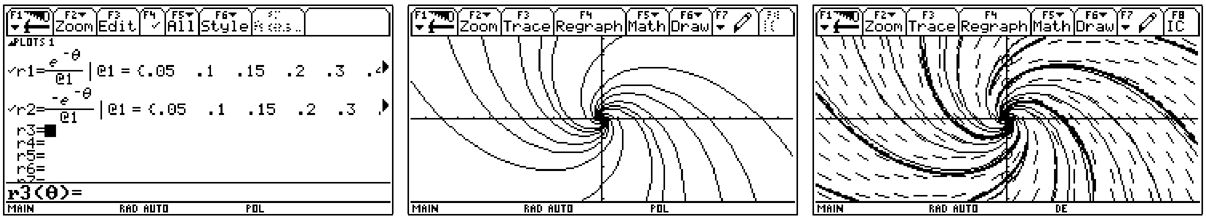


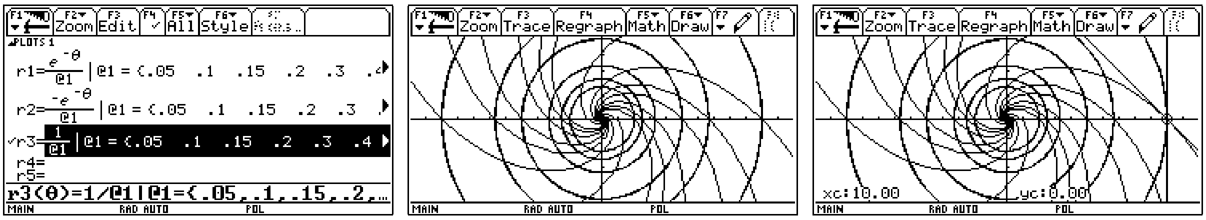






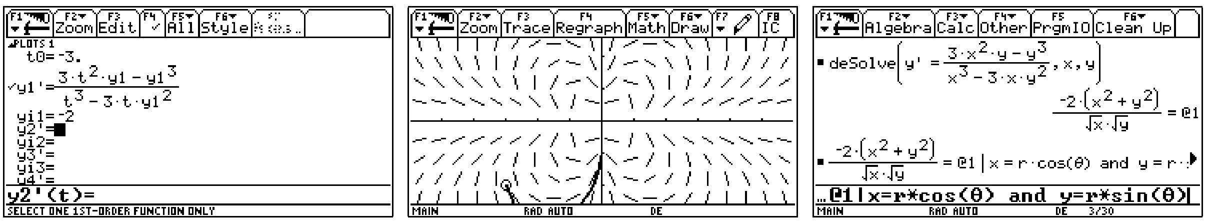
We find a family of logarithmic spirals intersecting the circles. As you can see we switch to polar coordinates. Performing this task requires a lot of mathematical and technological competence.



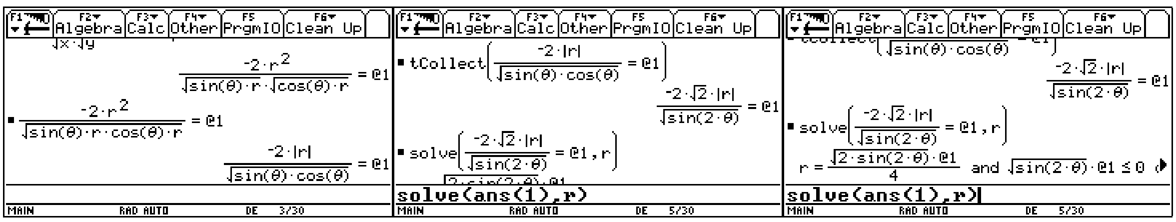


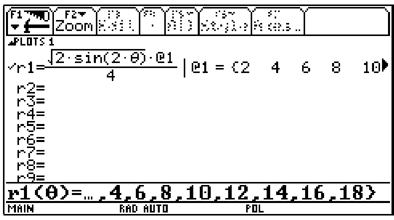
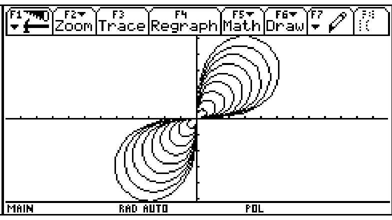
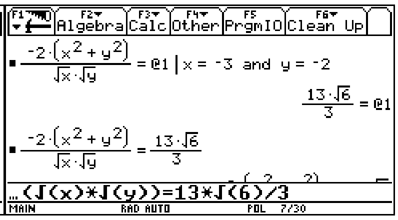
Ex 12: A point moves on a curve in the x-y-plane in such a way that the angle formed by the tangent of the curve with the x-axis is three times the angle between the radius vector and the x-axis. What is the Cartesian equation of the family of curves satisfying this condition.

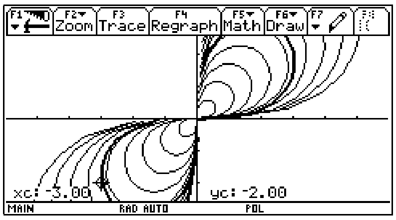
Plot a family of solution curves, especially this one which contains $P(-3|-2)$.



The first – graphical – approach is not really satisfying. Again application of polar coordinates proves to be a successful means to obtain better results. More explanations can be found in DNL#3.





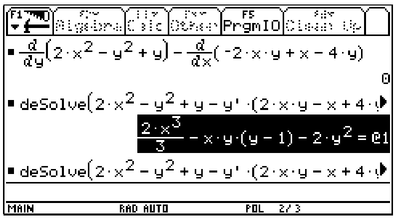
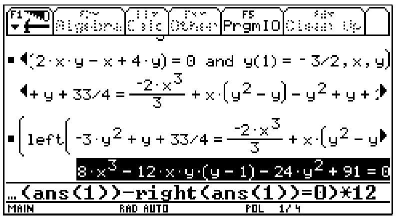
The solution curves are lemniscates.

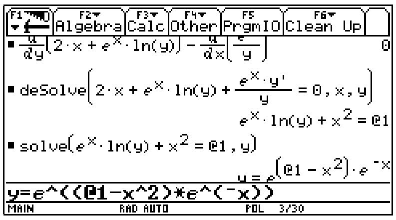
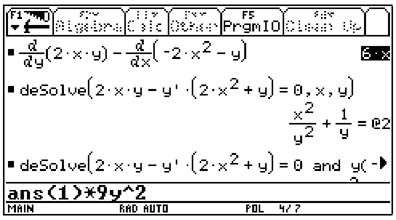
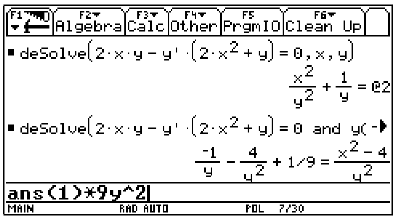
Ex 13: Show that the given DEs are of exact form. Give their general solutions and to each one the special solution if there is a point given:

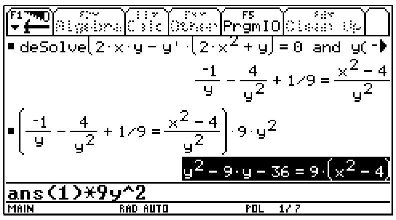
a) $2x^2 - y^2 + y - y'\cdot(2x\,y - x - 4y) = 0; \quad P(1\,|\,-1.5)$

b) $2x + e^x \ln y + \dfrac{e^x y'}{y} = 0$

c) $2x\,y - (2x^2 + y)y' = 0; \quad Q(-2\,|\,3)$



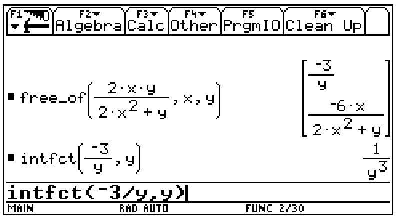
Ex 14: Try to find a solution of the differential equations given below. Manually they are solved by means of an integrating factor. Try to find this integrating factor and prove it.

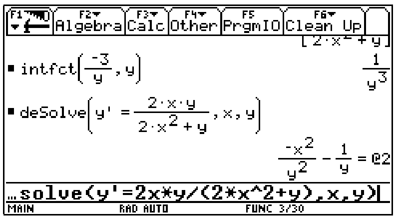
a) $2x\,y - (2x^2 + y)\,y' = 0$

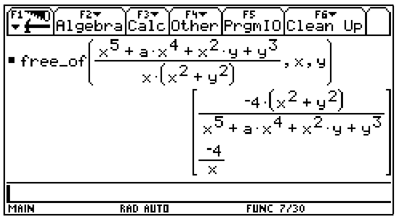
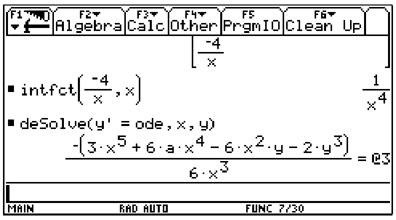
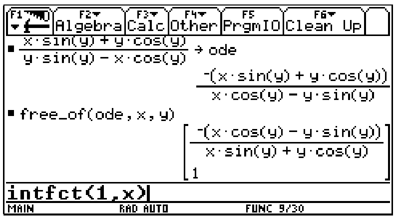
b) $(x^2 + y^2)\,(x\,dy - y\,dx) = x^4\,(a + x)\,dx$

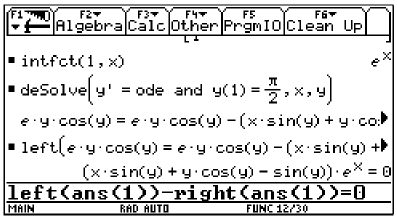
c) $(x\cos y - y\sin y)\,dy + (x\sin y + y\cos y)\,dx = 0;\quad P\left(1\middle|\frac{\pi}{2}\right)$

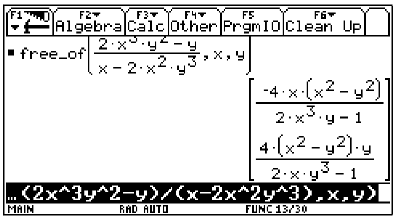
d) $(2x^3 y^2 - y)\,dx = (x - 2x^2 y^3)\,dy$

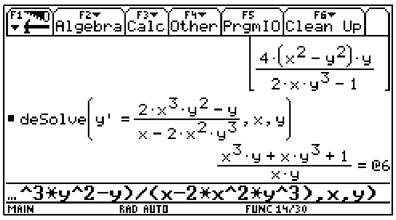










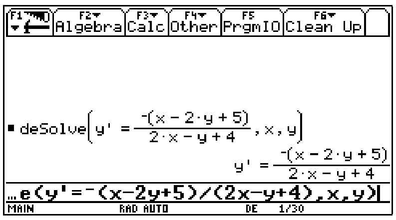
free_of and **intfct** are functions according to the functions which could be found in earlier Derive´s ODE1.MTH. (download!)

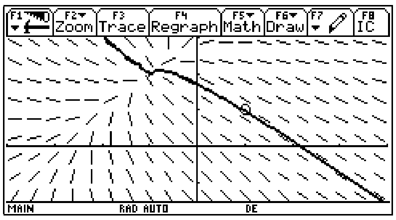
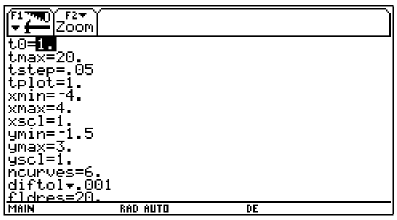
Ex 15: $(x - 2y + 5)\,dx + (2x - y + 4)\,dy = 0$

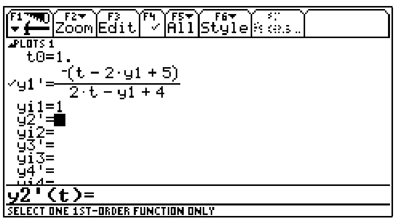
a) Find the general solution using $(x_0|y_0)$ and using a parameter c.

b) Find the special solution with $y(1) = 1$.

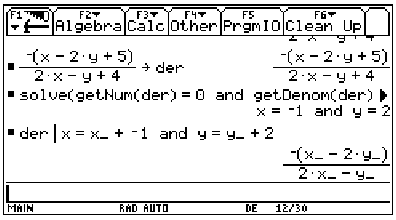
c) Sketch the integral curve and the direction field.

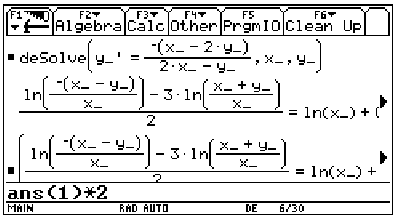


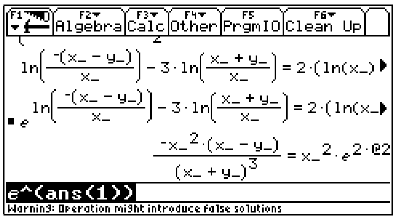



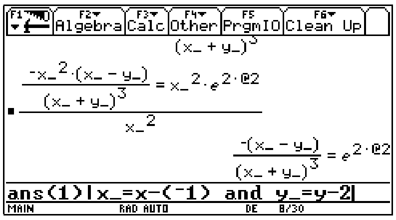


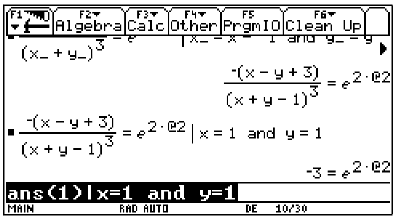
deSolve does not work. I investigated how Derive does and first solved the equation step by step to pack later the solving procedure it into one function **linfrac**.
(download).

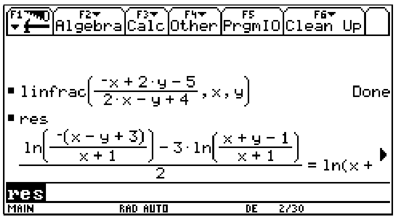


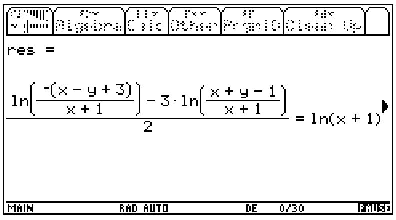


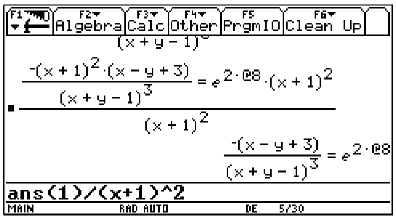


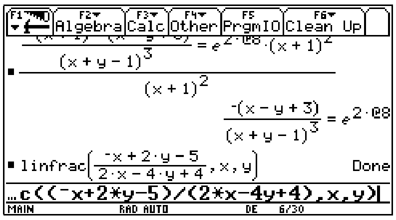


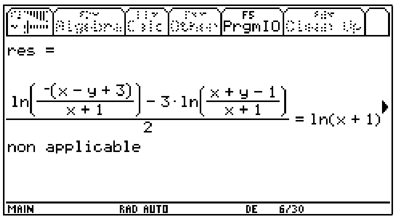












$$\text{LIN\_FRAC}\left(\frac{-x+2\cdot y-5}{2\cdot x-4\cdot y+4},\ -1,\ 2,\ -5,\ 2,\ -4,\ 4,\ x,\ y,\ 1,\ 1\right)$$

?

$$\text{FUN\_LIN\_CCF\_GEN}\left(\frac{-x+2\cdot y-5}{2\cdot x-4\cdot y+4},\ 2,\ -4,\ 4\right)$$

$$\frac{x-2\cdot(y-1)}{2}-\frac{3\cdot \text{LN}(2\cdot x-4\cdot y+7)}{4}=x+c$$

$$\left(\frac{x-2\cdot(y-1)}{2}-\frac{3\cdot \text{LN}(2\cdot x-4\cdot y+7)}{4}=x+c\right)\cdot 8$$

$$4\cdot(x-2\cdot(y-1))-6\cdot \text{LN}(2\cdot x-4\cdot y+7)=8\cdot(x+c)$$

$$-6\cdot \text{LN}(2\cdot x-4\cdot y+7)+4\cdot x-8\cdot y+8=8\cdot x+8\cdot c$$

$$(-6\cdot \text{LN}(2\cdot x-4\cdot y+7)+4\cdot x-8\cdot y+8=8\cdot x+8\cdot c)-8\cdot x$$

The final solution:

$$-6\cdot \text{LN}(2\cdot x-4\cdot y+7)-4\cdot x-8\cdot y+8=8\cdot c$$

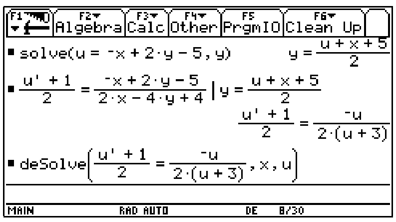
Special case if "numerator line" and "denominator line" don´t intersect.

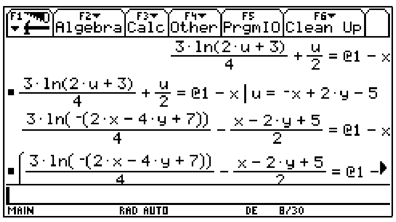
Then substituting u = numerator leadds to a DE which can be treated by separation of variables. You can follow this below.

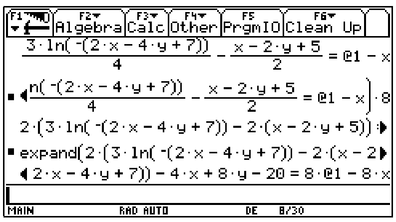
You might extend the TI-program **linfrac** in such a way that this special case is included instead of answering "non applicable".

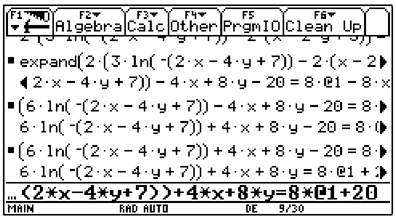
(download)

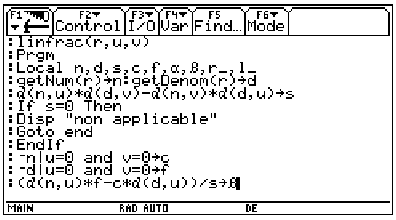
Derive provides a special utility function for this case.







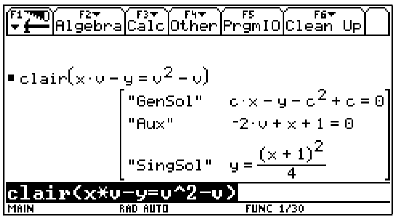


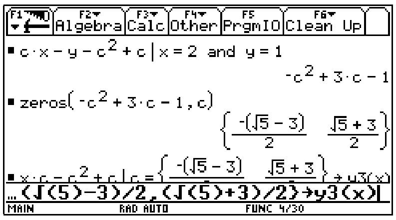


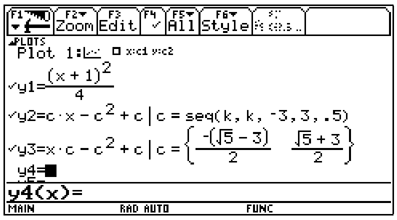
Ex 16: Clairaut equation $y = x\,y' + y' - y'^2$

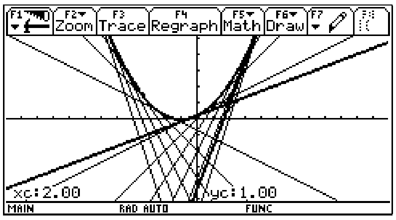
Find the general solution, give the singular solution and plot the respective graph.

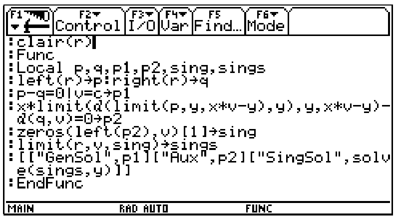
Find the special solution(s) through $P(2|1)$.







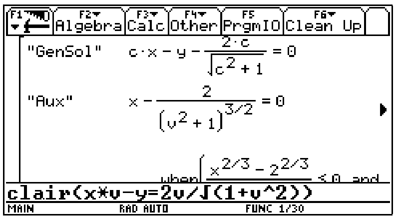


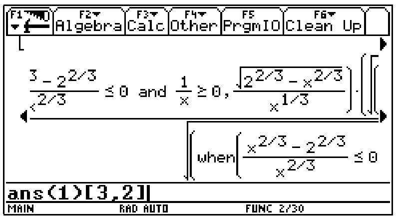


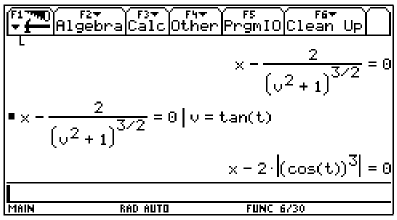
deSolve cannot be applied. Help yourself and write a function **clair**. (Replace y' by v.)

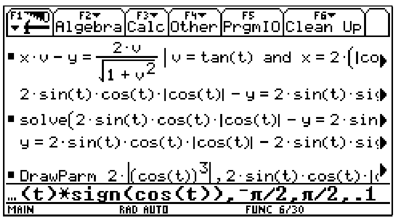
Ex 17: Which is the curve with its tangent´s segment between the axes having constant length $a = 2$?

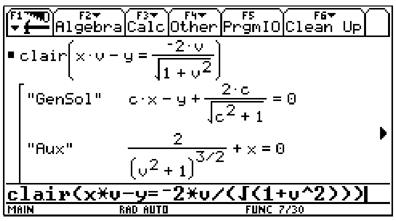
Give both general and singular solutions.

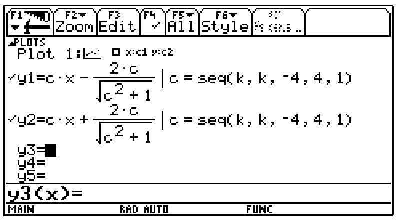


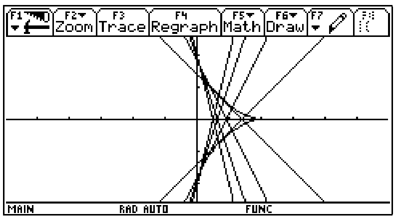


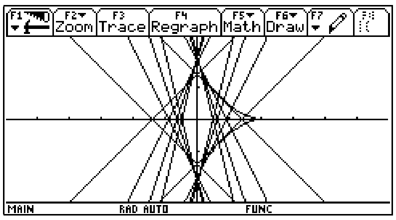












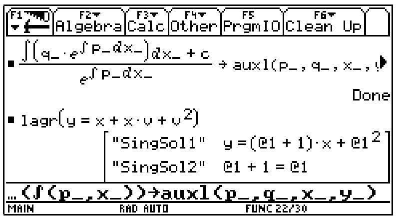
More explanations can be found in DNL#3.

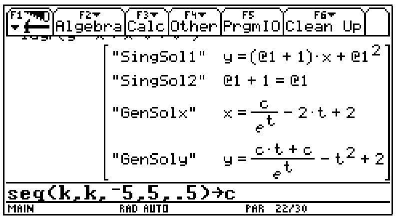
Ex 18: The following equations of form $y = x \cdot p(y') + q(y')$ are called LAGRANGE DEs.

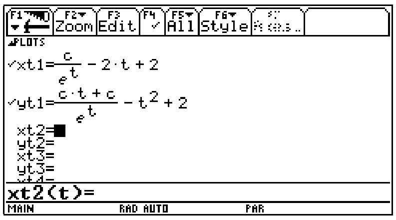
a) $y = x \cdot (1 + y') + y'^2$

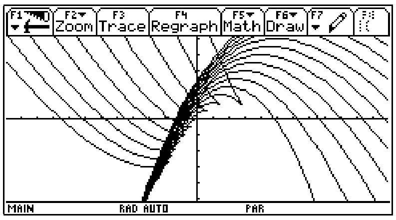
b) $y'^3 - 3y' = y - x$

c) $(x\,y' + y)^2 = y^2 \cdot y'$



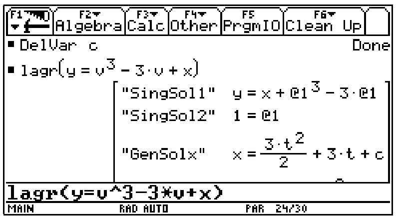


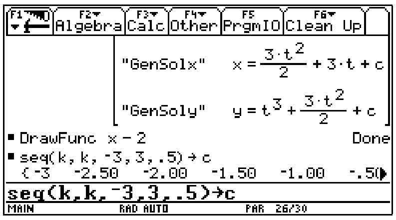


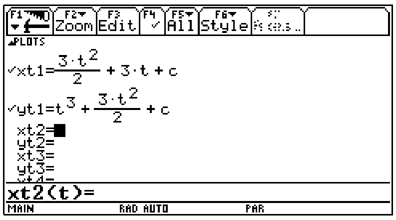


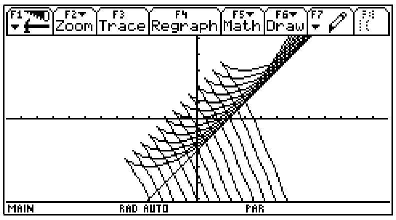
lagr(equ) with v for y' follows the Derive function from ODE1.MTH. You find the explanation in DNL#3. (download)

No singular solution, a family of special soultions.

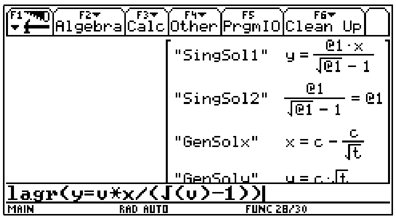


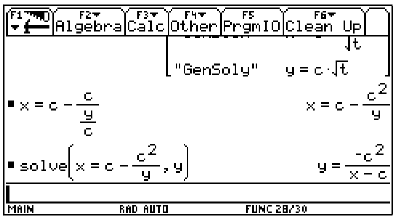


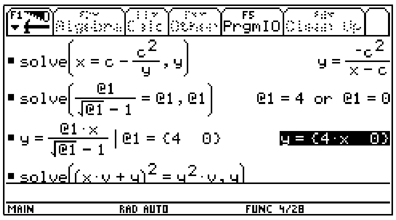


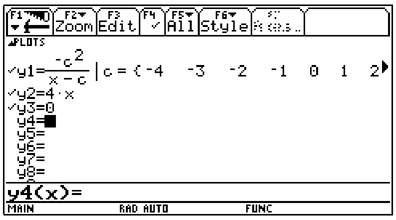


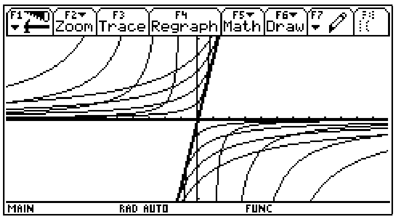
Singular solution $y = x - 2$ together with a family of special solutions.

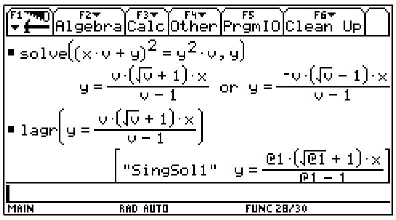










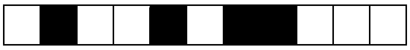


[1] Günter-Kusmin, Aufgabensammlung zur Höheren Mathematik, Berlin 1964
[2] The Calculus Problem Solver, Research and Education Association 1985
[3] Büktas, Aufgabensammlung, Diesterweg 1978
[4] Günter Redl, T^3-Course Materials

Cellulära automater / Rule 30

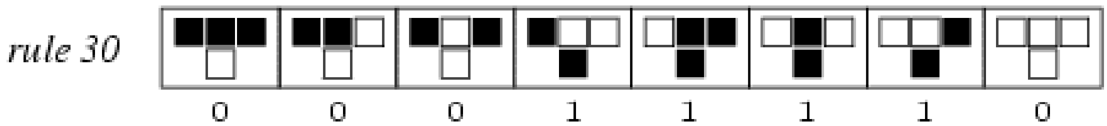
I leave David´s part in Swedish, because I beleive that it is more or less self explanatory. Some additional comments are in English (red).

En cellulär automat är en samling “färgade” celler i ett rutnät. Färgerna uppdateras enligt en speciell regel.
Vi tänker oss en rad, med obegränsad utsträckning åt såväl vänster som höger, med rutor eller celler, som är fyllda med antingen svart eller vit färg.



Vi tänker oss även att varje cell i raden ändrar färg enligt en regel beroende på dess och de närmaste grannarnas färg. Färgändringen sker samtidigt i alla celler. Regler för färgändring kan anges på en del olika sätt. På detta sätt ger regeln upphov det olika generationer.
Vi kommer att visa att en undersökning av några cellulära automater med t.ex. *DERIVE*, Excel, Voyage 200, Mathematica and Wolfram Research´s *A New Kind of Science Explorer* utgör ett intressant projektarbete inom ramen för kursen Matematik Diskret vad gäller bl.a. binära tal och satslogik.
Josef Böhm är ordförande i Derive User Group och har i många år verkat som gymnasielärare i Österrike.
David Sjöstrand, lektor i matematik vid Elof Lindälvs gymnasium, Kungsbacka.

En känd och intressant regel är rule 30, som kan definieras så här:



Övning. Det finns 256 regler av denna typ. Visa detta.
På http://mathworld.wolfram.com/ElementaryCellularAutomaton.html finns alla de 256 elementära endimensionella cellulära automaterna åskådliggjorda.
Övning. Bestm antalet funktioner $f: A \to B$, där antalet element i A är a och antalet element i B är b.
Om man identifierar vitt med 0 och svart med 1 kan man formulera en regel av denna typ med en funktion $f:\{0,1\}\times\{0,1\}\times\{0,1\}\to\{0,1\}$ på följande sätt:
Färgen på en cell bestäms av $f(p,q,r)$ där p, q, och r färgen på den vänstra grannen, cellen själv respektive den högra grannen. För rule 30 gäller således att

$f(1,1,1)=0,\ f(1,1,0)=0,\ f(1,0,1)=0,\ f(1,0,0)=1,\ f(0,1,1)=1,\ f(0,1,0)=1,\ f(0,0,1)=1,\ f(0,0,0)=0$

Vi ser att (1, 1, 1), (1, 1, 0), (1, 0, 0), ... (0,0,1), (0, 0, 0) svarar mot talen 7, 6, 5, ... 1, 0 i binär form.
Vi ser även att högerleden bildar talet 00011110, vilket är 30 i binär form, därav namnet rule 30.
Man kan lyckligtvis bestämma ett polynom i p, q och r, som ger värdena av f.

A. Ansätt ett polynom

Ett sätt att göra detta är att ansätta

$f(p,q,r) = apqr + bpq + cqr + drp + ep + fq + gr + h$

samt att sedan bestämma de 8 koefficienterna , a, b, ..., h genom att lösa det ekvationssystem som består av de 8 likheterna ovan. Man kan med fördel använda *DERIVE*.

```
InputMode := Character

ff(p, q, r) := a·p·q·r + b·p·q + c·q·r + d·r·p + e·p + f·q + g·r + h

SOLVE([ff(1, 1, 1) = 0, ff(1, 1, 0) = 0, ff(1, 0, 1) = 0, ff(1, 0, 0) = 1, ff(0, 1, 1) =
  1, ff(0, 1, 0) = 1, ff(0, 0, 1) = 1, ff(0, 0, 0) = 0], [a, b, c, d, e, f, g, h])

[a = 2 ∧ b = -2 ∧ c = -1 ∧ d = -2 ∧ e = 1 ∧ f = 1 ∧ g = 1 ∧ h = 0]

ff30(p, q, r) := 2×p×q×r - 2×p×q - q×r - 2×r×p + p + q + r
```

Vi ser att $f(p,q,r) = 2pqr - 2pq - qr - 2rp + p + q + r$.

We can try another approach, interpreting the triple p,q,r as a binary number pqr, taking its decimal equivalence and finding the respective polynomial dependent on only one variable:
$f(7) = 0, f(6) = 0, f(5) = 0, f(4) = 1, f(3) = 1, f(2) = 1, f(1) = 1, f(0) = 0$

$$fb(x) := a\cdot x^7 + b\cdot x^6 + c\cdot x^5 + d\cdot x^4 + e\cdot x^3 + f\cdot x^2 + g\cdot x + h$$

```
SOLVE([fb(7) = 0, fb(6) = 0, fb(5) = 0, fb(4) = 1, fb(3) = 1, fb(2) = 1, fb(1) = 1, fb(0)
  = 0], [a, b, c, d, e, f, g, h])
```

$$\left[a = -\frac{1}{360} \wedge b = \frac{23}{360} \wedge c = -\frac{41}{72} \wedge d = \frac{89}{36} \wedge e = -\frac{481}{90} \wedge f = \frac{1787}{360} \wedge g = -\frac{7}{12} \wedge h = 0\right]$$

$$fb30(x) := -\frac{x^7}{360} + \frac{23}{360}\times x^6 - \frac{41}{72}\times x^5 + \frac{89}{36}\times x^4 - \frac{481}{90}\times x^3 + \frac{1787}{360}\times x^2 - \frac{7}{12}\times x$$

$$\mathrm{TABLE}(fb30(x),\ x,\ 7,\ 0,\ -1) = \begin{bmatrix} 7 & 0 \\ 6 & 0 \\ 5 & 0 \\ 4 & 1 \\ 3 & 1 \\ 2 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

B. Logik

Om vi identifierar svart med SANT och vitt med FALSKT ser vi att vi kan uttrycka rule 30 på detta sätt

$$(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)\,. \ (*)$$

Om vi gör följande definitioner i *DERIVE*, kan vi bestämma ett polynom, som ger rule 30.

```
and_(p, q) := p·q

or_(p, q) := p + q - and_(p, q)

not_(p) := 1 - p

or__(p, q, r, s) := or_(p, or_(q, or_(r, s)))

and__(p, q, r) := and_(p, and_(q, r))
```

Now we translate Rule 30 from above (*):

```
or__(and__(p, not_(q), not_(r)), and__(not_(p), q, r), and__(not_(p), q, not_(r)),
  and__(not_(p), not_(q), r))
```

$$p^4 \cdot q^2 \cdot r^2 \cdot (q - 1)^2 \cdot (r - 1)^2 + p^3 \cdot q \cdot r \cdot (1 - r) \cdot (q - 1) \cdot (3 \cdot q^2 \cdot r \cdot (r - 1) + q \cdot r \cdot (5 - 3 \cdot r) - 1) +$$
$$p^2 \cdot (3 \cdot q^4 \cdot r^2 \cdot (r - 1)^2 + q^3 \cdot r^2 \cdot (1 - r) \cdot (6 \cdot r - 11) + q^2 \cdot (3 \cdot r^4 - 11 \cdot r^3 + 6 \cdot r^2 + 4 \cdot r - 1) +$$
$$q \cdot (4 \cdot r^2 - 6 \cdot r + 1) - r \cdot (r - 1)) - p \cdot (q^4 \cdot r^2 \cdot (r - 1)^2 + 2 \cdot q^3 \cdot r^2 \cdot (1 - r) \cdot (r - 3) + q^2 \cdot (r^4 -$$
$$6 \cdot r^3 + 5 \cdot r^2 + 3 \cdot r - 1) + q \cdot (3 \cdot r^2 - 8 \cdot r + 3) - r^2 + 3 \cdot r - 1) + q^3 \cdot r^2 \cdot (r - 1) + q^2 \cdot r^2 \cdot (2 -$$
$$r) + q \cdot (1 - 2 \cdot r) + r$$

After expanding we find a very bulky expression, which can be "simplified", because alle powers of p, q and r are 1. We use the asterisk as TimesOperator, because we intend to transfer this formula to Excel.

$$p^4 \cdot q^4 \cdot r^4 - 2 \cdot p^4 \cdot q^4 \cdot r^3 + p^4 \cdot q^4 \cdot r^2 - 2 \cdot p^4 \cdot q^3 \cdot r^4 + 4 \cdot p^4 \cdot q^3 \cdot r^3 - 2 \cdot p^4 \cdot q^3 \cdot r^2 + p^4 \cdot q^2 \cdot r^4 -$$
$$2 \cdot p^4 \cdot q^2 \cdot r^3 + p^4 \cdot q^2 \cdot r^2 - 3 \cdot p^3 \cdot q^4 \cdot r^4 + 6 \cdot p^3 \cdot q^4 \cdot r^3 - 3 \cdot p^3 \cdot q^4 \cdot r^2 + 6 \cdot p^3 \cdot q^3 \cdot r^4 - 14 \cdot p^3 \cdot q^3 \cdot r^3$$
$$+ 8 \cdot p^3 \cdot q^3 \cdot r^2 - 3 \cdot p^3 \cdot q^2 \cdot r^4 + 8 \cdot p^3 \cdot q^2 \cdot r^3 - 4 \cdot p^3 \cdot q^2 \cdot r^2 - p^3 \cdot q^2 \cdot r - p^3 \cdot q \cdot r^2 + p^3 \cdot q \cdot r +$$
$$3 \cdot p^2 \cdot q^4 \cdot r^4 - 6 \cdot p^2 \cdot q^4 \cdot r^3 + 3 \cdot p^2 \cdot q^4 \cdot r^2 - 6 \cdot p^2 \cdot q^3 \cdot r^4 + 17 \cdot p^2 \cdot q^3 \cdot r^3 - 11 \cdot p^2 \cdot q^3 \cdot r^2 +$$
$$3 \cdot p^2 \cdot q^2 \cdot r^4 - 11 \cdot p^2 \cdot q^2 \cdot r^3 + 6 \cdot p^2 \cdot q^2 \cdot r^2 + 4 \cdot p^2 \cdot q^2 \cdot r - p^2 \cdot q^2 + 4 \cdot p^2 \cdot q \cdot r^2 - 6 \cdot p^2 \cdot q \cdot r + p^2 \cdot q$$
$$- p^2 \cdot r^2 + p^2 \cdot r - p \cdot q^4 \cdot r^4 + 2 \cdot p \cdot q^4 \cdot r^3 - p \cdot q^4 \cdot r^2 + 2 \cdot p \cdot q^3 \cdot r^4 - 8 \cdot p \cdot q^3 \cdot r^3 + 6 \cdot p \cdot q^3 \cdot r^2 -$$
$$p \cdot q^2 \cdot r^4 + 6 \cdot p \cdot q^2 \cdot r^3 - 5 \cdot p \cdot q^2 \cdot r^2 - 3 \cdot p \cdot q^2 \cdot r + p \cdot q^2 - 3 \cdot p \cdot q \cdot r^2 + 8 \cdot p \cdot q \cdot r - 3 \cdot p \cdot q + p \cdot r^2 -$$
$$3 \cdot p \cdot r + p + q^3 \cdot r^3 - q^3 \cdot r^2 - q^2 \cdot r^3 + 2 \cdot q^2 \cdot r^2 - 2 \cdot q \cdot r + q + r$$

```
p·q·r - 2·p·q·r + p·q·r - 2·p·q·r + 4·p·q·r - 2·p·q·r + p·q·r - 2·p·q·r + p·q·r - 3·p·q·r
  + 6·p·q·r - 3·p·q·r + 6·p·q·r - 14·p·q·r + 8·p·q·r - 3·p·q·r + 8·p·q·r - 4·p·q·r - p·q·r
  - p·q·r + p·q·r + 3·p·q·r - 6·p·q·r + 3·p·q·r - 6·p·q·r + 17·p·q·r - 11·p·q·r + 3·p·q·r
  - 11·p·q·r + 6·p·q·r + 4·p·q·r - p·q + 4·p·q·r - 6·p·q·r + p·q - p·r + p·r - p·q·r +
  2·p·q·r - p·q·r + 2·p·q·r - 8·p·q·r + 6·p·q·r - p·q·r + 6·p·q·r - 5·p·q·r - 3·p·q·r +
  p·q - 3·p·q·r + 8·p·q·r - 3·p·q + p·r - 3·p·r + p + q·r - q·r - q·r + 2·q·r - 2·q·r + q
  + r
```

```
TimesOperator := Asterisk

p*(q*(2*r - 2) - 2*r + 1) + q*(1 - r) + r
```

Compare with expression ff30 from above

```
ff30(p, q, r) = p*(q*(2*r - 2) - 2*r + 1) + q*(1 - r) + r

rule30(p, q, r) := 2*p*q*r - 2*p*q - 2*p*r + p - q*r + q + r
```

Denna formel är onödigt stor. Man göra ersätta den med en mycket enklare genom att utnyttja att $p^n = p, q^n = q, r^n = r, n > 1$.

Övning. Enligt *S. Wolfram, A New Kind of Science* sid 884 ges rule 30 av

$$p \veebar (q \vee r).$$

$\veebar$ står för XOR (exlusive or).

a) Visa att $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \veebar (q \vee r)$ genom att använda funktionen TRUTH_TABLE i *DERIVE*.

b) Använd uttrycket $p \veebar (q \vee r)$ för att skapa ett polynom för rule 30. Använd *DERIVE* eller räkna för hand.

```
expr1 := (p ∧ ¬ q ∧ ¬ r) ∨ (¬ p ∧ q ∧ r) ∨ (¬ p ∧ q ∧ ¬ r) ∨ (¬ p ∧ ¬ q ∧ r)

expr2 := p XOR (q ∨ r)
```

TRUTH_TABLE(p, q, r, expr1 IFF expr2) =

p	q	r	expr1 IFF expr2
true	true	true	true
true	true	false	true
true	false	true	true
true	false	false	true
false	true	true	true
false	true	false	true
false	false	true	true
false	false	false	true

				1				
			1	1	1			
		1	1	0	0	1		
	1	1	0	1	1	1	1	
1	1	0	0	1	0	0	0	1

C. Rule 30 med Excel

Vi kan nu åskådliggöra ett antal generationer av Rule 30 i ett kalkylblad i Excel. I den första generationen finns endast en svart cell eller en cell innehållande värdet 1.

Anvisningar

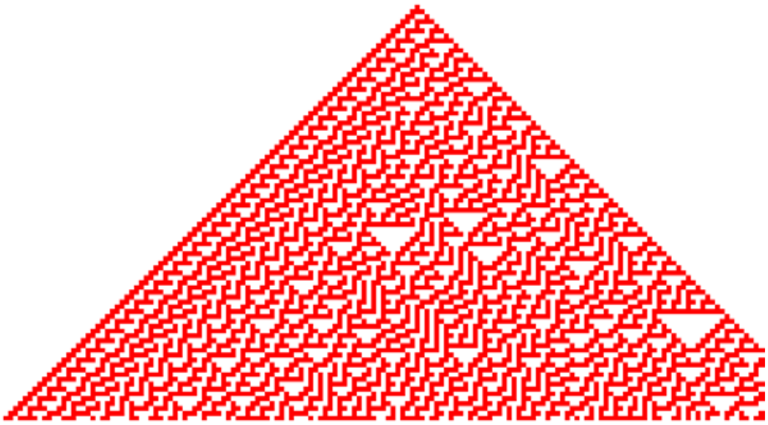
1. Skapa en egendefinierad funktion $\text{rule30}(p,q,r) = 2pqr - 2pq - qr - 2rp + p + q + r$.
   a) Öppna Visual Basic Editor genom att välja `Verktyg Makro Visual Basic Editor`.
   b) Välj sedan `Infoga Modul`.
   c) Skriv in följande

```
Function rule30(p, q, r)
    rule30 = p + q + r - q * r - 2 * p * q - 2 * p * r + 2 * p * q * r
End Function
```

Funktionen Rule30 kan nu användas i ett kalkylblad.

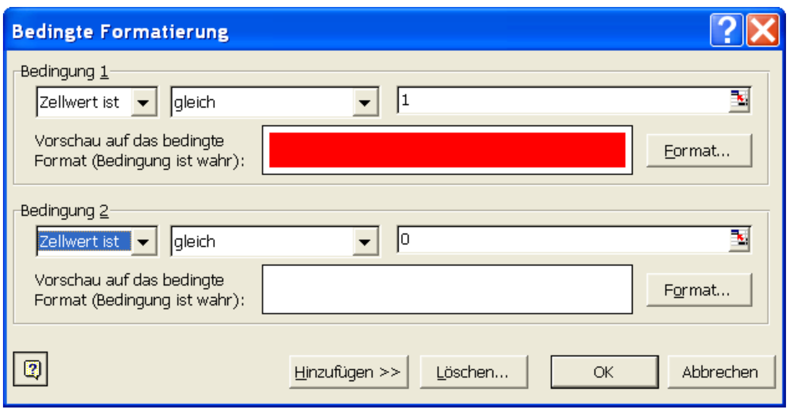
2. Skriv in 1 i cell `DG1` samt `=Rule30(DF1;DG1;DH1)` i cell `DG2`. Kopiera sedan formler så att du får ett kalkylblad av den typ som fins i figuren ovan.
3. Du kan sedan modifiera cellernas storlek med `Format Kolumn Bredd, Format Rad Höjd`. Man kan lätt man göra de celler, som innehåller 1:or svarta.

   I figuren här bredvid har vi åskådliggjort 78 generationer av Rule 30.



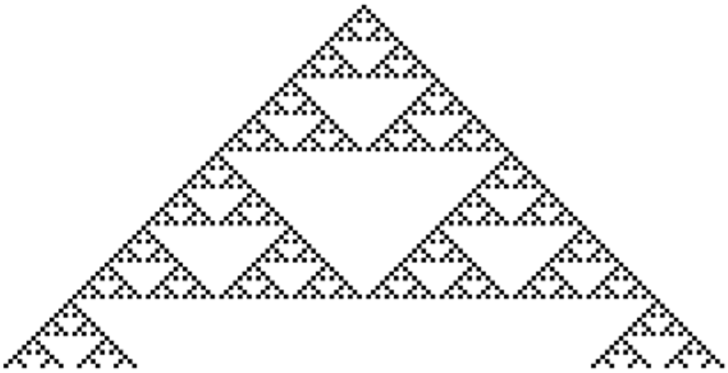
You can format the cells dependend on their contents.

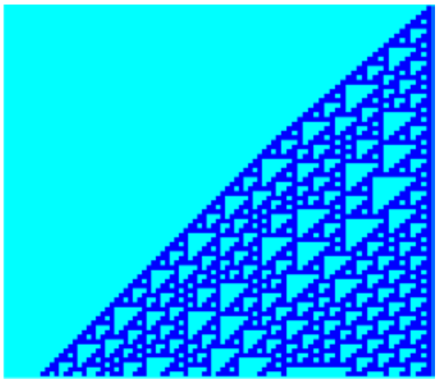
All cells with content 1 are red (font and pattern), all cells with content 0 are full white.



Övning. Åskådligör ett antal generationer av rule 90 och rule 110 med Excel.

```
ff(p, q, r) := a*p*q*r + b*p*q + c*q*r + d*r*p + e*p + f*q + g*r + h

SOLVE([ff(1, 1, 1) = 0, ff(1, 1, 0) = 1, ff(1, 0, 1) = 0, ff(1, 0, 0) = 1, ff(0, 1, 1) =
  1, ff(0, 1, 0) = 0, ff(0, 0, 1) = 1, ff(0, 0, 0) = 0], [a, b, c, d, e, f, g, h])

[a = 0 ∧ b = 0 ∧ c = 0 ∧ d = -2 ∧ e = 1 ∧ f = 0 ∧ g = 1 ∧ h = 0]

- 2*r*p + p + r
```



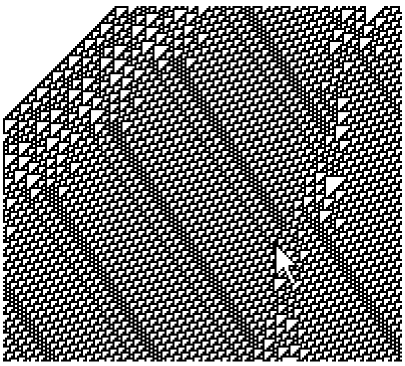


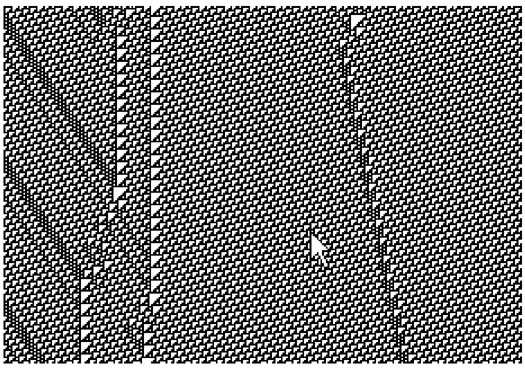
This are Excel-creations

Med hjälp av *DERIVE* kan man exempelvis åstadkomma dessa figurer. Den ursprungliga generationen består av flera 1:or och 0:or.

A New Kind of Science Explorer

A New Kind of Science Explorer är en programvara från Wolfram Research med vilken man kan göra alla de experiment med cellulära automater vilka beskrivs i [1.] Man kan "zooma" var som helst i en cellulär automat.





Med programmet kan man även undersöka cellulära automater av högre dimension och med mer komplicerade uppdateringsregler än vad vi beskrivit ovan samt sådana som är uppbyggda av fler färger.

Litteratur:

[1.] S. Wolfram, A New Kind of Science

Länkar:

http://mathworld.wolfram.com/ElementaryCellularAutomaton.html
http://www.math.usf.edu/~eclark/ANKOS_reviews.html

D. A possible Derive implementation:

```
       binar(n, n_, i_, j_) :=
         Prog
           OutputBase := 2
           n_ := STRING(n)
           OutputBase := 10
           i_ := 1
#1:        j_ := 8 - DIM(n_)
           Loop
             If i_ > j_
                RETURN VECTOR(k_ - 48, k_, NAME_TO_CODES(n_))
             n_ := APPEND("0", n_)
             i_ :+ 1
```

$$\#2:\quad [\text{binar}(30),\ \text{binar}(110),\ \text{binar}(90)] = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

```
       cell_aut(r, n, st_, stt_, i_, k_, l_, pts_, r_, ant) :=
         Prog
           i_ := 1
           pts_ := [[0, 100]]
           l_ := -1
           stt_ := [1]
           Loop
             st_ := APPEND([0, 0], stt_, [0, 0])
             stt_ := []
             k_ := 1
#3:          Loop
               r_ := st_↓k_·4 + st_↓(k_ + 1)·2 + st_↓(k_ + 2)
               If (binar(r))↓(8 - r_) = 1
                  pts_ := APPEND(pts_, [[k_ + l_ - 1, 100 - i_]])
               stt_ := APPEND(stt_, [(binar(r))↓(8 - r_)])
               k_ :+ 1
               If k_ > DIM(st_) - 2 exit
             If i_ = n
                RETURN pts_
             l_ :- 1
             i_ :+ 1
```

The initial number is 1. For finding the next generation I append a double zero in front and at the end of each sequence (in red).

Plotting starts at (0|100) and moves downwards – like a curtain.

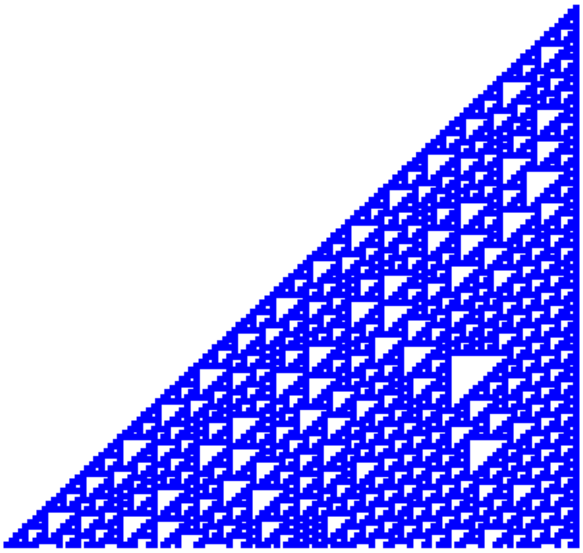
All following generalisations base on `cell_aut(rule#,#ofGenerations)`.

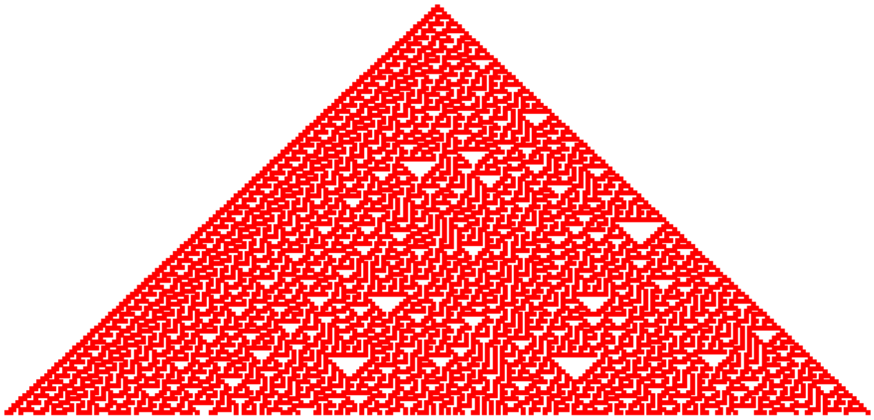
```
            0 0 1 0 0
          0 0 1 1 1 0 0
        0 0 1 1 0 0 1 0 0
      0 0 1 1 0 1 1 1 1 0 0
    0 0 1 1 0 0 1 0 0 0 1 0 0
  0 0 1 1 0 1 1 1 1 0 1 1 1 0 0
  0 1 1 0 0 1 0 0 0 0 1 0 0 1 0
```

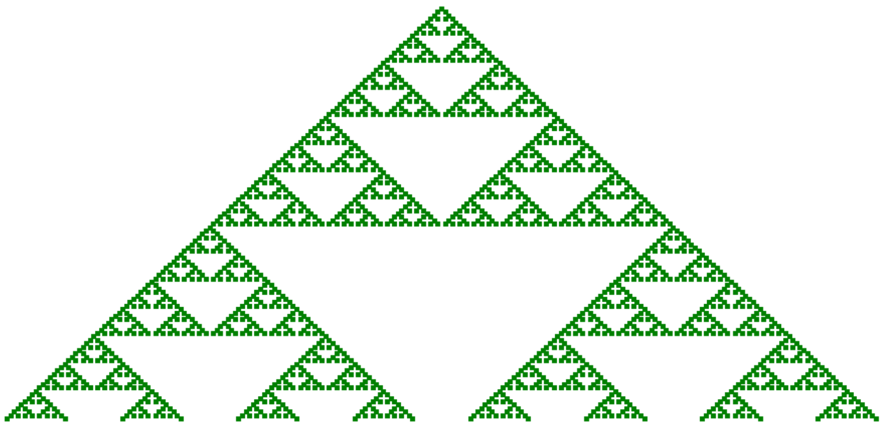
#4: cell_aut(30, 120)

#5: cell_aut(90, 120)

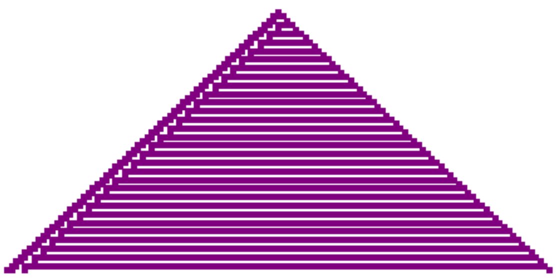
#6: cell_aut(110, 120)







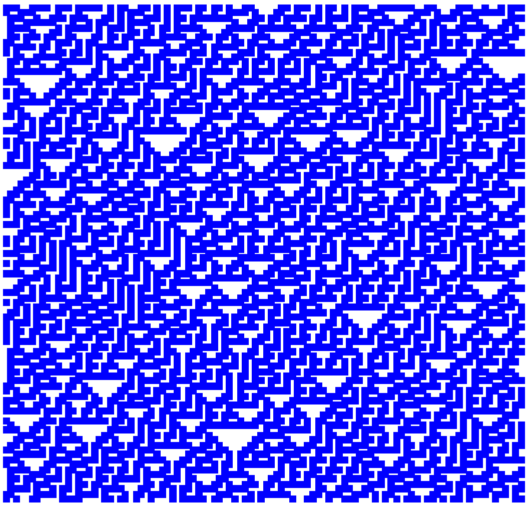
#7: cell_aut(31, 60)



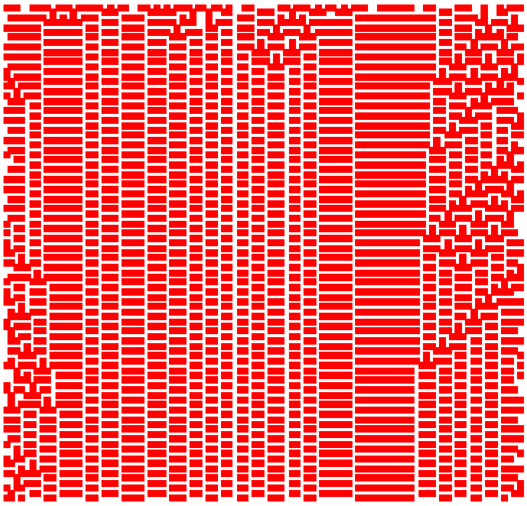
In my first generalisation I start with an initial binary sequence – self written or produced randomly.

```
VECTOR(RANDOM(2), k, 100)

[1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1,
  0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1,
  0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1,
  1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1]

test1 := [1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1,
  1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0,
  1, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0,
  0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1]
```

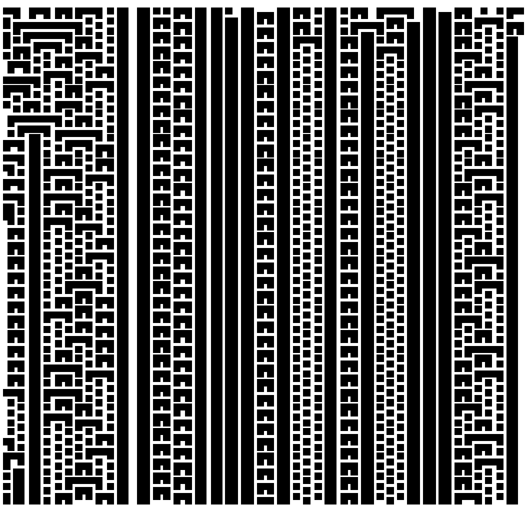
`cell_aut_gen(30, test1, 100)`



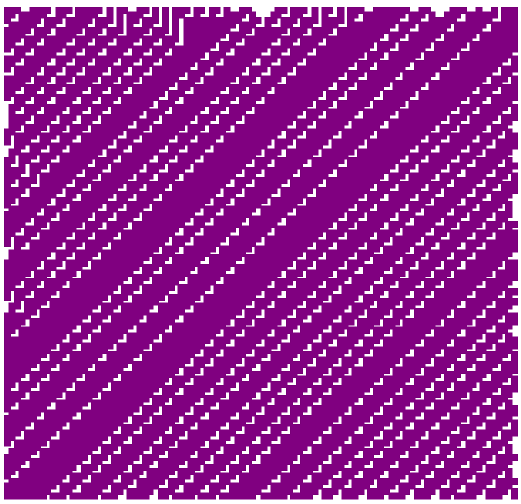
`cell_aut_gen(37, test1, 100)`



`cell_aut_gen(73, test1, 100)`

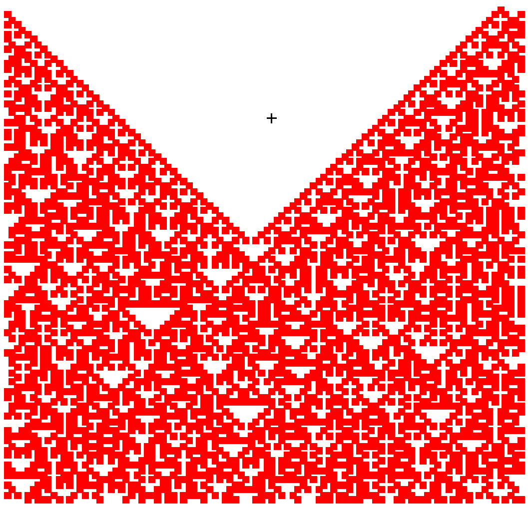


`cell_aut_gen(158, test1, 100)`

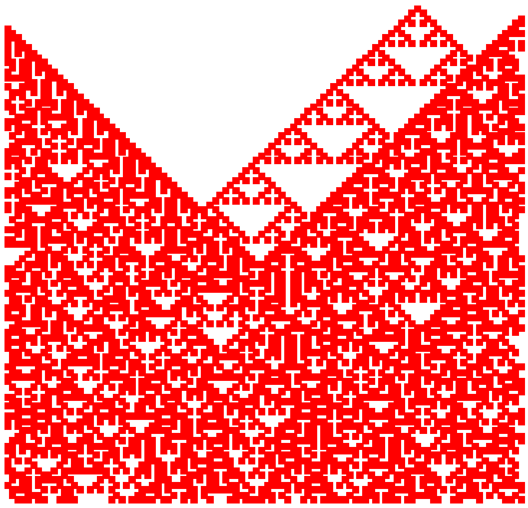


test3 and test4 consist both of 99 zeros and one 1 on different places. See the difference applying Rule90 (next two pictures).

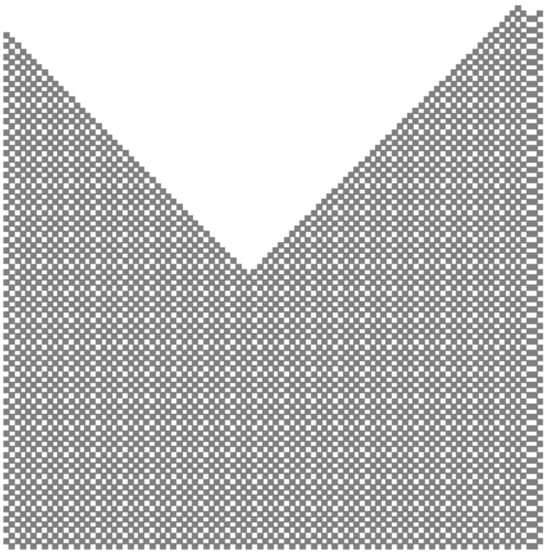
```
test3 := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0]

test4 := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
```

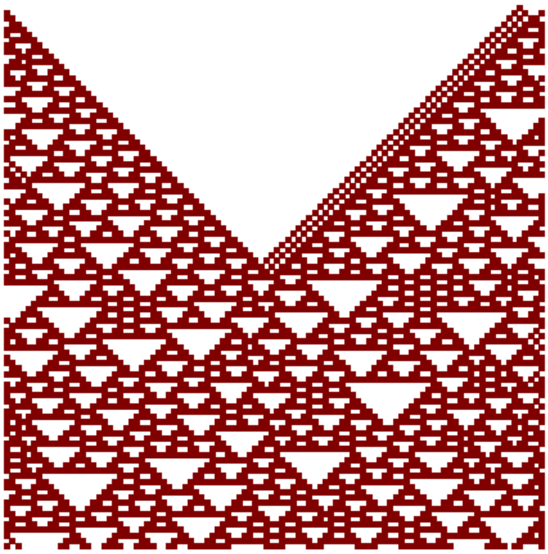


`cell_aut_gen(50, test3, 100)`



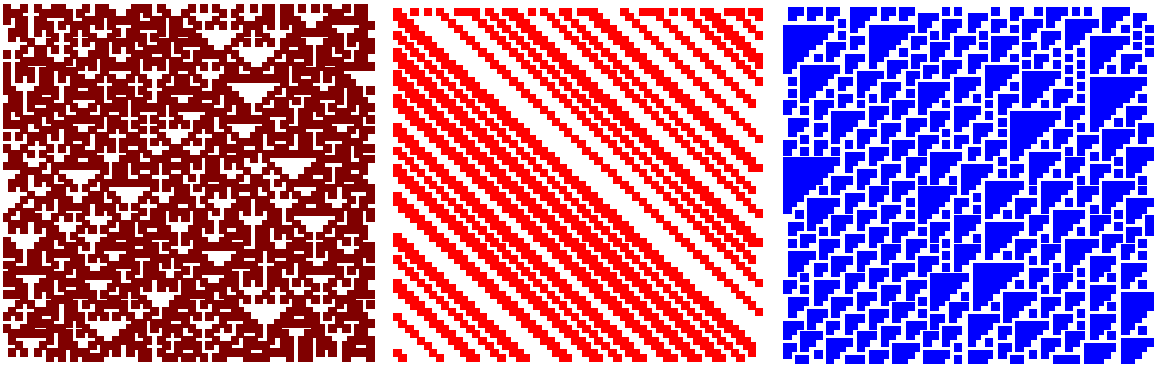
`cell_aut_gen(122, test3, 100)`





In my last generalisation of this kind of cellular automates I include generating the first generation randomly in the program. `cell_aut_rnd(rule#,#ofColumns,#ofGenerations)` produces surprising results.

Rules 90, 80 and 137



Try special rules and observe the various kinds of the created patterns:

rules 8, 136

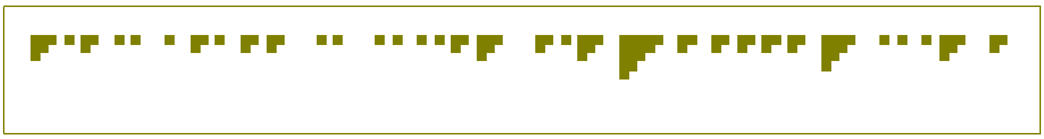
rules 4, 37, 56, 73

rules 18, 45, 146

rule 110

....

Go on and design your own wallpaper.



References

[1] Peitgen, Jürgens, Saupe, Fractals for the Classroom, Springer, 1992

[2] Clifford A. Pickover, Mit den Augen des Computers (Computers and the Imagination), Markt und Technik, 1992

[3] Hans Lauwerier, Fraktale verstehen und selbst programmieren, Wittig, 1992

[4] Frank Piefke, Simulation mit dem Personalcomputer, Hüthig, 1991

One example to convert from DERIVE6 to DERIVE5

(No responsibility for possible mistakes can be taken.)

The Derive 6 mth-file `trrules.mth`
(containing some greek letters!)

Sine Rule

#1: $\frac{a}{b} = \frac{SIN(\alpha)}{SIN(\beta)}$

#2: $SOLVE\left(\frac{a}{b} = \frac{SIN(\alpha)}{SIN(\beta)}, \alpha\right)$

#3: $\alpha = -ASIN\left(\frac{a \cdot SIN(\beta)}{b}\right) - \pi \vee \alpha = \pi - ASIN\left(\frac{a \cdot SIN(\beta)}{b}\right) \vee \alpha = ASIN\left(\frac{a \cdot SIN(\beta)}{b}\right)$

#4: Cosine Rule, given a,b and γ_

#5: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot COS(\gamma\_)$

as it appears in Derive 5

#1: $\frac{a}{b} = \frac{SIN(\Delta \cdot 3 \cdot b1)}{SIN(\Delta \cdot 3 \cdot b2)}$

#2: $SOLVE\left[\frac{a}{b} = \frac{SIN(\Delta \cdot 3 \cdot b1)}{SIN(\Delta \cdot 3 \cdot b2)}, \Delta \cdot 3 \cdot b1\right]$

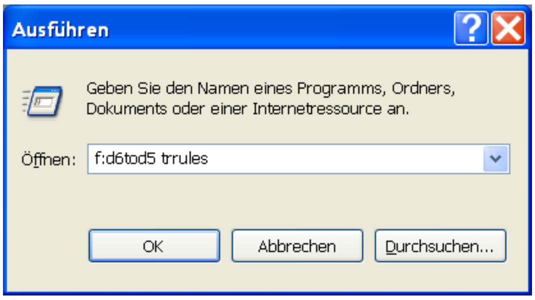
#3: $\Delta \cdot 3 \cdot b1 = -ASIN\left[\frac{a \cdot SIN(\Delta \cdot 3 \cdot b2)}{b}\right] - \pi \vee \Delta \cdot 3 \cdot b1 = \pi - ASIN\left[\frac{a \cdot SIN(\Delta \cdot 3 \cdot b2)}{b}\right] \vee \Delta \cdot 3 \cdot b1 = ASIN\left[\frac{a \cdot SIN(\Delta \cdot 3 \cdot b2)}{b}\right]$

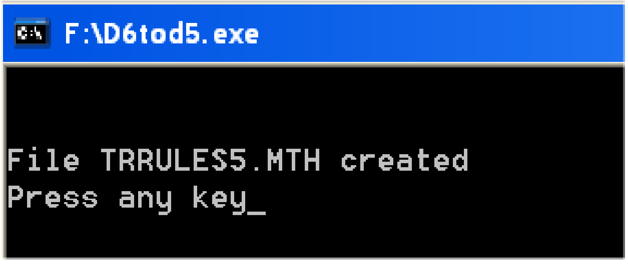
#4: Cosine Rule, given a,b and ∆03b3_

#5: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot COS(\Delta \cdot 3 \cdot b3\_)$

You can see that the greek letters are replaced by strange other characters – cuased by using Unicode in Derive 6.

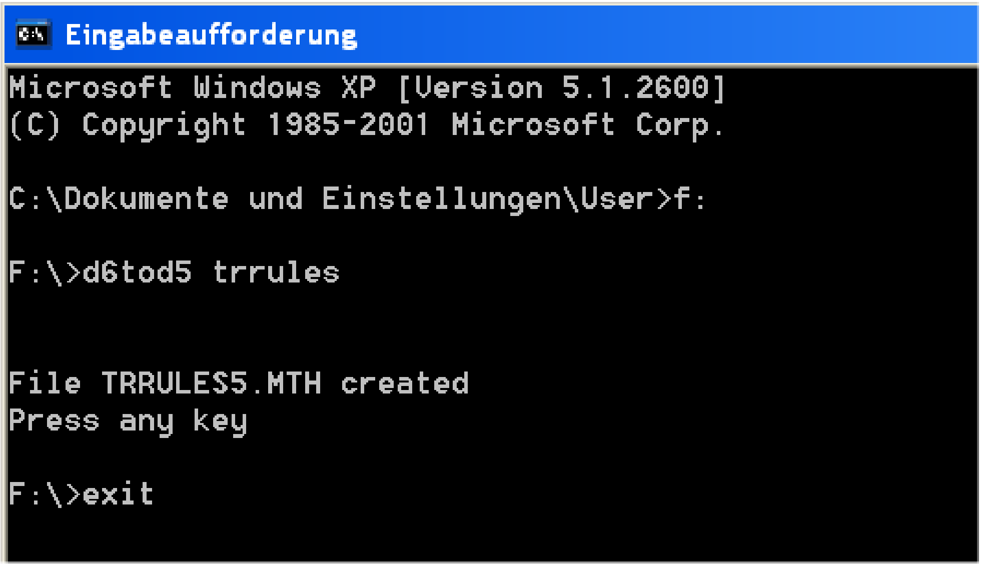
I have `d6tod5.exe` in the same folder as `trrules.mth` and run `d6tod5 trrules`:



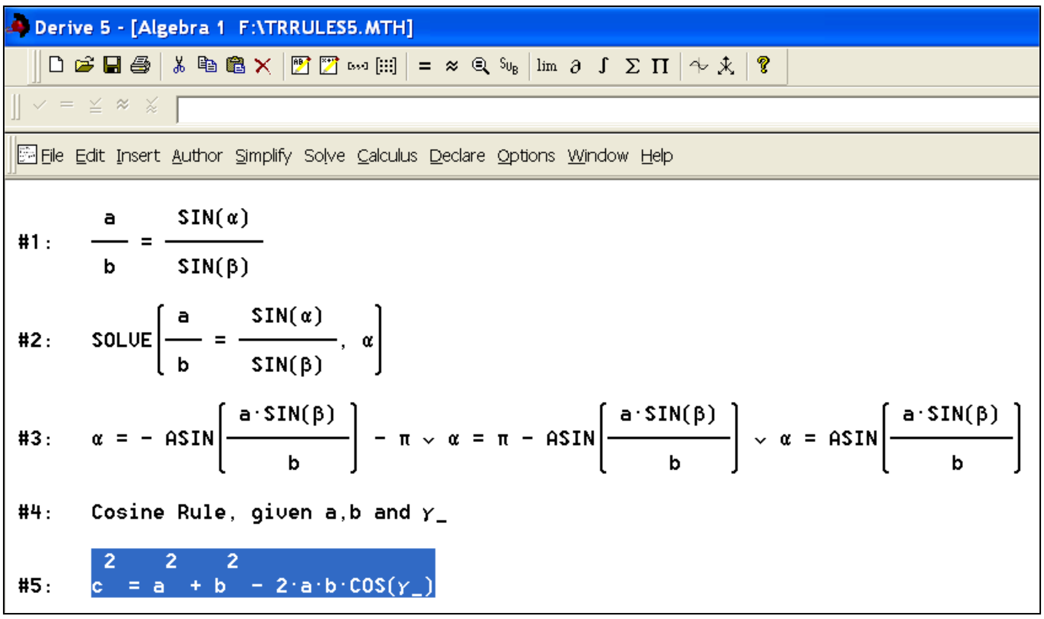


I could also open the DOS-Window and then call the program.

The next key press should terminate the little step into DOS past.



I leave DOS by typing exit.
We can see that `TRRULES5.MTH` has been created. Opening this file with Derive 5 shows the converted file, containing the correct greek letters.



Albert Rich

Hello Valeriu,

Thank you for pointing out the problem with Derive's trig functions when approximating large magnitude arguments. I have resolved the problem as you suggested by temporarily increasing the precision while reducing the argument modulo 2*pi. Now SIN(10^11) approximates to 0.9286936604 at 10 digits of precision.

Since 10^11+0.5 justifiably approximates to 10^11 at 10 digits of precision, both SIN(10^11+0.5) and your sin_(10^11+0.5) approximate to 0.9286936604. However, in Mixed precision mode, SIN(10^11+0.5) correctly approximates to 0.9927992642. In fact, this is the best application of Mixed precision mode that I have ever encountered!

I will answer the other issues raised in your email in a subsequent email.

Aloha,
Albert

Hello Valeriu,

I have implemented the 3 Euler Substitutions used to rationalize integrands of the form F(x,SQRT(a+b*x+c*x^2)) as given by formulas in Section 2.251 of Gradshteyn&Ryzhik's "Table of Integrals, Series, and Products", Fifth Edition.

Derive is now able to integrate 1/(x*SQRT(x^2+x+1)+2*x+2) to the sum of two logs and an arctangent. Mathematica's on-line integrator gives an inexplicable antiderivative of this expression in terms of Rootsum. Maple V, version 4 is unable to integrate it. I would appreciate any readers of this email having a more recent version of Maple, try integrating this expression using Maple.

Used indescriminantly the Euler Substitutions results in high degree integrands that give more complicated results or even hang Derive. This is because partial fraction expansion of the original integrand is easier than expansion of the rationalized integrand. Therefore, as a hack, I have restricted the use of the Euler substitutions to integrands that are pure reciprocals and b is non-zero. In an effort to narrow the scope of this restriction, I would appreciate any examples that:

1. The Euler substitutions can get;

2. Derive 6.01 cannot get; and

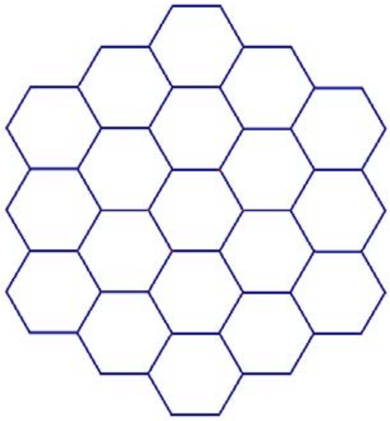
3. are pure reciprocals or have a b=0.

Finally, is there a better substitution to rationalize square roots of binomials of the form F(x,SQRT(a+b*x^2)) or even better F(x,SQRT(a+b*x^n)) where n is a positive integer?

Thanks again for your help in furthering the education of Derive.

Aloha,
Albert

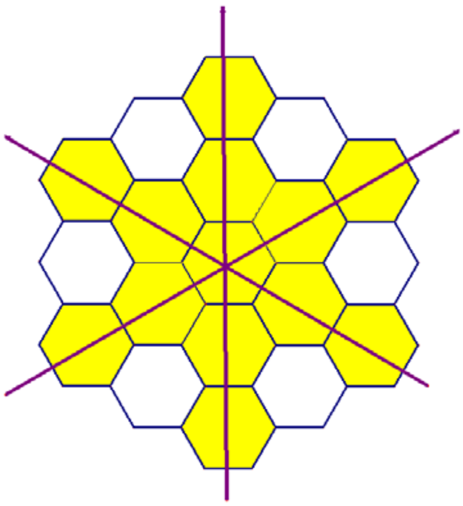
Dear Derive and TI-Community

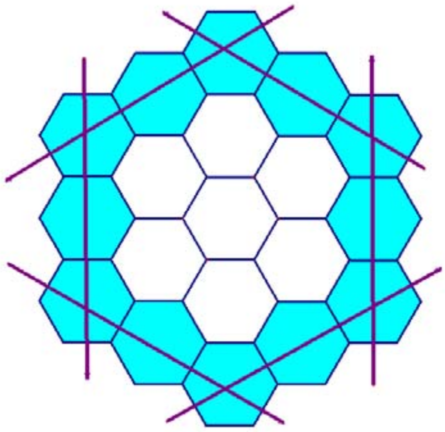


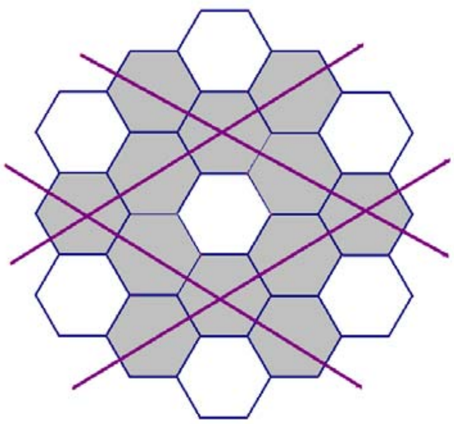
It is a really difficult problem.
It does not have solution?

We have to put the numbers 1,2,3,…..19 in the diagram.

The following sums have to be 38. In the verticals, diagonals shown.







As you see, we need

Three sums of five summands equal to 38
Six sums of three summands equal to 38
Four sums of four summand equal to 38.

Milton Lesmes, Bogotá, Colombia, miltonlesmes@hotmail.com