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DERIVE®

USER GROUP

+ CAS-TI

Contents:

March 2012

Neu: ACDCA-Mitgliedschaft

Das ACDCA (Austrian Center for Didactics of Computer Algebra) wurde Ende der 80er Jahre gegründet und ist seither eine der treibenden Kräfte für den Einsatz von CAS in der Mathematikausbildung. Dieser Verein wurde kürzlich umstrukturiert und bietet nun allen Interessierten die Mitgliedschaft und damit die Gelegenheit zur Mitwirkung an. Bitte informieren Sie sich auf der Homepage des ACDCA über die Aufgaben und Ziele des ACDCA.
Info und Beitrittserklärung finden Sie auf

http://rfdz.ph-noe.ac.at/acdca.html

Hier steht auch die verbesserte und ergänzte Version der „Dynamischen Systeme“ zum Download bereit:

http://rfdz.ph-noe.ac.at/acdca/materialien.html

Latest News:

You are invited to download

Dynamic Systems on Various Platforms - An Excursion from Environment & Tourism to Strange Attractors! (113 pages)

http://rfdz.ph-noe.ac.at/acdca/materialien.html

Prof. Paditz from HTW Dresden informed that the complete proceedings of two MEC21-Conferences (Dresden 2009 with 147 contributions and Grahamstown 2011 with 73 contributions) are available as download at

http://nbn-resolving.de/urn:nbn:de:bsz:14-qucosa-79236
http://nbn-resolving.de/urn:nbn:de:bsz:14-qucosa-79243

STATISTICS with TI-Nspire

I am very happy that *Guido Herweyers* – our Flemish Statistics expert – permitted publication of his 50 pages paper. I tried to translate from Flemish (Dutch) into English (my wife is from Dutch origin) and you will find his paper divided in some parts in this and in the next DNLs. TI-Nspire has some very nice features especially for statistics.

Attend TIME2012 in Tartu, Estonia
July 10-14, 2012
http://time2012.ut.ee/

List of lectures and workshops, which have been accepted until now is on pages 45 and 46. Don’t miss the opportunity to meet CAS-experts from all continents and register!

Liebe DUG-Mitglieder,

Leider bin ich mit diesem DNL etwas zu spät dran weil ich die Arbeit an einigen Artikeln unterschätzt habe und die Vorbereitung der TIME 2012 immer wieder Zeit in Anspruch genommen hat.

Im letzten DNL wurde das TI-Nspire-Programm TRIGO vorgestellt. Herr van Lantschoot hat eine verbesserte Version geschickt, herzlichen Dank dafür.

Es freut mich besonders, dass Philip Todd einen Artikel beigetragen hat, in dem er das Zusammenwirken seines schönen Programms *Geometry Expressions* mit *TI-NspireCAS* demonstriert. Dass dies auch mit *DERIVE* (u.a. CAS) möglich ist, habe ich zu zeigen versucht. Philip wird auch an der TIME 2012 mitwirken.

Ludwig Heinrichs Cubus-Simus-Aufsatz hat auch für mich eine Eigendynamik bekommen. Viel Information war im Internet zu finden. Einige „Challenges" für Sie sind noch offen!

Ich kann mir denken, dass Dietmar Oertel schon ungeduldig auf die Veröffentlichung seiner umfangreichen Untersuchungen im Bereich der Primzahlen und der Taylorreihen wartet. Seine Unterlagen umfassen jeweils ca 60-70 Seiten und eine sinnvolle Zusammenfassung braucht seine Zeit, aber ich bin schon dabei.

Der im letzten DNL angekündigte Vorschlag von Klaus Körner muss auf den DNL#86 warten wie auch interessante Informationen für mu-Math-Kenner.

Von der Webseite des ACDCA können Sie eine verbesserte und erweiterte Version meines Aufsatzes zu den *Dynamischen Systemen* (ca 110 Seiten) beziehen. Beachten Sie bitte unsere Einladung auf der Info-Seite.

Schließlich finden Sie die Liste der bisher angenommenen Vorträge für die TIME 2012, in deren Rahmen auch ein DUG-Treffen stattfinden wird.

Dear DUG Members,

It is really a pity that I am a little bit late with this DNL. Actually I underestimated work on some contributions and preparation of TIME 2012 needed some time, too

The TI-Nspire-program TRIGO was presented in the last DNL. Erik van Lantschoot, the author of TRIGO sent an improved version, many thanks for this.

I am very happy that Philip Todd provided an article demonstrating cooperation of his excellent *Geometry Expressions* program with *TI-NspireCAS*. I try to show that this is also possible with *DERIVE* (and other CAS). Philip will present a lecture and a workshop at TIME 2012.

Ludwig Heinrich's Cubus Simus contribution got an interesting self dynamics for me. Much information could be found in the web. Some "challenges" are still remaining for you.

I am quite sure that Dietmar Oertel is is anxious for publication of his extensive investigations in the field of prime numbers and Taylor series. His papers comprise about 60-70 pages each, so preparing a meaningful summary needs time, but I am busy …

I announced a proposal from Klaus Körner in the last DNL. It must wait for DNL#86 – together with interesting information for my-Math-experts.

You are invited to download a paper on *Dynamic Systems on Various Platforms* (~100 pages) from the ACDCA-website. Its URL is given on the information page.

Finally you can find the list of the accepted lectures and workshops for TIME 2012 where we will have a DUG-Meeting, too.

Viele Grüße, Best regards

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a content of 40 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE*, *TI*-CAS and other CAS as well to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

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Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: June 2012

Preview: Contributions waiting to be published

Some simulations of Random Experiments, J. Böhm, AUT, Lorenz Kopp, GER
Wonderful World of Pedal Curves, J. Böhm, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
Hill-Encription, J. Böhm, AUT
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
Do you know this? Cabri & CAS on PC and Handheld, W. Wegscheider, AUT
An Interesting Problem with a Triangle, Steiner Point, P. Lüke-Rosendahl, GER
Overcoming Branch & Bound by Simulation, J. Böhm, AUT
Graphics World, Currency Change, P. Charland, CAN
Cubics, Quartics – Interesting features, T. Koller & J. Böhm, AUT
Logos of Companies as an Inspiration for Math Teaching
Exciting Surfaces in the FAZ / Pierre Charland´s Graphics Gallery
BooleanPlots.mth, P. Schofield, UK
Old traditional examples for a CAS – what´s new? J. Böhm, AUT
Truth Tables on the TI, M. R. Phillips, USA
Where oh Where is It? (GPS with CAS), C. & P. Leinbach, USA
Embroidery Patterns, H. Ludwig, GER
Mandelbrot and Newton with *DERIVE*, Roman Hašek, CZK
Tutorials for the NSpireCAS, G. Herweyers, BEL
Some Projects with Students, R. Schröder, GER
Structures in the Set of Prime Numbers, D. Oertel, GER
Dirac Algebra, Clifford Algebra, D. R. Lunsford, USA
Treating Differential Equations (M. Beaudin, G. Piccard, Ch. Trottier), CAN
A New Approach to Taylor Series, D. Oertel, GER
Cesar Multiplication, G. Schödl, AUT
Henon & Co; Find your very own Strange Attractor, J. Böhm, AUT
Rational Hooks, J. Lechner, AUT
Simulation of Dynamic Systems with various Tools, J. Böhm, AUT
Using DERIVE to simulate the basic steps of the Q. Shor's algorithm, N. Urrego, COL

and others

Impressum:
Medieninhaber: *DERIVE* User Group, A-3042 Würmla, D´Lust 1, AUSTRIA
Richtung: Fachzeitschrift
Herausgeber: Mag. Josef Böhm

I am very happy that Philip kept his promise to send a contribution for the DNL. I appreciate his *Geometry Expressions* as an extremely powerful geometry tool which has a strong built-in CAS. *GE* can cooperate with all main Computer Algebra Systems. Come and attend Philip's lecture and workshop at TIME 2012.

Josef

Approximating Circular Arcs with Cubic Splines

Philip Todd, Saltire Software
phil@saltire.com

We use Geometry Expressions and the TI nSpire CAS to find the best cubic spline approximation to a quarter circle.

1. Introduction

A problem came from a user of Geometry Expressions recently, which nicely illustrates the power of a symbolic geometry system used in conjunction with CAS. The user wanted to find the control points of a Bezier spline which approximated a circular arc of a given angle. Why would he want to do such a thing? Some software and some machine tools have Bezier spline curves as primitives, but do not have circular arcs. If you want a circle, you need to piece an approximation together out of splines.

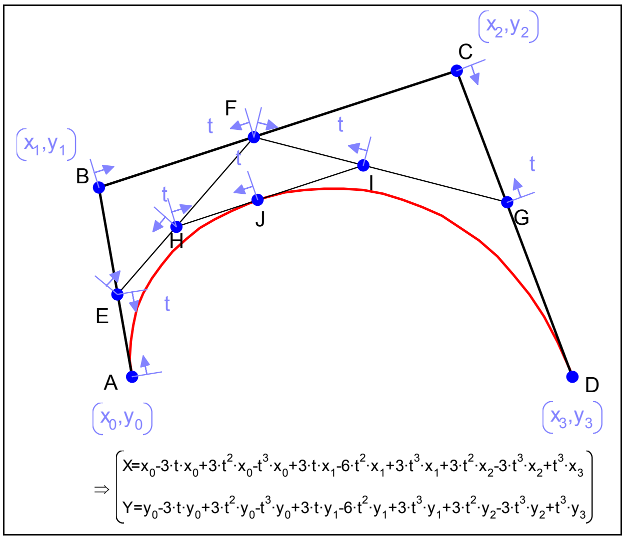


Figure 1: A cubic spline constructed in Geometry Expressions. The spline's parametric equation is computed by the software

A Bezier spline can be readily modeled in Geometry Expressions [1] using the geometric version of de Casteljau's algorithm [2]. We start with control polygon ABCD. These points are constrained to have coordinates (x_0,y_0), (x_1,y_1), (x_2,y_2), (x_3,y_3). Point E is constrained to be proportion t along AB. Point F is constrained to be proportion t along BC, and point G is constrained to be proportion t along CD. Points H and I are now added and constrained to be proportion t along segments EF and FG. Finally, point J is added and constrained to be proportion t along segment HI. The spline curve is the locus of J as t runs from 0 to 1.

An important property of the spline curve is that it is tangential to the control polygon at its ends. That is at t=0, the spline curve is tangential to AB, and at t=1 it is tangential to CD.

This property can be readily verified using the TI-Nspire CAS. Geometry Expressions will automatically generate the parametric equation of the curve (figure 1), and this can be copied (using Copy As / TI Nspire) into a TI-Nspire CAS worksheet (figure 2).

$$[x=x[0]+x[0]\cdot t\cdot -3+x[0]\cdot t^2\cdot 3+x[0]\cdot t^3\cdot -1+x[1]\cdot t\cdot 3+x[1]\cdot t^2\cdot -6+x[1]\cdot t^3\cdot 3+x[2]\cdot t^2\cdot 3+x[2]\cdot t^3\cdot -3+x[3]\blacktriangleright$$

$$[x=t^3\cdot(x[3]-3\cdot x[2]+3\cdot x[1]-x[0])+t^2\cdot(3\cdot x[2]-6\cdot x[1]+3\cdot x[0])+t\cdot(3\cdot x[1]-3\cdot x[0])+x[0]\quad y=t^3\cdot(y[3]-3\blacktriangleright$$

$$\frac{d}{dt}\left(t^3\cdot(x[3]-3\cdot x[2]+3\cdot x[1]-x[0])+t^2\cdot(3\cdot x[2]-6\cdot x[1]+3\cdot x[0])+t\cdot(3\cdot x[1]-3\cdot x[0])+x[0]\right)|t=0$$

$$3\cdot x[1]-3\cdot x[0]$$

$$\frac{d}{dt}\left(t^3\cdot(y[3]-3\cdot y[2]+3\cdot y[1]-y[0])+t^2\cdot(3\cdot y[2]-6\cdot y[1]+3\cdot y[0])+t\cdot(3\cdot y[1]-3\cdot y[0])+y[0]\right)|t=0$$

$$3\cdot y[1]-3\cdot y[0]$$

$$\frac{d}{dt}\left(t^3\cdot(x[3]-3\cdot x[2]+3\cdot x[1]-x[0])+t^2\cdot(3\cdot x[2]-6\cdot x[1]+3\cdot x[0])+t\cdot(3\cdot x[1]-3\cdot x[0])+x[0]\right)|t=1$$

$$3\cdot x[3]-3\cdot x[2]$$

$$\frac{d}{dt}\left(t^3\cdot(y[3]-3\cdot y[2]+3\cdot y[1]-y[0])+t^2\cdot(3\cdot y[2]-6\cdot y[1]+3\cdot y[0])+t\cdot(3\cdot y[1]-3\cdot y[0])+y[0]\right)|t=1$$

$$3\cdot y[3]-3\cdot y[2]$$

5/99

Figure 2: differentiating the spline curve at t=0 and t=1 shows that it is tangential to the control polygon

The derivatives of the x coordinate and y coordinate with respect to t evaluated at t = 0 show that the tangent aligns itself with the segment AB. Again, the derivatives evaluated at t=1 show that this tangent aligns itself with the segment CD.

#1: $$\left[x = x_0 + x_0\cdot t\cdot(-3) + x_0\cdot t^2\cdot 3 + x_0\cdot t^3\cdot(-1) + x_1\cdot t\cdot 3 + x_1\cdot t^2\cdot(-6) + x_1\cdot t^3\cdot 3 + x_2\cdot t^2\cdot 3 + x_2\cdot t^3\cdot(-3) + x_3\cdot t^3,\ y = y_0 + y_0\cdot t\cdot(-3) + y_0\cdot t^2\cdot 3 + y_0\cdot t^3\cdot(-1) + y_1\cdot t\cdot 3 + y_1\cdot t^2\cdot(-6) + y_1\cdot t^3\cdot 3 + y_2\cdot t^2\cdot 3 + y_2\cdot t^3\cdot(-3) + y_3\cdot t^3\right]$$

#2: $$\frac{d}{dt}\left[x_0 + x_0\cdot t\cdot(-3) + x_0\cdot t^2\cdot 3 + x_0\cdot t^3\cdot(-1) + x_1\cdot t\cdot 3 + x_1\cdot t^2\cdot(-6) + x_1\cdot t^3\cdot 3 + x_2\cdot t^2\cdot 3 + x_2\cdot t^3\cdot(-3) + x_3\cdot t^3,\ y_0 + y_0\cdot t\cdot(-3) + y_0\cdot t^2\cdot 3 + y_0\cdot t^3\cdot(-1) + y_1\cdot t\cdot 3 + y_1\cdot t^2\cdot(-6) + y_1\cdot t^3\cdot 3 + y_2\cdot t^2\cdot 3 + y_2\cdot t^3\cdot(-3) + y_3\cdot t^3\right]$$

#3: $$\left[3\cdot t^2\cdot x_3 + (6\cdot t - 9\cdot t^2)\cdot x_2 + (9\cdot t^2 - 12\cdot t + 3)\cdot x_1 - (3\cdot t^2 - 6\cdot t + 3)\cdot x_0,\ 3\cdot t^2\cdot y_3 + (6\cdot t - 9\cdot t^2)\cdot y_2 + (9\cdot t^2 - 12\cdot t + 3)\cdot y_1 - (3\cdot t^2 - 6\cdot t + 3)\cdot y_0\right]$$

#4: $$\left[3\cdot x_1 - 3\cdot x_0,\ 3\cdot y_1 - 3\cdot y_0\right]$$

#5: $$\left[3\cdot x_3 - 3\cdot x_2,\ 3\cdot y_3 - 3\cdot y_2\right]$$

DERIVE screen containing expressions imported from GE and further calculations.

2. Center Point on Circle

In this article, we'll use a spline to approximate a quarter circle, we'll refer to general arcs briefly in the conclusion.

We model a quarter circle of unit radius in the first quadrant. Point A is (0,1) and point D is (1,0). AB is constrained to be perpendicular to the y axis (hence the spline will be horizontal at A). CD is constrained to be perpendicular to the x-axis, and hence the spline is vertical at D.

It also seems reasonable that we should require our spline to be symmetric about x=y. We enforce this by constraining AB and BC to be the same length: k.

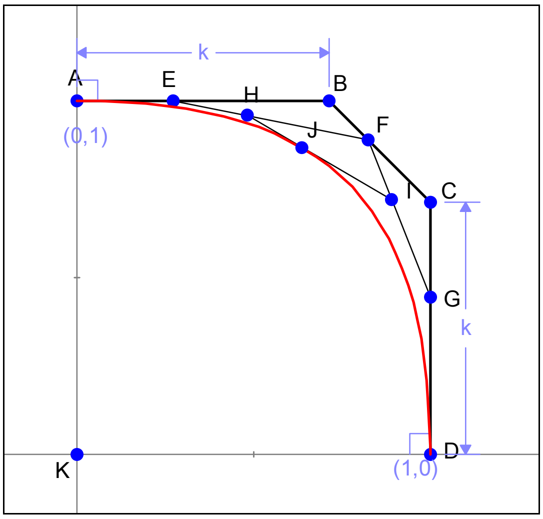


Figure 3: Spline curve with control polygon constrained to guarantee tangency with the unit circle at A and D and symmetry about y=x (proportional constraints are hidden for clarity).

We would like to find a value for k which provides a close approximation to a quarter circle. One approach would be to require the center point of the spline curve to lie on the circle. The center point is the point with parametric location on the curve ½. This can be easily created in *Geometry Expressions* by constraining a point's parametric location on the curve to be ½ (figure 4).

Geometry Expressions can supply the distance of this point from the origin. It hardly needs a CAS to find a value k such that this distance is 1. However, as we have one available, we'll use it. The resulting value of k can be copied from the TI-Nspire CAS back into geometry Expressions. Comparison with a unit circle (figure 5) shows that the fit is quite good. How good we will see soon.

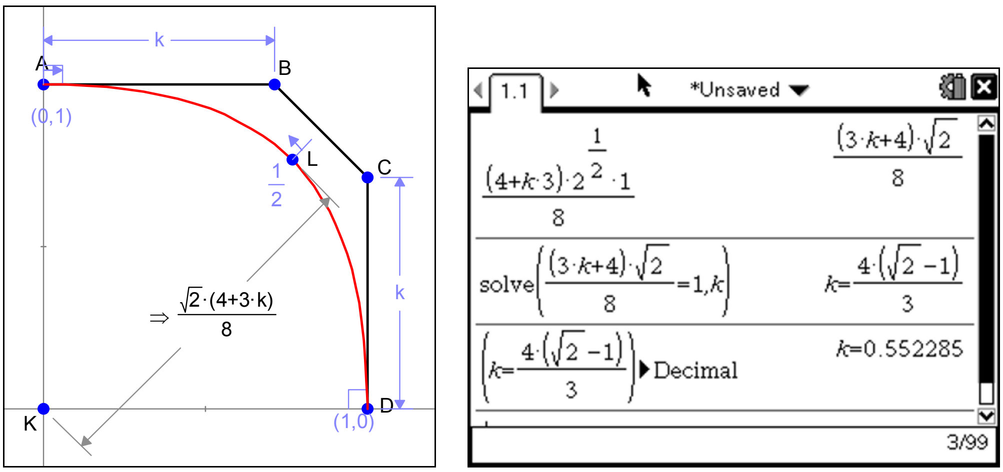


Figure 4: Distance of the center of the spline from the origin is measured in Geometry Expressions. The CAS is used to find the value of k which makes this distance 1.

#6: $$\frac{(4 + k\cdot 3)\cdot 2^{1/2}\cdot 1}{8}$$

#7: $$\text{SOLUTIONS}\left(\frac{(4 + k\cdot 3)\cdot 2^{1/2}\cdot 1}{8} = 1,\ k\right) = \left[\frac{4\cdot\sqrt{2}}{3} - \frac{4}{3}\right]$$

#8: [0.5522847498]

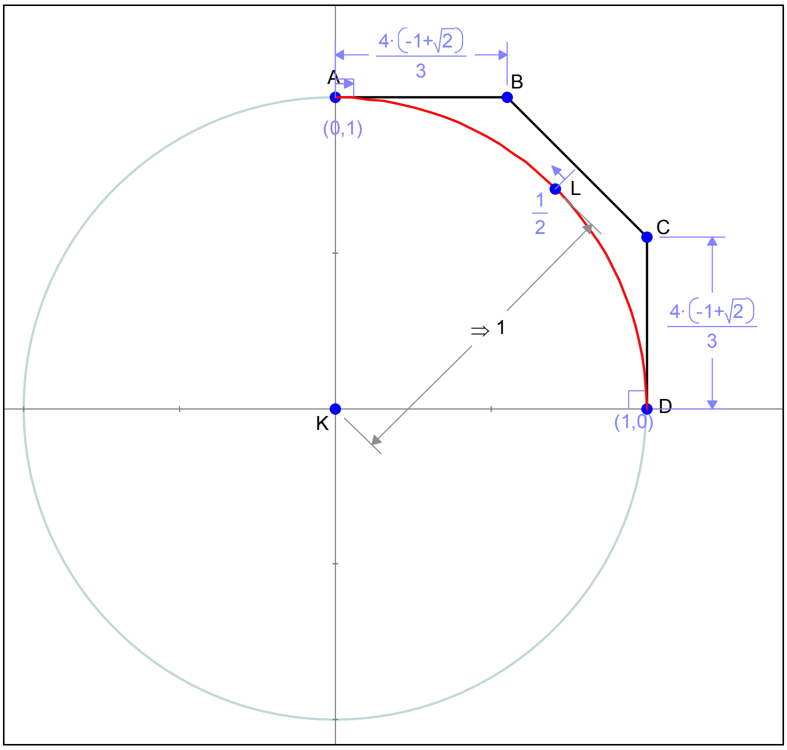
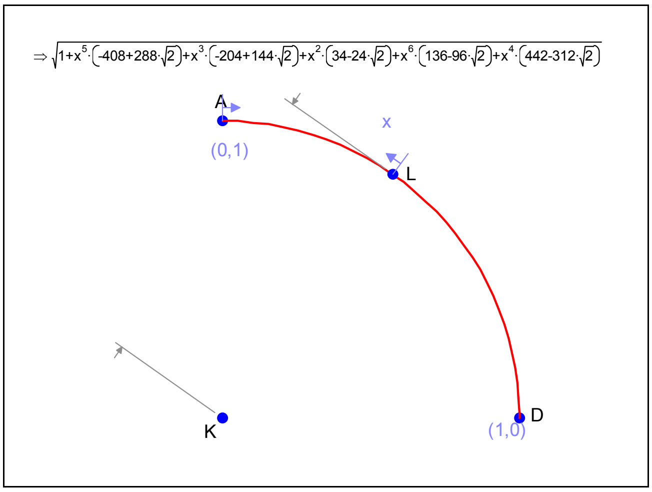


Figure 5: Values of k which make point L lie on the unit circle are copied from the CAS into Geometry Expressions allowing a visual inspection of the solution.

2.1 Error Analysis

To see how far away from the circle our curve lies, we change the location of L from ½ to a generic parametric location x (we're using x for convenience graphing in the TI-Nspire, but at the risk of confusion …x is not the x coordinate of L, but its parametric location on the curve). *Geometry Expressions* generates an expression for the distance of this generic point from the origin (figure below).



$$\left(1+\left(-408+2^{\frac{1}{2}}\cdot 288\right)\cdot x^5+\left(-204+2^{\frac{1}{2}}\cdot 144\right)\cdot x^3+\left(34+2^{\frac{1}{2}}\cdot -24\right)\cdot x^2+\left(136+2^{\frac{1}{2}}\cdot -96\right)\cdot x^6+\left(442+2^{\frac{1}{2}}\cdot -312\right)\cdot x^4\right)^{\frac{1}{2}}$$

$$\sqrt{-8\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^6+24\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^5-26\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^4+12\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^3-2\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^2+1}$$

$$\left(\sqrt{-8\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^6+24\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^5-26\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^4+12\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^3-2\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^2+1}\right)^2$$

$$-\left(8\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^6-24\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^5+26\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^4-12\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^3+2\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^2-1\right)$$

$$\frac{d}{dx}\left(-\left(8\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^6-24\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^5+26\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^4-12\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^3+2\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x^2-1\right)\right)$$

$$-4\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x\cdot\left(12\cdot x^4-30\cdot x^3+26\cdot x^2-9\cdot x+1\right)$$

$$\text{solve}\left(-4\cdot\left(12\cdot\sqrt{2}-17\right)\cdot x\cdot\left(12\cdot x^4-30\cdot x^3+26\cdot x^2-9\cdot x+1\right)=0,x\right)$$

$$x=0 \text{ or } x=\frac{-\left(\sqrt{3}-3\right)}{6} \text{ or } x=\frac{1}{2} \text{ or } x=\frac{\sqrt{3}+3}{6} \text{ or } x=1$$

1/4

Figure 6: The distance from the origin of point at parametric location x on the spline curve is copied from Geometry Expressions into the TI-nSpire CAS. The expression is squared, differentiated and solved.

To find the values of x for which this distance is maximal or minimal, we first copy the expression into our CAS. Optimizing the distance is the same as optimizing its square, so our first step inside the CAS is to square the expression, thus getting rid of the square root, and making it more likely that the derivative will fit on the page. Next, differentiate, then solve (figure 6).

$$d := \left(1 + \left(-408 + 2^{1/2}\cdot 288\right)\cdot t^5 + \left(-204 + 2^{1/2}\cdot 144\right)\cdot t^3 + \left(34 + 2^{1/2}\cdot(-24)\right)\cdot t^2 + \left(136 + 2^{1/2}\cdot(-96)\right)\cdot t^6 + \left(442 + 2^{1/2}\cdot(-312)\right)\cdot t^4\right)^{1/2}$$

$$\text{SOLUTIONS}\left(\frac{d}{dt}\, d^2 = 0,\ t\right)$$

$$\left[0,\ 1,\ \frac{1}{2},\ \frac{\sqrt{3}}{6}+\frac{1}{2},\ \frac{1}{2}-\frac{\sqrt{3}}{6}\right]$$

We observe that we have extrema at 0, 1/2 (guaranteed by symmetry), 1 and at two other values. By construction, the curve lies on the unit circle at 0, ½ and 1, leaving two points of interest, which are symmetric about x = ½ and should, by the symmetry of the figure, lie at the same distance from the origin. This then is the quantity we are interested in.

We simply copy it from the CAS and paste into the *Geometry Expressions*, replacing x as the parametric location of point L (figure 7).

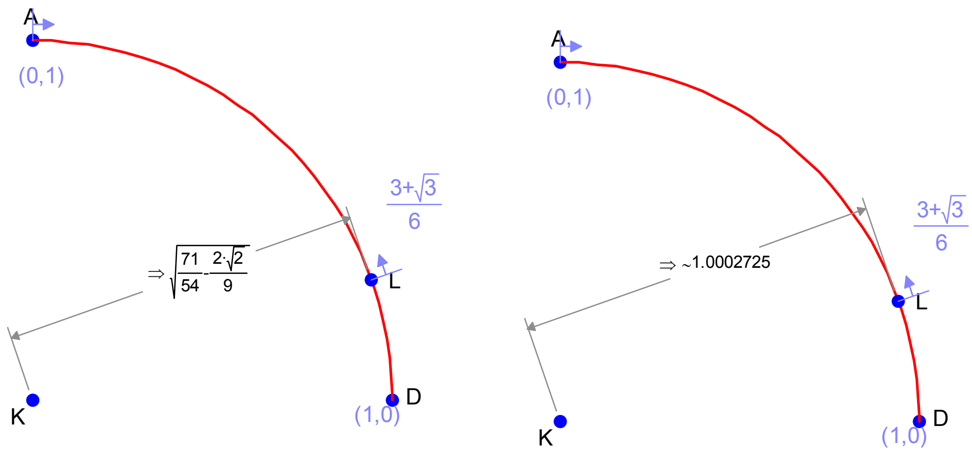


Figure 7: Maximal distance of point L is obtained by pasting the solution for x into the parametric location constraint for point L. Numeric display shows the error to be less that 3/10,000

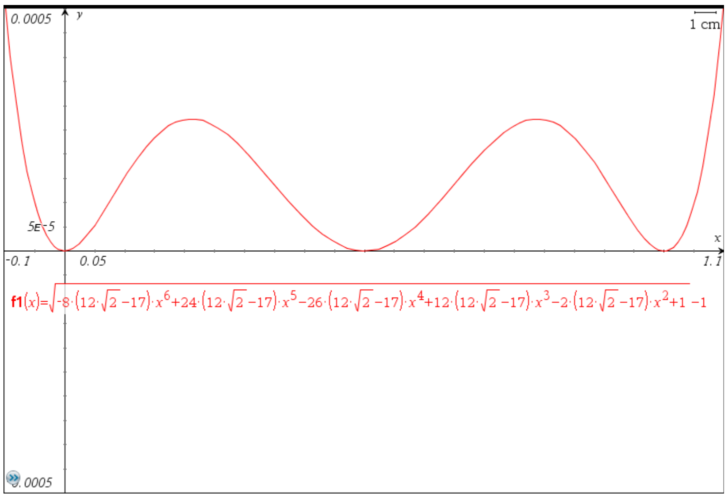
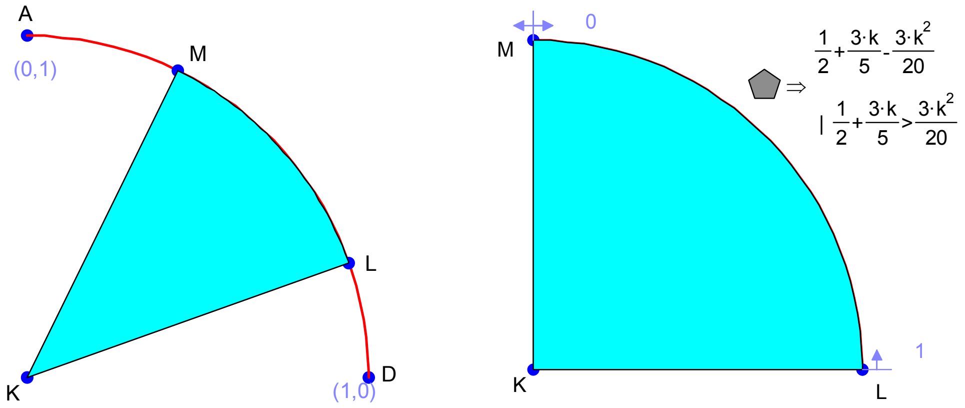


Figure 8: Error graphed as a function of parametric location on the curve.

An error function may be obtained by subtracting 1 (the radius of the circle) from the distance function measured in figure 6. This is displayed in figure 8. We observe that this spline curve always errs on the side of being too large. For some applications this may be a benefit, however, we might ask ourselves whether it would be possible to improve our estimate if we allowed the spline to err both inside and outside the circle. We'll look at one such curve in the next section.

3. Equal Area

One way of ensuring that the spline errs both on the inside and on the outside of the unit circle is to insist that the area under the spline is the same as the area under the quarter circle. To measure the area under the spline in *Geometry Expressions*, you need to create first an arc LM on the spline, then draw line segments LK and KM (where K is the origin). You can then select the segments and the arc and create a polygon. As a final step, you will make sure L and M lie at the start and end of the curve by constraining their parametric locations to 0 and 1.



$$\left[\frac{1}{2}+\frac{k\cdot 3}{5}+k^2\cdot\frac{-3}{20} \quad \frac{1}{2}+\frac{k\cdot 3}{5}+k^2\cdot\frac{-3}{20}>0\right]$$
$$\left[\frac{-3\cdot k^2}{20}+\frac{3\cdot k}{5}+\frac{1}{2} \quad \frac{-3\cdot k^2}{20}+\frac{3\cdot k}{5}+\frac{1}{2}>0\right]$$
$$\text{solve}\left(\frac{1}{2}+\frac{k\cdot 3}{5}+k^2\cdot\frac{-3}{20}=\frac{\pi}{4},k\right)$$
$$k=\frac{-\left(\sqrt{-3\cdot(5\cdot\pi-22)}-6\right)}{3} \text{ or } k=\frac{\sqrt{-3\cdot(5\cdot\pi-22)}+6}{3}$$
$$\left(k=\frac{-\left(\sqrt{-3\cdot(5\cdot\pi-22)}-6\right)}{3} \text{ or } k=\frac{\sqrt{-3\cdot(5\cdot\pi-22)}+6}{3}\right)\blacktriangleright\text{Decimal}$$
$$k=0.551778 \text{ or } k=3.44822$$
3/99

Figure 9: The area under the spline is measured in Geometry Expressions by creating an arc LM on the spline, connecting L and M to the origin and creating a curve-sided polygon KLM. Finally, L and M are constrained to lie at parametric locations 1 and 0 on the curve. In our CAS, we can find the value of k which makes the area under the spline equal to the area under a quarter unit circle.

Copying the area value into the CAS we can equate it with π/4 (the area under the quarter unit circle), and solve for k. We have two solutions. The first is numerically close to the value of k which puts the center point on the unit circle. The second is too far removed to be reasonable. Figure 10 shows what they look like:

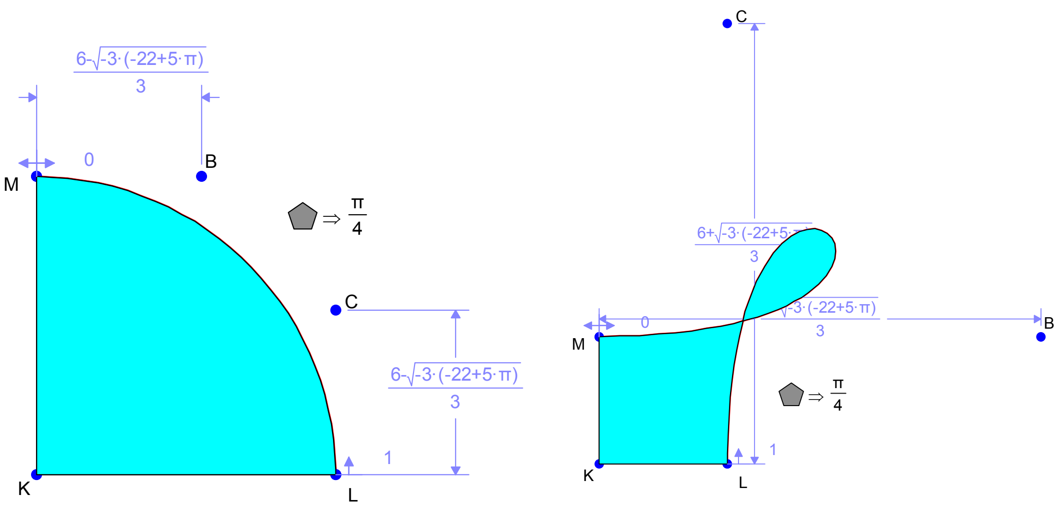


Figure 10: two solutions yielding the same area under the spline. Only one is a good approximation for the quarter circle!

We can paste this value back into *Geometry Expressions* for k, and let the software derive the distance from the origin to the point on the curve at parametric location x (figure 11. This can in turn be copied into TI-Nspire for graphing (figure 12).

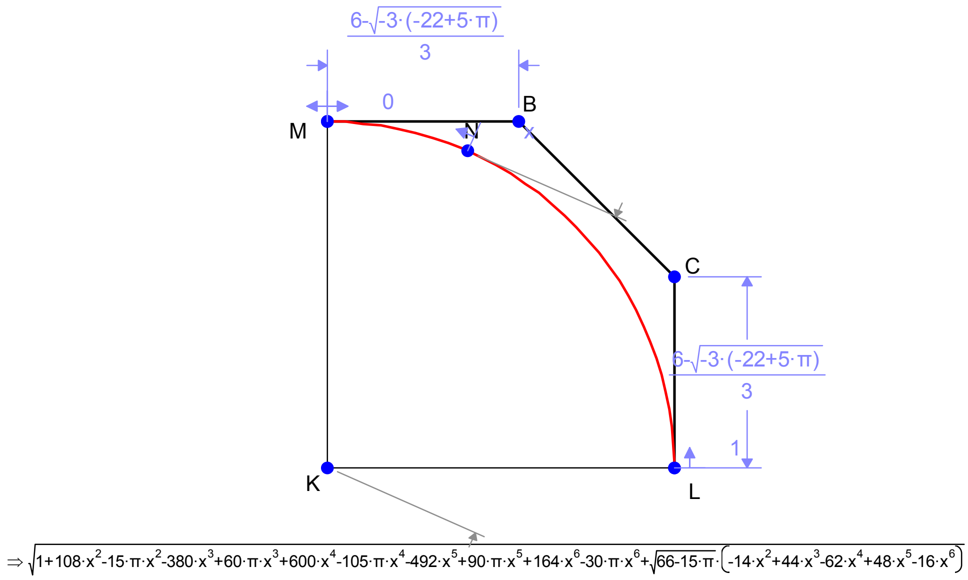


Figure 11: distance to the origin for point at parametric location x on the equiareal approximation

Analysis of the graph shows that while we have succeeded in reducing the extent to which the approximate curve overshoots the circle, we have done so at a cost of undershooting it by almost as much at the center point.

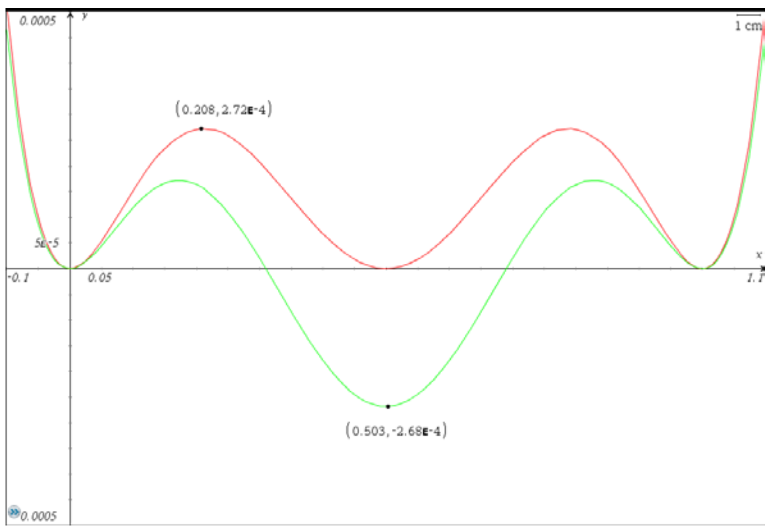


Figure 12: Radius for the equiareal approximation is copied into TI-Nspire from Geometry Expressions, and plotted next to the area for the central-point approximation.

However, this suggests an approach for finding a better solution. Can we find a value of k such that the error function undershoots by the same amount that it overshoots?

4. Minimizing the maximum error

Using *Geometry Expressions*, we can generate a distance function for a point at parametric location x, copy this into TI-Nspire CAS and differentiate to find the extrema (figure 13,14).

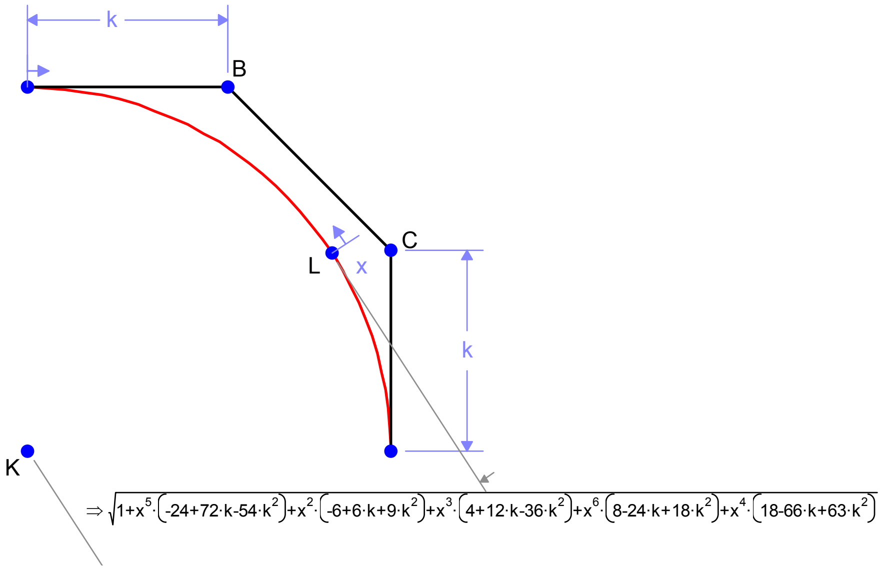


Figure 13: Distance from the origin of the generic point on the spline with parameter k

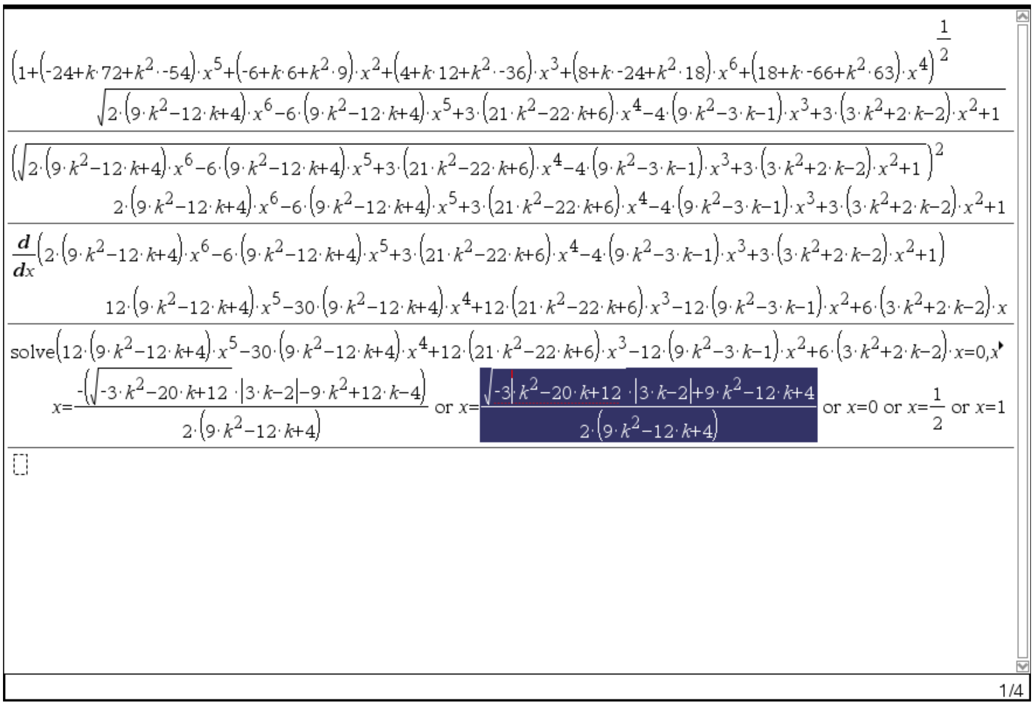


Figure 14: Extrema for the curve of figure 12 calculated in the TI-nSpire CAS

The extrema lie at x = ½ and at two more complicated points. We can copy and paste the complicated points into *Geometry Expressions* and let it worry about the algebra. The distance value (figure 15) is surprisingly simple.

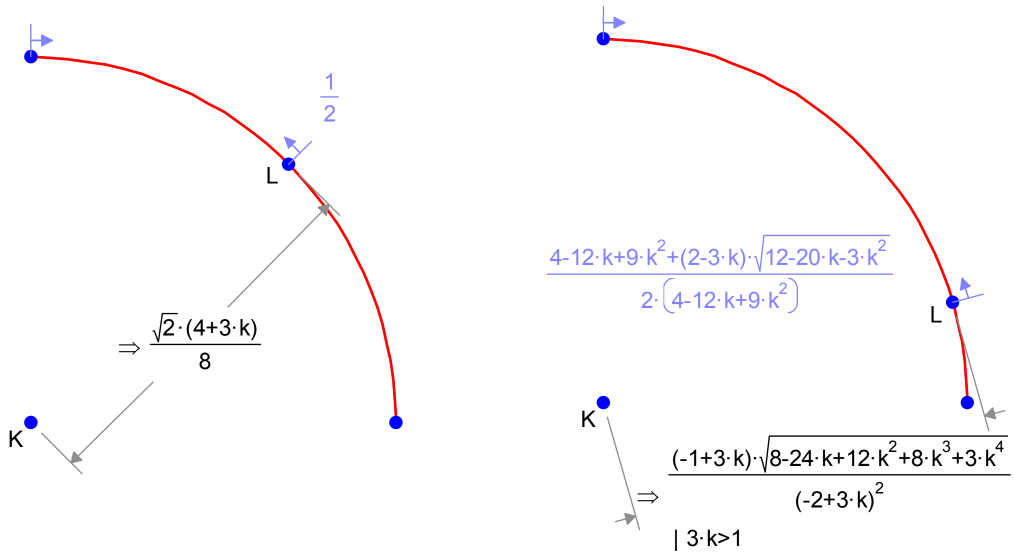


Figure 15: Extreme values for the distance from the origin on the spline curve with parameter k

In the TI-Nspire CAS we define a variable error1 to be the distance at the midpoint subtracted from 1. We define error2 to be the distance at the more complicated extremum minus 1. The CAS will find, numerically, a value of k which equates these errors (figure 16).

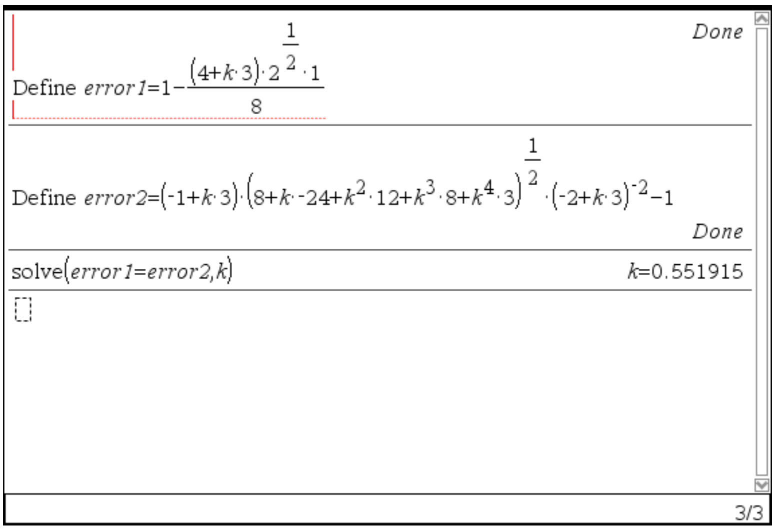


Figure 16: Numerical solution for k, equating the absolute values of the two errors

A plot of the error functions (figure 17) reveals that the minimax solution reduces the maximal error by over 25% compared with either of the other approximations.

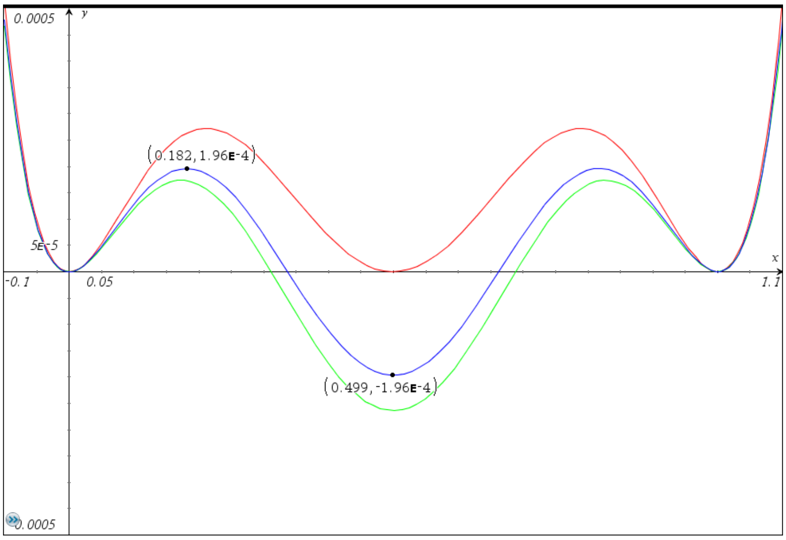


Figure 17: Error curve for the minimax solution lies between the error curves for the central point solution and the equiareal solution.

5. Conclusion

We have looked at modeling a quarter circle. It is also useful to be able to approximate a general portion of a circular arc. The development of spline approximations for general arcs yields more complicated formulas, but you can use a similar process, and the CAS can take care of the details. You do need to be careful with the minimax solution as the location of the extremum is complicated enough that computing its value can cause problems both for *Geometry Expressions* and the TI-Nspire.

A straightforward, but interesting analysis would be to investigate the improvement in accuracy to be obtained by approximating a quarter circle by two 45 degree spline approximations.

Another question would be this: let's say we are approximating a full circle by splines, each of which approximates one quarter of the circle. It is a little artificial to insist that the spline meets the circle at the axes. Why not allow the spline to start and finish at a radius slightly below that of the circle being approximated. In this way we could essentially translate the error curve for the center point approximation vertically down by half its amplitude and achieve a 50% reduction in error.

The huge benefit of having such powerful software tools as *Geometry Expressions* and the TI-Nspire CAS working together under your direction is that they allow you to explore many more interesting variants of a problem than you might be inclined to undertake working only by hand.

The beauty of *Geometry Expressions* is that it assists in the mathematical modeling phase of problem formulation, allows you to copy the mathematics into your CAS, and then assists in interpreting the results, by displaying the solution. In this way degenerate or nonsensical solutions can be immediately differentiated visually from sensible solutions. Such was the case in figure 10, where a picture immediately showed us that one solution involved a spline with a loop.

References

[1] www.geometryexpressions.com

[2] Farin, Gerald & Hansford, Dianne (2000). The Essentials of CAGD.: A K Peters, Ltd. ISBN 1-56881-123-3

Here are some screenshots from my *Geometry Expressions – DERIVE* collaboration:

$$\left[x_0 + x_0\cdot t\cdot(-3) + x_0\cdot t^2\cdot 3 + x_0\cdot t^3\cdot(-1) + x_1\cdot t\cdot 3 + x_1\cdot t^2\cdot(-6) + x_1\cdot t^3\cdot 3 + x_2\cdot t^2\cdot 3 + x_2\cdot t^3\cdot(-3) + x_3\cdot t^3,\ y_0 + y_0\cdot t\cdot(-3) + y_0\cdot t^2\cdot 3 + y_0\cdot t^3\cdot(-1) + y_1\cdot t\cdot 3 + y_1\cdot t^2\cdot(-6) + y_1\cdot t^3\cdot 3 + y_2\cdot t^2\cdot 3 + y_2\cdot t^3\cdot(-3) + y_3\cdot t^3\right]$$

$$\left[0 + 0\cdot t\cdot(-3) + 0\cdot t^2\cdot 3 + 0\cdot t^3\cdot(-1) + k\cdot t\cdot 3 + k\cdot t^2\cdot(-6) + k\cdot t^3\cdot 3 + 1\cdot t^2\cdot 3 + 1\cdot t^3\cdot(-3) + 1\cdot t^3,\ 1 + 1\cdot t\cdot(-3) + 1\cdot t^2\cdot 3 + 1\cdot t^3\cdot(-1) + 1\cdot t\cdot 3 + 1\cdot t^2\cdot(-6) + 1\cdot t^3\cdot 3 + k\cdot t^2\cdot 3 + k\cdot t^3\cdot(-3) + 0\cdot t^3\right]$$

$$\left[k\cdot(3\cdot t^3 - 6\cdot t^2 + 3\cdot t) - 2\cdot t^3 + 3\cdot t^2,\ k\cdot(3\cdot t^2 - 3\cdot t^3) + 2\cdot t^3 - 3\cdot t^2 + 1\right]$$

$$\text{error1} := 1 - \frac{(4 + k\cdot 3)\cdot 2^{1/2}\cdot 1}{8}$$

$$\text{error2} := \frac{(-1 + k\cdot 3)\cdot(8 + -24\cdot k + 12\cdot k^2 + 8\cdot k^3 + 3\cdot k^4)^{1/2}}{(3\cdot k - 2)^2} - 1$$

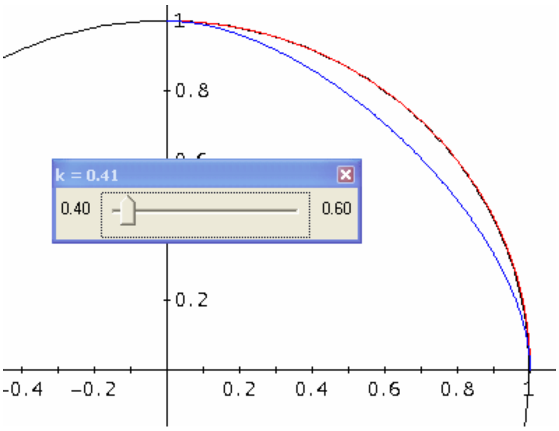
NSOLVE(error1 = error2, k)

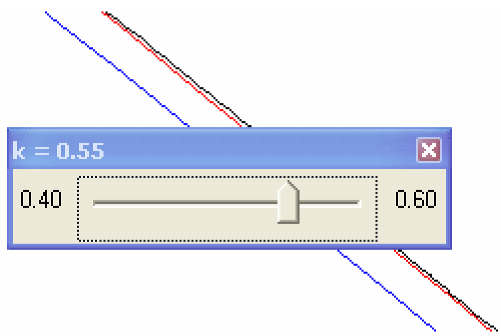
$k = 0.5519150238$

$x^2 + y^2 = 1$

$$\left[0.5519150238\cdot(3\cdot t^3 - 6\cdot t^2 + 3\cdot t) - 2\cdot t^3 + 3\cdot t^2,\ 0.5519150238\cdot(3\cdot t^2 - 3\cdot t^3) + 2\cdot t^3 - 3\cdot t^2 + 1\right]$$

I introduce a slider bar in order to vary the parameter *k*.





The blue curve changes when moving the slider, the black line is the circle line and the spline is plotted in red color.

I don't show plots of the error curves, it is too obvious that this is no problem for *DERIVE* (and other CAS, of course).

Diophantine Polynomials[1] (Part 2)

Duncan E. McDougall, duncanemcdougall@hotmail.com

(1)

In the pursuit of families of polynomials which are factorable in the set of integers and whose derivatives are factorable in the set of rational numbers, it is amazing how much easier the process is when we incorporate an old trick from the remainder theorem. I refer to taking the sum of all the numerical coefficients. If this sum is zero, then the number *1* is an automatic root; if not, it is not. In essence, we plan to have one as a root by making sure that the selection of numerical coefficients add up to zero in any of the cubic or quartic forms. In terms of the quintic, it does simplify this form to some extent, but Galois Theory promises that we will have irrational numbers in exact form.

To begin, examine all three degrees in various forms with one as an automatic root and obtain new families of curves.

Cubic Polynomials: Form $y = ax^3 - bx^2 + (b-a)x$

Quickly, $a - b + (b-a)$ is zero and so 1 is root (remainder Theorem). Using synthetic division gives

1	*a*	*-b*	*b-a*
		a	*a-b*
	a	*a-b*	*0*

or

$$(x-1)\cdot(ax^2 + (a-b)x)$$
$$= x\cdot(x-1)\cdot(ax + (a-b))$$

For integral roots, "*b*" needs to be a multiple of "*a*" and this will guarantee that the original polynomial is factorable. Now take the derivative and look for rational roots; this gives

$$y' = 3ax^2 - 2bx + (b-a)$$
$$0 = 3ax^2 - 2b\cdot x + (b-a)$$

(2)

Now test the discriminant $B^2 - 4AC$ to determine values for which the polynomial is factorable. Here $B = -2b, A = 3a,$ and $C = b - a$ and this gives

$$B^2 - 4AC = (-2b)^2 - 4(3a)(b-a)$$
$$= 4b^2 - 12ab + 12a^2$$
$$= 4(b^2 - 3ab + 3a^2)$$

Here 4 is a perfect square, but $b^2 - 3ab + 3a^2$ is a perfect square for a limited number of values.

Fixing values of *a* in order to obtain values of *b* to make the expression $b^2 - 3ab + 3a^2$, a perfect square, the following table is produced:

[1]Diophantine Polynomials (Part 1) appeared in DNL#81.

TABLE I

Value of *a*	Resulting Polynomial	Corresponding Values of *b*
1	b^2-3b+3	1, 2
2	$b^2-6b+12$	2, 4
3	$b^2-9b+27$	-2, 3, 6, 11
4	$b^2-12b+48$	4, 8
5	$b^2-15b+75$	-11, 2, 5, 10, 13, 26
6	$b^2-18b+108$	-4, 6, 12, 22
7	$b^2-21b+147$	-26, -1, 7, 14, 22, 47
8	$b^2-24b+192$	1, 8, 11, 13, 16, 23
9	$b^2-27b+243$	-47, -6, 9, 18, 33
10	$b^2-30b+300$	-22, 4, 10, 20, 26

In the interest of discovering single roots only, those values of b which are multiples of *a* are eliminated from consideration because we get double roots. Thus no values of *b* work for $a=1$ or $a=2$, but we do get values for *b* when $a=3$ namely $b=-2$ and $b=11$. There are obviously more values for *b* but do not consider them in order to keep the factors relatively small and easy to calculate. The above values are easy to spot on the TI graphing calculator by entering $y=x^2-3x+3$, and pressing 2nd Function Table.

Simply look for those values of y which are perfect squares.

(3)

An example of a cubic can be taken from TABLE I where *a* = 3 and *b* = 11. This gives

$$y=3x^3-11x^2+8x$$
$$y=x\cdot(3x^2-11x+8)$$
$$0=x\cdot(3x^2-11x+8)$$
$$0=x\cdot(3x^2-3x-8x+8)$$
$$0=x\cdot\left(3x\cdot(x-1)-8\cdot(x-1)\right)$$
$$0=x\cdot(x-1)\cdot(3x-8)$$

where $x=0, x=1$ and $x=\frac{8}{3}$

and for the first derivative see next page.

$y = 3x^3 - 11x^2 + 8x$
$y' = 9x^2 - 22x + 8$
$y' = 9x^2 - 18x - 4x + 8$
$0 = 9x \cdot (x-2) - 4 \cdot (x-2)$
$0 = (9x-4) \cdot (x-2)$

where $x = \frac{4}{9}, x = 2$ *

* This gives rise to a new family of cubics whose original form and derivative form are both factorable.

Polynomial	Derivative
$x \cdot (x-1) \cdot (3x-8)$	$(x-2) \cdot (9x-4)$
$(x-1) \cdot (x-2) \cdot (3x-11)$	$(x-3) \cdot (9x-13)$
$(x-2) \cdot (x-3) \cdot (3x-14)$	$(x-4) \cdot (9x-22)$
$(x-3) \cdot (x-4) \cdot (3x-17)$	$(x-5) \cdot (9x-31)$

Quartic Form

Moving on to the quartic form, examine $y = ax^4 - bx^2 + (b-a)$. (We do not worry about missing consecutive powers of *x* because we can translate the above form either right or left and all the consecutive powers of *x* will appear.)

Again examine the discriminate knowing that 1 is an automatic root.
This becomes $(-b)^2 - 4(a)(b-a) = b^2 - 4ab + 4a^2 = (b-2a)^2$. Thus, the expression $ax^4 - bx^2 + (b-a)$ is always factorable regardless of the values for *b* and *2a*.

(4)

Take the derivative, in order to see what values of *a* and *b* would make it factorable; this gives

$y = a \cdot x^4 - b \cdot x^2 + (b-a)$
$y' = 4a \cdot x^3 - 2b \cdot x$
$0 = 2x \cdot (2a \cdot x^2 - b)$
$2x = 0, 2a \cdot x^2 - b = 0$
$2a \cdot x^2 = b$
$x^2 = \frac{b}{2a}$

In order to get integral roots, $\frac{b}{2a}$ must equal a perfect square such as 1, 4, 9, 16…

Thus

$\frac{b}{2a} = 1, 4, 9, 16, 25...$
$b = 2a, 8a, 18a, 32a, 50a...$

So going back to the original $y = ax^4 - bx^2 + (b-a)$, substitute $b = 2a$ and obtain
$y = ax^4 - 2ax^2 + a = a(x^4 - 2x^2 + 1)$ which gives double roots (disregard); letting $b = 8a$,
$y = ax^4 - 8ax^2 + 7a = a(x^4 - 8x^2 + 7) = a(x^2 - 7)(x^2 - 1)$ which contains irrational roots.

Continue the search until rational roots appear (TABLE II).

TABLE II

Value of *b*	Resulting Polynomial	Factors of Polynomial
2a	ax^4-2ax^2+a	$a(x^4-2x^2+1)=a(x^2-1)^2$
8a	ax^4-8ax^2+7a	$a(x^4-8x^2+7)=a(x^2-1)(x^2-7)$
18a	ax^4-18ax^2+17a	$a(x^4-18x^2+17)=a(x^2-1)(x^2-17)$
32a	ax^4-32ax^2+31a	$a(x^4-32x^2+31)=a(x^2-1)(x^2-31)$
50a	ax^4-50ax^2+49a	$a(x^4-50x^2+4a)=a(x^2-1)(x^2-49)$

(5)

Here $a\cdot(x^2-1)\cdot(x^2-49)=a\cdot(x+1)\cdot(x-1)\cdot(x-7)\cdot(x+7)$ and the ensuing sequence for TABLE II 1, 7, 17, 31, 49, … $2n^2-1$ does not yield any perfect squares for relatively small values but a new family of quartics is acquired because the derivative is also factorable at *n = 7*.

or,

$$y=a\cdot(x^4-50x^2+49)$$
$$y'=a\cdot(4x^3-100x)$$
$$y'=a\cdot(4x)\cdot(x^2-25)$$
$$y'=a\cdot(4x)\cdot(x-5)\cdot(x+5)$$

thus for

$$0=a\cdot(4x)\cdot(x-5)\cdot(x+5)$$
$$x=0, x=5, x=-5$$

TABLE III reflects the new family of quartics and some of the family members. It suffices to say that translation left or right can go on indefinitely, but the selection is limited to the list below because the numbers become large.

TABLE III:

Family	Translated Form	Derivative
$(x-1)(x+1)(x-7)(x+7)$	$x(x+2)(x-6)(x+9)$	$4(x+1)(x+6)(x-4)$
	$(x+1)(x+3)(x-5)(x+9)$	$4(x+2)(x+7)(x-3)$
	$(x+2)(x+4)(x-4)(x+10)$	$4(x+3)(x+8)(x-2)$
	$(x+3)(x+5)(x-3)(x+11)$	$4(x+4)(x+9)(x-1)$
	$(x+4)(x+6)(x-2)(x+12)$	$4x(x+5)(x+10)$
	$(x+5)(x+7)(x-1)(x+13)$	$4(x+6)(x+11)(x+1)$
	$x(x+6)(x+8)(x+14)$	$4(x+7)(x+12)(x+2)$

(6)

Quintic Form

The best we can hope for here is a mix of rational and irrational numbers (single roots), but in exact form (Galois Theory). Again planting the root of 1, by making all the numerical coefficients add up to zero, examine $y = ax^5 - bx^3 + (b-a)x.$. A serendipitous result here is that -1 is also a root just like in the quartic family of polynomials, (quartic form $ax^4 - bx^2 + (b-a)$).

In polynomial form, this yields

$$y = ax^5 - bx^3 + (b-a)x$$
$$y = x(ax^4 - bx^2 + (b-a))$$

and

$$\begin{aligned} B^2 - 4AC &= (-b)^2 - 4a(b-a) \\ &= b^2 - 4ab + 4a^2 \\ &= (b-2a)^2 \quad \text{always factorable} \end{aligned}$$

thus

$$y = x(x-1)(x+1)\left(ax^2 - (b-a)\right)$$
$$y = x(x-1)(x+1)\cdot a\left(x^2 - \left(\sqrt{\frac{b}{a}-1}\right)^2\right)$$
$$y = a\cdot x(x-1)(x+1)\left(x - \sqrt{\frac{b}{a}-1}\right)\left(x + \sqrt{\frac{b}{a}-1}\right)$$

(7)

So for integral roots, $\frac{b}{a} - 1$ must be a perfect square or

$$\frac{b}{a} - 1 = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100...$$
$$\frac{b}{a} = 2, 5, 10, 17, 26, 37, 50, 65, 82, 101...$$

or $\quad b = 2a, 5a, 10a, 17a, 26a, 37a, 50a, 65a, 82a, 101a...$

For an example: let $b = 26a$ then

$$y = ax^5 - 26ax^3 + 25ax = ax(x^4 - 26x^2 + 25) = ax(x^2 - 25)(x^2 - 1) = ax(x-5)(x+5)(x-1)(x+1)$$

In terms of the derivative

$$y = ax^5 - bx^3 + (b-a)x$$
$$y' = 5ax^4 - 3bx^2 + b - a$$

and $B^2 - 4AC = (-3b)^2 - 4(5a)(b-a)$

$$= 9b^2 - 20ab + 20a^2$$

Finding values *a* and *b* for which the above is factorable gives rise to TABLE IV.

(8)

TABLE IV

Value of *a*	Resulting Polynomial	Values of *b*
1	$9b^2 - 20b + 20$	-1, 1, 2
2	$9b^2 - 40b + 80$	-2, 1, 2, 4, 11
3	$9b^2 - 60b + 180$	-3, 3, 6
4	$9b^2 - 80b + 320$	-4, 2, 4, 8, 11, 22
5	$9b^2 - 100b + 500$	-22, -5, 5, 10
6	$9b^2 - 120b + 720$	-6, 3, 6, 12, 33
7	$9b^2 - 140b + 980$	-19, -7, -2, 7, 11, 14, 29
8	$9b^2 - 160b + 1280$	4, 8, 16, 22, 44
9	$9b^2 - 180b + 1620$	2, 9, 11, 18, 29
10	$9b^2 - 200b + 2000$	5, 10, 20, 38
etc.		

Unfortunately, negative values of *b* in TABLE IV create complex roots in the original polynomial and multiples of *"a"* create double or triple roots. Thus our choices are quite limited when stating new families. Hence, there are no monic polynomials (*a* = 1*)* but there are families where $a \neq 1 ... \rightarrow$. A good example here is *a = 9* and *b = 29* and the analysis gives the following results:

Here

$$y = 9x^5 - 29x^3 + 20x = x(9x^4 - 29x^2 + 20) = x(9x^4 - 9x^2 - 20x^2 + 20) = x(9x^2(x^2-1) - 20(x^2-1))$$

and the roots are

$$0 = x(x^2-1)(9x^2-20) = x(x-1)(x+1)(3x-2\sqrt{5})(3x+2\sqrt{5})$$
$$x = 0,\ x = 1,\ x = -1,\ x = \frac{2\sqrt{5}}{3},\ x = \frac{-2\sqrt{5}}{3}$$

(9)

for the derivative

$$
\begin{aligned}
y &= 9x^5 - 29x^3 + 20x \\
y' &= 45x^4 - 87x^2 + 20 \\
y' &= 45x^4 - 75x^2 - 12x^2 + 20 \\
0 &= 15x^2(3x^2 - 5) - 4(3x^2 - 5) \\
0 &= (3x^2 - 5)(15x^2 - 4) \\
0 &= \left(x\sqrt{3} - \sqrt{5}\right)\left(x\sqrt{3} + \sqrt{5}\right)\left(x\sqrt{15} - 2\right)\left(x\sqrt{15} + 2\right)
\end{aligned}
$$

$$x_1 = \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{3} \qquad x_2 = \frac{-\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{15}}{3}$$

$$x_3 = \frac{2}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{2\sqrt{15}}{15} \qquad x_4 = \frac{-2}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{-2\sqrt{15}}{15}$$

The reader is left to develop other such families where 2, 3, 4 … are automatic roots. These can all be achieved using the remainder theorem.

Summary:

The sole objective of this paper was to present both families of polynomials which are easily factored and methods by which to determine them. The process is even easier if we plant 1, or 2, or 3 as roots making the original function factorable. And, since the only two numerical coefficients are *a* and *b*, we can determine values of *b* easily while holding *"a"* constant. Limiting the tables to small numbers is another effort to make successful curve sketching of polynomial functions more accessible to students of mathematics.

Last summer started an extended email exchange between Heinrich Ludwig and me. His mails are given below together with a summary in English. His Snub Cube turned out to be the starting point of an exciting journey into the world of polyhedrons. You are friendly invited to follow us.

Von: Heinrich Ludwig [heinrich.ludwig@gym-raubling.de]
Gesendet: Sonntag, 24. Juli 2011 11:27
Betreff: DERIVE: Gefüllte Darstellung eines Vielecks im 3D-Fenster

Lieber Herr Böhm,

meine Frage klingt simpel, ich habe aber in allen DNLs, die in pdf-Version verfügbar sind, keinen Hinweis auf eine Antwort gefunden.

Vielleicht könnten Sie mir weiterhelfen?

Wie stellt man mit DERIVE 6 ein n-Eck (Eckenzahl n > 4) dessen Ecken dreidimensionale Koordinaten haben, als Fläche gefärbt im 3D-Fenster dar? (Natürlich müssen die Eckpunkte in einer Ebene liegen.)

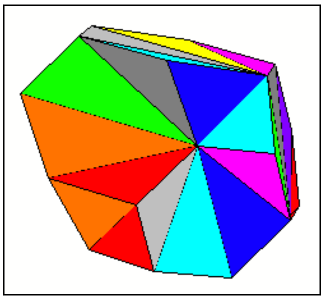
Für n=3 und n=4 habe ich eine Möglichkeit gefunden. Man bildet aus den Seiten des n-Ecks einen Vektor, wobei der Endpunkt der einen Strecke der Anfangspunkt der folgenden Strecke ist. Für n=5 klappt das nicht mehr, denn intern behandelt DERIVE dieses Konstrukt so, dass zwei aufeinander folgende Strecken als Dreieck aufgefasst werden, welches flächig gefärbt dann dargestellt wird.

Ich habe auch die Funktion POLY_FILL(u) ausprobiert, aber leider wird damit aus jedem Vieleck ein "Zebra", denn POLY_FILL unterteilt das Vieleck in Dreiecke, färbt diese verschieden ein und zeichnet auch die Diagonalen des Vielecks, welche es in Dreiecke unterteilen, als schwarze Striche ein.

Weil der Rechenaufwand zur Prüfung, ob ein Vieleck eben ist, beträchtlich sein dürfte (vermutlich n – 2 Kreuzprodukte und n – 3 Tests auf Kollinearität der Normalenvektoren) nehme ich nach all meinem Ausprobieren an, dass DERIVE nur auf das Einfärben von Dreiecken eingerichtet ist. Ich wollte archimedische Körper nicht nur mit ihrem Kantengerüst, sondern als gefüllte Körper darstellen. Da wäre es schön gewesen, wenn man auch 4-, 5-, 6-, 8- und 10-Ecke hätte einfärben können.

Eine Stelle im DNL fand ich, von der ich mir Hilfe versprochen hatte: Im DNL #77 haben Sie die Abituraufgabe mit DERIVE gelöst, die später den medienwirksamen Titel "Oktaeder des Grauens" erhalten hat. Das abgestumpfte Oktaeder haben Sie dann 3-dimensional dargestellt.

Ich habe die Datei horror.dfw, die in gezippter Form dem Newsletter beilag, mit DERIVE geladen und die entsprechenden Zeilen, die die POLYGON_FILL-Funktion enthalten, zum Zeichnen verwendet. Und jetzt kommt das, was mir unverständlich ist: Bei Ihnen zeichnet POLYGON_FILL schöne einfarbige Polygone, bei mir aber diese "Zebras". Was läuft da verkehrt? Gibt es eine Einstellung, die DERIVEs Farbwechselschema steuert? Ich finde nirgends etwas Geeignetes in den Menüs. Damit Sie sehen, was bei mir passiert, hänge ich ein Bild zu meinem Ergebnis an dieses Mail.



The Zebra

Ich höre jetzt auf mit meinen Nachforschungen und würde mich recht freuen, wenn Sie mir den entscheidenden Tipp geben könnten.

Mit besten Grüßen
Ihr Heinrich Ludwig

Heinrich Ludwig's problem from 24 July 2011 summarized:

How can I represent a planar polygon (number of vertices n > 4) in 3D-space as a coloured area?

I found a way for $n = 3$ and $n = 4$:

One has to form a vector consisting of the edges of the n-gon with the endpoint of one side being the start point of the next one. But this does not work for n > 4, because DERIVE interprets two subsequent segments as a triangle.

Then I tried the function POLY_FILL(u) but unfortunately every polygon appears as a "zebra". POLY_FILL divides the polygon into triangles, dyes them in different colours and plots all sides of all triangles in black.

I assume that DERIVE needs some calculation for testing if the points are lying in a common plane etc (which needs some cross products) and that the function is only intended for colouring triangles. I wanted to represent Archimedean solids not only as a wireframe model but as a filled solid. It would be great to dye 4-, 5-, 6- 8- and 10-gons.

I found one article in a DNL which promised some support.

In DNL#77 you presented a truncated octahedron ("Octahedron of Horror") using function POLYGON_FILL. I tried this function and I cannot understand what happened then: Your polygons are pretty one-coloured – and I received again "zebras". What am I doing wrong? Is there a DERIVE setting which controls the colour change.

I will stop now my investigations and would be happy if you could provide any hint.

Sincerely Yours

Heinrich Ludwig

See my answer

Dear Mr. Ludwig,

Your "zebra" will change to a "grey donkey" or a "brown stallion" if you switch off option "Automatically change color of new plots" under Options > Display > Color. Plotting the borders needs some "manual work":

Perform a right mouse click on each triangle and the Edit-popup window will open: Choose Scheme: Custom, then choose the colour for the polygon and set the same colour for the mesh lines.

Performing this procedure for all triangles should lead to the desired result.

Best regards

Josef

Von: Heinrich Ludwig [mailto:heinrich.ludwig@gym-raubling.de]
Gesendet: Dienstag, 26. Juli 2011 23:15
Betreff: kleines Dankeschön für Ihre Hilfe

Lieber Herr Böhm,

als kleines Dankeschön für Ihre freundliche Hilfestellung schicke ich Ihnen, wofür ich das Einfärben von Flächen verwenden wollte: einen archimedischen Körper. Mein Liebling ist der Cubus Simus. Er tanzt so schön aus der Reihe, ist eben nicht zentralsymmetrisch wie fast alle anderen. Außerdem zerfallen die Dreiecke unter den Seitenflächen des Cubus Simus topologisch in zwei Klassen, auch eine Besonderheit, die nur noch ein weiterer archimedischer Körper (das Dodekahedron Simum) hat.

In den DNL-Heften, sowohl in den alten, die Sie neu als pdf aufgelegt haben, wie auch in den neueren, habe ich nichts Explizites zu archimedischen Körpern gefunden, insbesondere keine Koordinatensätze für die Ecken. Ich kann mir nicht vorstellen, dass es keinen schönen Beitrag dazu gegeben hat. Er wird wohl in einem der Hefte 26 bis 52 stehen, für die es keine pdf-Version gibt und die ich darum nicht kenne. Weil so ein Beitrag sehr wahrscheinlich schon erschienen ist, arbeite ich die Sache nicht weiter aus. Für Schüler ist der Cubus Simus eine zu harte Nuss. Nicht einmal meinem Schüler, der sich in seiner Seminararbeit mit der 3D-Darstellung verschiedener Körper beschäftigt, traue ich das zu. (Es gibt ja genügend viele einfachere Körper auch noch.) Aber weil das Wetter in den letzten Tagen gar so bedauerlich war, habe ich selber ein Objekt zum Spielen gebraucht.

Der erste Schritt zur Konstruktion des Körpers war es, die Koordinaten der Ecken anzugeben. Ich wusste nur soviel über sie, dass sie auf die Seitenflächen eines Würfels gelegt werden können und dazu kannte ich die Symmetrien des Cubus Simus. Mit Hilfe zweier Parameter k und m, deren Werte noch zu bestimmen waren, konnte ich die Koordinaten formulieren. Der zweite Schritt bestand dann darin, die Ecken richtig miteinander zu verbinden. Dazu muss der Graph des Polyeders aufgestellt werden. Hat man beides, dann ist es nur noch eine Fleißaufgabe, die 38 Seitenflächen einzutippen.

Da ich's ohnehin nur für mich aufbewahre, genügt mir eine handschriftliche Skizze, wie man zu den Eckpunktskoordinaten kommen kann. Den Graphen habe ich mit Herrn Hohenwarters Geogebra gezeichnet (siehe Seite 25, Josef).

Es wird an unserer Schule gern verwendet und ich selber nutze es auch recht gerne.

Herzliche Grüße

Heinrich Ludwig

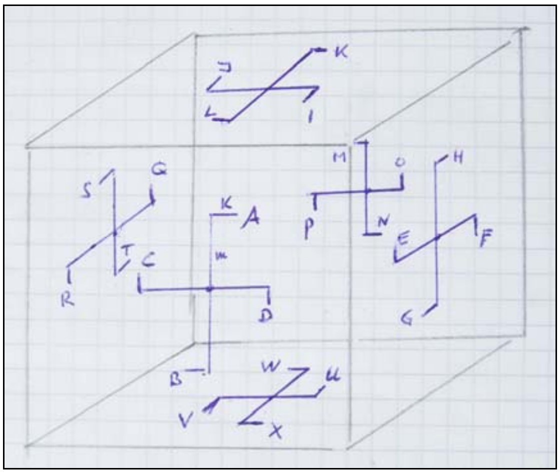
Heinrich Ludwig's mail from 26 July 2011 summarized:

As a little expression of thank I am sending an Archimedean solid. This was what I needed colouring areas for. My favourite solid is the Cubus Simus. It is pretty extraordinary because it is not central symmetric like almost all others. Additionally the faces are divided topologically in two classes. This is also a special feature only shared with one other Archimedean solid, the Dodecahedron Simum.

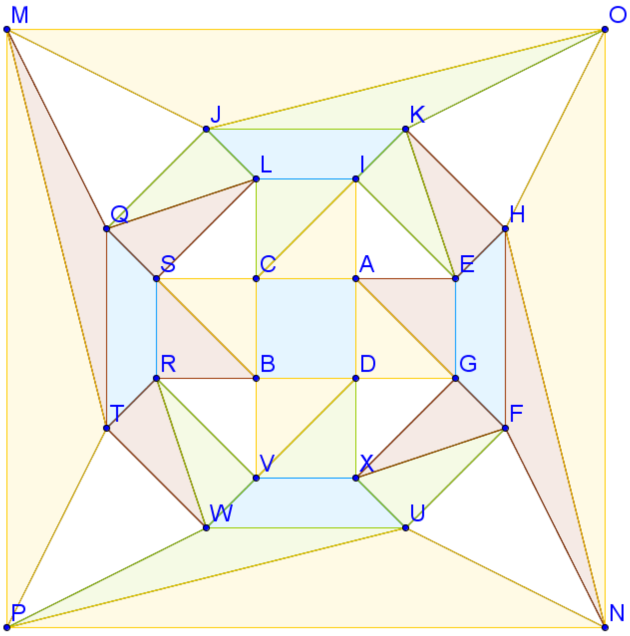
I didn't find any contribution on Archimedean Solids in earlier DNLs, in particular no sets of coordinates for the vertices. I wonder if there has no respective article appeared up to now. The Cubus Simus is a hard nut for students. Not even one of my students who works on a paper containing 3D-representations of various solids would be able to do this. (Fortunately enough there are many other easier solids, too.)

Because of bad weather in the last days I needed an object to play with.

The first step for constructing the solid was to express the coordinates of its vertices. All what I knew was that they are located on the faces of a cube – and I knew the symmetries of the Cubus Simus. Using two parameters k and m (their values had to be found) I could express the coordinates. (see my sketch).



The second step was to connect the vertices (24) in the right way (60 edges forming 6 squares and 32 triangles). This needs composing the graph of the polyhedron (see the GeoGebra graph). The rest is some diligent work: entering the 38 faces of the solid.



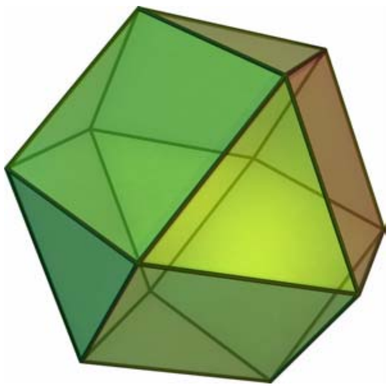
The GeoGebra Graph

Der kronrandige Würfel, das abgeschrägte Hexaeder, Cubus Simus, Snub Cube

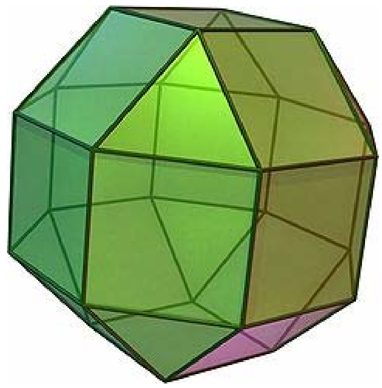
Heinrich Ludwig, Germany

Der Körper ist dem Würfel mit den Ecken [±1, ±1, ±1] einbeschrieben. Alle Ecken des Körpers liegen auf den Seitenflächen des Würfels. Die Koordinaten der 24 Ecken A bis X sind ±1, ±k oder ±m. Die genauen Werte für k und m werden unten hergeleitet. Es ist auch interessant zu sehen, wie sich der Körper verändert, wenn man k und m mit Schiebern variiert. Man kann auch ein Kubooktaeder oder ein kleines Rhombikubooktaeder erzeugen.

The solid is inscribed a cube with vertices [±1, ±1, ±1]. All 24 vertices – A through X – lie in the faces of the cube with coordinates ±1, ±k or ±m. The exact values for k and m are derived below. It is interesting to observe how the solid changes varying k and m by applying sliders. It is possible to create a cuboctahedron or a small rhombicuboctahedron.



Cuboctahedron



Small Rhombicuboctahedron

Here are the 24 vertices:

```
#6:   [A := [1, k, m], B := [1, -k, -m], C := [1, -m, k], D := [1, m, -k]]
#7:   [E := [m, 1, k], F := [-m, 1, -k], G := [k, 1, -m], H := [-k, 1, m]]
#8:   [I := [k, m, 1], J := [-k, -m, 1], K := [-m, k, 1], L := [m, -k, 1]]
#9:   [M := [-1, -k, m], N := [-1, k, -m], O := [-1, m, k], P := [-1, -m, -k]]
#10:  [Q := [-m, -1, k], R := [m, -1, -k], S := [k, -1, m], T := [-k, -1, -m]]
#11:  [U := [-k, m, -1], V := [k, -m, -1], W := [-m, -k, -1], X := [m, k, -1]]
```

These are the 6 squares followed by 32 triangles:

$$\left[ADBC := \begin{bmatrix} A & D \\ D & B \\ B & C \\ C & A \end{bmatrix}, EHFG := \begin{bmatrix} E & H \\ H & F \\ F & G \\ G & E \end{bmatrix}, ILJK := \begin{bmatrix} I & L \\ L & J \\ J & K \\ K & I \end{bmatrix}\right]$$

$$\left[MPNO := \begin{bmatrix} M & P \\ P & N \\ N & O \\ O & M \end{bmatrix}, QTRS := \begin{bmatrix} Q & T \\ T & R \\ R & S \\ S & Q \end{bmatrix}, UXVW := \begin{bmatrix} U & X \\ X & V \\ V & W \\ W & U \end{bmatrix}\right]$$

$$\left[AEI := \begin{bmatrix} A & E \\ E & I \\ I & A \end{bmatrix}, CAI := \begin{bmatrix} C & A \\ A & I \\ I & C \end{bmatrix}, CIL := \begin{bmatrix} C & I \\ I & L \\ L & C \end{bmatrix}, IEK := \begin{bmatrix} I & E \\ E & K \\ K & I \end{bmatrix}\right]$$

$$\left[EHK := \begin{bmatrix} E & H \\ H & K \\ K & E \end{bmatrix}, AGE := \begin{bmatrix} A & G \\ G & E \\ E & A \end{bmatrix}, DGA := \begin{bmatrix} D & G \\ G & A \\ A & D \end{bmatrix}, LCS := \begin{bmatrix} L & C \\ C & S \\ S & L \end{bmatrix}\right]$$

$$\left[JQM := \begin{bmatrix} J & Q \\ Q & M \\ M & J \end{bmatrix}, KOH := \begin{bmatrix} K & O \\ O & H \\ H & K \end{bmatrix}, DGX := \begin{bmatrix} D & G \\ G & X \\ X & D \end{bmatrix}, RVB := \begin{bmatrix} R & V \\ V & B \\ B & R \end{bmatrix} \right]$$

$$\left[PWT := \begin{bmatrix} P & W \\ W & T \\ T & P \end{bmatrix}, RWV := \begin{bmatrix} R & W \\ W & V \\ V & R \end{bmatrix}, RTW := \begin{bmatrix} R & T \\ T & W \\ W & R \end{bmatrix}, PUW := \begin{bmatrix} P & U \\ U & W \\ W & P \end{bmatrix} \right]$$

$$\left[PNU := \begin{bmatrix} P & N \\ N & U \\ U & P \end{bmatrix}, XUF := \begin{bmatrix} X & U \\ U & F \\ F & X \end{bmatrix}, UNF := \begin{bmatrix} U & N \\ N & F \\ F & U \end{bmatrix}, FNH := \begin{bmatrix} F & N \\ N & H \\ H & F \end{bmatrix} \right]$$

$$\left[HNO := \begin{bmatrix} H & N \\ N & O \\ O & H \end{bmatrix}, JKO := \begin{bmatrix} J & K \\ K & O \\ O & J \end{bmatrix}, JOM := \begin{bmatrix} J & O \\ O & M \\ M & J \end{bmatrix}, QMT := \begin{bmatrix} Q & M \\ M & T \\ T & Q \end{bmatrix} \right]$$

$$\left[TMP := \begin{bmatrix} T & M \\ M & P \\ P & T \end{bmatrix}, VDB := \begin{bmatrix} V & D \\ D & B \\ B & V \end{bmatrix}, VXD := \begin{bmatrix} V & X \\ X & D \\ D & V \end{bmatrix}, GFX := \begin{bmatrix} G & F \\ F & X \\ X & G \end{bmatrix} \right]$$

$$\left[LJQ := \begin{bmatrix} L & J \\ J & Q \\ Q & L \end{bmatrix}, QSL := \begin{bmatrix} Q & S \\ S & L \\ L & Q \end{bmatrix}, SBC := \begin{bmatrix} S & B \\ B & C \\ C & S \end{bmatrix}, RBS := \begin{bmatrix} R & B \\ B & S \\ S & R \end{bmatrix} \right]$$

Calculation of appropriate values for *k* and *m*:

The edges CA, AI and IC must be of equal length.

$$|C - A|^2 = 2\cdot(k^2 + m^2)$$

$$|A - I|^2 = 2\cdot(k^2 - k\cdot(m + 1) + m^2 - m + 1)$$

$$|I - C|^2 = 2\cdot(k^2 - 2\cdot k + 2\cdot m^2 + 1)$$

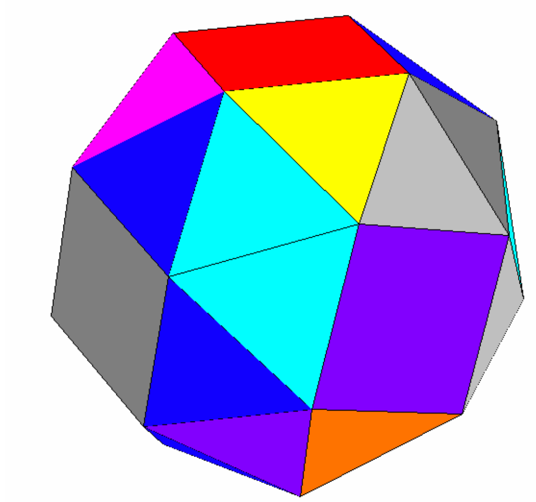
$$\mathrm{SOLVE}(|A - I|^2 = |C - A|^2 \wedge |I - C|^2 = |C - A|^2, [k, m], \mathrm{Real})$$

$$\text{SOLVE}(|A - I|^2 = |C - A|^2 \wedge |I - C|^2 = |C - A|^2, [k, m], \text{Real})$$

$$k = \frac{2\cdot(233 - 39\cdot\sqrt{33})^{1/3} + 2\cdot(39\cdot\sqrt{33} + 233)^{1/3} + 2\cdot(6\cdot\sqrt{33} - 26)^{1/3} - 2\cdot(6\cdot\sqrt{33} + 26)^{1/3} - 6}{18} \wedge m = -\frac{(6\cdot\sqrt{33} - 26)^{1/3}}{3} + \frac{(6\cdot\sqrt{33} + 26)^{1/3}}{3} - \frac{1}{3}$$

$$k = 0.5436890126 \wedge m = 0.2955977425$$

Plotting all squares and triangles gives a wonderful representation of the snub cube.



Hint: You can highlight and plot all expressions from ADBC:= to RBS:= in one single step.

Comment: The values for *k* and *m* and closely related with the *Tribonacci Constant c*, which is the real solution of equation $x^3 -- x^2 - x - 1 = 0$. ($c \approx 1.839287$)

(Additional Comment: The *Tribonacci Sequence* is defined as $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 4$ with $a_1 = a_2 = 1$, $a_3 = 2$.

1, 1, 2, 4, 7, 13, 24, 44, …

The ratio of adjacent elements tends to the Tribonacci Constant and the numbers of the sequence increase asymptotically to $a_n \sim c^n$.

(See also my additional comments at the end of this contribution, Josef)

SOLVE($X^3 - X^2 - X - 1 = 0$, X)

$$X = -\frac{(19 - 3\cdot\sqrt{33})^{1/3}}{6} - \frac{(3\cdot\sqrt{33} + 19)^{1/3}}{6} + \frac{1}{3} + i\cdot\left(\frac{(27\cdot\sqrt{11} + 57\cdot\sqrt{3})^{1/3}}{6} - \frac{(57\cdot\sqrt{3} - 27\cdot\sqrt{11})^{1/3}}{6}\right) \vee X = -\frac{(19 - 3\cdot\sqrt{33})^{1/3}}{6} - \frac{(3\cdot\sqrt{33} + 19)^{1/3}}{6} + \frac{1}{3} + i\cdot\left(\frac{(57\cdot\sqrt{3} - 27\cdot\sqrt{11})^{1/3}}{6} - \frac{(27\cdot\sqrt{11} + 57\cdot\sqrt{3})^{1/3}}{6}\right) \vee X = \frac{(19 - 3\cdot\sqrt{33})^{1/3}}{3} + \frac{(3\cdot\sqrt{33} + 19)^{1/3}}{3} + \frac{1}{3}$$

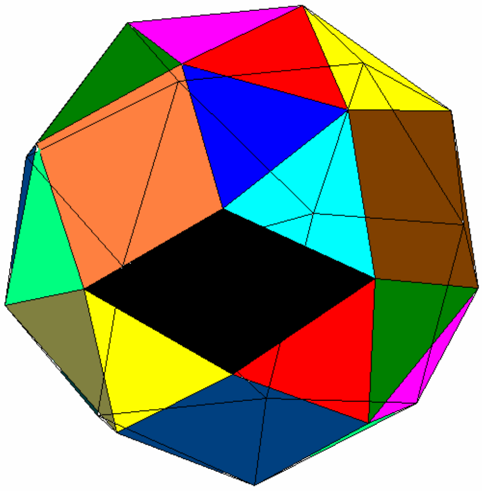
$X = -0.4196433776 + 0.6062907292\cdot i \vee X = -0.4196433776 - 0.6062907292\cdot i \vee X = 1.839286755$

Von: Josef Böhm [nojo.boehm@pgv.at]
Gesendet: Samstag, 30. Juli 2011 13:25

Dear Mr. Ludwig,

I couldn't resist colouring your pretty Cubus Simus, too.

I did it in another way: First of all I produced the wireframe model of the solid (using your data together with GeoGebra-Graph). Then I copied and pasted the figure from the 3D-Plot Window into the Algebra Window. I converted this picture to a Paintbrush graph (Edit > Convert Picture Object > Paintbrush Picture). Then I could fill all the faces in Paintbrush. The edges remain visible. As you can see I chose the visibility from the other side.



JB

Von: Heinrich Ludwig [mailto:heinrich.ludwig@gym-raubling.de]
Gesendet: Dienstag, 9. August 2011 13:11

Lieber Herr Böhm,

es freut mich, dass meine Spielereien Sie angeregt haben, damit weiter zu experimentieren. Ich habe auch schon daran gedacht, das Einfärben einem Programm zu überlassen, das sich auf graphische Bearbeitung besser versteht als Derive. Liefert Derive das Kantenmodell des Körpers, dann hat ein Graphikprogramm alle Information, um die weitere Gestaltung zu übernehmen. Natürlich verliert man dabei, dass man auf einfache Weise den Augenpunkt wandern lassen kann, aber für eine statische Abbildung des Körpers ist das nicht wichtig.

Den von Ihnen beschriebenen Weg kann man noch ein wenig abkürzen, indem man den Graphikinhalt des 3D-Fensters mit Strg+C unmittelbar in die Windows-Zwischenablage kopiert und deren Inhalt dann vom Graphikprogramm einliest. Als solches bevorzuge ich GIMP. Es ist das am meisten ausgereifte OpenSource-Produkt auf dem Markt.

Wussten Sie schon, dass

$$\mathrm{ATAN}\left(\frac{2^{1/4}\cdot(\sqrt{5}-1)}{8\cdot\mathrm{COS}\left(\frac{11\cdot\pi}{120}\right)\cdot\mathrm{COS}\left(\frac{29\cdot\pi}{120}\right)\cdot\sqrt{\left(-4\cdot\sqrt{2}\cdot\mathrm{SIN}\left(\frac{3\cdot\pi}{20}\right)-\sqrt{(5-\sqrt{5})}+3\cdot\sqrt{2}\right)}}\right)=\frac{3\cdot\pi}{8}$$

Für Derive war die Vereinfachung des Terms leider ein zu harter Brocken. Wie kommt man aber auf so einen verrückten Term? Mich hat die Frage beschäftigt (das Wetter war allzu traurig in letzter Zeit), wie gut archimedische Körper rollen können. Das ist sicher eine eher physikalische Frage. Mein mathematischer Ansatz war, den Raumwinkel an der Ecke des Polyeders zu betrachten. Nirgends konnte ich dazu eine Auflistung der Werte für die 5+13 archimedischen Körper finden, also habe ich Derive anpacken lassen. Derive musste die Formel von L'Huilier auswerten. Und dann kam für das Hexakisikosaeder (4,6,10) das o.g. Ungetüm heraus. Jetzt wäre es freilich interessant, was die großen Computeralgebrasysteme (Mathematica, Maple, etc.) leisten.

Schaffen diese die Vereinfachung? Und was leistet der Derive-Nachfolger Nspire?

Herzliche Grüße

Heinrich Ludwig

Heinrich Ludwig's mail from 9 August 2011 summarized:

Dear Mr. Böhm

The procedure which you described can be shortened by copying the contents of the 3D-plot window into the clipboard (Ctrl+C) and then pasting into a graphics program. I prefer GIMP. For me it seems to be the best available open source program on the market.

BTW, did you know that

$$\mathrm{ATAN}\left(\frac{2^{1/4}\cdot(\sqrt{5}-1)}{8\cdot\mathrm{COS}\left(\frac{11\cdot\pi}{120}\right)\cdot\mathrm{COS}\left(\frac{29\cdot\pi}{120}\right)\cdot\sqrt{\left(-4\cdot\sqrt{2}\cdot\mathrm{SIN}\left(\frac{3\cdot\pi}{20}\right)-\sqrt{(5-\sqrt{5})}+3\cdot\sqrt{2}\right)}}\right)=\frac{3\cdot\pi}{8}$$

It was a work too hard for DERIVE to simplify. But does one encounter such a crazy expression? I was busy treating the problem (weather was too bad in the last time) how good Archimedean solids can roll along. I know that this is more a physics question. My mathematical approach was to investigate the space angle in an edge of the solid. As I could not find any list of these angles for the 5 + 13 Archimedean solids I engaged DERIVE to do the job. DERIVE had to evaluate the formulae of L'Huilier. And then there was the Hexakis Icosahedron. It is the source of the onster from above. I would be very interested what the „big" CAS like MATHEMATICA and Maple are able to do with it.

Can they perform the simplification? And what about TI-Nspire?

Dear Mr. Ludwig,

Many thanks for your extended letter. Don't underestimate DERIVE. It needs only setting the right mode for treating trig expressions:

$$\mathrm{ATAN}\left(\frac{2^{1/4}\cdot(\sqrt{5}-1)}{8\cdot\mathrm{COS}\left(\frac{11\cdot\pi}{120}\right)\cdot\mathrm{COS}\left(\frac{29\cdot\pi}{120}\right)\cdot\sqrt{\left(-4\cdot\sqrt{2}\cdot\mathrm{SIN}\left(\frac{3\cdot\pi}{20}\right)-\sqrt{(5-\sqrt{5})}+3\cdot\sqrt{2}\right)}}\right)$$

Trigonometry := Collect

$$\mathrm{ASIN}\left(\frac{\sqrt{(\sqrt{2}+2)}}{2}\right)$$

$$\frac{3\cdot\pi}{8}$$

It is also interesting simplifying this expression piecewise:

$$8\cdot\mathrm{COS}\left(\frac{11\cdot\pi}{120}\right)\cdot\mathrm{COS}\left(\frac{29\cdot\pi}{120}\right) = 4\cdot\mathrm{COS}\left(\frac{3\cdot\pi}{20}\right)+2$$

$$\sqrt{\left(\left(4\cdot\mathrm{COS}\left(\frac{3\cdot\pi}{20}\right)+2\right)^2\cdot\left(-4\cdot\sqrt{2}\cdot\mathrm{SIN}\left(\frac{3\cdot\pi}{20}\right)-\sqrt{(5-\sqrt{5})}+3\cdot\sqrt{2}\right)\right)}$$

$$\sqrt{5}\cdot 2^{3/4} - 50^{1/4} - 2^{3/4} + 2^{1/4}$$

$$\frac{2^{1/4}\cdot(\sqrt{5}-1)}{\sqrt{5}\cdot 2^{3/4} - 50^{1/4} - 2^{3/4} + 2^{1/4}} = \sqrt{2}+1$$

$$\mathrm{ATAN}(\sqrt{2}+1) = \frac{3\cdot\pi}{8}$$

or:

$$\left(-4\cdot\sqrt{2}\cdot\text{SIN}\left(\frac{3\cdot\pi}{20}\right)-\sqrt{(5-\sqrt{5})}+3\cdot\sqrt{2}\right)\cdot\left(4\cdot\text{COS}\left(\frac{3\cdot\pi}{20}\right)+2\right)^2$$

$$-6\cdot\sqrt{10}+8\cdot\sqrt{5}+18\cdot\sqrt{2}-24$$

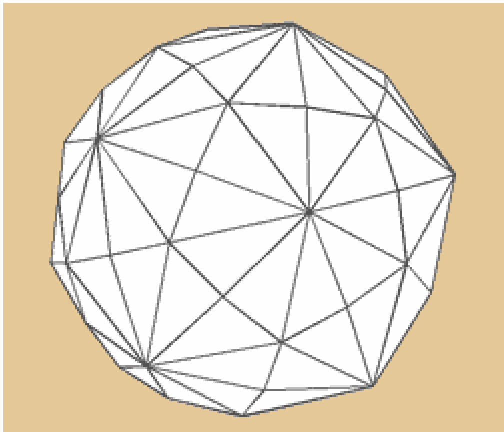
$$\frac{2^{1/4}\cdot(\sqrt{5}-1)}{\sqrt{(-6\cdot\sqrt{10}+8\cdot\sqrt{5}+18\cdot\sqrt{2}-24)}}=\sqrt{2}+1$$

You could also perform "stepwise simplification" in order to get some insight how DERIVE works internally.

I add picture of the Hexakis Icosahedron. It is one of the Catalan solids which are dual to the Archimedean solids.

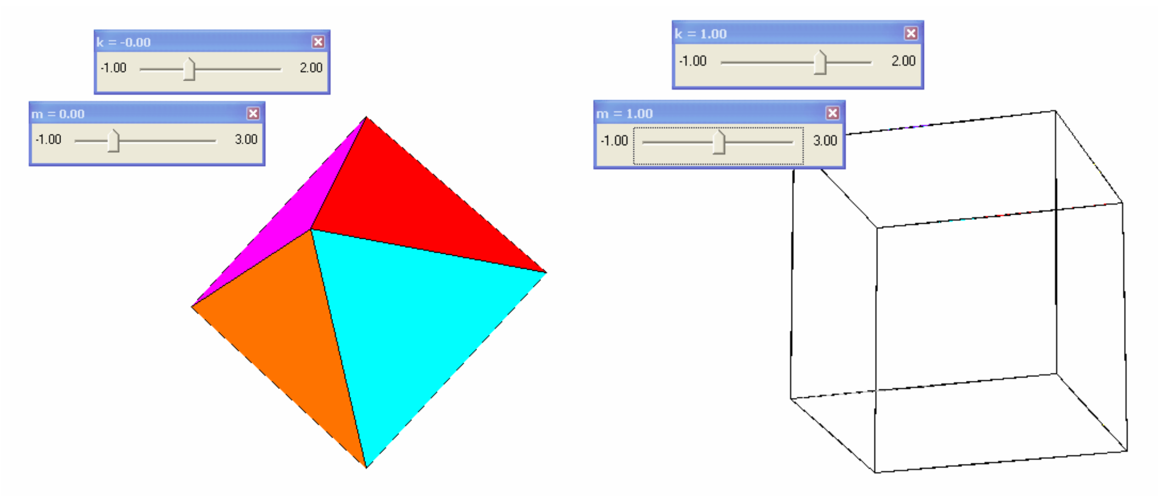
The H.I. is the dual of the Great Rhombicosidodecahedron.

(Source: the interesting kauernet-website from below.)



Some additional comments:

(1) *Inserting sliders for k and m provides more insight in the influence of the parameters.*



Challenge 1: Which combination of k and m create the solids given on page 26?

(2) *Another definition of the snub cube:*

The solid whose vertices are the 24 points on the surface of a sphere for which the smallest distance between any two is as great as possible.

(3) *Heinrich Ludwig had in the head line of his DERIVE file the expression **Pentagonal Icositetrahedron**. This is the "dual" solid to the snub cube. What are **dual polyhedra**?*

The dual of a polyhedron can be found by connecting the midpoints of the edges surrounding each vertex to a polygon and then constructing its tangent polygon (of the circumcircle). The numbers of vertices and faces interchange. The P.I. has 24 faces and 38 vertices.

*The dual solids of the Archimedean solids are the **Catalan solids**.*

Challenge 2: Find the DERIVE model of the Pentagonal Icositetrahedron.

(4) *For our German readers: a snub nose is a "Stupsnäschen".*
For all readers: "Simus" is a latin word for "abgeplattet" (German) and "flattened, oblate" (English).

(5) *The Tribonacci-sequence*

trib_c(20, 1, 1, 2) = [1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, 10609, 19513, 35890, 66012]

$$(\text{trib\_c}(100, 1, 1, 2))_{100} = 98079530178586034536500564$$

$$c := \frac{(19 - 3\cdot\sqrt{33})^{1/3}}{3} + \frac{(3\cdot\sqrt{33} + 19)^{1/3}}{3} + \frac{1}{3}$$

c := 1.839286755214161132551852564653286600424

$$\frac{(\text{trib\_c}(21, 1, 1, 2))_{21}}{(\text{trib\_c}(20, 1, 1, 2))_{20}} = 1.839286796340059383142458946858147003575$$

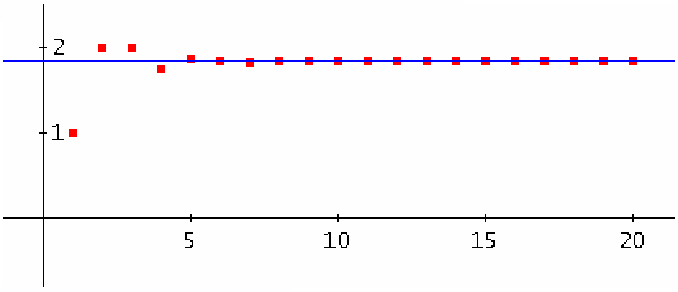
$$\frac{(\text{trib\_c}(101, 1, 1, 2))_{101}}{(\text{trib\_c}(100, 1, 1, 2))_{100}} =$$

1.839286755214161132551852564653286600424

The next page illustrates the properties of the Tribonacci constant. Finding a function for generating the Tribonacci sequence is left for the reader.

```
      ⎛ (trib_c(x + 1, 1, 1, 2))                  ⎞
      ⎜                          x + 1            ⎟
TABLE ⎜ ─────────────────────────────────, x, 1, 20⎟
      ⎜      (trib_c(x, 1, 1, 2))                  ⎟
      ⎝                          x                 ⎠

c
```

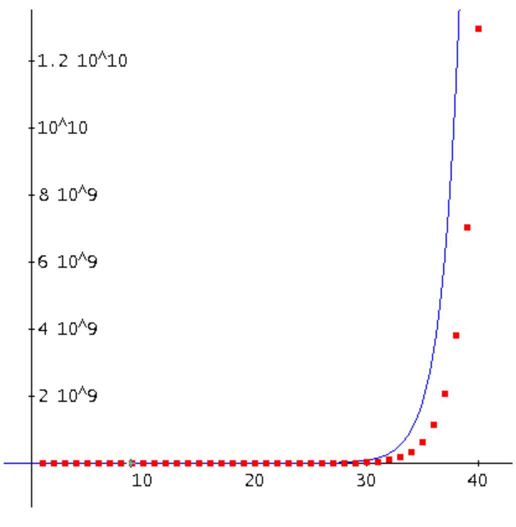


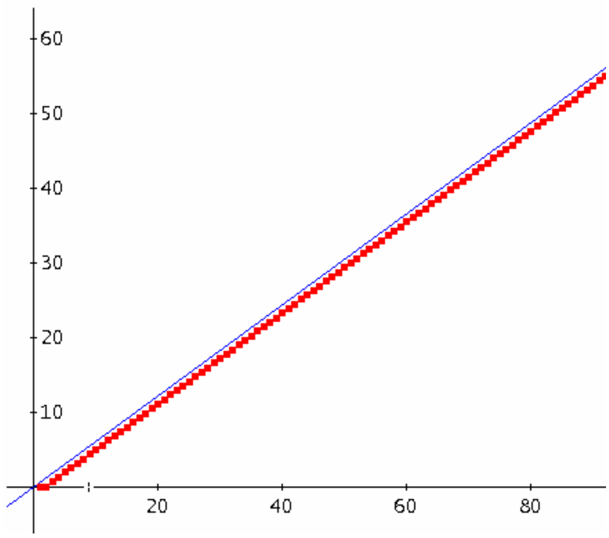

```
 x
c

TABLE((trib_c(x, 1, 1, 2)) , x, 1, 50)
                          x
```

```
      x
LN(c )

TABLE((LN(trib_c(x, 1, 1, 2))) , x, 1, 100)
                              x
```





(6) *A (small) selection of (many) interesting websites. (I didn't add the Wikipedia sites which are also highly recommended.)*

http://arxiv.org/pdf/1202.0172v1.pdf

http://www.ewetel.net/~neubauer/raumgeometrie/dateien/basteln/cubus%20simus%20basteln.pdf

http://mathworld.wolfram.com/PentagonalIcositetrahedron.html

http://kauernet.de/hendes/html/h306_catal.htm

http://mathworld.wolfram.com/LHuiliersTheorem.html

http://www.calculus.com/konfuciy.asp?tda=dt&t=966&fs=l%27huilier%27s+theorem

(7) *Finally, many thanks to Heinrich Ludwig for his inspiring contribution.*

(8) *Most of my "knowledge" and some pictures presented above are from the internet.*

Josef

Statistics

Visualising and Simulating Dynamically with TI-Nspire 3.1

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Introduction

"Measuring is Knowing" is more important than ever nowadays. Wednesday 20 October 2010 was the first "World Statistics Day" initiated by the UN. At this occasion the NIS (Nationaal Instituut voor Statistiek)[1] published a special extended version of the core numbers 2009 with Belgium placed into an European perspective[2].

Technology proceeds and offers new possibilities for teaching. This is for sure for statistics. Actual data can be visualised quickly in various ways, simulations of random samples and experiments can be investigated. Doing so statistical concepts can be introduced early placing the data into the foreground and the formulae in the background. The intention of this paper is to demonstrate a possible way offering concrete examples. The statistical basic ideas are not defined here. The necessary background can be found in numerous statistics textbooks.

Part 1 of this paper is to introduce the possibilities of TI-Nspire (version 3.1)[3], containing mainly descriptive statistics (2nd and 3rd grade in Belgium with two to four lessons mathematics a week). We are working with data from the web, too.

Part 2 deals with simulating with TI-Nspire. How to select a simple random sample from a population? How does the normal distribution appear as probability distribution for the sample mean observing graphically the variability of more than 1000 sample means?

Part 3 shows how to discover new probability distributions by simulation, starting with the uniform distribution on the interval [0,1]. We also treat the t-distribution.

Part 4 demonstrates a possible approach for hypothesis testing supported by simulations.

This paper can be downloaded (in its Flemish original) from the T³ Vlaanderen website,

www.t3vlaanderen.be,

together with the TI-Nspire files and all the data appearing in this paper.

---

[1] **NIS** (Nationaal Instituut voor Statistiek) is the biggest statistical organisation of the Belgian government: http://statbel.fgov.be/nl

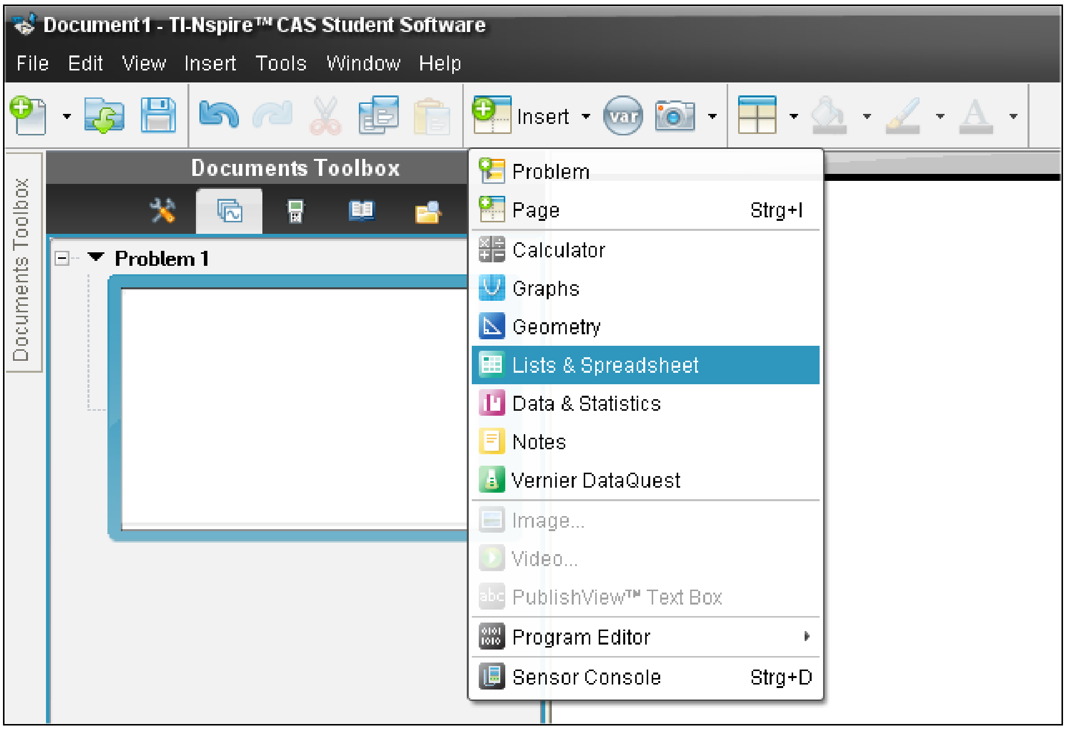
[2] Kerncijfers 2009, België in een Europees perspectief http://economie.fgov.be/nl/modules/pressrelease/statistieken/generale/world_statistics_day.jsp

[3] Information about TI-Nspire: www.education.ti.com

Part 1: Descriptive Statistics

(1) Qualitative ungrouped data

Open TI-Nspire, close the welcome screen and insert a page with the **Lists & Spreadsheet Application** (which is one of seven applications offered):

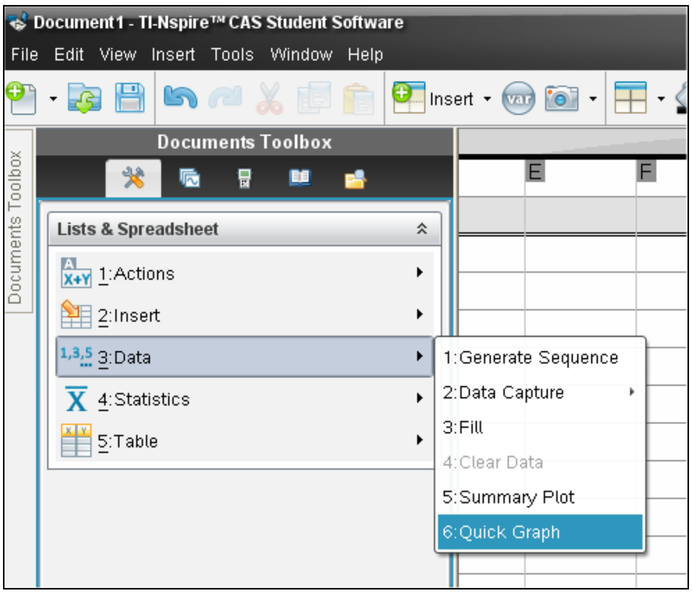


Example 1:

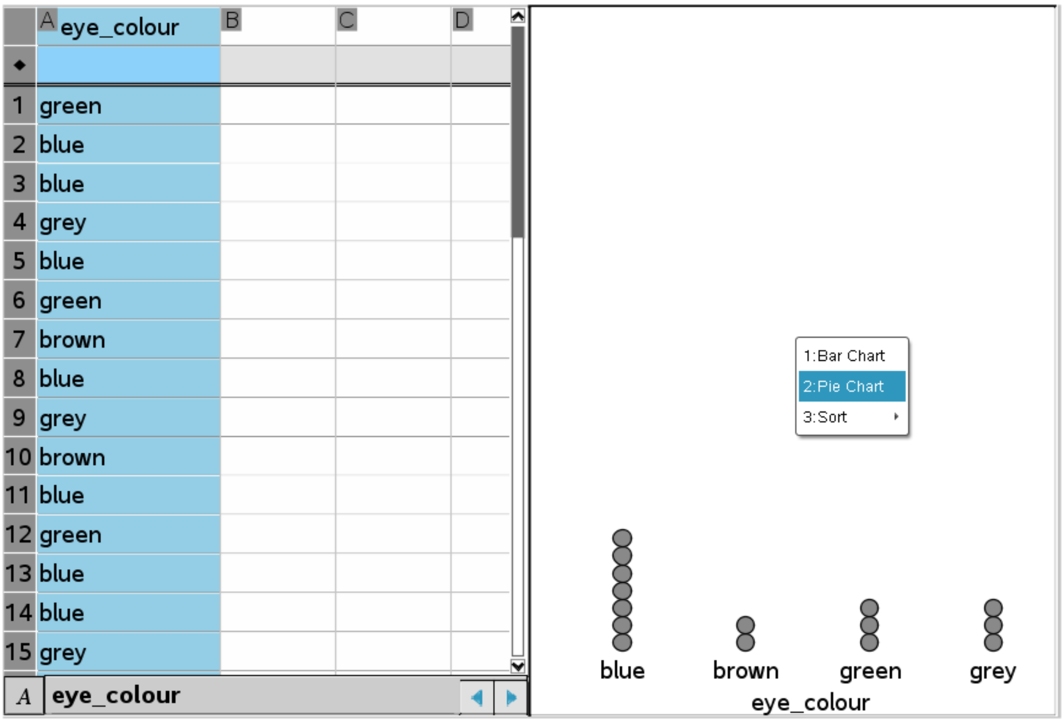
Given are the eye colours of 15 persons: green, blue, blue, grey, blue, green, brown, blue, grey, brown, blue, green, blue, blue and grey.

→ Name the first column A as eye_colour.

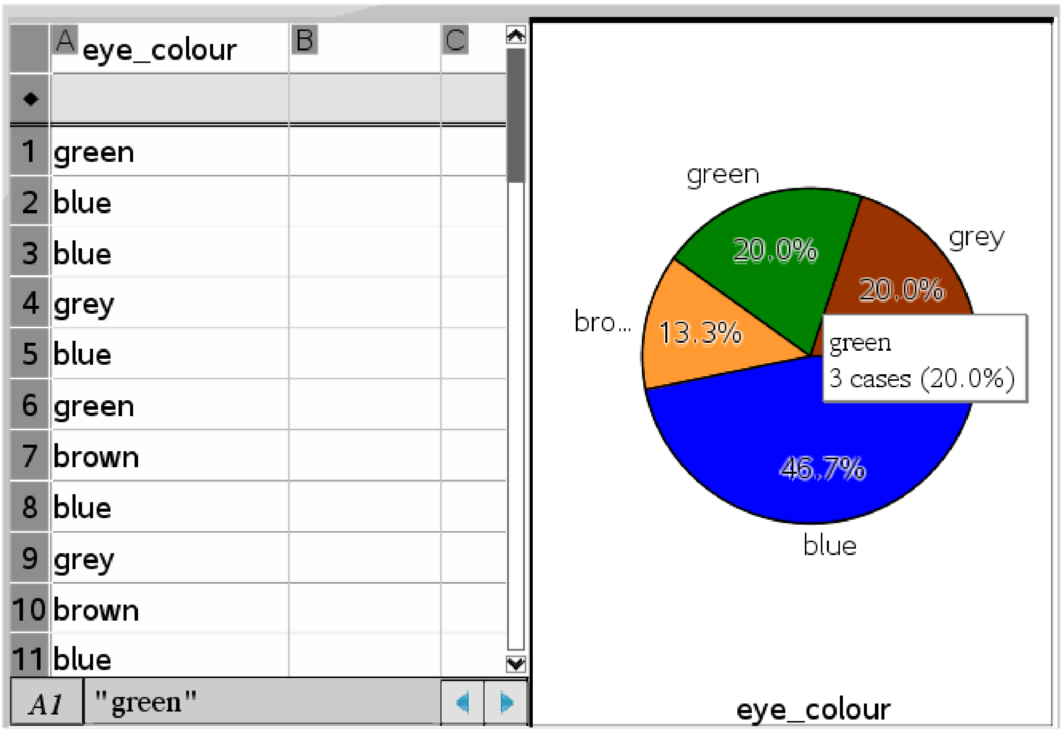
→ Enter the colours (begin in cell *A1*) one after another – under quotes because the data are strings.



→ Select the whole column (left mouse click on letter A), then choose the first menu (**Document Tools**) in the **Document Toolbox**. Choose option **3: Data** > **Quick Graph**. The window will split and you will receive a **Dot Plot** in the right window.



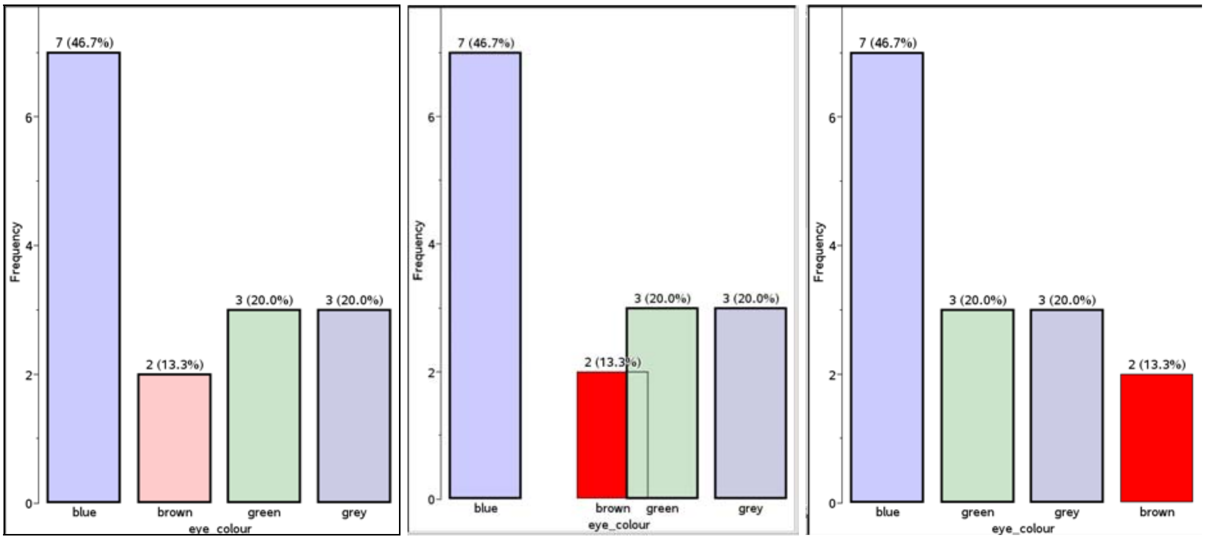
→ Choose the right window (left mouse click on the window) and change the diagram to a **Pie Chart**. Another (right) mouse click on the pie offers presenting the percentages (**Show all Labels**). Hovering the mouse over the sectors of the pie opens a small window showing details.



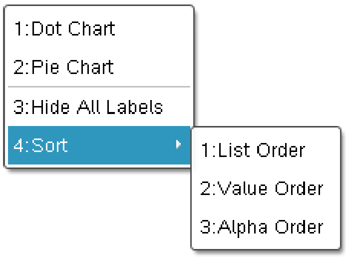
The **context menu** (right mouse click) enables quick performing appropriate actions. You can close a window by selecting it (left mouse click), pressing Ctrl + K followed by pressing the Del(ete)-key.

→ Change the diagram to a **Bar Chart** (right mouse click in the right window).

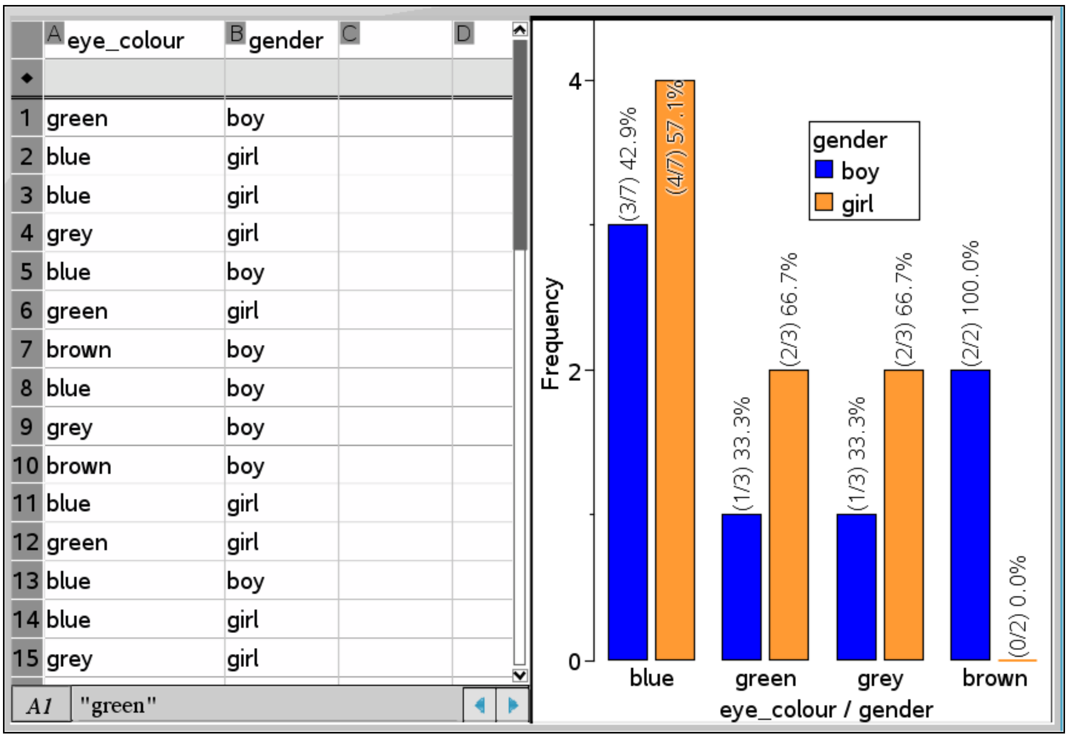
Shift the category brown to the right (left mouse click on the label brown and drag it to the right).



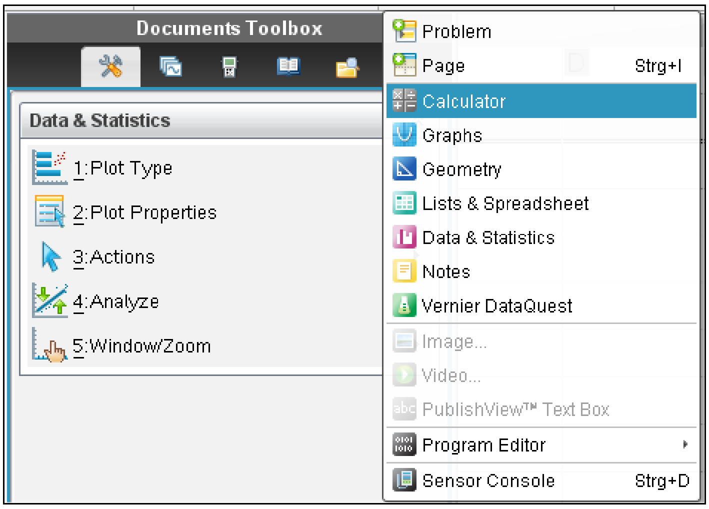
Right mouse click pop up menu offers three ways of sorting the bars.



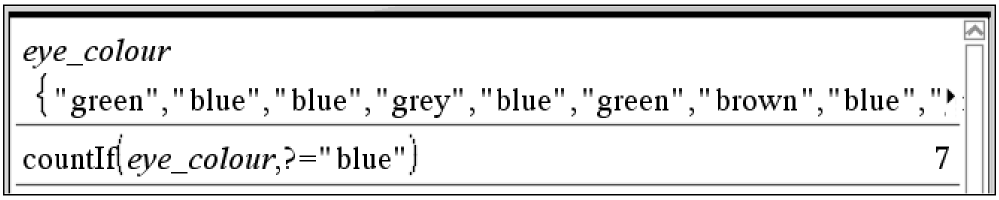
→ Add another column containing the gender of the persons. and then split the category of the eye colour with respect to the second category gender. (Right click on the X-variable eye_colour, select option **4: Split Categories per Variable** and select gender.)



→ Insert a **Calculator** page via **Insert** from the Menu bar.



→Type eye_colour followed by pressing the ENTER-key. The variable eye_colour is a list of strings. The command countif(eye_colour,?=,"blue") gives the number of appearances of "blue" in this list.



Hint: The size of the characters can be adapted by using the slider bar on the bottom of the screen: 

(2) Qualitative grouped data

Example 2:

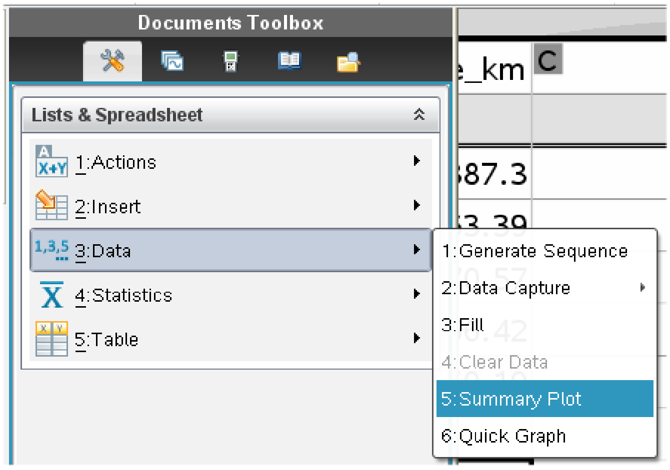
On the NIS website one can find the distribution of the use of ground in Belgium (2009)

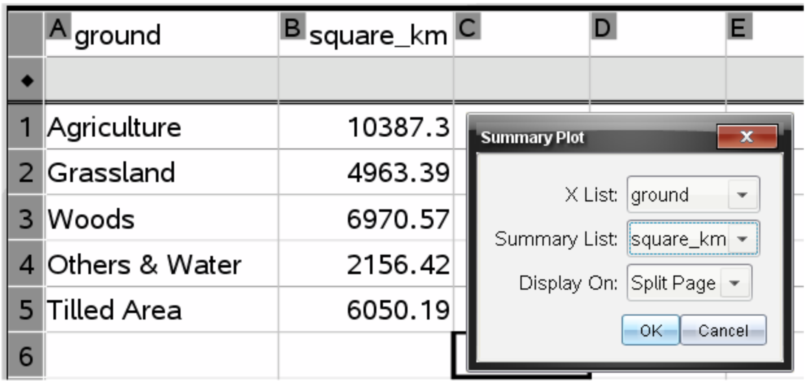
Bodemgebruik België 2009 (in km²) bron: NIS	
Landbouw	10387,34
Permanente weiden en grasland	4963,39
Bossen	6970,57
Andere en water	2156,42
Bebouwde percelen	6050,19

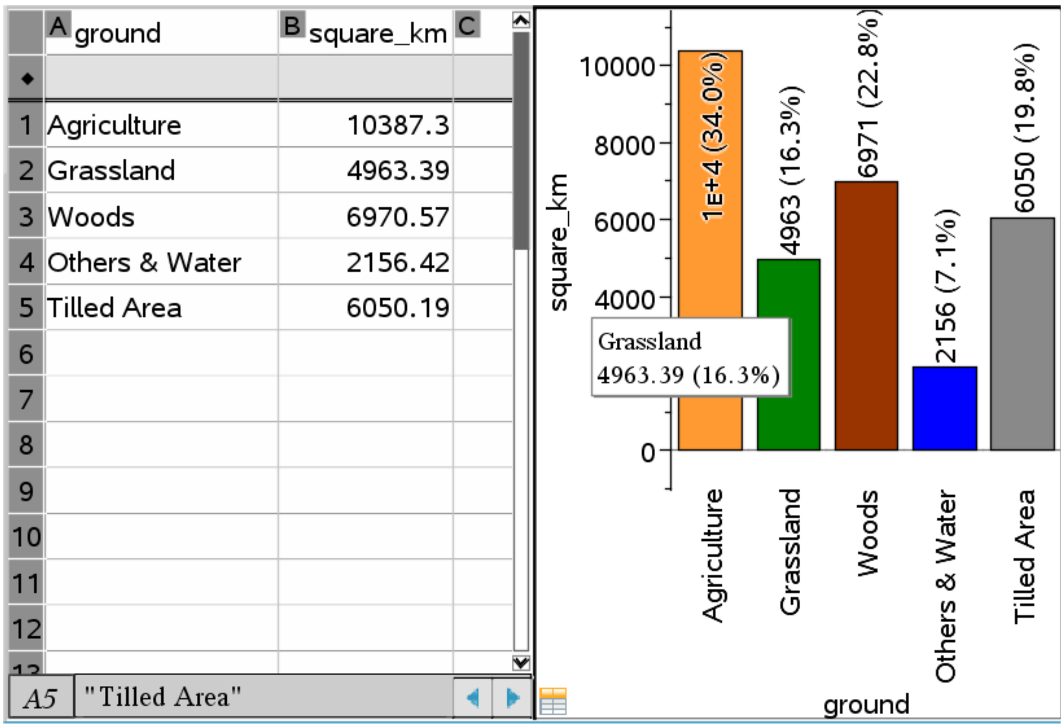
(agriculture, permanent pastures and grassland, woods, others and water, tilled areas)

→ Insert a new spreadsheet page and fill in the data.

→ Insert a new spreadsheet page and fill in the data.

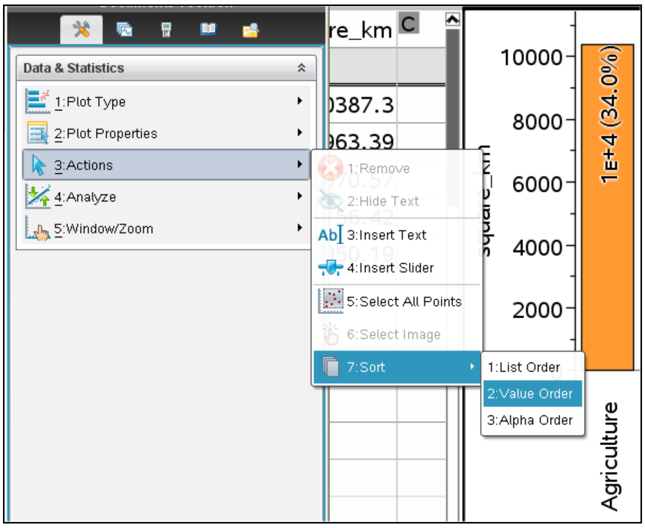


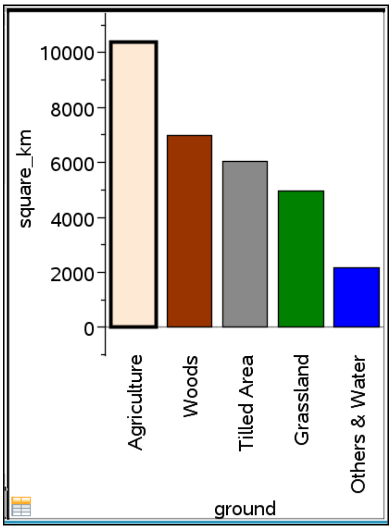




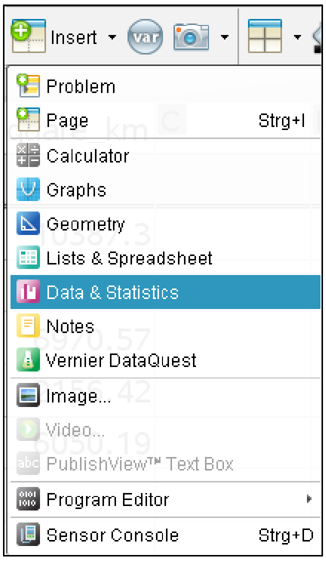
→ Activate the right window. Bring the bars to a decreasing order by the menu **Actions** > **Sort** > **Value Order**. You can also shift the bars respectively.

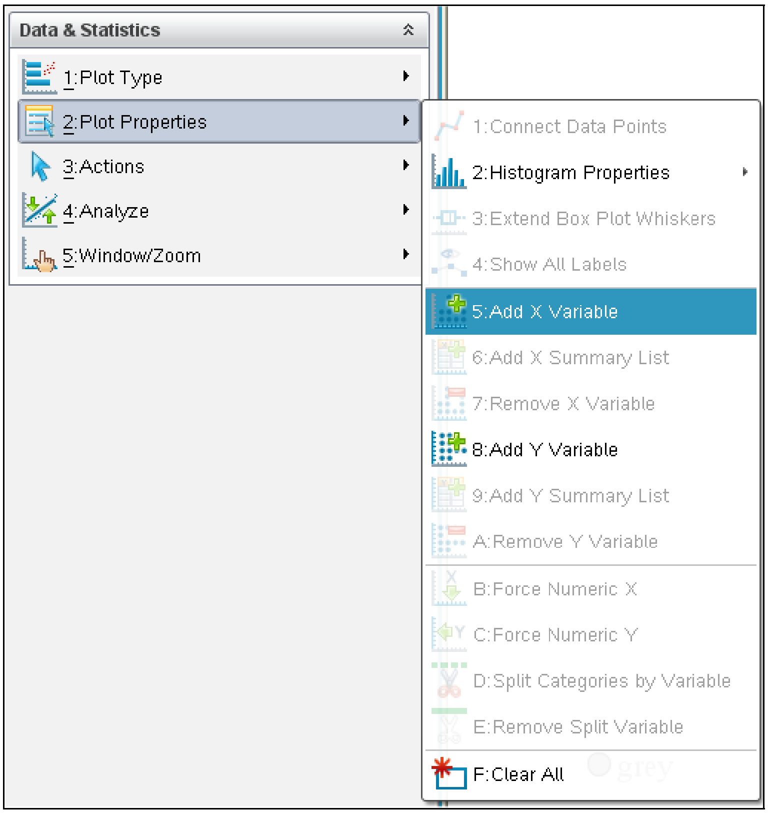
Then change the bar chart to a pie chart via the **Plot Type** menu (or by a right mouse click on one of the bars).



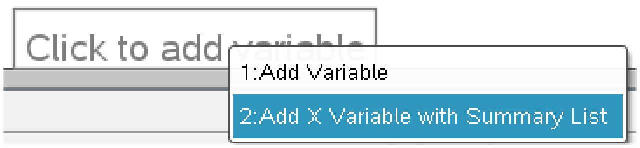


The statistics diagrams appear in a **Data & Statistics Application**. You could also start by inserting this application into a separate page, then use the **Plot Properties** menu, select **Add X Variable** (ground) and finally select **Add Y Summary List** (square_km).



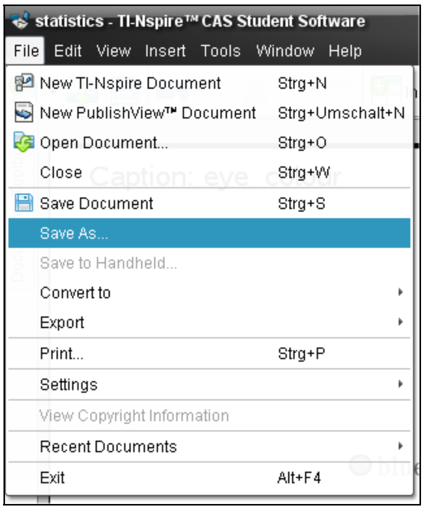


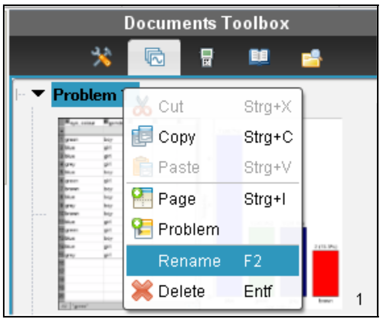
But it works really faster by a rmcl (right mouse click) on **Click to add variable** on the window bottom and then choosing **Add X Variable with Summary List**.

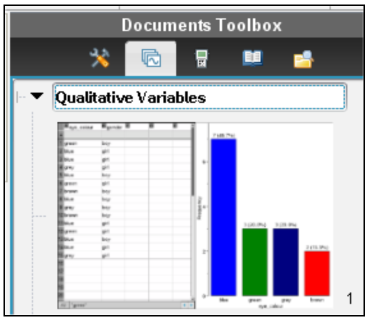


→ Save the file as **statistics** (Menu **File** > **Save as**).

Open menu **Page Sorter**  in the **Documents Toolbox**. Rename the standard name **Problem 1** as **Qualitative Variables** by a rmcl on **Problem 1**.







One file can contain various problems, with each problem containing more pages (which can be selected in the Page Sorter ). Each page can consist of maximum four applications (seven applications are available). Each application has its own menu which is available under  in the Documents Toolbox.

Within one problem a variable has the same value on all pages. Variable *x* in one problem is independent of variable *x* in another problem.

Example 3:

Belgium consists of three districts (constituent states): the Flemish district (Vlaams Gewest), the Brussels Capital Town district (Brussels Hoofstedelijk Gewest) and the Wallonian district (Waals district). The German speaking minority (Duitstalige gemeenschap) is part of the Waals district.

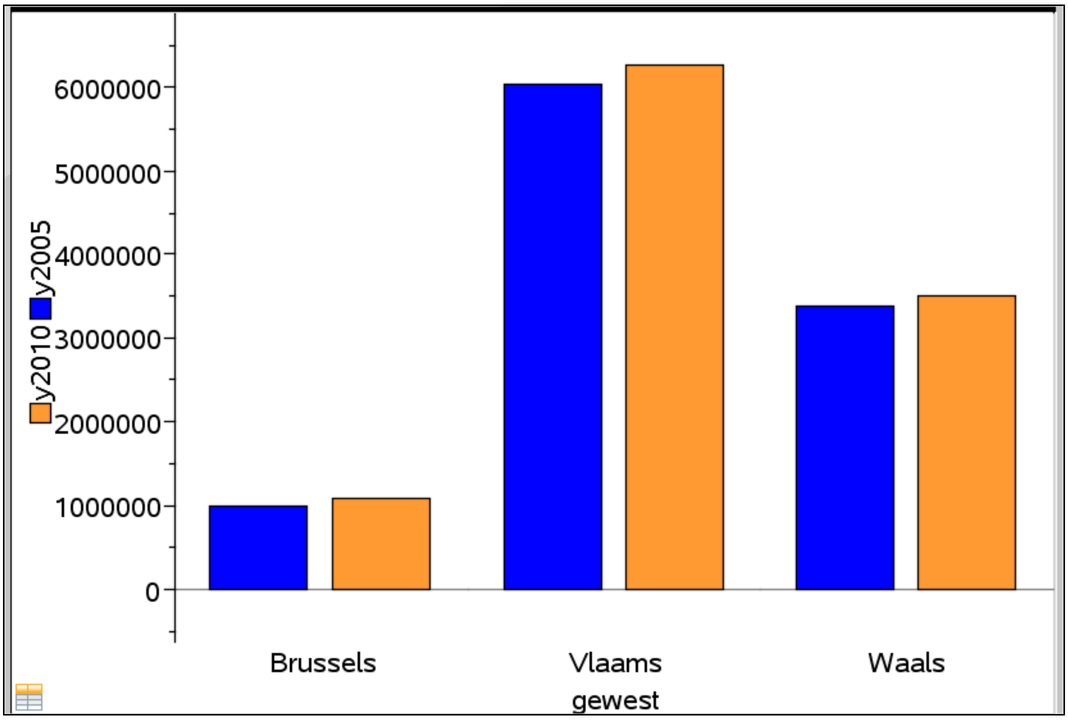
The population numbers from 2005 to 2010 were (according NIS):

Gewest	2005	2006	2007	2008	2009	2010
Brussels H. Gew.	1.006.749	1.018.804	1.031.215	1.048.491	1.048.491	1.089.538
Vlaams Gewest	6.043.161	6.078.600	6.117.440	6.161.600	6.208.877	6.251.983
Waals Gewest	3.395.942	3.413.978	3.435.879	3.456.775	3.475.671	3.498.384
waarvan Duitstalige gem.	75.512	73.119	73.675	74.169	74.540	75.222

→ Create a bar chart comparing district populations of the years 2005 and 2010.

Hint: you can add different **Add Y Summary Lists** with one X-variable given (via Plot Type in the Data & Statistics Documents Toolbox .

	A gewest	B y2005	C y2006	D y2007	E y2008	F y2009	G y2010
◆							
1	Brussels	1006749	1018804	1031215	1048491	1068532	1089538
2	Vlaams	6043161	6078600	6117440	6161600	6208877	6251983
3	Waals	3395942	3413978	3435879	3456775	3475671	3498384



Our USER Forum

On 07/01/12 13:53, Giuseppe Ornaghi wrote:
I am using Derive 6.10 for Windows and I would like to know if it is possible to plot the absolute value of the zeta function in a range.
Thank you very much.
Giuseppe

Hi Giuseppe,

try ABS(ZETA(x+i*y)) as a 3-d plot. You will get a surface in R^3 defined by [x,y,ABS(ZETA(x+i*y))]. I hope this is what you were looking for.

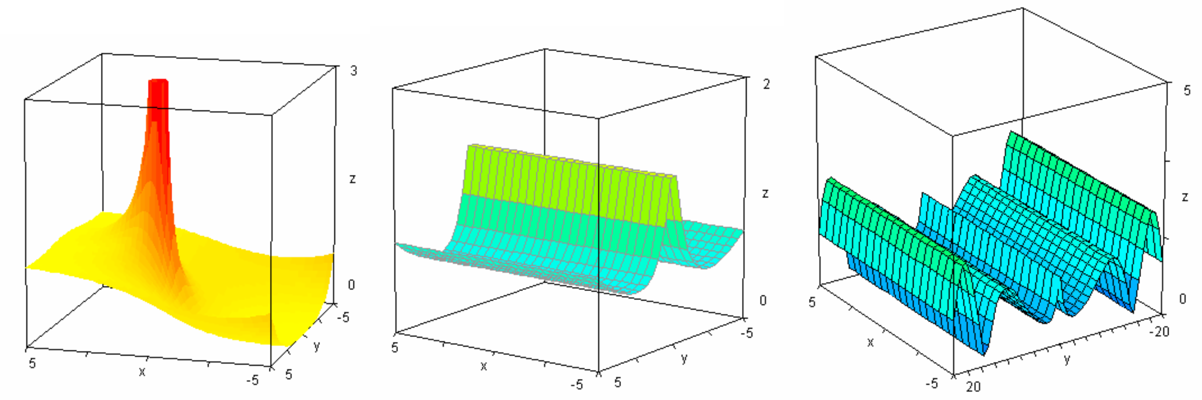
Stefan Welke

With ABS(ZETA(x+i*y)) I get the error message:Too many variables.With z=ABS(ZETA(0.5+i*y)) I get a surface in 3D. I would like to plot ABS(ZETA(0.5+i*y)) on a xy plane (x-axis corresponds to the values of y and y-axis to the absolute value of ZETA(0.5+i*y)).
Is it possible?
Thank you very much.
Giuseppe

Hello Giuseppe,

did you use the "i" from the menu in the bottom? If not, then "i" from the keyboard will be interpreted as a variable, and then there are three, which is one too much. Alternatively try ABS(ZETA(x+#i*y)) as input. This will work correctly. I assume, that you will now see a surface different from that of your previous attempt.

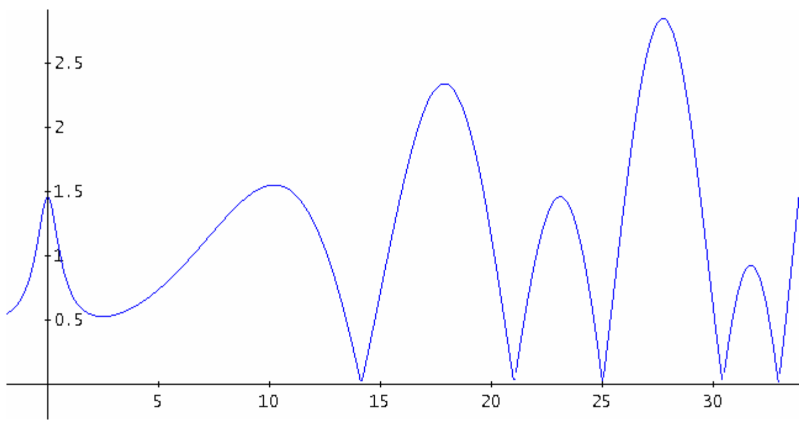
Cordiali saluti, Stefan Welke



$|\zeta(x + i\cdot y)|$ $|\zeta(0.5 + i\cdot y)|$

I see a surface different from previous examples. Thank you.
And for my attempt to plot a graph in xy-plane?
Thank you very much.
Saluti dall'Italia.
Giuseppe

$|\zeta(0.5 + i\cdot x)|$



(symmetric wrt y-axis)

TIME 2012 – The Keynotes, Lectures and Workshops

List of presentations (in alphabetical order) of accepted proposals. It is very likely that more will follow. All continents will be present in Estonia in July.

A Stepwise Graphical Approach for Teaching Automated Theorem Proving in Engineering

A Tool for Evaluation of Rationality of Algebraic Transformations

Adapting the Assessment to the Instrumental Genesis

Adopting TI-Nspire CAS Technology (Handheld and Software), a Campus-Wide Experience

An Evaluation of Students: Experiences in Technology-based Courses in Mathematics and Statistics

Attracted by (Strange) Attractors

Being a Math Teacher – Looking through the Blog Posts of Student Teachers (Keynote)

CAS in the Classroom: Yesterday, Today, and Tomorrow (Keynote)

CAS or not CAS? that is the question when using technology in math education

Changing Assessment Methods: New Rules, New Roles

Compound Damped Pendulum: an Energy Comparison

Computer Algebra as an Educational Tool

Critical Issues of Effective CAS Utilization (Keynote)

Deriving Big Formulas with Derive and What Happened Then

Determining the Proof Skills of 2nd Year Preservice Mathematics Teachers by Means of GeoGebra

Development of Pedagogical Tools for Teaching Mathematics from the PC Lab to Smartphone (Keynote)

Dynamic Systems – From Simulation Software to CAS (WS)

Dynamic Systems - Many Competences - Presented Dynamically

Experiences of 15 Years Using Graphic Calculators in Saxonia

Finding Bioinformatics Data and DERIVE Programs to Process it (WS)

Find your very own "attractive" Attractor

Finnish Upper Secondary Students' Collaborative Processes in Learning Statistics in CSCL Environment

From Inductive Reasoning to Proof by Induction with Geometry Expressions (WS)

From Vienna to Tartu: A 10 Years Tour using DERIVE in TIME (Keynote)

Hidden Markov Models and an Introduction to Their Uses in Bioinformatics

High School Students' Acquisition of Knowledge and Skills through Self-Organization and Collaboration

Integration of Digital Tools into the Mathematics Classroom: A Challenge for Preparing and Supporting the Teacher

Invariant Region Method (IRM) Applied to Two Dimensional Predator-Prey Systems

Learning Mathematics through the Surrounding World

Making Learning Math Addictive - Localization of the Famous Open Source Website Khan Academy in Estonia

Mathematical Induction and CAS

Mathematical Modelling with Geometry Expressions and TI-Nspire CAS

Math eTextbooks

Media Diversity Project – E-Learning Paths in Maths Education

Modeling the Probability of a Binary Outcome

Motion Sensors in Mathematics Teaching: Learning Tools for Understanding General Math Concepts?

Notice Mathematics around You

Programming-like Activities for Calculus and Differential Equations Classes

Prospective Teachers' Curriculum Strategies in CAS-based Written Lessons

Representations of Solids and Surfaces within the TI-Nspire Environment

Systems of Computer Algebra and Dynamic Geometry as Tools of Mathematical Investigation

Teaching conics through mathematical modelling (WS)

Technology in Maths (a Queensland Perspective)

The Effects of Using a Calculator on Reducing Mathematics Anxiety

Types of Mistakes and Dynamics of the Occurrence of Mistakes Solving Linear Equations Using Computer Program T-algebra

Using Derive To Generate DNA Fractal Representations

Using Dynamic Documents to Enhance Student Understanding

Using Fourier Series to Analyse Mass Imperfections in Vibratory Gyroscopes

Using TI-Nspire CAS Technology in Teaching Engineering Mathematics: ODE's

Using TI-Nspire CAS Technology in Teaching Engineering Mathematics: Calculus

Visualizing and Understanding Statistical Concepts with TI-Nspire

What was Special in DERIVE?

When do Computer Algebra Systems Offer Unexpected Answers to School Equations?

Why Trying to Avoid Using Complex Numbers?

DUG-Meeting 2012

For more Information and Registration:

http://www.time2012.ut.ee/