THE DERIVE - NEWSLETTER #97

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THE BULLETIN OF THE

DERIVE

USER GROUP

+ CAS-TI

Contents:

March 2015

Interesting websites:

You can find a list of (all?) available Dynamic Geometry Systems (2D & 3D) on

http://en.wikipedia.org/wiki/List_of_interactive_geometry_software

David Stoutemyer (one of the fathers of DERIVE and the TI-92) wrote many papers on Computer Algebra. Some of his articles can be downloaded from:

http://arxiv.org/

(This is an archive which offers open access to 1,024,684 e-prints in Physics, Mathematics, Computer Science, Quantitative Biology, Quantitative Finance and Statistics.)

David's papers are:

A computer algebra user interface manifesto

Representation, simplification and display of fractional powers of rational numbers in computer algebra (together with Albert D. Rich)

Series Crimes

Series misdemeanors

Simplifying products of fractional powers of powers

Subtotal ordering -- a pedagogically advantageous algorithm for computing total degree reverse lexicographic order

Can the Eureqa symbolic regression program, computer algebra and numerical analysis help each other?

More recommended resources:

Numerous links to school mathematics related world wide websites can be found at (provided by Douglas Butler):

http://www.tsm-resources.com/mlink.html

Mathematical Imagery by Jos Leys:

With among many other fascinating images:

4D Polychora: http://www.josleys.com/show_gallery.php?galid=341

Hybrid 3D Fractals: http://www.josleys.com/show_gallery.php?galid=347

Biomorphic Fractals: http://www.josleys.com/galleries.php?catid=3

Dear DUG Members,

I am very happy that DNL#97 is almost in time. Looking at the contents of this newsletter I'd like to add some comments.

Take Robert Hawkes' contribution on the US Civil War: He gave this talk at the *DERIVE* & *TI-92* Conference at Gettysburg in 1998. This was a very emotional and passionate keynote lecture. Unfortunately I cannot include the movie parts which he presented during his talk in the DNL. This keynote is not included into the Conference CD, so I decided to publish it in our DNL. I added some private pictures from a visit of the Gettysburg Battlefield and some pictures which I found in the web.

On 9 April 1865 – which is exact 150 years ago – US Civil War ended.

The "Megalomorphs" offer another opportunity to transfer *DERIVE* objects to *TI-Nspire*-technology. Thanks a very efficient communication with Alfred Roulier we can present LUA-scripts in order to produce interesting graphs on the *TI-Nspire* screen. We make use of CAS for calculating envelopes. I hope that you will enjoy the patterns and pictures and that you feel inspired to create your own "spider traces".

Walter Wegscheider, our very busy webmaster, is one of the authors of the "Deshpande-article". It is great that he – more than busy at all times – does not miss the occasion to work on maths problems. Thanks to Walter and to Thomas Himmelbauer as well. I am very glad that I could add a *Geometry Expressions* application and demonstrate its collaboration with *DERIVE* and *TI-Nspire*.

Wolfgang Alvermann gave a great workshop at TIME 2014 sharing his rich experience as a teacher at a vocational school. He presented technical problems which could be solved by using SEC 2 mathematics. All his tasks are based on problems posed by the industry. I asked Wolfgang to provide other examples for the DNL. And he did without hesitating, many thanks and enjoy your well deserved retirement.

Finally I'd like to draw your attention to the links presented on the Information Page. I am quite sure that you can find valuable materials.

Best regards until next DNL in summer.

Always Yours

Josef

Download all *DNL*-DERIVE- and TI-files from

http://www.austromath.at/dug/

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a content of 40 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE*, *TI*-CAS and other CAS as well to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

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Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: June 2015

Preview: Contributions waiting to be published

Some simulations of Random Experiments, J. Böhm, AUT, Lorenz Kopp, GER
Wonderful World of Pedal Curves, J. Böhm, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
Hill-Encryption, J. Böhm, AUT
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
An Interesting Problem with a Triangle, Steiner Point, P. Lüke-Rosendahl, GER
Graphics World, Currency Change, P. Charland, CAN
Cubics, Quartics – Interesting features, T. Koller & J. Böhm, AUT
Logos of Companies as an Inspiration for Math Teaching
Exciting Surfaces in the FAZ / Pierre Charland´s Graphics Gallery
BooleanPlots.mth, P. Schofield, UK
Old traditional examples for a CAS – what´s new? J. Böhm, AUT
Truth Tables on the TI, M. R. Phillips, USA
Where oh Where is It? (GPS with CAS), C. & P. Leinbach, USA
Mandelbrot and Newton with *DERIVE*, Roman Hašek, CZK
Tutorials for the NSpireCAS, G. Herweyers, BEL
Some Projects with Students, R. Schröder, GER
Dirac Algebra, Clifford Algebra, D. R. Lunsford, USA
A New Approach to Taylor Series, D. Oertel, GER
Henon & Co; Find your very own Strange Attractor, J. Böhm, AUT
Rational Hooks, J. Lechner, AUT
Simulation of Dynamic Systems with various Tools, J. Böhm, AUT
Statistics of Shuffling Cards, Charge in a Magnetic Field, H. Ludwig, GER

and others

Impressum:
Medieninhaber: *DERIVE* User Group, A-3042 Würmla, D´Lust 1, AUSTRIA
Richtung: Fachzeitschrift
Herausgeber: Mag. Josef Böhm

Dear Josef:

In order to find the permutations of the elements of a set where some elements are repeated I am using the procedures in the attached file. The strategy seems to work but I am not sure if it is sufficiently reliable and it is not elegant at all. Perhaps somebody may be interested in developing a better set of procedures.

Best regards,

Marcelo

```
      tperms(N, m := 2, s := [[1]]) :=
        Loop
          If m > N
             RETURN SORT(s)
#1:       t := []
          MAP(t := APPEND(t, VECTOR(INSERT(m, v, k), v, s)), k, m, 1, -1)
          s := t
          m :+ 1
```

#2: $\text{tperms}(3) = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

I am interested in the distinct permutations of the elements of a set when some elements are repeated:

#3: $[1,\ 1,\ 2]_{\text{tperms}(3)} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

This function removes identical rows when they are placed consecutively:

```
      limpia(v, v1, j) :=
        Prog
          v1 := [v↓1]
          j := 2
          Loop
#4:         If v↓j ≠ v1↓DIM(v1)
               v1 := APPEND(v1, [v↓j])
            j :+ 1
            If j > DIM(v)
               RETURN v1
```

In order to remove the identical rows I propose

#5: $\text{limpia}(\text{SORT}([1,\ 1,\ 2]_{\text{tperms}(3)})) = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

```
                                                        ⎡ 1  1  2  2 ⎤
                                                        ⎢            ⎥
                                                        ⎢ 1  2  1  2 ⎥
                                                        ⎢            ⎥
                                                        ⎢ 1  2  2  1 ⎥
#6:   limpia(SORT([2, 2, 1, 1]          )) =            ⎢            ⎥
                              tperms(4)                 ⎢ 2  1  1  2 ⎥
                                                        ⎢            ⎥
                                                        ⎢ 2  1  2  1 ⎥
                                                        ⎢            ⎥
                                                        ⎣ 2  2  1  1 ⎦
```

However, I am sure that this calculation can be done in a more reliable and elegant way.

Dear Marcelo,

1. I collected your very clever auxiliary functions into one program.
2. And I generalized for non numerical elements. The result is a set of permutations with strings.

```
      perms_rep(set, N, m := 2, s := [[1]], s_, j) :=
        Prog
          set := VECTOR(NAME_TO_CODES(STRING(v)), v, set)'↓1
          N := DIM(set)
          Loop
            If m > N exit
            t := []
            MAP(t := APPEND(t, VECTOR(INSERT(m, v, k), v, s)), k, m, 1, -1)
            s := t
#7:         m :+ 1
          s_ := SORT(set↓s)
          se_ := [s_↓1]
          j := 2
          Loop
            If s_↓j ≠ se_↓DIM(se_)
               se_ := APPEND(se_, [s_↓j])
            j :+ 1
            If j > DIM(s_) exit
          VECTOR(VECTOR(CODES_TO_NAME(se_↓i_↓j_), i_, DIM(se_)), j_, DIM(se_↓1))'
```

```
                                   ⎡ 4  4  5  5 ⎤
                                   ⎢            ⎥
                                   ⎢ 4  5  4  5 ⎥
                                   ⎢            ⎥
                                   ⎢ 4  5  5  4 ⎥
#8:   perms_rep([4, 5, 4, 5]) =    ⎢            ⎥
                                   ⎢ 5  4  4  5 ⎥
                                   ⎢            ⎥
                                   ⎢ 5  4  5  4 ⎥
                                   ⎢            ⎥
                                   ⎣ 5  5  4  4 ⎦

                                   ⎡ a  a  b  b ⎤
                                   ⎢            ⎥
                                   ⎢ a  b  a  b ⎥
                                   ⎢            ⎥
                                   ⎢ a  b  b  a ⎥
#9:   perms_rep([a, a, b, b]) =    ⎢            ⎥
                                   ⎢ b  a  a  b ⎥
                                   ⎢            ⎥
                                   ⎢ b  a  b  a ⎥
                                   ⎢            ⎥
                                   ⎣ b  b  a  a ⎦

#10:  DIM(perms_rep([a, a, b, b, 1, 1])) = 90
```

#11: $\frac{6!}{2!\cdot 2!\cdot 2!}$

I enjoyed our transatlantic conversation.
I also wanted to use perms() but I had no idea how to proceed. Great ideas from Argentina.

Best regards and many thanks for the great communication.

Josef

Dear Joseph,

I have also enjoyed this conversation. Your perms_rep() is extremely elegant, more general and shorter than the other functions. It works for characters and numbers but does not work for words like perms_rep([n1,n2,n3]). Fortunately I can avoid such a case, but I tried it just for curiosity.

Best regards,

Marcelo

Same day, 2 hours later:

Dear Joseph,

If we use words the strategy is to write, for example, [n1,n2,n2] sub perms_rep([1,2,2]). My earlier objection is void.

Best regards,

Marcelo

Dear Marcelo,

I am not quite sure if this works in all cases.

$$[n1,\ n2,\ n2]_{\text{perms\_rep}([1,\ 2,\ 2])} = \begin{bmatrix} n1 & n2 & n2 \\ n2 & n1 & n2 \\ n2 & n2 & n1 \end{bmatrix}$$

$$[blue,\ red,\ red]_{\text{perms\_rep}([1,\ 2,\ 2])} = \begin{bmatrix} blue & red & red \\ red & blue & red \\ red & red & blue \end{bmatrix}$$

$$[red,\ red,\ blue]_{\text{perms\_rep}([1,\ 1,\ 2])} = \begin{bmatrix} red & red & red \\ red & red & red \\ red & red & red \end{bmatrix}$$

Dear Joseph,

I tried to solve the problem as simply as possible and at first sight that idea seemed to work, but you proved it not to be reliable. The next simple approach appears to be to substitute numbers for the words, then apply your function, and then substitute back to words. What do you think?

Marcelo

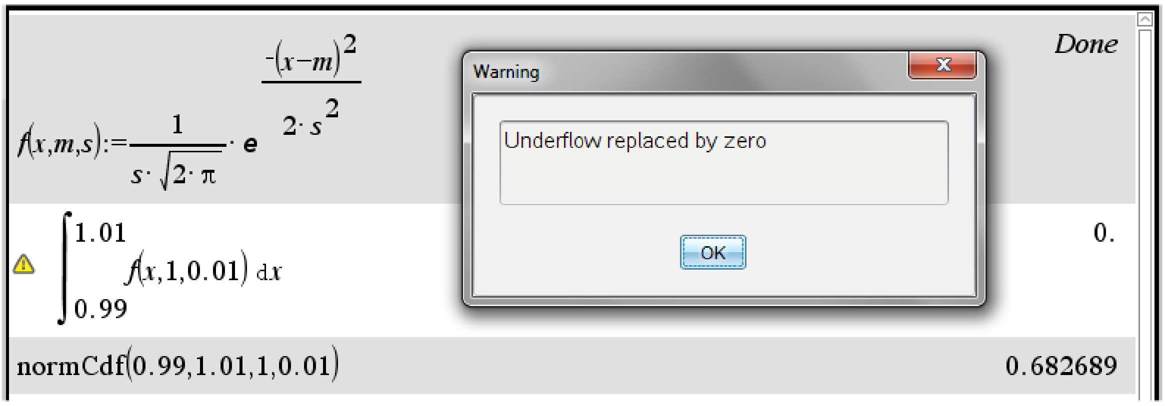
Dear DUGers, what do you think?
Regards, Josef

Hubert Langlotz, Germany

Yet another bug with normal distribution?

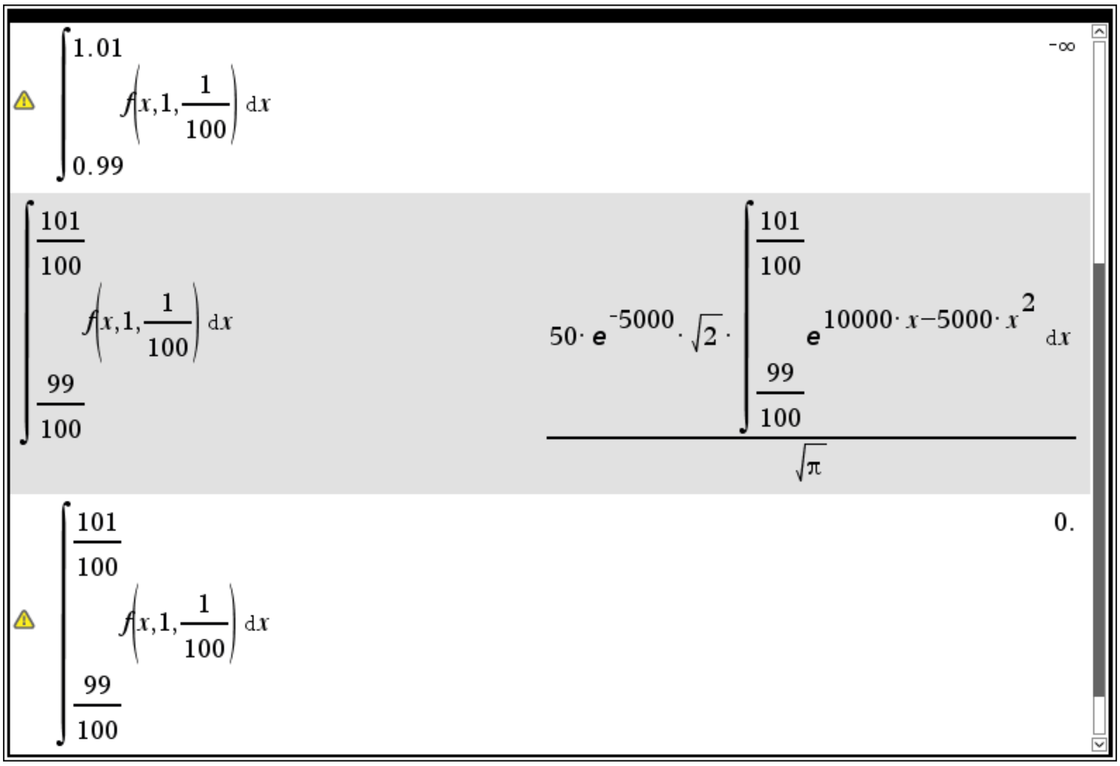
Problem: The filling quantity *X* of orange juice bottles is normally distributed with mean *m* = 1l and standard deviation *σ* = 0.01l. What is the percentage of bottles with a filling quantity of 0.98l ≤ *X* ≤ 1.02l and 0.991l ≤ *X* ≤ 1.01l.

The attached file shows that there is a problem in solving the integral belonging to the normal distribution without using normcdf().

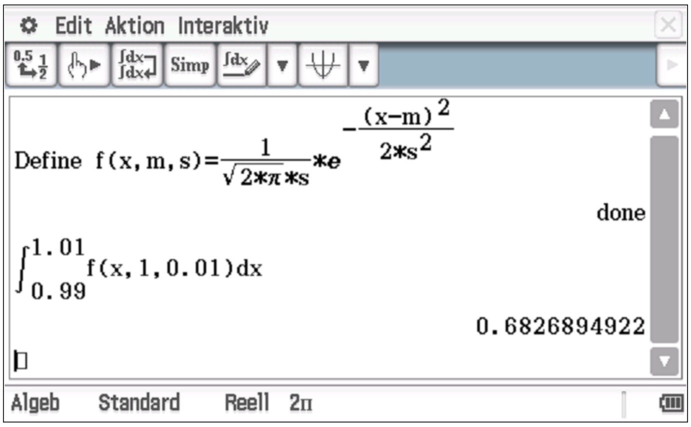


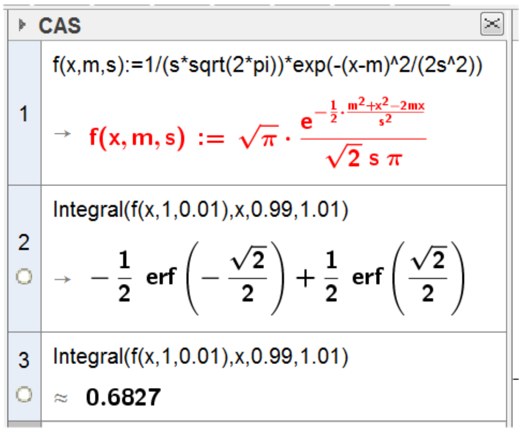
Same happens with 0.98 ≤ *X* ≤ 1.02.

Because I think, that normcdf() uses a certain kind of approximation modus (like binomcdf) the mistake (zero overflow) appears. But when you switch from decimals to fractions instead of the wrong solution "0" the solution "-inf" appears, which is also wrong.



As you can see on the next screen shot ClassPad has no problems calculating the integral:





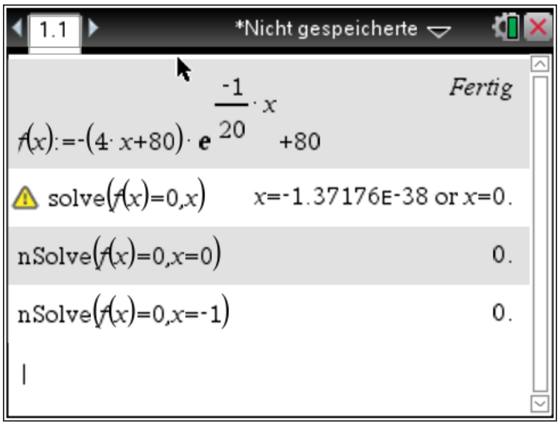
And there is no problem with the CAS of GeoGebra (look at the nice exact result, and the a little bit strange “simplification” of the input. Only DERIVE’s exact result is nicer:

$$\int_{0.99}^{1.01} f(x,\ 1,\ 0.01)\ dx = \mathrm{ERF}\left(\frac{\sqrt{2}}{2}\right)$$

Josef

And Hubert Langlotz, Germany, again:

Although there was no reaction to my last bug here is another one:



Especially the operator “or” is cute.

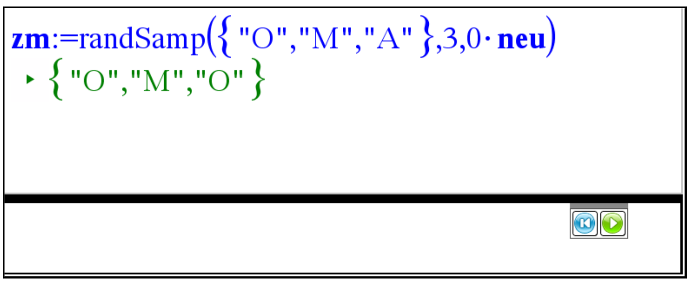
And there was a third mail from Hubert:

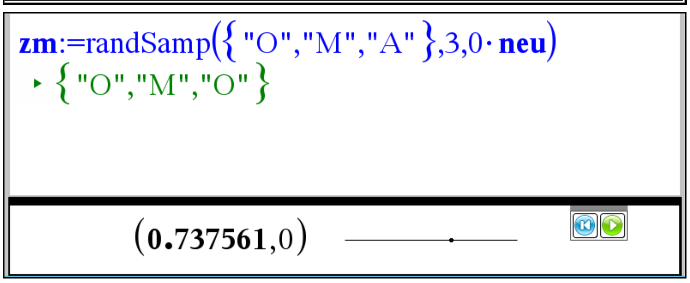
Silly Question: Does this “Play Button“ still exist? Or have I to apply a slider? (see right below)

Benno Grabinger’s answer:

This button appears if you set a point on a segment or a circle and then animate the point. (Context menu – right mouse click).

You can view the hidden objects by 1: Actions > 3: Hide/Show. You will see that the point is animated.





(Gertrud Aumayr knew the right answer, too. Unfortunately there is no hint or advice in the manual about this possibility – and it’s a little bit tricky, Josef.)

What if General Lee Had a Graphing Calculator at Gettysburg?

Robert Hawkes
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South Brunswick, ME
TeacherBob@aol.com

Every teacher these days feels the need to make the math in his courses become real and, dare we hope, even exciting to our students. It was this desire and a happy coincidence that prompted me to combine a newly-found enthusiasm for a milestone event in the history of the United States with the mathematics we study at the pre-calculus and calculus levels. One day as I stood at the green board expounding the virtues of the parabola to model real events, I found myself chatting about artillery fire. An idea then began to take shape. What if I could present a project that would allow my students to look at a key military battle and model it on the graphing calculator? Of course I could, but which battle? Well, it just so happened that the movie Gettysburg was making the rounds of selected theatres and the head of our history department was taking the entire US History class to view it. In desperate need of chaperones, he turned to me to call in a favor. This was the battle I was looking for. The movie was great and gave me a hero whose name should have been important to me much earlier. Joshua Chamberlain was a Maine man, governor of the state, president of the college I attended and a Medal of Honor winner. The movie brought out his humanism and heroism. I had a new hero and I intended to find out more.

This is probably a very good time for me to offer a serious and wide-ranging disclaimer. I am not an historian in any sense of the word. My enthusiasm for history comes late in life. It is more a product of A&E, Ken Burns, The Killer Angels by Michael Sharra and Ted Turner's movie Gettysburg. I am a very visual learner and now had at my disposal the VCR and a very knowledgeable colleague. At one time, I mentioned to this friend that I had to go stand on Little Round Top before I died. He looked at me standing there after a long day in front of the classroom and said "We better hurry and get that done." Three trips to Gettysburg and many books and videos later, I stand ready to share this presentation with you.

On one of these trips, we took a tour along Cemetery Ridge at what has been called the "high water" mark of the Confederacy. Our young guide was very animated as he described the artillery assault just prior to Pickett's Charge. His point, presented so enthusiastically, was that following the initial volleys, the trajectory of the cannon fire was altered and the shells began to find their way over the crest of the ridge. This point was later confirmed to me in Stars In Their Courses , by Shelby Foote. I could see it plainly now. Trajectory, vectors, parametric equations, distances, elevations, gravity, graphing calculator, physics, math and more math all joined with the spectacle of Gettysburg.

I present this workshop to students at the pre-calculus and calculus levels. A knowledge of the graphing calculator, vectors, matrices, and various other algebra and geometry skills are required.

I start my school workshop with a Hyperstudio Presentation which establishes the battlefield placement, the overall situation, the key people on both sides and the following premise: The weather conditions and soft ground at the battlefield contributed to a change in the cannon fire trajectory and a lack of observational capability that rendered the greatest artillery barrage ever on the American continent to be ineffective.

This presentation will eventually be available on the Internet.

For the benefit of the readers of this paper who will not view it, I present a list of the topics in this presentation and a brief idea of the content:

A) **Date**: July 3, 1863

B) **Location**: Gettysburg, PA

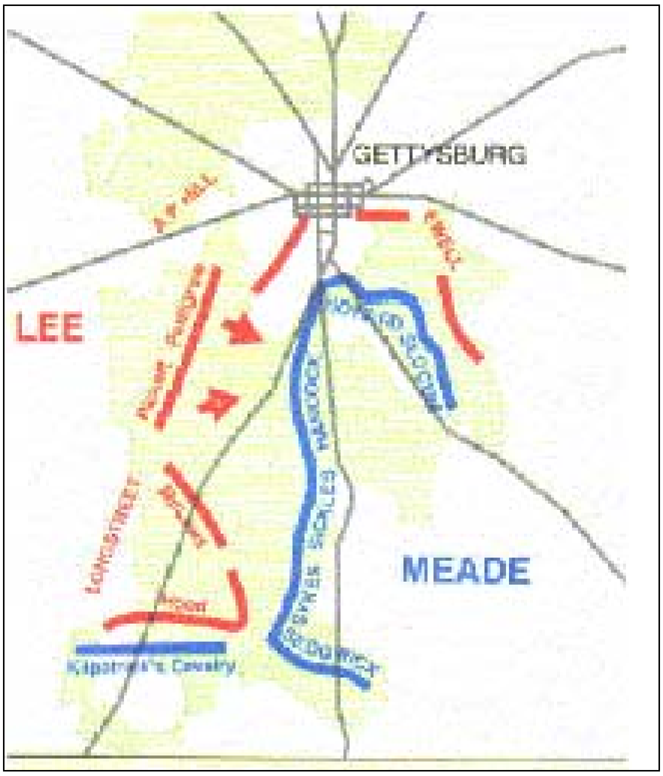
C) **Previous Action**: Two days previous, a group of Union cavalry had encountered a small portion of the entire Confederate Army. A battle ensued in which the aroused Confederates chased the Union soldiers through the town of Gettysburg to a defensive position on the heights on Cemetery Ridge and Culp's Hill.

D) **Weather Report**: There had been some rain, in previous days, to soften the ground but, as the hour for Pickett's Charge approached the day was very hot and humid. The hot, humid air and the softened ground play a large role in this battle.

E) **Placement of the troops**: The Confederates were situated on Seminary Ridge. The Union troops were on a slightly higher ridge called Cemetery Ridge. Between them lay a shallow valley of undulating ground crossed by the Emmitsburg Road and some farm fences. The distance between the ridges is almost a mile.

F) **Key people in the battle**:
Confederates: General Robert E. Lee, General James Longstreet, Col. Edwin P Alexander, General George E. Pickett, General Lewis A. Armistead.

Union: General George G. Meade, General Winfield S. Hancock.



G) **The plan of attack**: A concentrated artillery barrage is to be concentrated on the center of the Union position. Lee feels that Meade has strengthened his flanks and weakened his center position. He plans to weaken the Union in the center or even drive them from the ridge altogether. The infantry could then assault the weakened center and split the Union troops into two very much weakened forces.

H) **Longstreet talks to Lee**: Longstreet listens to Lee's plan but feels that it will fail. He would prefer to wait for the Union to attack so they could fight a defensive battle. Lee disagrees. Longstreet comes close to insubordination when he asks that someone else be placed in command of the assault.

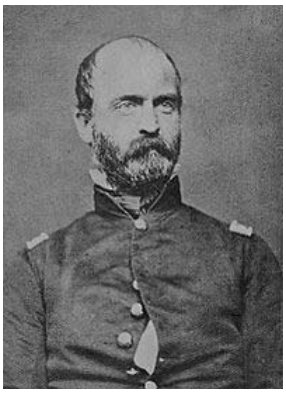
I) **Longstreet talks to Alexander:** Col. Alexander is the best artillery man in the Confederate Army. Longstreet directs him to place some fire on the Union batteries situated on Little Round Top. However, most of his fire, more than 150 cannon is to be directed on the ridge at the center of the Union position. He is to fire all cannon and only save enough ammunition to support the infantry on its heroic assault of the ridge almost a mile away.

J) **Longstreet talks to Pickett:** Pickett's division has arrived on the battlefield weary from the long march but ready for a fight. Longstreet tries to be optimistic as he tells Pickett that he will lead the charge. However, he is less than confident as he asks, "George, can you take the ridge?" Pickett replies with a war whoop.

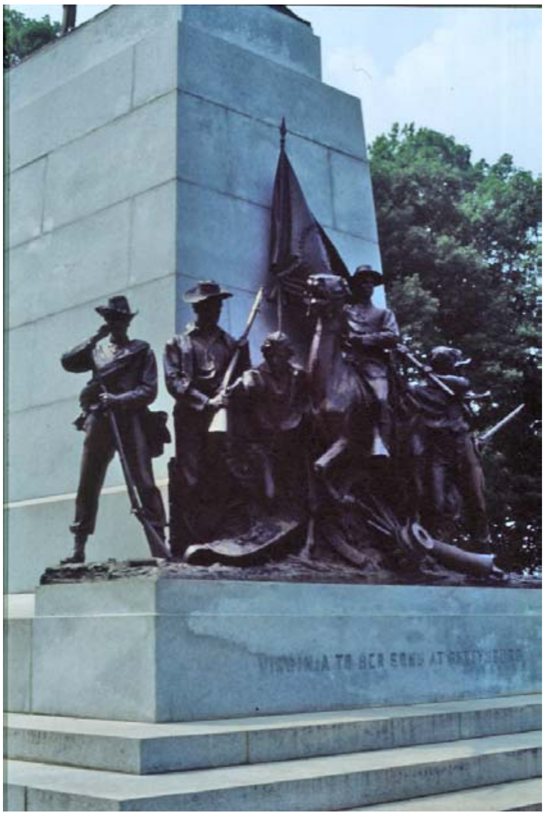
K) **Longstreet talks to Generals:** Longstreet meets with Pickett, Trimble and Pettigrew to outline strategy. There will 15,000 men spread out over a mile. In a series of turning maneuvers, the advance will pinpoint a small clump of trees on Cemetery Ridge.

L) **The barrage starts:** At about 1:00 in the afternoon, a two-gun signal begins what is to be the largest artillery action ever on the American continent. The Union batteries answer. Smoke hangs heavily over the battlefield, making observation difficult at best. The softened ground allows the cannon trail to sink slightly into the ground. This changes the angle of elevation of the cannon. The Union commanders decide to curtail their return fire in order to save ammunition for the infantry charge that they know will follow. They also hope that the Confederates will interpret this as a sign of withdrawal or weakness, a ruse that does have some effect. The barrage will last almost two hours.

M) **The Confederate Infantry moves forward:** The Confederate Infantry emerges from the cover of the woods, bravely advancing through the line of cannon. My presentation concentrates on General Armistead of Virginia. He is an inspirational leader who leads his troops over the Union protective wall. He is mortally wounded by the troops commanded by his friend and companion of earlier days, General Hancock.



General L. A. Armistead



VIRGINIA TO HER SONS AT GETTYSBURG (private)

N) **The Union Cannon begins its effective return fire:** As the Confederate troops come into range, a murderous fire commences from the Union cannon.

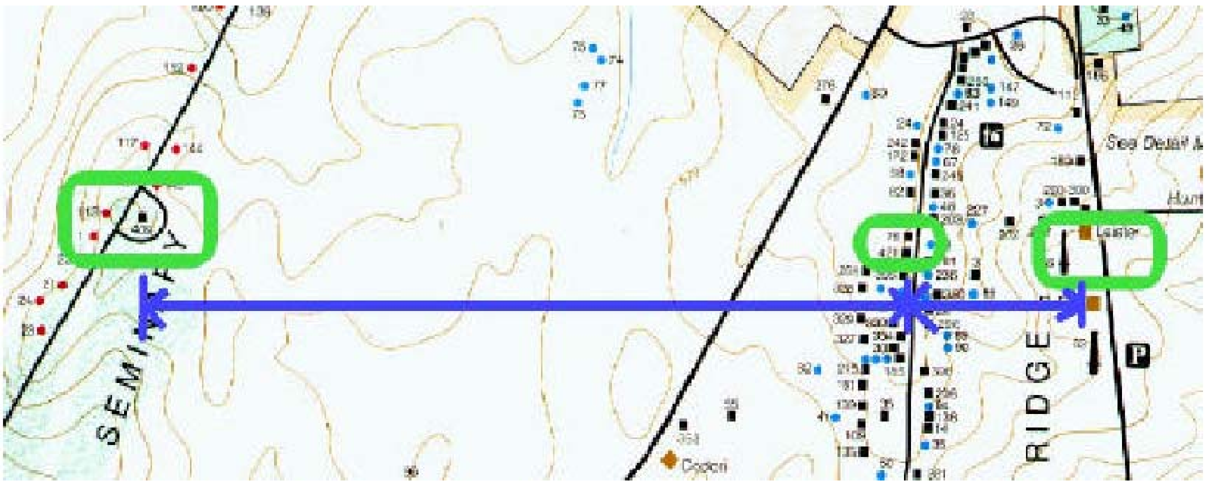
The Battlefield (private)

The Hyperstudio presentation is followed by video clips of the movie, illustrating many of the events mentioned here. So where is the math? Well, it is time now. The map below is used for establishing the some key distances and elevations. We use three key spots on the map:

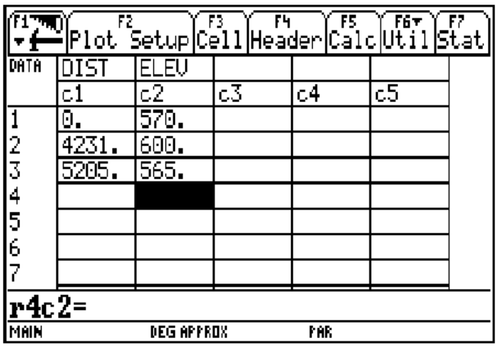
A) The location of the Virginia monument on Seminary Ridge. This will represent the Confederate position.

B) The small box labeled "76" on Cemetery Ridge. This represents the Union position. It denotes the position of the 14th Connecticut Infantry.

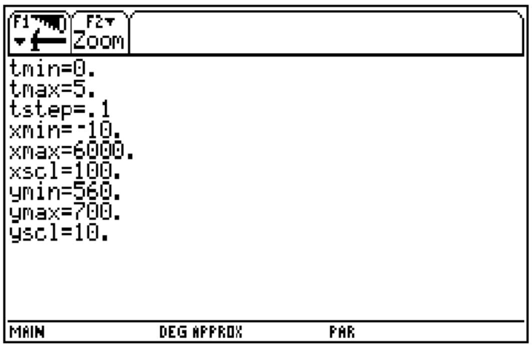
C) The small house on the Taneytown road behind and below Cemetery Ridge. It is the Leister House and was the location of Meade's Headquarters.



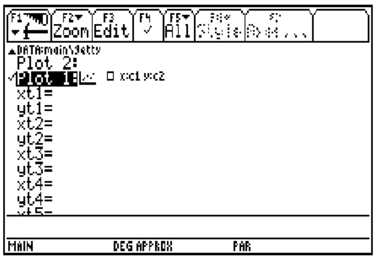
The students draw a line through these three positions and use a ruler and unit conversion techniques to establish distances in feet. By using the contour lines, we establish approximate elevations. For the purposes of this paper, we will use the values shown in these calculator windows:



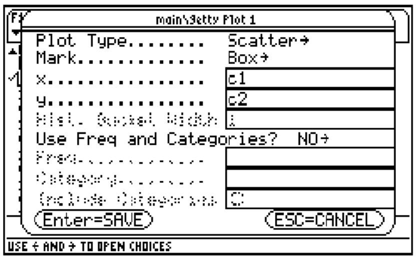
The lists



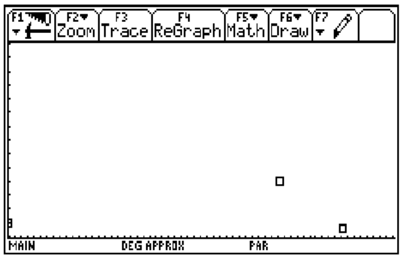
The WINDOW-settings



The Y= Screen



The Plot Setup



The Resulting Plot

In our previous course work we will have established all the foundations for the math we will do next. Of prime importance is work in vectors, matrices and parametric equations.

To establish the plot of cannon fire a pair of parametric equations needs to be established. In their general form the look like this:

$$x_{iT} = v_0 \cdot t \cdot \cos\alpha$$

$$y_{iT} = -\frac{g}{2} \cdot t^2 + v_0 \cdot t \cdot \sin\alpha + h_0$$

where:

v_0 is the Initial Velocity of the Muzzle Velocity

g is the coefficient of acceleration due to gravity

h_0 is the Initial Height (Elevation)

α is the Angle of Elevation for the Cannon

In my research, I could not find a muzzle velocity for any of the cannon. I did find, however, enough data to solve for a good approximation for this muzzle velocity. A 12-lb cannon ball has a range of 2100 yards at 4 degrees of elevation and a time-in-the-air of 5 seconds. Using the equation for x_{iT}, we can solve for v_0:

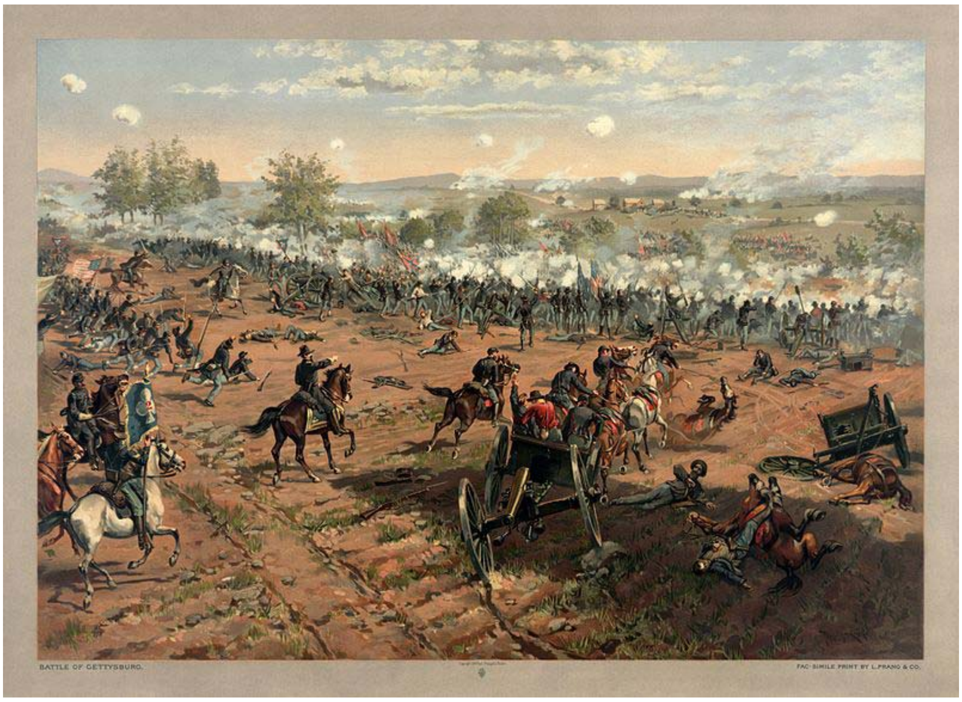
$$x_{iT} = v_0 \cdot t \cdot \cos\alpha$$

$$2100 \cdot 3 = v_0 \cdot 5 \cdot \cos(4°) \qquad \text{(1yard = 3 feet)}$$

$$v_0 = \frac{6300}{5 \cdot \cos(4°)} \approx 1263\,\text{ft} \cdot \text{sec}^{-1}$$

If we accept the gravity factor to be – 32 ft·sec^{-2}, we can now establish our trajectory equations:

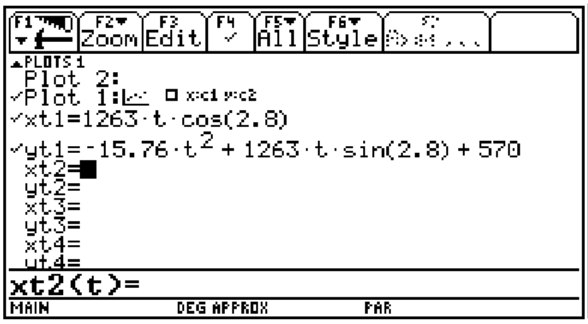
$$x_{iT} = 1263 \cdot t \cdot \cos\alpha$$
$$y_{iT} = -\frac{32}{2} \cdot t^2 + 1263 \cdot t \cdot \sin\alpha + 570$$



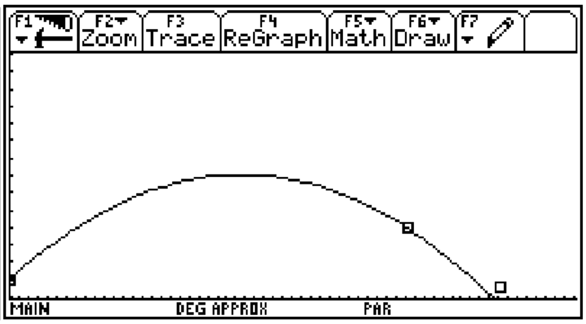
Thure de Thulstrup's *Battle of Gettysburg*, showing Pickett's Charge

At one point a leader of a workshop I was attending gave me an idea of how to expand this with a project using the Calculator Based Lab or Ranger. He very pointedly asked me if the battle was fought in a vacuum. This prompted many experiments dropping various sized balls on the CBR to allow us to compute the gravity factors related to size and weight. This culminated in a ritual dropping of a cannon ball onto the CBR, in a protected case of course. In addition to that, we borrowed a projectile launcher from our friends in the science department. They have all the nicest academic toys. By the time we are ready to do this workshop, all of this math and calculator work is second nature.

After our experimentation with the CBR we settled on a value for - g/2 as – 15.76. This value will change slightly with each class. Students are now encouraged to find an angle of elevation which will land a cannon ball squarely on the Union position. This often is an exercise in what an appropriate estimation is. The occasional estimation of 80° is quickly discouraged even if it lands close.

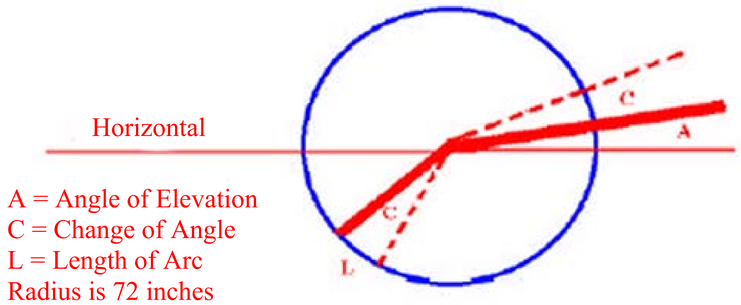


The Y= Screen



The First Trajectory Plot

Now comes the unexpected factor that may have changed the course of history: The trail, rear stabilizing portion of the cannon, sinks almost imperceptibly into the soft soil. The cannon balls disappear into the smoke created by extensive cannon fire from opposing ridges of the battlefield. Taking a good deal of academic license, I suggest that this change is the tiny value of 0.175 inches. Time to translate this to the angular change in the elevation of the cannon. The cannon trail is 72 inches. Therefore, the circumference of this circle used here as a model is 144π inches.



Computation of Change of Elevation:

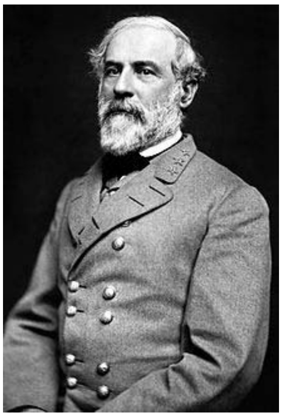
$$0.175 = \frac{C}{360} \cdot 144\pi \;\rightarrow\; C = 0.175 \cdot \frac{360}{144\pi} \;\rightarrow C \approx 0.14°$$

Students will be very eager to enter this new angle of elevation into their calculators. The results become obvious. This slight increase in elevation has caused the cannon fire to overshoot the Union troops and fall ineffectively on the backside of the slope. Indeed, the fire was so devastating at the Leister House that General Meade was forced to move his headquarters.

The most important troop leaders in the Battle at Gettysburg (Wikipedia)

General Meade

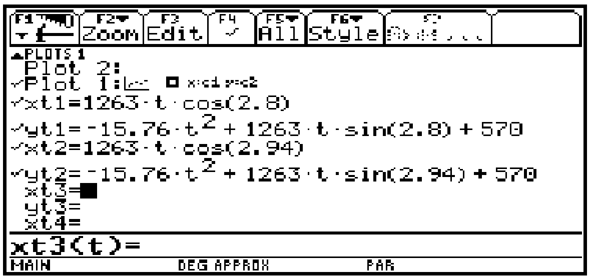
General Hancock



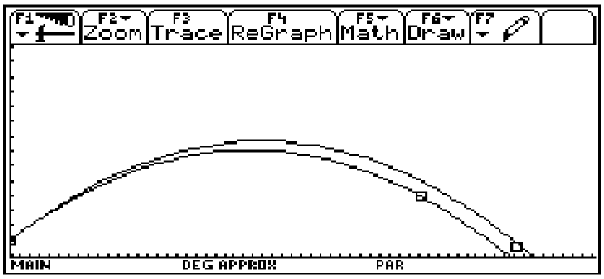
General Lee

General Longstreet

General Pickett



The new Y= Screen



The new Trajectory Screen

This proves to be very unfortunate for the Confederate troops. As they enter onto the open battlefield, they soon realize that their enemy has not been driven from the ridge nor has it been weakened significantly. Pickett's Charge, while heroic in effort and proportion, is destined to be a terrible defeat from which the Confederacy never recovers.

All that remains for us is some video of the advance of Pickett's men. The presentation concludes when the Union guns begin their murderous fire.

References

The Killer Angles by Michael Sharra
Stars In Their Courses by Shelby Foote
Gettysburg, Ted Turner Pictures

Memorial on the Battlefield (private)

The private pictures were taken at the Conference Tour at the 3rd DERIVE & TI-92 Conference at Gettysburg in 1998. Find below more interesting websites, Josef.

http://www.brotherswar.com/Gettysburg-3e.htm
http://www.history.com/topics/american-civil-war/winfield-scott-hancock

Megalomorphs

From DERIVE to LUA-programs for TI-Nspire

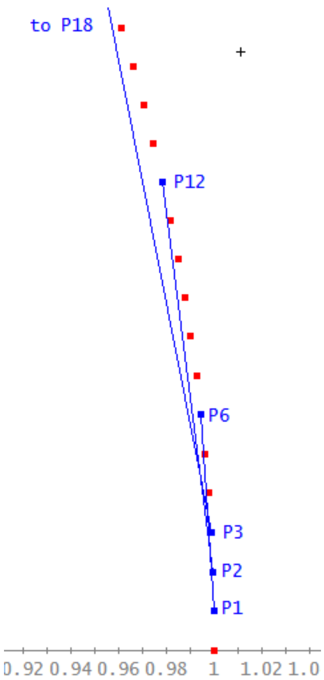
Alfred Roulier and Josef Böhm

Many years ago I (Josef) came across a short paper in *Mathematical Recreations with MuPAD, 2003*, written by Miroslaw Majewski, Zayed University, UAE, entitled *Spider Nets*. MM called his function "Megalomorph". I "googled" and found out that MM had tarantulas (bird-eating spiders) in mind.

The ***Mygalomorphae*** *(also called the Orthognatha) are an infraorder of spiders. The scientific name comes from the orientation of the fangs which point straight down and do not cross each other (as opposed to araneomorph).*

Consider a race of spider-beings named Mygalomorphs who spend their days spinning webs upon circular frames. Status in their society is based on the beauty of their webs. To create the web patterns, the spiders string a straight piece of web from one point on the circle to another. Usually the patterns are dull and uninspiring, and therefore most spiders are relegated to lower societal classes.

One day, a rather intelligent Mygalomorph let a straight web piece amble around the circumference of the circle, the front end going six times as fast as the rear. In other words, every time the rear of the straight web moved one space, the front end moved six. After a few moments' contemplation, the Mygalomorph realized that by the time the fast end has completed one trip around the circle, the slow end had travelled just a sixth of the way around.

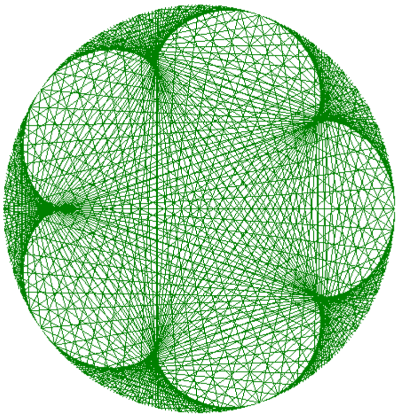


http://sprott.physics.wisc.edu/pickover/mygal.html

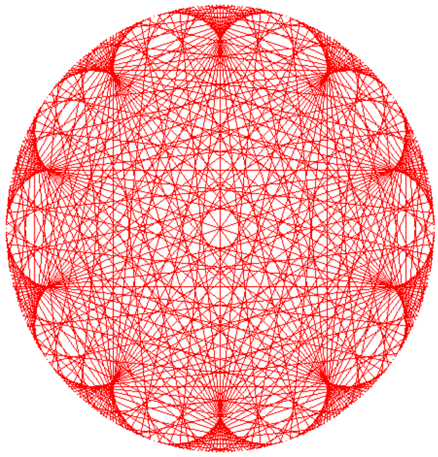
The program code for this kind of spider nets is an easy one. Many of the appearing patterns of the segments are well known, but one can create new and exciting figures.

```
mygalo(r, mov, i, θ, l) :=
  Prog
    i := 1
    l := []
    Loop
      If i > 360 exit
      θ := i·π/180
      l := APPEND(l, [[r·COS(θ), r·SIN(θ); r·COS(mov·θ), r·SIN(mov·θ)]])
      i :+ 1
    l
```

mygalo(1,6)



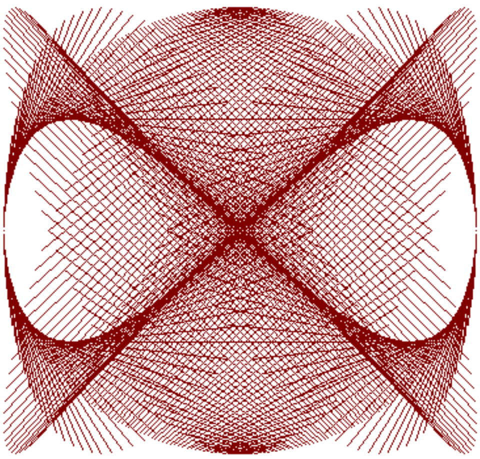
mygalo(1,11)



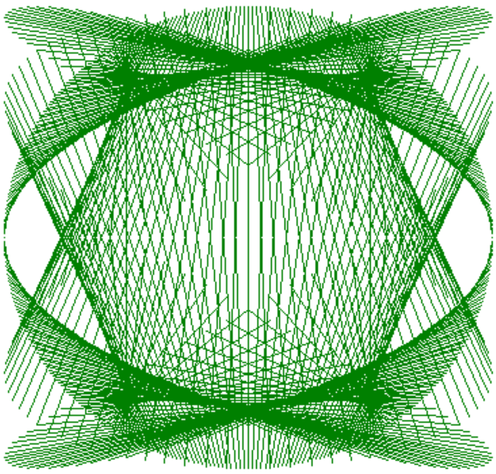
Now we can invent some variants of this basic pattern. We introduce two different velocities for moving both points or the end point only. Compare `mygalo1` and `mygalo2`:

```
mygalo1(r, mov1, mov2, i, θ, l) :=
  Prog
    i := 1
    l := []
    Loop
      If i > 360 exit
      θ := i·π/180
      l := APPEND(l, [[r·COS(θ), r·SIN(θ); r·COS(mov1·θ), r·SIN(mov2·θ)]])
      i :+ 1
    l
```

mygalo1(1,3,5)

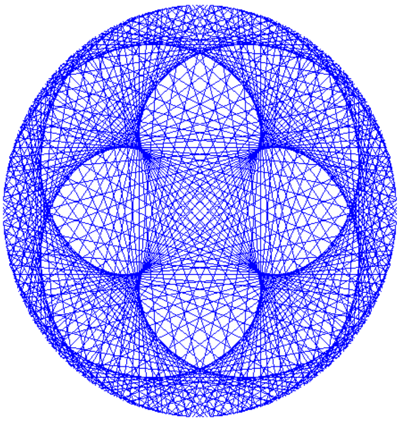


mygalo1(1,5,3)

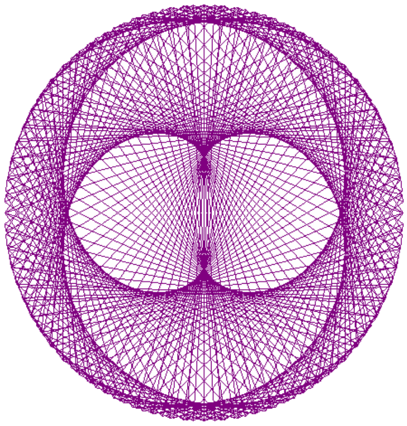


```
mygalo2(r, mov1, mov2, i, θ, l) :=
  Prog
    i := 1
    l := []
    Loop
      If i > 360 exit
      θ := i·π/180
      l := APPEND(l, [[r·COS(mov1·θ), r·SIN(mov1·θ); r·COS(mov2·θ), r·SIN(mov2·θ)]])
      i :+ 1
    l
```

mygalo2(1, 3, 7)

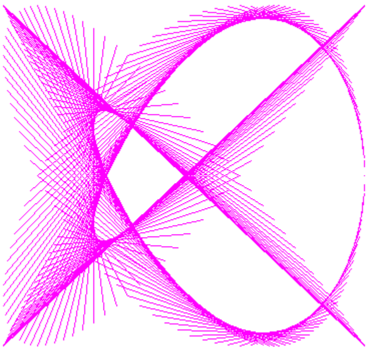


mygalo2(1, 111, 185)

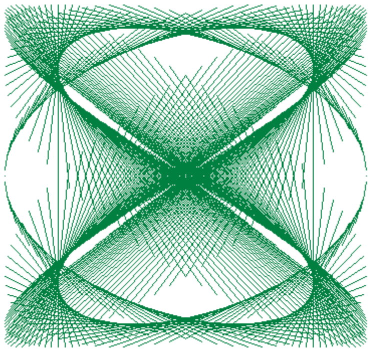


Why not introducing four parameters for the arguments of the trig functions?

mygalo3(1,2,3,4,5)



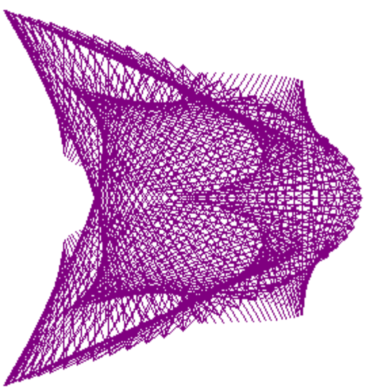
mygalo3(1,5,4,3,2)



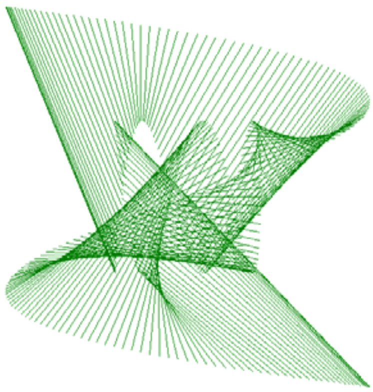
Next possible generalisation: the spider is running along a Lissajous Curve:

```
lissalo(r1, r2, a, b, c, d, i, θ, l) :=
  Prog
    i := 1
    l := []
    Loop
      If i > 360 exit
      θ := i·π/180
      l := APPEND(l, [[r1·COS(a·θ), r1·SIN(b·θ); r2·COS(c·θ), r2·SIN(d·θ)]])
      i :+ 1
    l
```

lissalo(2, 3, 4, 1, 6, 3)



lissalo(5, 2, 3, 1, 2, 7)



I wanted to produce these pretty graphs – and others which I had in my mind – on the Nspire screen, too. But there were – again - the problems plotting families of segments. As the segments were not connected but isolated from each other it was not possible – at least for me – to apply scatter plots.

Then I read an article in the TI-News written by Dr.Alfred Roulier. He produced great plots of Lindenmayer Systems (see DNLs #25, #51 and #52). He mentioned that he had used LUA-scripts. I sent a mail together with my Lindenmayer paper and asked for the tns-file (including the LUA script). Alfred Roulier answered very friendly and sent the Nspire-file (see page 27).

Very glad about the good working communication I sent the Mygalo-DERIVE plots together with the respective functions and asked if he could "translate" the files into an appropriate LUA-Script.

And it really worked. This is Alfred's answer:

Dear Mr. Böhm,

This is my first attempt with a spider's net. The lists of coordinates of the endpoints of the segments are produced in a notes page. Then they are transformed by LUA into segments to be plotted. The script is very simple. I am very fascinated by these patterns and would like to proceed.

Best regards
Alfred Roulier

Spinnennetz_original.tns

n:=360 ▸ 360 Anzahl Schritte

r:=250 ▸ 250 Radius

Parameter

mov1:=8 ▸ 8 **mov2**:=3 ▸ 3

$\mathbf{x1}:=\mathrm{seq}\left(\mathbf{r}\cdot\cos\left(\frac{\mathbf{mov1}\cdot i\cdot\pi}{180.}\right),i,1,\mathbf{n}\right)$

▸ {248.,240.,228.,212.,192.,167.,140.,110.,77.3,43.4,8.72,-26.1,-60.5,-93.7,-125.,-154.,-180.,-202

$\mathbf{y1}:=\mathrm{seq}\left(\mathbf{r}\cdot\sin\left(\frac{\mathbf{mov1}\cdot i\cdot\pi}{180.}\right),i,1,\mathbf{n}\right)$

▸ {34.8,68.9,102.,132.,161.,186.,207.,225.,238.,246.,250.,249.,243.,232.,217.,197.,174.,147.,117.

$\mathbf{x2}:=\mathrm{seq}\left(\mathbf{r}\cdot\cos\left(\frac{\mathbf{mov2}\cdot i\cdot\pi}{180.}\right),i,1,\mathbf{n}\right)$

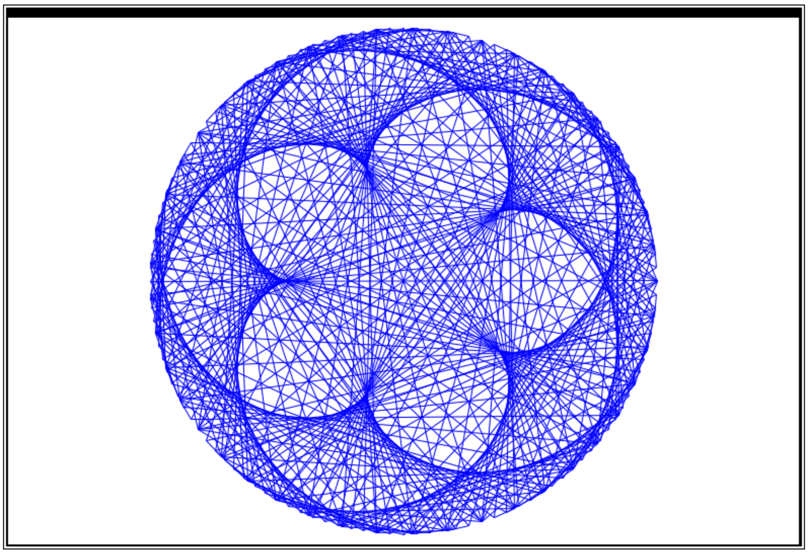
▸ {250.,249.,247.,245.,241.,238.,233.,228.,223.,217.,210.,202.,194.,186.,177.,167.,157.,147.,136.

$\mathbf{y2}:=\mathrm{seq}\left(\mathbf{r}\cdot\sin\left(\frac{\mathbf{mov2}\cdot i\cdot\pi}{180.}\right),i,1,\mathbf{n}\right)$

▸ {13.1,26.1,39.1,52.,64.7,77.3,89.6,102.,113.,125.,136.,147.,157.,167.,177.,186.,194.,202.,210.,

The next page shows the LUA-script for `megalo` together with one result.

In a second problem Alfred provided the adapted script for presenting the `lissalo`-patterns, too.



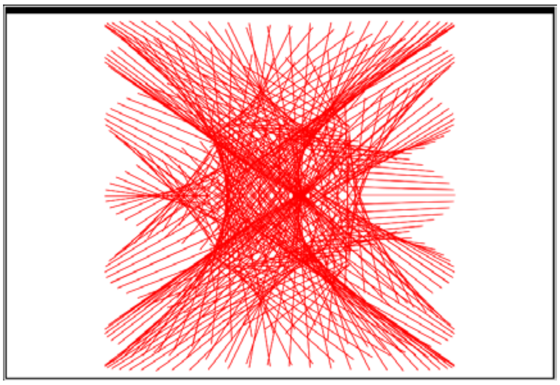
```
platform.apilevel = '1.0'

function on.create()
  h=platform.window:height()
  w=platform.window:width()
end

function
on.resize(width,height)
  h=height
  w=width
end

function on.enterKey()
 platform.window:invalidate()
end

function on.paint(gc)
-- import the number of steps
n=(var.recall("n") or  1)
-- define the lists of coordinates
x1 = {} ; x2 = {};  y1 = {} y2 = {} ;
   for i=1,n do
      x1[i] = 0 ; x2[i] = 0 ; y1[i] = 0 ; y2[i] = 0
   end
-- import the lists of coordinates
   x1=(var.recall("x1" or 2)) ; x2=(var.recall("x2" or 2))
   y1=(var.recall("y1" or 2)) ; y2=(var.recall("y2" or 2))
-- plot the segments
-- plot colour is blue
  gc:setColorRGB(0,0,255)
  for i=1,n do
    gc:drawLine(w/2+x1[i],h/2-y1[i],w/2+x2[i],h/2-y2[i])
  end
 end
```

The lissalo – pattern:

n:=720 ▸ 720 Number of Steps

r1:=250 ▸ 250 r2:=100 ▸ 100 Radii

Parameters: a:=7 ▸ 7 b:=2 ▸ 2 c:=11 ▸ 11 d:=7 ▸ 7

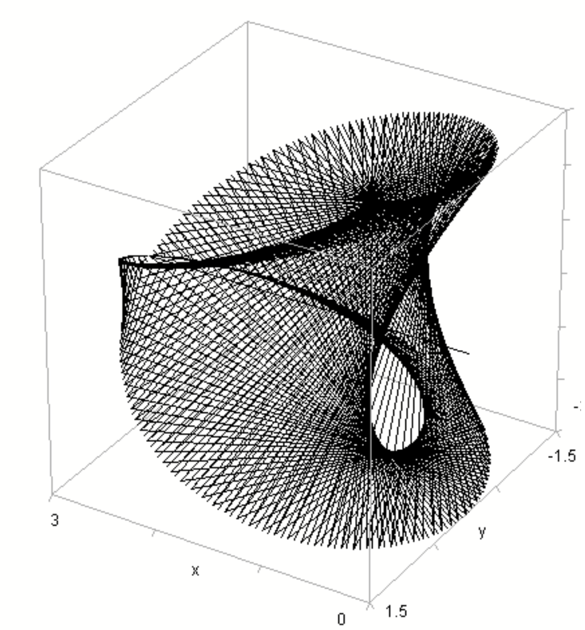


I wrote to Alfred:

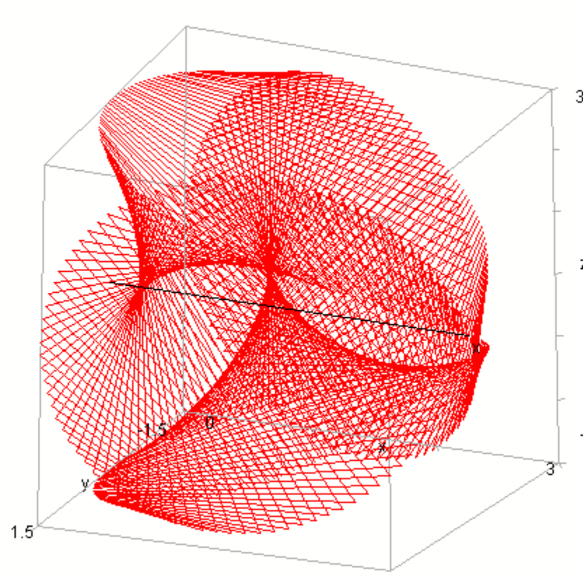
3-D objects formed by segments cannot be presented in TI_Nspire's 3D pages. In DERIVE we can. I produce 3D-spider nets. I will try to use a matrix transformation to the 3D-object into the 2D-plane, i.e. on the Nspire Graph page.

```
spider3d(r, a, b, c, i, θ, l) :=
  Prog
    l := []
    i := 1
    Loop
      If i > 720 exit
      θ := i·π/180
      l := APPEND(l, [[[r·COS(θ)·COS(θ), r·SIN(θ)·COS(θ), r·SIN(θ);
           r·COS(a·θ)·COS(a·θ), r·SIN(b·θ)·COS(b·θ), r·SIN(c·θ)]]])
      i :+ 1
```

spider3d(3, 2, 2, 2)



spider3d(3, 2, 1, 4)



I sent another mail to Alfred Roulier and attached a tns-file containing four problems using again his “all purpose” LUA-script:

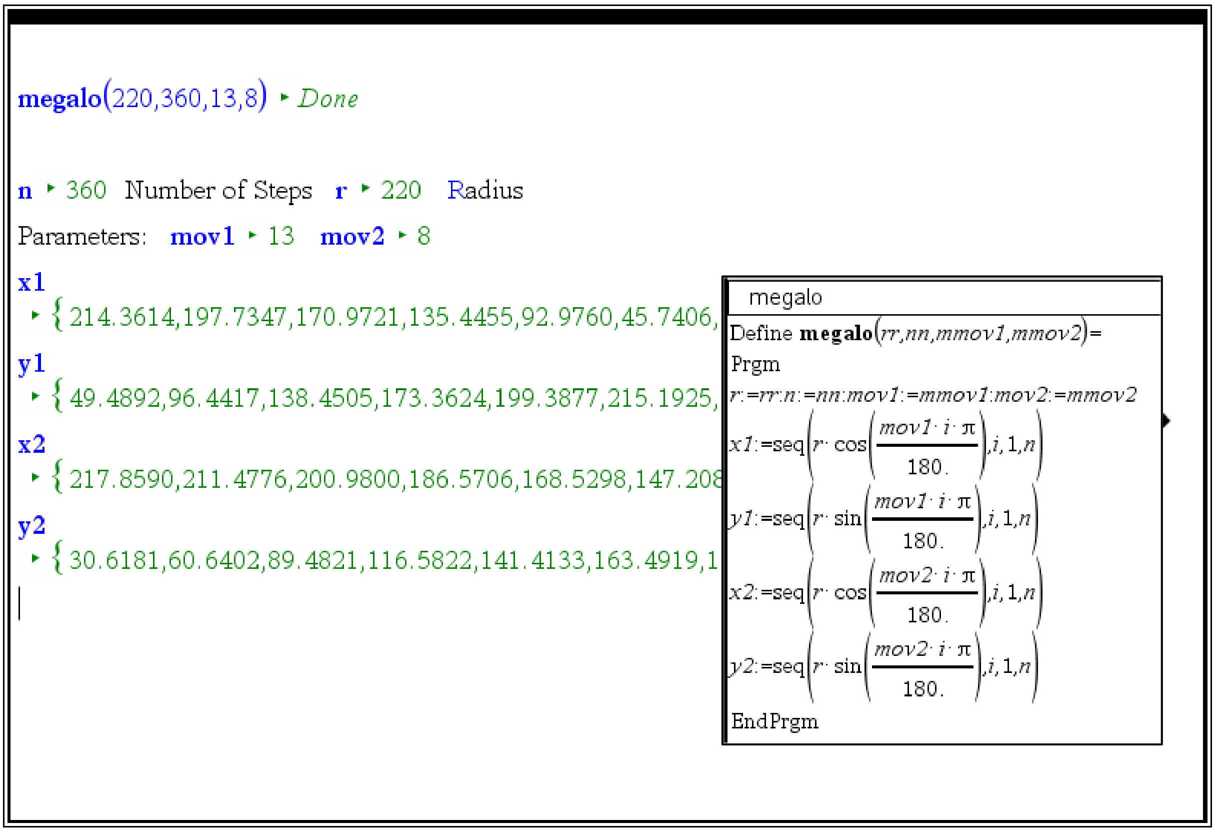
Problem 1: megalo_prog (the spider nets from above produced by a program)

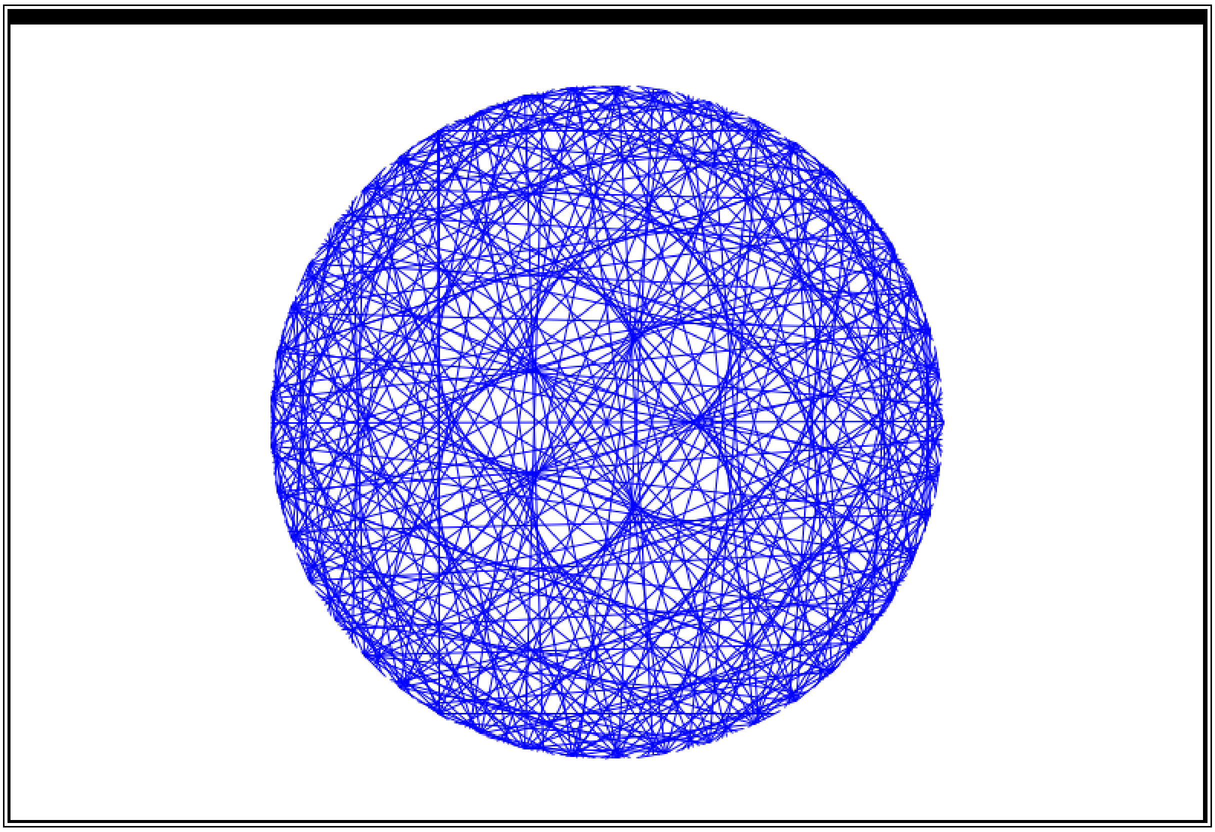
Problem 2: lissajou (without a program, created in math boxes in a notes page)

Problem 3: lissa_prog (the Lissajous-patters produced by a program)

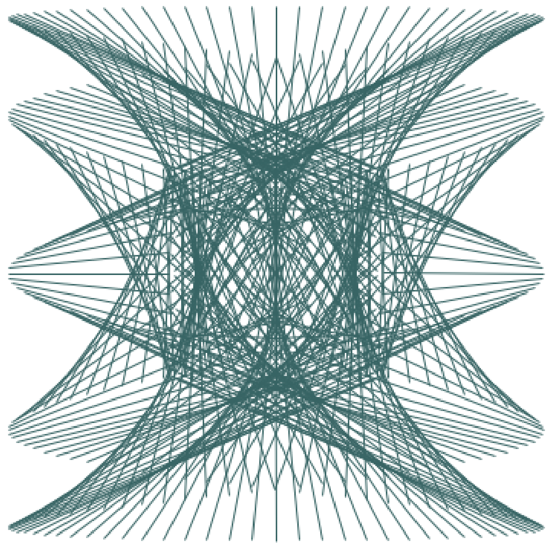
Problem 4: lissa_3D (the 3D-figures as an axonometric projection into the 2D-plane together with introducing sliders for interactive changing the colours.

Problem 1: megalo_prog





Next page shows problem 3: the program for creating the Lissajous-Mygalomorphs.



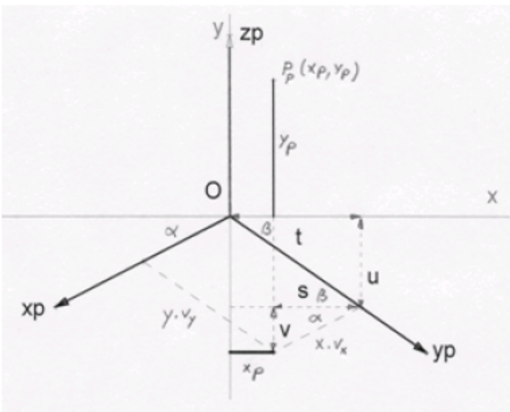
Enter the program and its parameters:

lissalo(radius1, a, b, radius2, c,d, steps)

lissalo(250· 0.7,5,1,100· 0.7,7,11,360) ▸ *Done*

Problem 4: the projection of the 3D objects:

Axonometric projection with ratios of contraction vx, vy and vz and the projection angles alpha and beta:



$$x_p = t - s = y \cdot v_y \cdot \cos\beta - x \cdot v_x \cdot \cos\alpha$$

$$y_p = -u - v + z \cdot v_z = -y \cdot v_y \cdot \sin\beta - x \cdot v_x \cdot \sin\alpha + z \cdot v_z$$

lissa_3D produces the projections of the objects presented with DERIVE using spider3d (see above) into the xy-plane. And these projections can be plotted with TI-Nspire.

This is the transformation matrix to find the 2D-coordinates (x_p,y_p) of a point in space (x,y,z).

$$(x_p, y_p) = (x, y, z) \cdot \begin{pmatrix} -v_x \cos\alpha & -v_x \sin\alpha \\ v_y \cos\beta & -v_y \sin\beta \\ 0 & v_z \end{pmatrix}$$

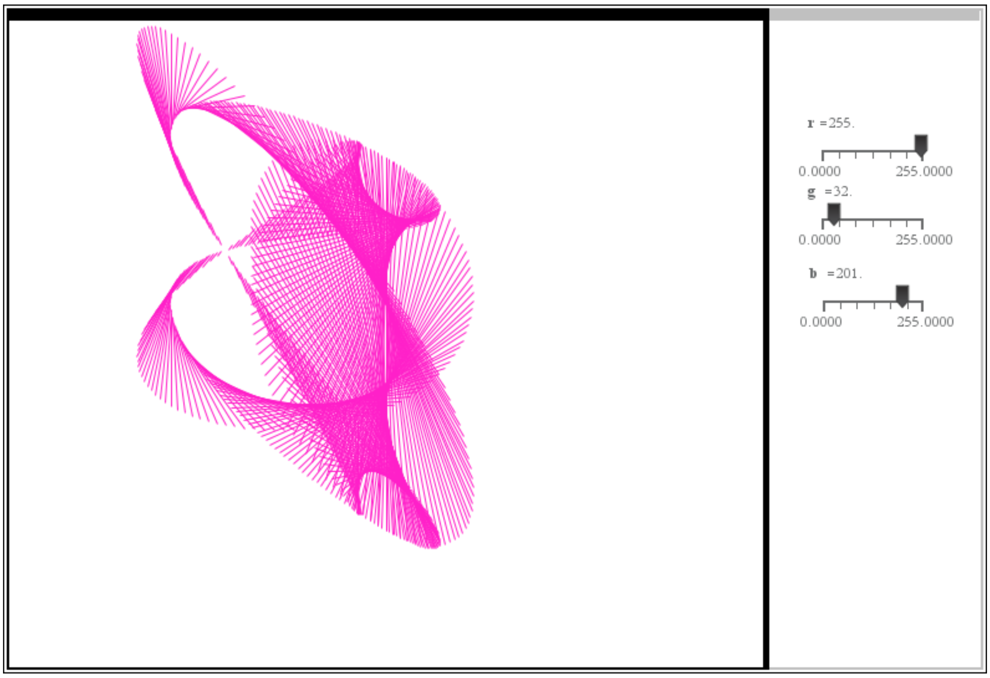
We can enter the projection parameters on a notes page and then call spider3d().

vx:=0.75 ▸ 0.7500 **vy**:=1 ▸ 1 **vz**:=0.75 ▸ 0.7500

α:=-30 ▸ -30 **β**:=30 ▸ 30

spider3d(200,1,2,3,360)

Use the sliders to change the colours.



Define **spider3d**(r,a,b,c,nn)=
Prgm
Local *pts*
$x1:=\{\,\}:y1:=\{\,\}:x2:=\{\,\}:y2:=\{\,\}$
$n:=nn$
For $i,1,n$

$$pts:=\begin{bmatrix} r\cdot\cos\left(\frac{i\cdot\pi}{180}\right)\cdot\cos\left(\frac{i\cdot\pi}{180}\right) & r\cdot\sin\left(\frac{i\cdot\pi}{180}\right)\cdot\cos\left(\frac{i\cdot\pi}{180}\right) & r\cdot\sin\left(\frac{i\cdot\pi}{180}\right) \\ r\cdot\cos\left(\frac{a\cdot i\cdot\pi}{180}\right)\cdot\cos\left(\frac{a\cdot i\cdot\pi}{180}\right) & r\cdot\sin\left(\frac{b\cdot i\cdot\pi}{180}\right)\cdot\cos\left(\frac{b\cdot i\cdot\pi}{180}\right) & r\cdot\sin\left(\frac{c\cdot i\cdot\pi}{180}\right)\end{bmatrix}\cdot aff(vx,vy,vz,\alpha,\beta)$$

$x1:=$augment$(x1,\{pts[1,1]\}):y1:=$augment$(y1,\{pts[1,2]\})$
$x2:=$augment$(x2,\{pts[2,1]\}):y2:=$augment$(y2,\{pts[2,2]\})$
EndFor
EndPrgm

Define **aff**(vx,vy,vz,α,β)=
Func

$$\begin{bmatrix} -vx\cdot\cos\left(\frac{\alpha\cdot\pi}{180}\right) & -vx\cdot\sin\left(\frac{\alpha\cdot\pi}{180}\right) \\ vy\cdot\cos\left(\frac{\beta\cdot\pi}{180}\right) & -vy\cdot\sin\left(\frac{\beta\cdot\pi}{180}\right) \\ 0 & vz\end{bmatrix}$$

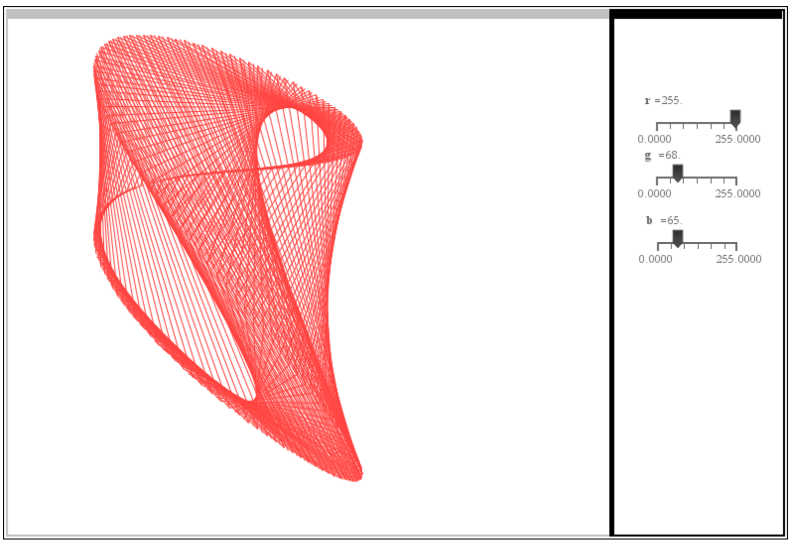
EndFunc

I neither print the program spider3d() nor the LUA-script for plotting the 3D-objects.

You can inspect both by running the file with TI-NspireCAS.

vx:=0.75 ▸ 0.7500   **vy**:=1 ▸ 1   **vz**:=0.75 ▸ 0.7500
α:=-30 ▸ -30   **β**:=30 ▸ 30
spider3d(250,2,2,2,360) ▸ *Done*

Use the sliders for changing the colours.



Mail from Alfred Roulier:

Dear Mr. Böhm,

… You worked a lot on our problem. In particular the 3D-supplementation is very interesting.

This is what I did: I summarized all 2D-forms to one function – the "simple" pictures are a subgroup of the Lissajous form. Then I included a "run of colours". It makes possible to see where the figure is starting and where it is ending. It is necessary to transfer the HSV colour space into the RGB colour space. …

(See Alfred's contribution on Julia sets in DNL#94/95 and the respective explications.)

n segments with end points on circles (radii r1 and r2) are plotted. The plane is completely used if the greater radius is 250.

The increments of the angles of four coordinates x1,y1,x2,y2 in radiant q·π/180 with q = a, b, c or d.

The coordinates and the colours of the segments are calculated by using TI-Nspire commands, the segments are plotted by using a LUA script.

A continuous run of colours can be defined easily in the HSV-colour space. But LUA supports the RGB-system. Program farbtransfer(h,s,v) translates the parameters h = hue, s = saturation and v = brightness into the RGB values for red, green and blue. Program farbgang(h1,h2,s,v) generates the lists red, green und blue for the colours of the respective segments between the boundary values for the hue h1 and h2.

(In order to save space I don't print the program codes of farbtransfer and farbgang. You are invited to inspect both programs and the LUA-script by yourself, Josef)

This is the notes page where one has to enter all necessary parameters, for the figure and for the colour setting as well:

Anzahl Schritte/Number of steps **n**:=360 ▸ Radien/Radii **r1**:=200 ▸ 200 **r2**:=250 ▸ 250

Parameters: **a**:=1 ▸ 1 **b**:=1 ▸ 1 **c**:=4 ▸ 4 **d**:=6 ▸ 6

Streckenendpunkte / endpoints of the segments (hidden output!)

$\mathbf{x1}:=\text{seq}\left(\mathbf{r1}\cdot\cos\left(\frac{\mathbf{a}\cdot\mathbf{i}\cdot\pi}{180.}\right),\mathbf{i},1,\mathbf{n}\right)$

$\mathbf{y1}:=\text{seq}\left(\mathbf{r1}\cdot\sin\left(\frac{\mathbf{b}\cdot\mathbf{i}\cdot\pi}{180.}\right),\mathbf{i},1,\mathbf{n}\right)$

$\mathbf{x2}:=\text{seq}\left(\mathbf{r2}\cdot\cos\left(\frac{\mathbf{c}\cdot\mathbf{i}\cdot\pi}{180.}\right),\mathbf{i},1,\mathbf{n}\right)$

$\mathbf{y2}:=\text{seq}\left(\mathbf{r2}\cdot\sin\left(\frac{\mathbf{d}\cdot\mathbf{i}\cdot\pi}{180.}\right),\mathbf{i},1,\mathbf{n}\right)$

Farbgang / Run of Colours

h1:=60 ▸ 60 **h2**:=350 ▸ 350 **s**:=1 ▸ 1 **v**:=0.9 ▸ 0.9 **farbgang(h1,h2,s,v)** ▸ *Done*

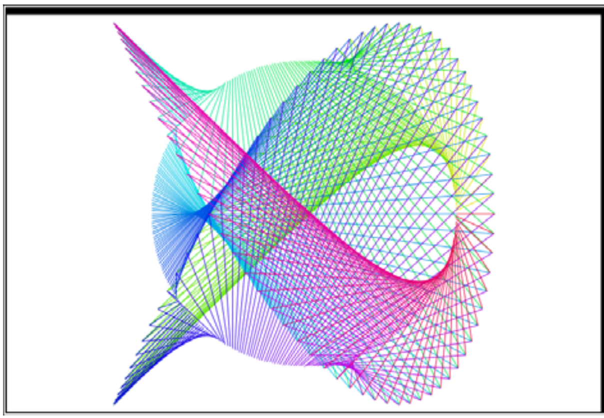
red

▸ {226.,223.,220.,217.,214.,211.,208.,205.,202.,199.,196.,193.,189.,186.,183.,180.,177.,174.,171.

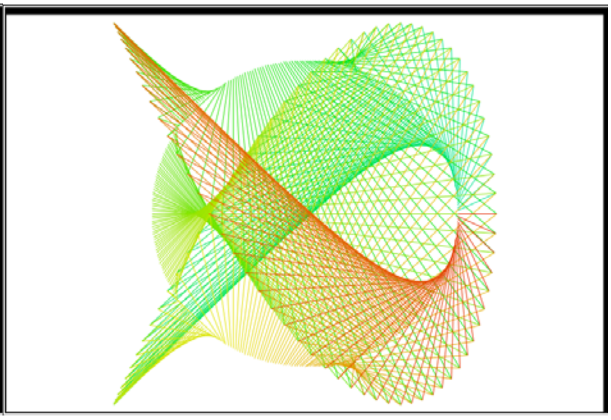
green

▸ {230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.,230.

blue



h1 = 60, h2 = 350



h1 = 160, h2 = 10

Dear Mr.Roulier,

I hope that you like this: I extended our spider nets by calculating and plotting the respective envelopes. The equations for the Lissa-envelopes are pretty bulky. I performed the calculation with DERIVE and transferred the equations to the Nspire.

It is always the same: such "playing around" develops very often a strange dynamics!

We try to find the envelope of the family of segments (lines).

The procedure is as follows:

The family of curves depending on the parameter, say t is given by f(x,y,t). We need the 1st derivative wrt the parameter. This gives f1(x,y,t) = dif(f(x,y,t),t). The system {f(x,y,t), f1(x,y,t)} must be solved for {x,y}. This gives the parameter representation of the envelope.

The family of lines is defined by two points P1 = (r*cos(m1*θ),r*sin(m1*θ)) and P2 = (r*cos(m2*θ, r*sin(m2*θ)) with θ = t*π/180.

$$f(x,y,t):=y-r\cdot\sin\left(\frac{m1\cdot t\cdot\pi}{180}\right)=\frac{r\cdot\sin\left(m2\cdot\frac{t\cdot\pi}{180}\right)-r\cdot\sin\left(m1\cdot\frac{t\cdot\pi}{180}\right)}{r\cdot\cos\left(m2\cdot\frac{t\cdot\pi}{180}\right)-r\cdot\cos\left(m1\cdot\frac{t\cdot\pi}{180}\right)}\cdot\left(x-r\cdot\cos\left(m1\cdot\frac{t\cdot\pi}{180}\right)\right) \blacktriangleright \text{Done}$$

$$f1(x,y,t):=\frac{d}{dt}(f(x,y,t)) \blacktriangleright \text{Done}$$

$$\text{solve}(f(x,y,t) \text{ and } f1(x,y,t),\{x,y\})$$

$$\blacktriangleright \cos\left(\frac{m2\cdot\pi\cdot t}{180}\right)-\cos\left(\frac{m1\cdot\pi\cdot t}{180}\right)\neq 0 \text{ and } x=\frac{r\cdot\left(m2\cdot\cos\left(\frac{m1\cdot\pi\cdot t}{180}\right)+m1\cdot\cos\left(\frac{m2\cdot\pi\cdot t}{180}\right)\right)}{m1+m2}$$

$$\text{and } y=\frac{r\cdot\left(m2\cdot\sin\left(\frac{m1\cdot\pi\cdot t}{180}\right)+m1\cdot\sin\left(\frac{m2\cdot\pi\cdot t}{180}\right)\right)}{m1+m2}$$

This is the parameter representation of the envelope.

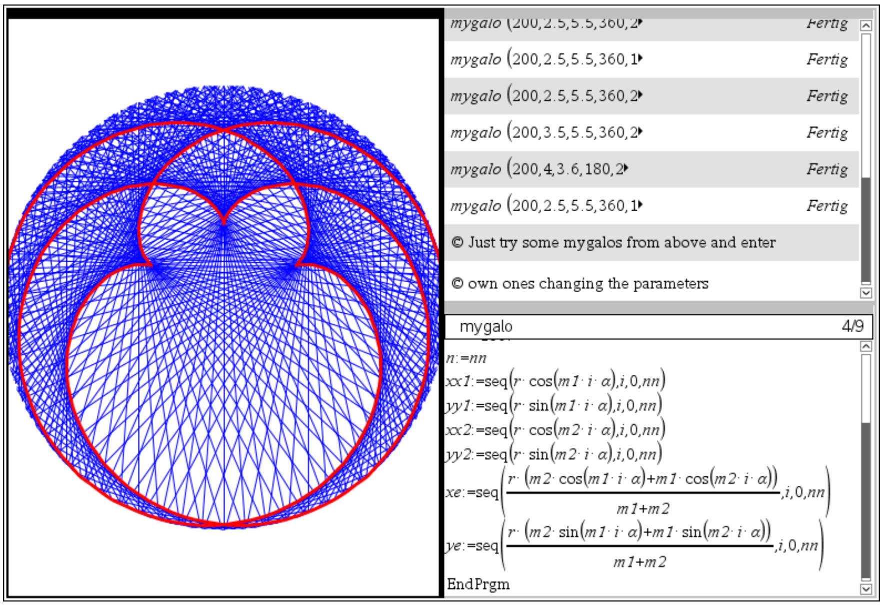
We use the derived equation to accomplish the program.

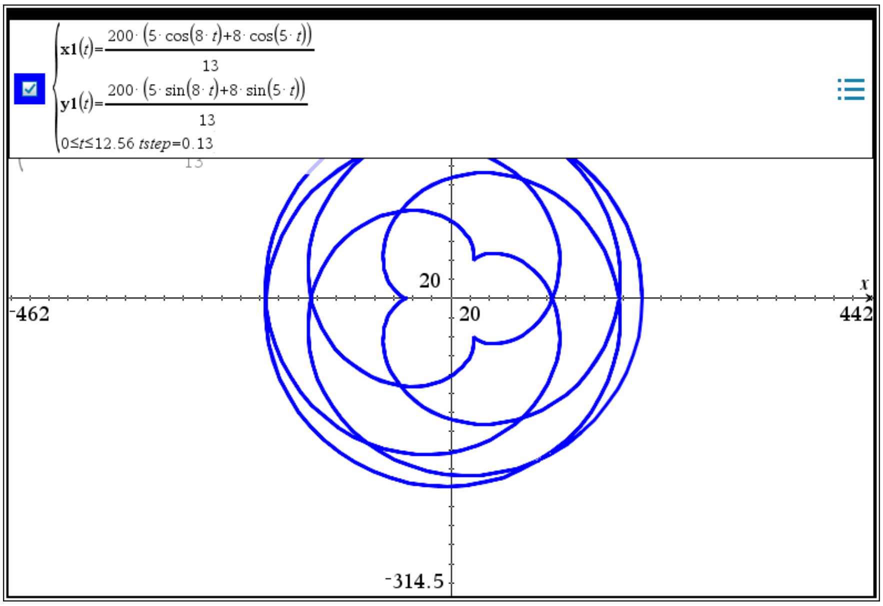
The envelope is presented in the Graph–Application

This one family of "spider tracks" together with the envelope of the family of segments.

Program mygalo generates the sets of coordinates which are then plotted by a LUA-script.

The plot colours are fixed in the script.





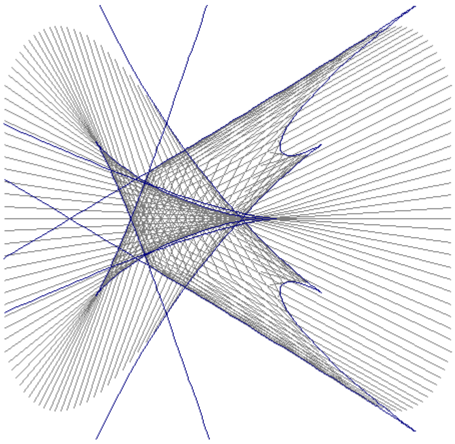
Here is another envelope plotted in its parameter form (r = 200, m1 = 5 and m2 = 8).

The DERIVE code below shows the derivation of the envelope (in its shortest form without displaying the bulky expressions for xt and yt.)

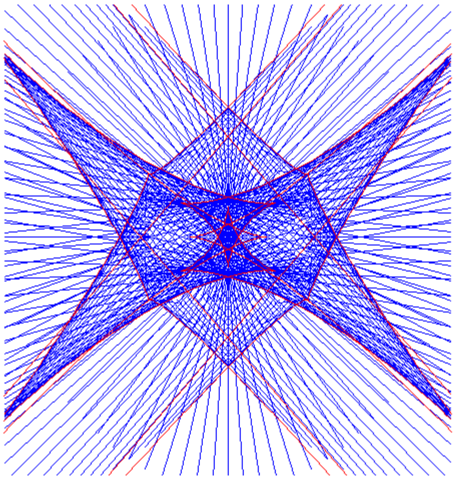
```
⎡ x = r1·COS(a·t) + u·(r2·COS(c·t) - r1·COS(a·t)) ⎤
⎣ y = r1·SIN(b·t) + u·(r2·SIN(d·t) - r1·SIN(b·t)) ⎦

SOLVE(x = r1·COS(a·t) + u·(r2·COS(c·t) - r1·COS(a·t)), u)

          r1·COS(a·t) - x
u = ─────────────────────────
     r1·COS(a·t) - r2·COS(c·t)

                                                                          r1·COS(a·t) - x
env1 := SUBST(y - (r1·SIN(b·t) + u·(r2·SIN(d·t) - r1·SIN(b·t))) = 0, u, ─────────────────────────)
                                                                      r1·COS(a·t) - r2·COS(c·t)

         d
env2 := ── env1
        dt

xt := (SOLUTIONS(env2, x))
                          1

yt := (SOLUTIONS(SUBST(env1, x, xt), y))
                                        1

lissa_env(r1, r2, a, b, c, d) := [xt, yt]
```

Lissa – Mygalo - Gallery

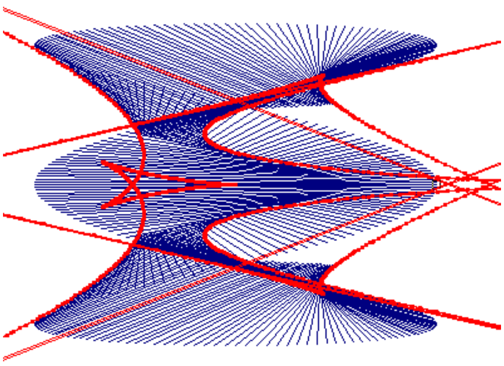
```
lissalo(1, 3, 8, 2, 2, 4)
lissa_env(1, 3, 8, 2, 2, 4)
```



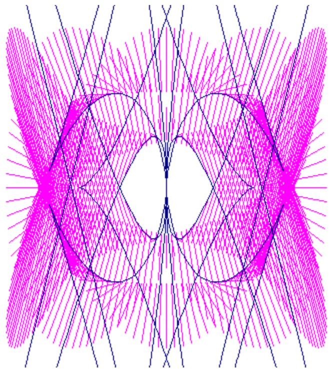
```
lissalo(1, 4, 5, 3, 3, 5)
lissa_env(1, 4, 5, 3, 3, 5)
```



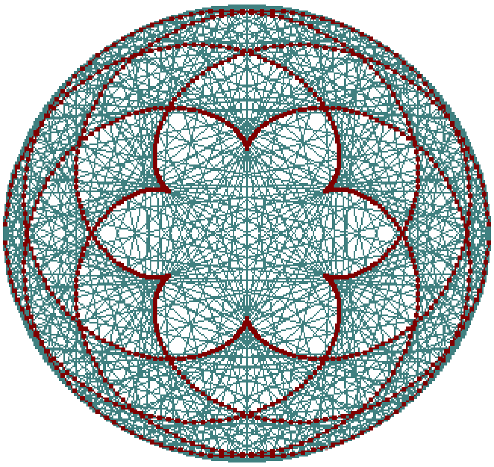
```
lissalo(2, 1, 3, 1, 4, 1)
lissa_env(2, 1, 3, 1, 4, 1)
```



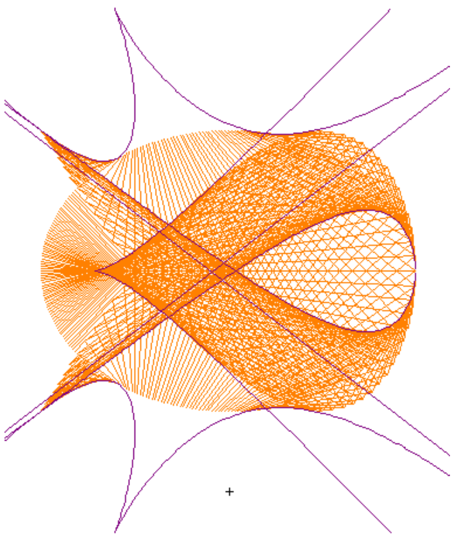
```
lissalo(5, 3, 3, 5, 3, 1)
lissa_env(5, 3, 3, 5, 3, 1)
```

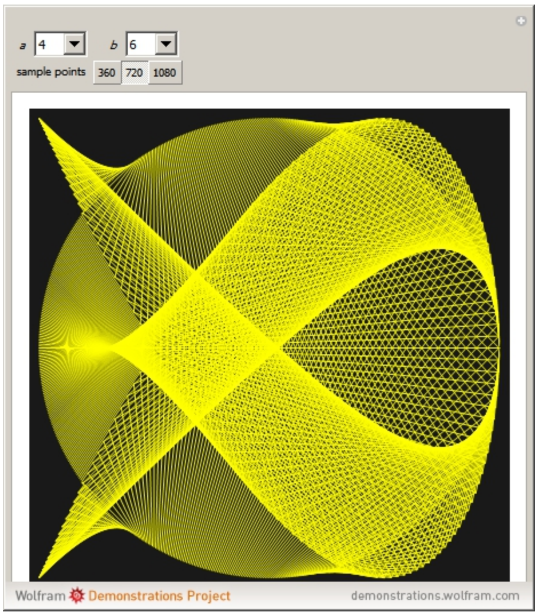


```
lissalo(1, 1, 5, 5, 11, 11)
lissa_env(1, 1, 5, 5, 11, 11)
```



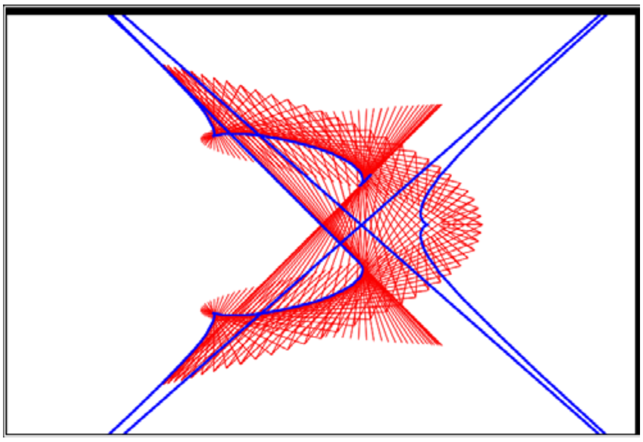
```
lissalo(2, 2, 1, 1, 4, 6)
lissa_env(2, 2, 1, 1, 4, 6)
```





A Mygalomorph Designer on [3] (left) and a Lissajous Mygalo together with its envelope (below) on the TI-NspireCAS screen.

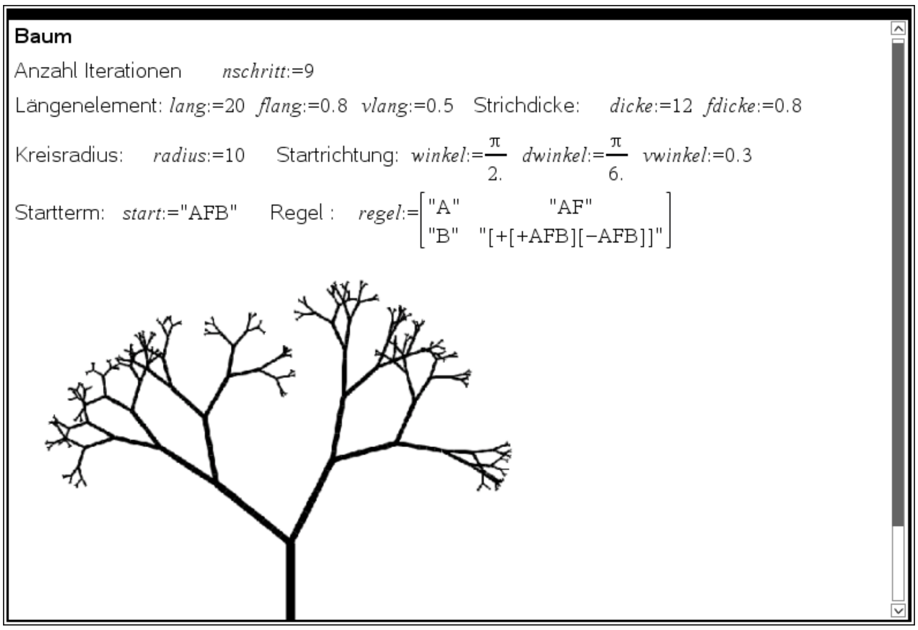
lissa_env(150,200,4,1,6,3,360,1)



There are so many possibilities to vary the *Mygalomorphs*. We would like to invite you experimenting and enjoying the wonderful graphs, Alfred and Josef.

References:

[1] J. S. Madachy, Madachy's Mathematical Recreations, New York, Dover
[2] C. A. Pickover, Keys to Infinity, New York, Wiley
[3] http://sprott.physics.wisc.edu/pickover/mygal.html
[4] Dr. A. Roulier, Colour Gradient and LUA Scripts with TI-Nspire, DERIVE Newsletter 95/95, 2014
[5] Dr. A. Roulier, Strukturen mit TI-Nspire™ konstruieren und zeichnen, TI-Nachrichten 2/13
[6] Private communication with Stephen Arnold who provided very valuable advice

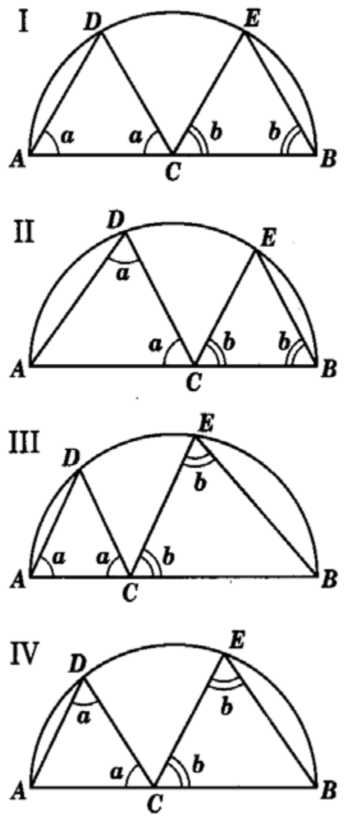


A fruitful Cooperation between Dynamic Geometry and Computer Algebra

Walter Wegscheider, Thomas Himmelbauer & Josef Böhm

Several years ago I (Josef) came across an interesting contribution in the *Reader Reflections* of the *Mathematics Teacher*.

Solution: Four possible cases exist:



In cases I and IV, $\cos 2a + \cos 2b = -1$; and in cases II and III, $a = b$.

Fig. 1 (Deshpande)

Surprising result

I found the following observation and results interesting and think it would be a nice problem for students.

Let $\overline{AB}$ be a diameter of a semicircle, and let C be any point on the diameter. Further, let D and E be two points on the semicircle, such that triangles ADC and CEB are isosceles. If the angles of the triangles are (a, a, and $180 - 2a$) and (b, b, and $180 - 2b$), then show that either $a = b$ or that $\cos 2a + \cos 2b = -1$. See **figure 1 (Deshpande).**

Proofs are left to the reader.

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India

Think about using Deshpande's "Surprising result" to encourage students to think about proof and justification as part of problem solving. To let them investigate the problem, pose the following prompts:

- Are these four cases distinct? Are they the only four cases?

Mathematics Teacher, volume 96, number 8, November 2003

I produced a short paper using Cabri, *DERIVE* and the TI-92 and offered to add it to the ACDCA-teaching materials. Walter and Thomas accepted and extended the paper with valuable didactical goals and hints. I will present this paper adapting it for 2015, i.e. instead of using the CABRI software for the PC we will use the Geometry Page on TI-Nspire, and some other changes. But I will leave the original structure of the paper, Josef

Didactical Goal: Showing that different approaches for a mathematical problem are possible, presenting different forms of giving reasons and performing proofs.

Steps:

1) **Verification** supported by a Dynamic Geometry Software product.
Question (Josef): Why not verifying the observation using paper and pencil? This will give many results in the class room emerging from different figures.

1a) Confirmation of the hypothesis applying the draw mode – "proof" by a large number of arbitrary distributed single cases.

2) **Approach by calculation** using analytic geometry supported by a CAS.
   2a) Take any numbers for radius and distance AC and verify the hypothesis.
   2b) Find the general algebraic solution.

3) **Proof** using definitions and theorems – the classical proof.

1. Working with a Dynamic Geometry System

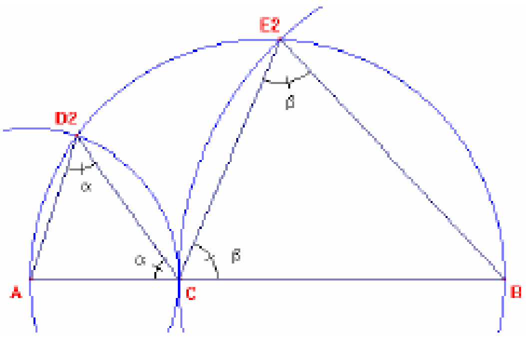
The hypothesis will be rejected very soon or confirmed with a high probability by creating many randomized instances. (The original paper referred to Cabri Geometre, because of a general license for all Austrian secondary schools.) I will mainly use the Geometry tools of TI-Voyage 200 (Cabri implementation) and TI-NspireCAS.

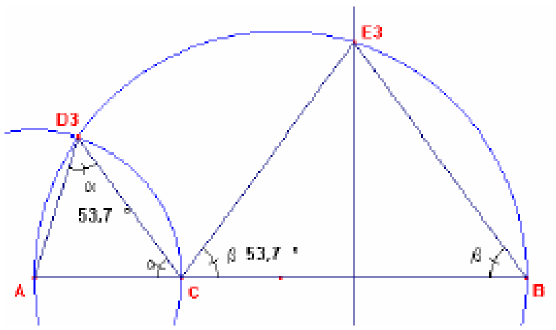
Alternative DGSs are:

Cabri Geométre (http://www.cabri.com/)
Cinderella (http://www.cinderella.de)
Geogebra (http://www.geogebra.org/download)
Euklid Dynageo (http://www.dynageo.de)
ZUL – Zirkel undLineal (http://zirkel-und-lineal.de.softonic.com/)
Geometers Sketchpad (http://www.dynamicgeometry.com/)
Geometry Expressions (http://www.geometryexpressions.com)
Geonext (http://geonext.de)

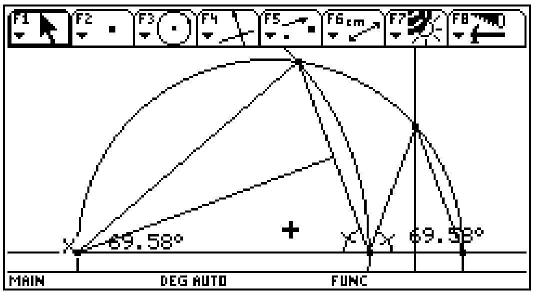
You can find a list of (all?) available Dynamic Geometry Systems (2D & 3D) on
http://en.wikipedia.org/wiki/List_of_interactive_geometry_software
(a very recommendable site!)

Here are two Cabri-figures from the original paper:

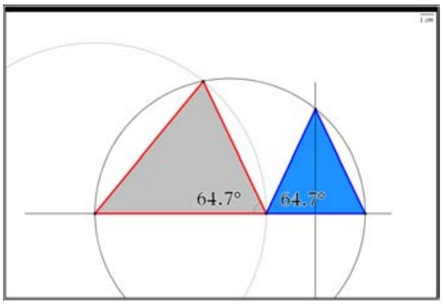




In the left figure one has to read off the angles and calculate $\cos 2\alpha + \cos 2\beta$. In the right figure is nothing more to do. We can move point C and observe that the statement remains true.



Case II on the TI-92 and Voyage 200



Case II on the TI-Nspire screen

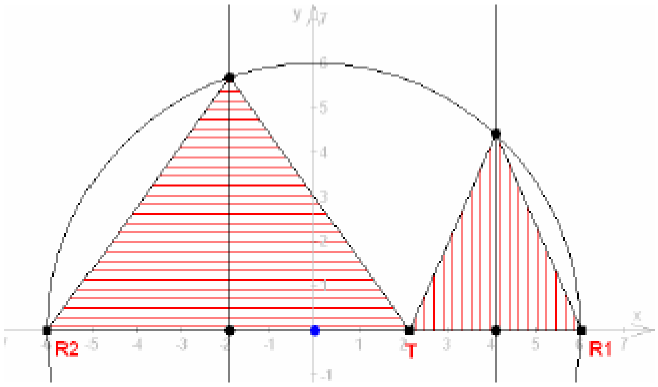
2. Clarification using a CAS

We will treat Case I using arbitrary numerical values.

We set points:

R2 (-r,0); T(t,0), R1(r,0).

Special values: r = 6, t = 2.5



$$\left[r := 6,\ t := \frac{5}{2}\right]$$

$$y(x) := \sqrt{(r^2 - x^2)}$$

$$\left[T := [t,\ 0],\ P1 := \left[\frac{r+t}{2},\ y\left(\frac{r+t}{2}\right)\right],\ P2 := \left[\frac{t-r}{2},\ y\left(\frac{t-r}{2}\right)\right],\ R1 := [r,\ 0],\ R2 := [-r,\ 0]\right]$$

#8: $(P1 - T)\cdot(R1 - T) = COS(\alpha)\cdot|P1 - T|\cdot|R1 - T|$

#9: $SOLVE((P1 - T)\cdot(R1 - T) = COS(\alpha)\cdot|P1 - T|\cdot|R1 - T|,\ \alpha,\ Real)$

#10: $$\alpha = ACOT\left(\frac{\sqrt{287}}{41}\right) \vee \alpha = ATAN\left(\frac{\sqrt{287}}{41}\right) + \frac{3\cdot\pi}{2} \vee \alpha = -ACOT\left(\frac{\sqrt{287}}{41}\right)$$

#11: $SOLVE((P2 - T)\cdot(R2 - T) = COS(\beta)\cdot|P2 - T|\cdot|R2 - T|,\ \beta,\ Real)$

#12: $$\beta = ACOT\left(\frac{\sqrt{527}}{31}\right) \vee \beta = ATAN\left(\frac{\sqrt{527}}{31}\right) + \frac{3\cdot\pi}{2} \vee \beta = -ACOT\left(\frac{\sqrt{527}}{31}\right)$$

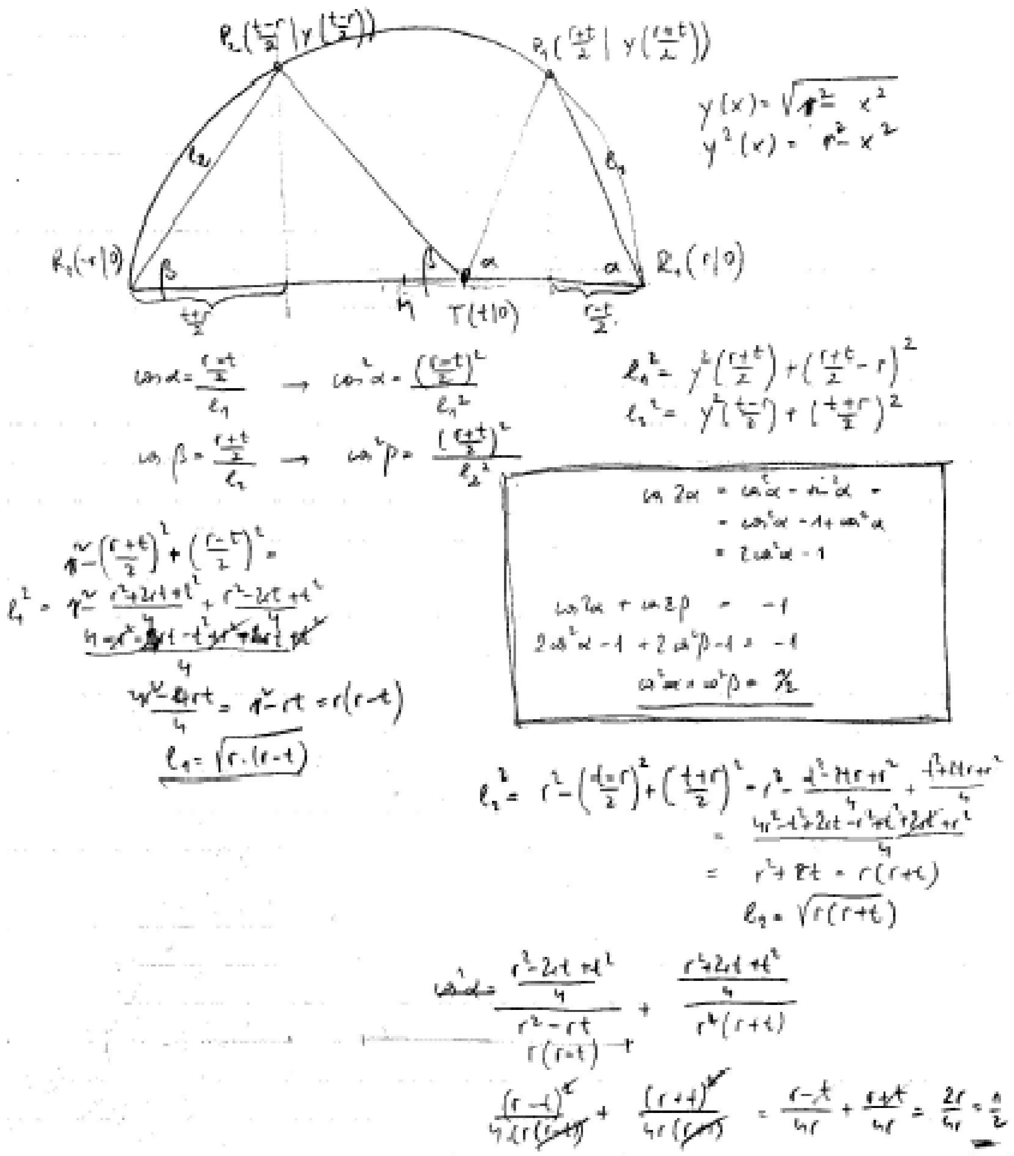
#13: $$COS\left(2\cdot ACOT\left(\frac{\sqrt{287}}{41}\right)\right) + COS\left(2\cdot ACOT\left(\frac{\sqrt{527}}{31}\right)\right) = -1$$

Generalization:

#14: $[r :=,\ t :=]$

In order to avoid huge expressions we "simplify" the trig expression first and then proceed with the generalized calculation.

Just for fun I include my paper and from 2003 (MAS supported, MAS = Mental Algebra System).



And now performed with the CAS:

```
[CaseMode := Sensitive, InputMode := Word, Trigonometry := Expand]
```

$y(x) := \sqrt{(r^2 - x^2)}$

$\left[T := [t, 0], P1 := \left[\dfrac{r + t}{2}, y\left(\dfrac{r + t}{2}\right)\right], P2 := \left[\dfrac{t - r}{2}, y\left(\dfrac{t - r}{2}\right)\right], R1 := [r, 0], R2 := [-r, 0]\right]$

$\cos(2\cdot\alpha) + \cos(2\cdot\beta) = 2\cdot\cos(\alpha)^2 + 2\cdot\cos(\beta)^2 - 2$

$2\cdot\cos(\alpha)^2 + 2\cdot\cos(\beta)^2 - 2 = -1$

$\dfrac{(2\cdot\cos(\alpha)^2 + 2\cdot\cos(\beta)^2 - 2 = -1) + 2}{2}$

$\cos(\alpha)^2 + \cos(\beta)^2 = \dfrac{1}{2}$

$$l1 := \sqrt{\left(\left(r - \frac{r+t}{2}\right)^2 + y\left(\frac{r+t}{2}\right)^2\right)}$$

$$l1 := \sqrt{(r\cdot(r-t))}$$

$$l2 := \sqrt{\left(\left(-r - \frac{t-r}{2}\right)^2 + y\left(\frac{t-r}{2}\right)^2\right)}$$

$$l2 := \sqrt{(r\cdot(r+t))}$$

Can you find a geometric interpretation of these two results?
(Leg Theorem in a right triangle!)

$$\left[\cos\alpha := \frac{r - \frac{r+t}{2}}{l1},\ \cos\beta := \frac{\frac{t-r}{2} + r}{l2}\right]$$

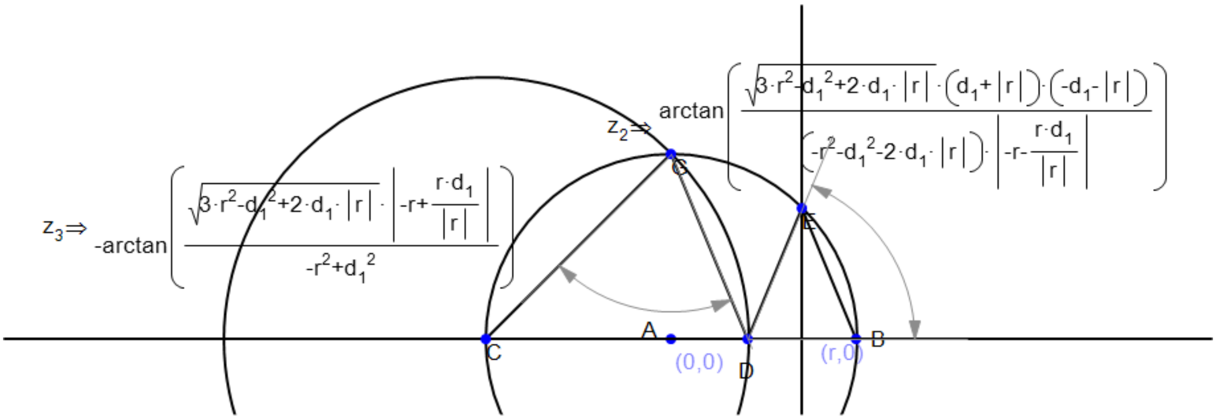
$$\cos\alpha^2 + \cos\beta^2 = \frac{1}{2}$$

This was to be proved!

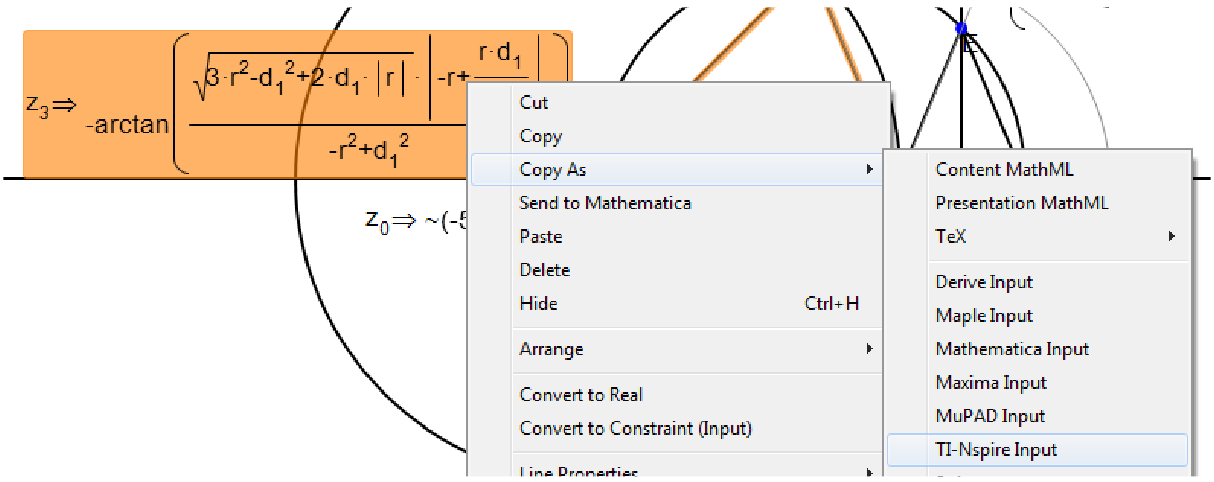
The other proofs are left for the reader.

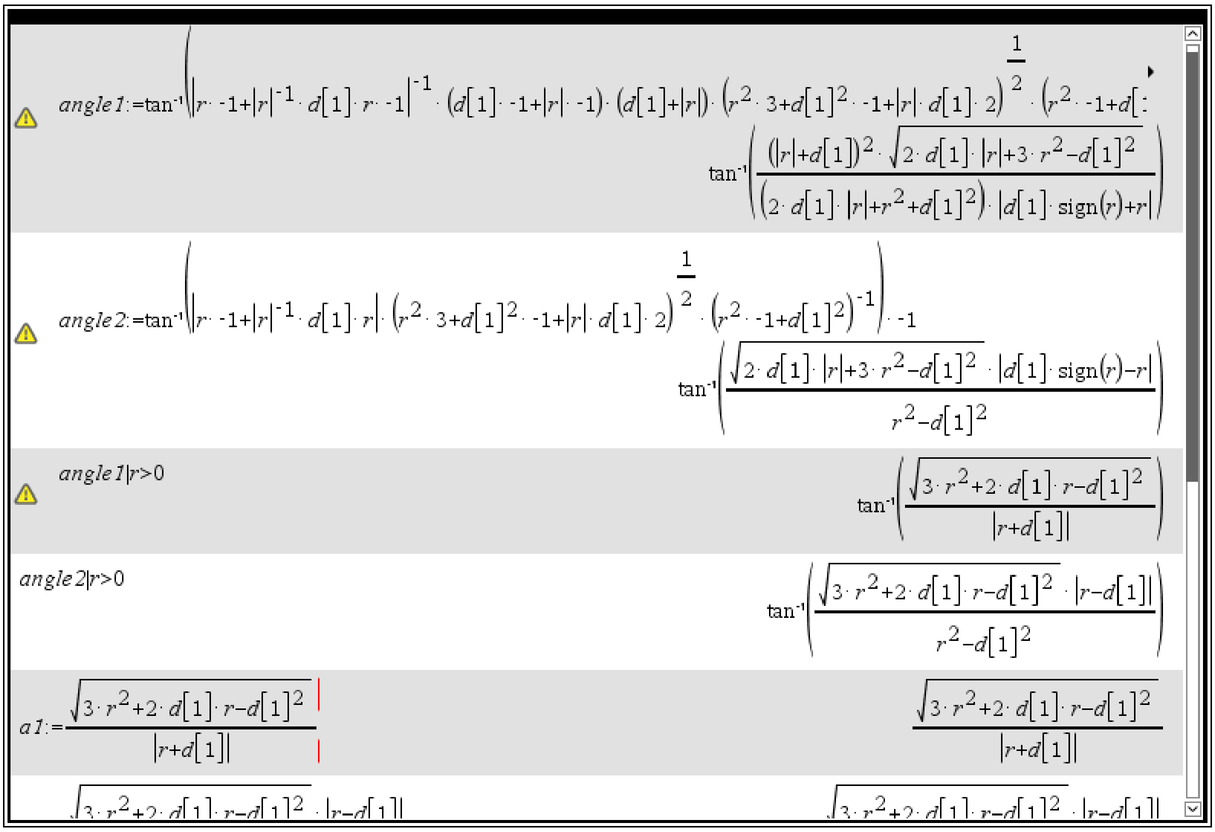
I will show another way to proof case II – which needs some competence working with a DGS and a CAS as well.

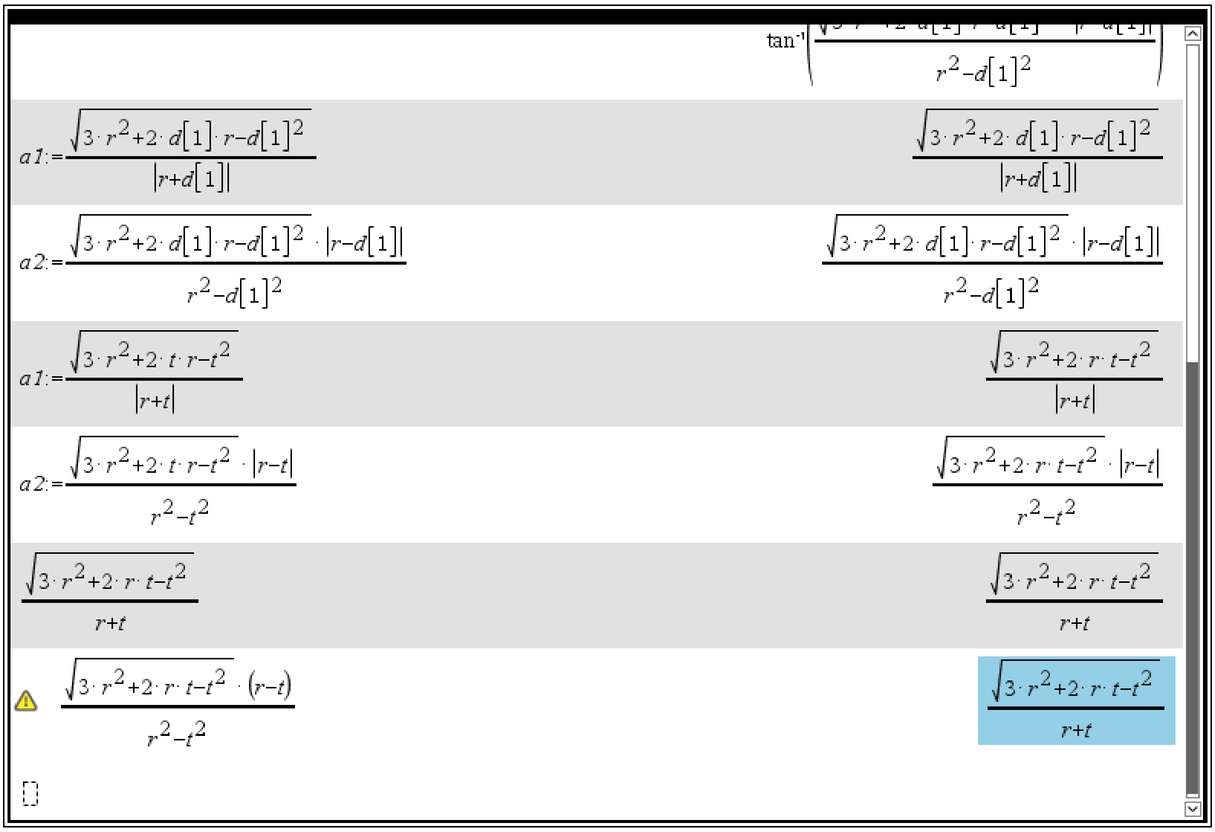
I performed the construction of case II with Geometry Epressions and let do the calculation of the two angles which should be equal by the software:



We can copy the results for further manipulations as input for several popular CAS with the TI-Nspire among them (*DERIVE* works also excellent).



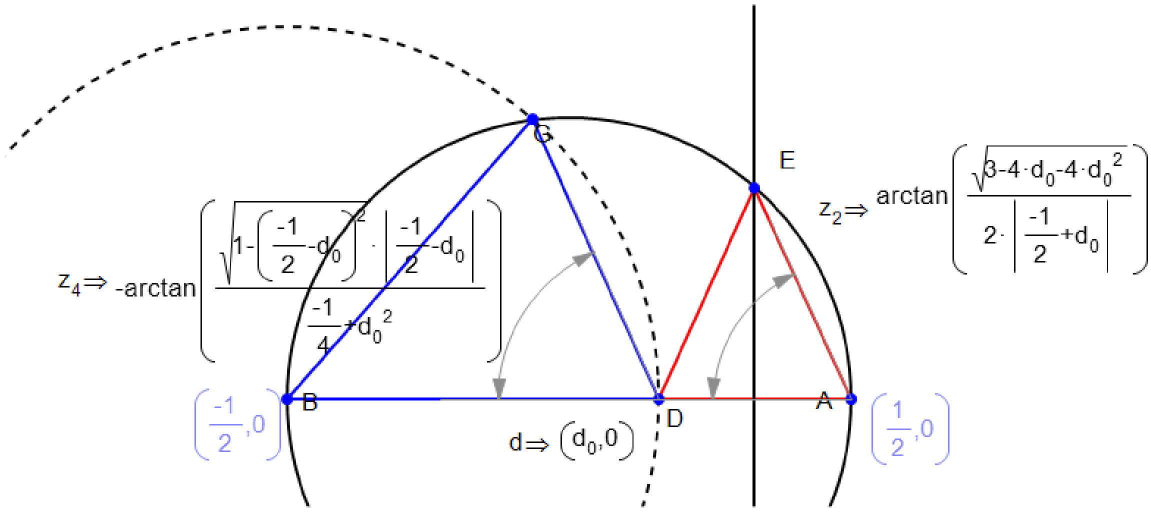




I replaced d[1] by t.

The expressions in the last two rows show the identity of the two angles.

The procedure is much easier if we fix the diameter of the semicircle with $2r = 1$.



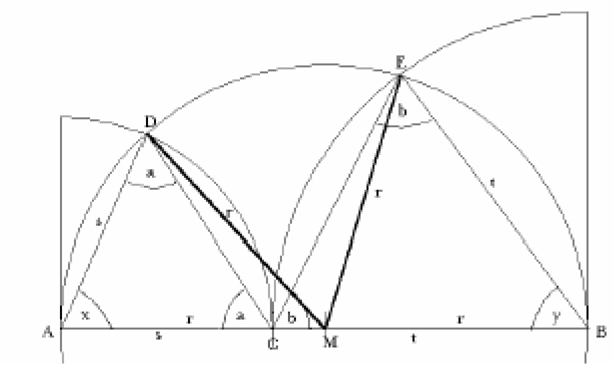
$angle3 := \tan^{-1}\left(\frac{\left\lvert\frac{-1}{2}+d\right\rvert^{-1}\cdot\left(3+d\cdot -4+d^2\cdot -4\right)^{\frac{1}{2}}\cdot 1}{2}\right)$	$\tan^{-1}\left(\frac{\sqrt{-4\cdot d^2-4\cdot d+3}}{\lvert 2\cdot d-1\rvert}\right)$
$angle4 := \tan^{-1}\left(\left\lvert\frac{-1}{2}+d\cdot -1\right\rvert\cdot\left(\frac{-1}{4}+d^2\right)^{-1}\cdot\left(1+\left(\frac{-1}{2}+d\cdot -1\right)^2\cdot -1\right)^{\frac{1}{2}}\right)\cdot -1$	$-\tan^{-1}\left(\frac{\sqrt{-4\cdot d^2-4\cdot d+3}\cdot\lvert 2\cdot d+1\rvert}{4\cdot d^2-1}\right)$
$\tan^{-1}\left(\frac{\sqrt{-4\cdot d^2-4\cdot d+3}}{\lvert 2\cdot d-1\rvert}\right)\Big\vert -0.5<d<0.5$	$-\tan^{-1}\left(\frac{\sqrt{-4\cdot d^2-4\cdot d+3}}{2\cdot d-1}\right)$
$-\tan^{-1}\left(\frac{\sqrt{-4\cdot d^2-4\cdot d+3}\cdot\lvert 2\cdot d+1\rvert}{4\cdot d^2-1}\right)\Big\vert -0.5<d<0.5$	$-\tan^{-1}\left(\frac{\sqrt{-4\cdot d^2-4\cdot d+3}}{2\cdot d-1}\right)$

Needless to say that one can perform similar manipulations with *DERIVE*.

Now I will return to the Wegscheider-Himmelbauer-Böhm-paper:

3. General proofs

Case IV: It is to show that $\cos(2a) + \cos(2b) = -1$



1) Cosine rule for ΔAMD:

$$r^2 = r^2 + s^2 - 2\cdot s\cdot r\cdot\cos(x)$$

$$0 = s - 2r\cdot\cos(x) \rightarrow \cos(x) = \frac{s}{2r}$$

2) Cosine rule for ΔMBE:

$$r^2 = r^2 + t^2 - 2\cdot t\cdot r\cdot\cos(y)$$

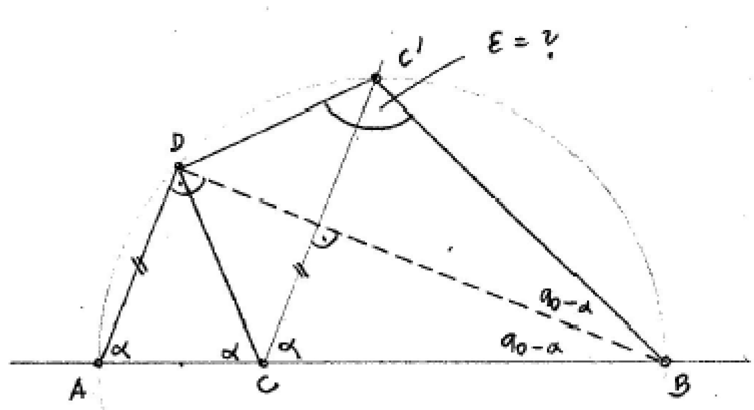
$$0 = t - 2r\cdot\cos(y) \rightarrow \cos(y) = \frac{t}{2r}$$

$$\Rightarrow \cos(x) + \cos(y) = \frac{s}{2r} + \frac{t}{2r} = \frac{s+t}{2r} = 1 \quad (1)$$

We see that: $x = 180° - 2a$ and $y = 180° - 2b$.
So according (1): $\cos(180° - 2a) + \cos(180° - 2b) = 1$.
As $\cos(180° - 2a) = -\cos(2a)$ and $\cos(180° - 2b) = -\cos(2b)$ we get:

$$-\cos(2a) - \cos(2b) = 1 \Rightarrow \cos(2a) + \cos(2b) = -1$$

Cases II and III:

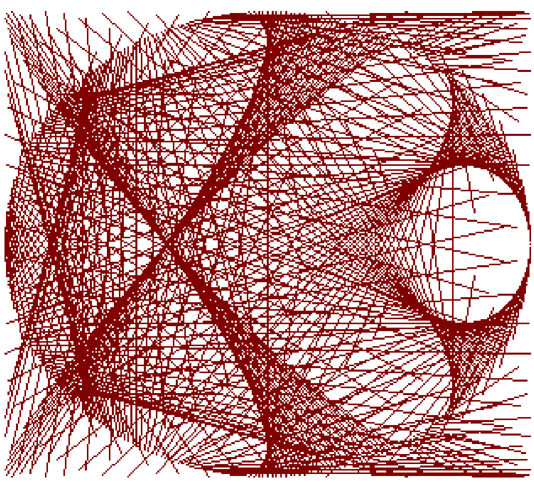


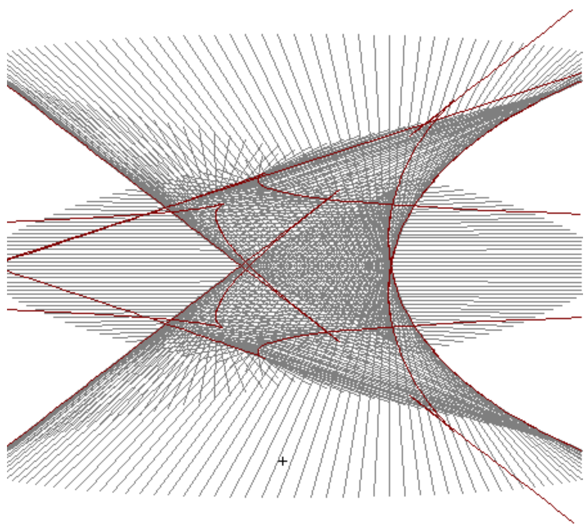
Given is the semicircle, point C on the diameter AB and the isosceles triangle ΔACD.

We add the segment DB and generate the right triangle ΔABD. Reflection of point C with respect to BD gives point C'. If we can show that angle ε = <DC'B = 180°– α then quadrilateral ABC'D is an inscribed quadrilateral and point C' is lying on the circle.

Theorem: The sum of opposite angles in an inscribed quadrilateral is always 180°.

Two more plots from my Mygalo - Gallery





Technical Problems – solved by Sec 2 Mathematics

Wolfgang Alvermann, Germany

Wolfgang gave a workshop on "Technical Problems" at TIME 2015. I invited him to present some more of his examples in our newsletter. Here they are, many thanks, Wolfgang.

Kinematics of a Point

1	Problem	1

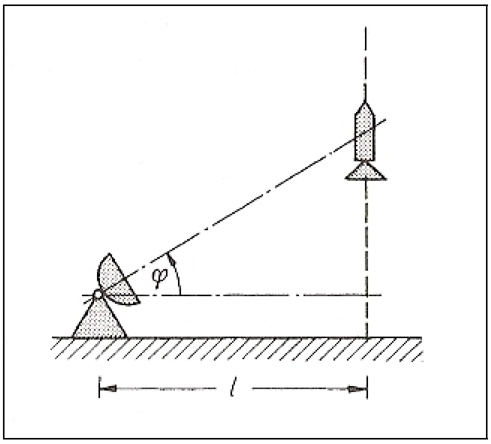
A radar screen follows a rocket which takes off vertically with constant velocity.

The rocket starts at time $t = 0$.

Find the angular velocity $\dot{\varphi}$ and the angular acceleration of the screen.

What is the maximum angular velocity and what is the respective angle?

Take $a = 5$ and $l = 50$. Provide a plot of the angular velocity vs time and



2	Proposed Solution	2

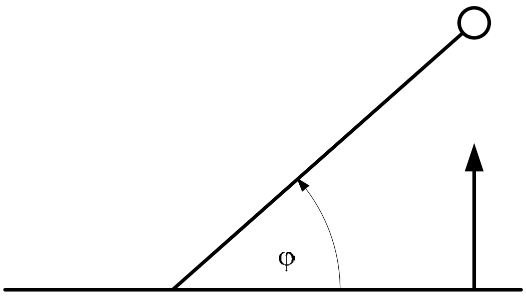
Velocity of the rocket: $v(t) = a \cdot t + v(0)$

Position of the rocket: $x(t) = \frac{a}{2} \cdot t^2 + t \cdot v(0) + x(0)$

Initial conditions:

$v(0) = 0 \Rightarrow \qquad v(t) = a \cdot t$

$x(0) = 0 \Rightarrow \qquad x(t) = \frac{a}{2} \cdot t^2$



Angle of the radar screen: $\tan(\varphi) = \frac{x(t)}{l} \Rightarrow \varphi(t) = \tan^{-1}\left(\frac{a \cdot t^2}{2l}\right)$

Angular velocity of the screen: $\dot{\varphi}(t) = \frac{a \cdot t}{l \cdot \left(1 + \left(\frac{a \cdot t^2}{2l}\right)^2\right)}$

Angular acceleration of the screen: $\ddot{\varphi}(t) = \left(\frac{a}{l} - \frac{3a^3 \cdot t^4}{4l^3}\right) \cdot \left(\frac{16l^4}{(4l^2 + a^2 \cdot t^4)^2}\right)$

We can find the derivatives using the CAS (compare the results!).

$$\phi(t) := \mathrm{ATAN}\left(\frac{a\cdot t^2}{2\cdot l}\right)$$

$$\frac{d}{dt}\phi(t) = \frac{4\cdot a\cdot l\cdot t}{a^2\cdot t^4 + 4\cdot l^2}$$

$$\left(\frac{d}{dt}\right)^2 \phi(t) = \frac{4\cdot a\cdot l\cdot(4\cdot l^2 - 3\cdot a^2\cdot t^4)}{(a^2\cdot t^4 + 4\cdot l^2)^2}$$

$$\varphi(t):=\tan^{-1}\left(\frac{a\cdot t^2}{2\cdot l}\right) \quad \textit{Done}$$

$$\frac{d}{dt}(\varphi(t)) \quad \frac{4\cdot a\cdot l\cdot t}{a^2\cdot t^4+4\cdot l^2}$$

$$\frac{d^2}{dt^2}(\varphi(t)) \quad \frac{-4\cdot a\cdot l\cdot(3\cdot a^2\cdot t^4-4\cdot l^2)}{(a^2\cdot t^4+4\cdot l^2)^2}$$

Time of maximum angular velocity: $\ddot{\varphi}(t) = 0$

[a :∈ Real (0, ∞), l :∈ Real (0, ∞)]

$$\mathrm{SOLUTIONS}\left(\left(\frac{d}{dt}\right)^2 \phi(t) = 0,\ t\right)$$

$$\left[\frac{\sqrt{2}\cdot 3^{3/4}\cdot\sqrt{l}}{3\cdot\sqrt{a}},\ -\frac{\sqrt{2}\cdot 3^{3/4}\cdot\sqrt{l}}{3\cdot\sqrt{a}},\ \infty,\ -\infty,\ \frac{\sqrt{2}\cdot 3^{3/4}\cdot i\cdot\sqrt{l}}{3\cdot\sqrt{a}},\ -\frac{\sqrt{2}\cdot 3^{3/4}\cdot i\cdot\sqrt{l}}{3\cdot\sqrt{a}}\right]$$

$$t\_ := \frac{\sqrt{2}\cdot 3^{3/4}\cdot\sqrt{l}}{3\cdot\sqrt{a}}$$

$$\mathrm{SUBST}\left(\frac{4\cdot a\cdot l\cdot t}{a^2\cdot t^4 + 4\cdot l^2},\ t,\ t\_\right) = \frac{\sqrt{2}\cdot 3^{3/4}\cdot\sqrt{a}}{4\cdot\sqrt{l}}$$

$$\phi(t\_) = \frac{\pi}{6}$$

$$\mathrm{zeros}\left(\frac{-4\cdot a\cdot l\cdot(3\cdot a^2\cdot t^4-4\cdot l^2)}{(a^2\cdot t^4+4\cdot l^2)^2},t\right) \quad \left\{\frac{\sqrt{2\cdot l}\cdot 3^{\frac{3}{4}}}{3\cdot\sqrt{a}},\frac{-\sqrt{2\cdot l}\cdot 3^{\frac{3}{4}}}{3\cdot\sqrt{a}}\right\}$$

$$\frac{4\cdot a\cdot l\cdot t}{a^2\cdot t^4+4\cdot l^2}\Big|t=\frac{\sqrt{2\cdot l}\cdot 3^{\frac{3}{4}}}{3\cdot\sqrt{a}} \quad \frac{\sqrt{2\cdot a}\cdot 3^{\frac{3}{4}}}{4\cdot\sqrt{l}}$$

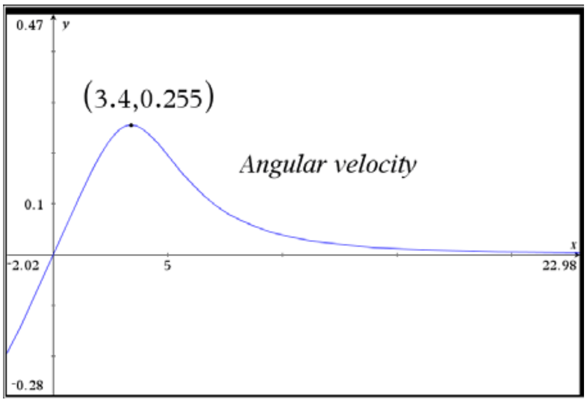
$$\varphi\left(\frac{\sqrt{2\cdot l}\cdot 3^{\frac{3}{4}}}{\ldots}\right) \quad \frac{\pi}{6}$$

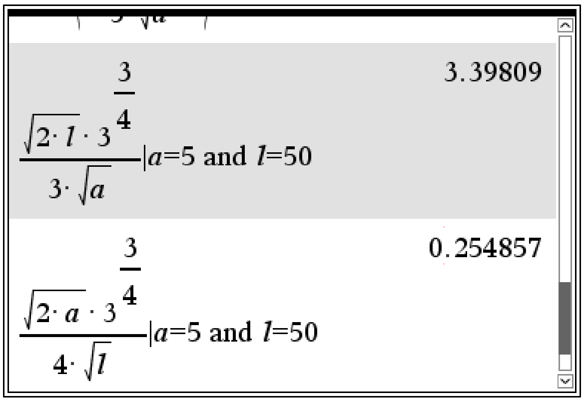
To solve: $\frac{a}{l} - \frac{3\cdot a^3\cdot t^4}{4\cdot l^3} = 0 \Rightarrow$ $\boxed{t^* = \sqrt[4]{\frac{4\cdot l^2}{3\cdot a^2}}}$

Maximum velocity: $\dot{\varphi}_{\max} = \dot{\varphi}(t^*) \Rightarrow$ $\boxed{\dot{\varphi}_{\max} = \sqrt{\frac{3\cdot\sqrt{3}}{8}\cdot\frac{a}{l}}}$

Angle at time t^*: $\varphi(t^*) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow$ $\boxed{\varphi(t^*) = 30°}$

Maximum velocity is reached after 3.4 seconds and is 0.255 rad/sec.





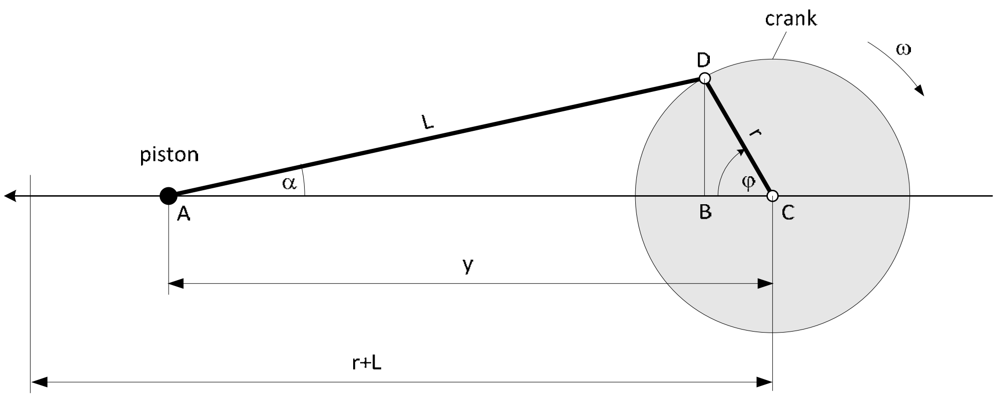
Wolfgang provided a second example for us:

The Crank Drive (Der Kurbeltrieb

1	Information	1

An example for a crank drive is the crankshaft which directs the pistons in the cylinders of a 4-cycle Otto-engine.

The crank drive given in the figure below transforms a circular motion into a periodical linear motion or vice versa.



2	Possible Tasks	2

a) Show that the way of the piston $y = \overline{CA}$ can be presented by:

$$y(\varphi) = r\cdot\cos(\varphi) + L\cdot\sqrt{1-\left(\frac{r}{L}\cdot\sin(\varphi)\right)^2}$$

b) With $\varphi = \omega \cdot t$ and $r = 1$ und $\omega = 1$ we obtain the time-distance-function:

$$y_L(t) = \cos(t) + L \cdot \sqrt{1 - \left(\frac{1}{L} \cdot \sin(t)\right)^2} \ .$$

Produce the graphs of $y_L(t)$ for three meaningful values for L!

c) For $L = 3$ plot in another system of coordinates the t-s-diagram, the t-v-diagram and the t-a-diagram for one period length. For which values of t do we find a zero velocity? When does acceleration disappear?
Give comments on the graph. Explain the positive and negative values for velocity and acceleration.

3	**Proposed Solutions**	3

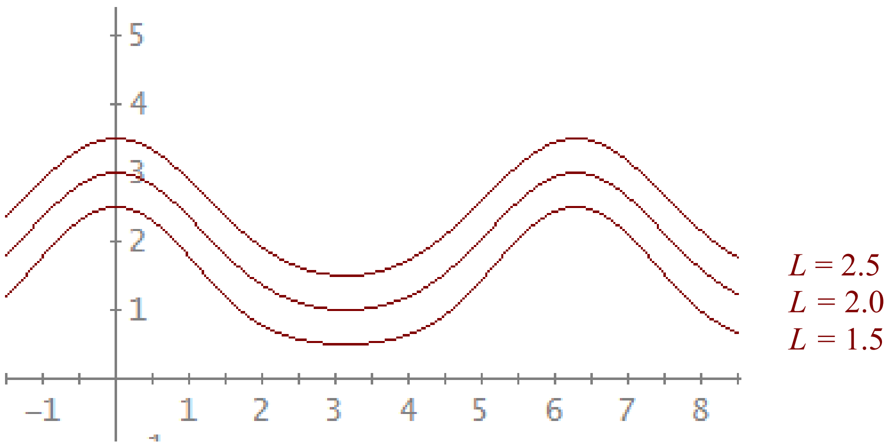
Task a)

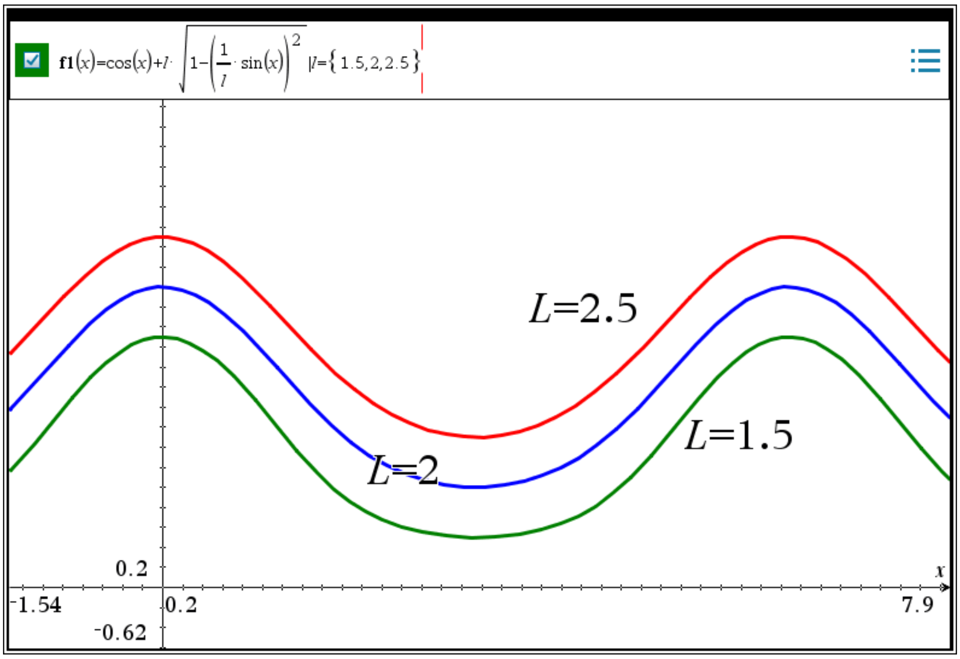
$$\left.\begin{array}{lll} y & = & \overline{CB} + \overline{BA} \\ \Delta BCD: & \overline{CB} & = r \cdot \cos(\varphi) \\ \Delta ABD: & \overline{BA} & = L \cdot \cos(\alpha) \\ \Delta ACD: & \dfrac{\sin(\alpha)}{r} = \dfrac{\sin(\varphi)}{L} & \\ & \cos(\alpha) = \sqrt{1 - \sin^2(\alpha)} & \end{array}\right\} \Rightarrow y(\varphi) = r \cdot \cos(\varphi) + L \cdot \sqrt{1 - \left(\frac{r}{L} \cdot \sin(\varphi)\right)^2} \qquad \text{q.e.d}$$

Task b)

$$y_L(t) = \cos(t) + L \cdot \sqrt{1 - \left(\frac{1}{L} \cdot \sin(t)\right)^2}$$

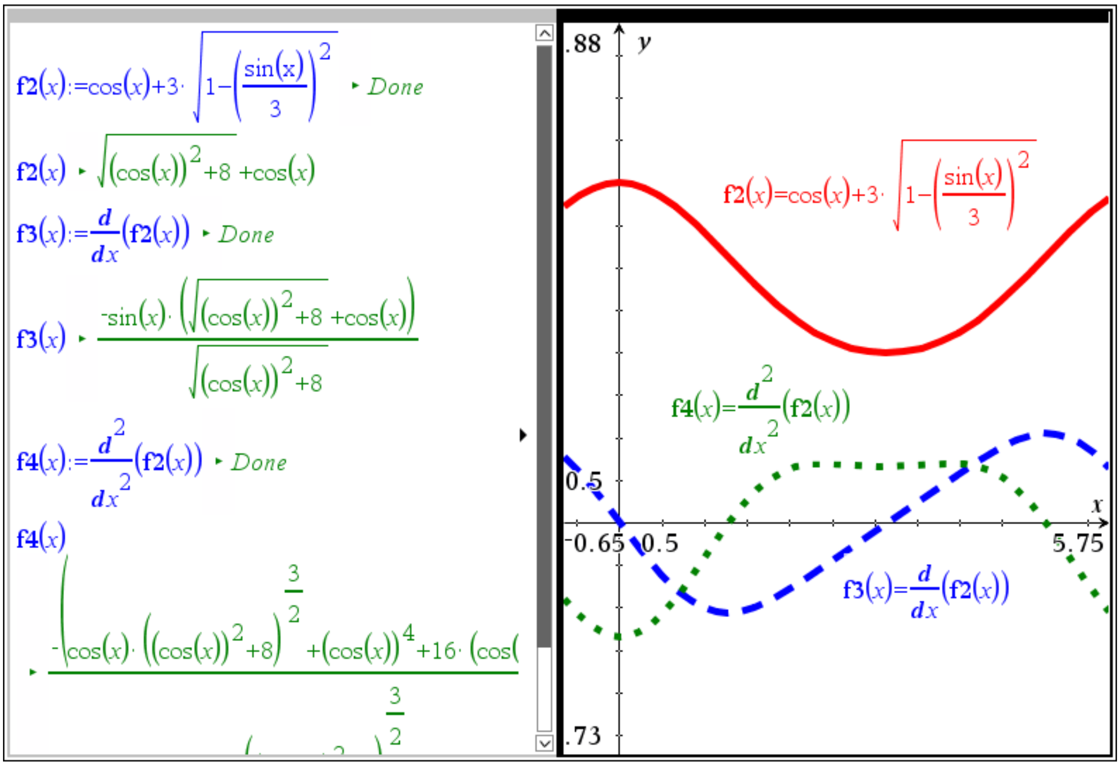
$$\text{VECTOR}\left(\text{COS(t)} + \text{L}\cdot\sqrt{\left(1 - \left(\frac{1}{\text{L}}\cdot\text{SIN(t)}\right)^2\right)}, \text{L}, [1.5, 2, 2.5]\right)$$





Task c)

Set $L = 3$ and define $s(t) = y_3(t)$, $v(t) = y_3'(t)$, and $a(t) = y_3''(t)$. Then plot the respective graphs:



$$y'(t) = v(t) = 0 \;\Rightarrow\; t = \{0, \pi, 2\pi\}$$
$$y''(t) = a(t) = 0 \;\Rightarrow\; t = \{1.277, 5.006\}$$

As one can observe on the sketch, the piston is moving to the right when point D is moving on the upper semicircle, hence v is negative. The semicircle below results in a positive velocity (v is a VECTOR).

At the turning points of v the sign of a is changing!