THE DERIVE - NEWSLETTER #99

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THE BULLETIN OF THE

DERIVE®

USER GROUP

+ CAS-TI

C o n t e n t s:

October 2015

I hope that one or the other URL might be of interest for you:

Articles for Educators:
http://www.articlesforeducators.com/directory.asp?fid=6

Calc101.com Automatic Calculus and Algebra Help
http://calc101.com/

Canadian Mathematical Society
http://cms.math.ca/crux/

An Internet Service for Mathematics Teachers and Students
http://mathcentral.uregina.ca/index.php

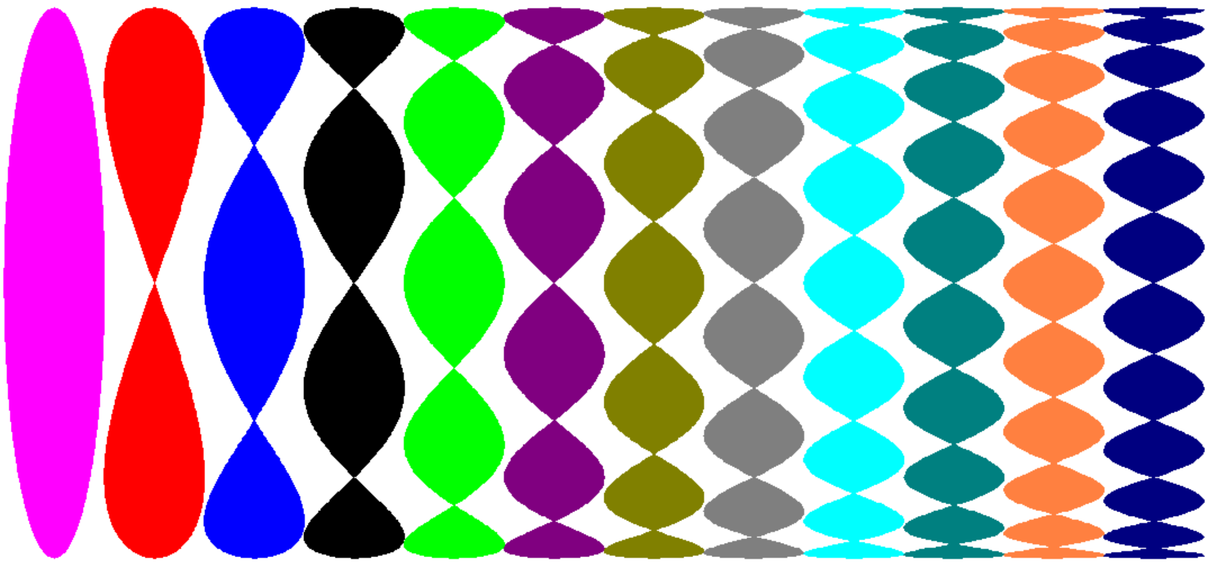
Custom Worksheets
http://www.mathfactcafe.com/worksheet/wordproblem/

Mathematics Contests + Solutions
http://mathleague.com/index.php/annualcontestinformation/samplecontests

A Gallery of Multimedia Learning Material (English & German)
http://www.univie.ac.at/future.media/moe/

A rich collection of Science Jokes
http://jcdverha.home.xs4all.nl/scijokes/

Many e-books (in German only) can be downloaded for free from:
http://bookboon.com/de/statistik-mathematik-ebooks



After a long while another nice mathematics painting created by Pierre Charland

Dear DUG Members,

I know, this DNL is very long overdue. I apologize and give the main reason which is documented by the picture below: my wife a I did a long planned travel to and through Brazil.

The Iguaçu Waterfalls was one the many highlights of our tour.

In July I participated at ACA 2015 which offered a very well organized Educational Session. You can find a list of the lectures plus links to the abstracts.

Rob Gough sent a very extended paper on Prime Pairs. This issue offers the first part accomplished by an advice how to transfer DERIVE Data to Excel.

My Polish colleague and friend Leon Magiera transmitted a some hundred pages book manuscript about Physics Examples treated by CAS. He left the decision what to do with the paper to me. I present some paragraphs. Maybe that we will offer the whole opus on the DUG or/and ACDCA website (after adding some DERIVE and Nspire related comments).

Best regards until next time

Josef

Download all *DNL*-DERIVE- and TI-files from
http://www.austromath.at/dug/

In December we will celebrate DNL#100 (= 25 Years DUG). It will be great if some of you – especially members from the early DUG Years – will contribute for this very exceptional issue. All articles, notes, comments, memories, … are very welcome, Josef.

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a content of 40 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE*, *TI*-CAS and other CAS as well to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

Editor: Mag. Josef Böhm
D´Lust 1, A-3042 Würmla, Austria
Phone: ++43-(0)660 3136365
e-mail: nojo.boehm@pgv.at

Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: December 2015

Preview: Contributions waiting to be published

Some simulations of Random Experiments, J. Böhm, AUT, Lorenz Kopp, GER
Wonderful World of Pedal Curves, J. Böhm, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
Hill-Encryption, J. Böhm, AUT
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
An Interesting Problem with a Triangle, Steiner Point, P. Lüke-Rosendahl, GER
Graphics World, Currency Change, P. Charland, CAN
Cubics, Quartics – Interesting features, T. Koller & J. Böhm, AUT
Logos of Companies as an Inspiration for Math Teaching
Exciting Surfaces in the FAZ / Pierre Charland´s Graphics Gallery
BooleanPlots.mth, P. Schofield, UK
Old traditional examples for a CAS – what´s new? J. Böhm, AUT
Where oh Where is It? (GPS with CAS), C. & P. Leinbach, USA
Mandelbrot and Newton with *DERIVE*, Roman Hašek, CZK
Tutorials for the NSpireCAS, G. Herweyers, BEL
Some Projects with Students, R. Schröder, GER
Dirac Algebra, Clifford Algebra, D. R. Lunsford, USA
A New Approach to Taylor Series, D. Oertel, GER
Henon & Co; Find your very own Strange Attractor, J. Böhm, AUT
Rational Hooks, J. Lechner, AUT
Simulation of Dynamic Systems with various Tools, J. Böhm, AUT
Statistics of Shuffling Cards, Charge in a Magnetic Field, H. Ludwig, GER
Pavement in Funchal, Th. Alvermann, GER
Next Number?, B. Grabinger, GER

and others

Impressum:
Medieninhaber: *DERIVE* User Group, A-3042 Würmla, D´Lust 1, AUSTRIA
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Computer Algebra in Education

at ACA'2015 to be held July 20-23, 2015 in Kalamata, Greece

Organizers:

Alkis Akritas, University of Thessaly, Greece
Michael Wester, University of New Mexico, USA
Michel Beaudin, ETS, Canada
José Luis Galán García, Universidad de Málaga, Spain
Elena Varbanova, Technical University of Sofia, Bulgaria

Overview:

Education has become one of the fastest growing application areas for computers in general and computer algebra in particular. Computer Algebra Systems (CAS) make for powerful teaching and learning tools within mathematics, physics, chemistry, biology, economics, etc. Among them are:
(a) the commercial "heavy weights" such as Casio ClassPad 330, Derive, Magma, Maple, Mathematica, MuPAD, TI NSpire CAS, and TI Voyage 200, and
(b) the free software/open source systems such as Axiom, Euler, Fermat, wxMaxima, Reduce, and the rising stars such as GeoGebra, Sage, SymPy and Xcas (the swiss knife for mathematics).

The goal of this session is to exchange ideas, discuss classroom experiences, and to explore significant issues relating to CAS tools/use within education. Subjects of interest for this session will include new CAS-based teaching/learning strategies, curriculum changes, new support materials, assessment practices from all scientific fields, and experiences of joint use of applied mathematics and CAS.

If you are interested in proposing a talk, please send an abstract to Michel Beaudin who will then redistribute it to the other organizers. Note that it is planned to compile an electronic (PDF) book of abstracts for the meeting. Please use this LaTeX template for your abstract and send both the LaTeX source and a compiled PDF version. If you do not work with LaTeX, the submission may be sent in Word format (.doc or .docx file) and later, once accepted, we will try to adapt the proposal to the LaTeX template.

Talks

1. About balanced application of CAS in undergraduate mathematics
   (Elena Varbanova, Technical University of Sofia, Bulgaria)
2. Some reflections about open vs. proprietary Computer Algebra Systems in mathematics teaching
   (F. Botana, University of Vigo, Spain)
3. Create SageMath Interacts for All Your Math Courses
   (Razvan A. Mezei, Lenoir-Rhyne University, Hickory, NC, USA)
4. Using SageMathCell and Sage Interacts to Reach Mathematically Weak Business Students
   (Gregory V. Bard, University of Wisconsin—Stout, Wisconsin, USA)
5. GINI-Coefficient, GOZINTO-Graph and Option Prices
   (Josef Böhm, Austria)

6. When Mathematics Meet Computer Software
(M. Beaudin and F. Henri, ÉTS, Montréal, Canada)
7. Revival of a Classical Topic in Differential Geometry: Envelopes of Parametrized Families of Curves and Surfaces
(Th. Dana-Picard and N. Zehavi, Israel)
8. Generating animations of JPEG images of closed surfaces in space using Maple and Quicktime
(G. Labelle, UQAM, Montréal, Canada)
9. Plotting technologies for the study of functions of two real variables
(David Zeitoun and Thierry Dana-Picard, Israel)
10. Some remarks on Taylor's polynomials visualization using Mathematica in context of function approximation
(Wlodzimierz Wojas and Jan Krupa, Warsaw University of Life Sciences, Poland)
11. Visualization of Orthonormal Triads in Cylindrical and Spherical Coordinates
(Jeanett López García, Jorge J. Jiménez Zamudio and Ma. Eugenia Canut Díaz Velarde, UNAM, Mexico)
12. Contemporary interpretation of a historical locus problem with an unexpected discovery
(R. Hasek, University of South Bohemia, Czech Republic)
13. A Constructive Proof of Feuerbach's Theorem Using a Computer Algebra System
(Michael Xue, Vroom Laboratory for Advanced Computing, USA)
14. Math Partner and Math Tutor
(Gennadi and Nastasha Malaschonok, Tambov State University, Russia)
15. Ideas for Teaching Using CAS
(Michel Beaudin, ETS, Montréal, Québec, Canada)
16. Solving Brain Teasers/Twisters - CAS Assisted
(Josef Böhm, Austria)
17. Various New Methods for Computing Subresultant Polynomial Remainder Sequences (PRS's)
(Alkiviadis G. Akritas, University of Thessaly, Volos, Greece)
18. Teaching improper integrals with CAS
(G. Aguilera, J.L. Galán, M.Á. Galán, Y. Padilla, P. Rodríguez, R. Rodríguez, Spain)
19. Application of wxMaxima System in LP problem of compound feed mass minimization
(Wlodzimierz Wojas and Jan Krupa, Warsaw University of Life Sciences, Poland)
20. The Use of CAS for Logical Analysis in Mathematics Education
(T. Takahashi, T. Sakai, F. Iwama, Japan)
21. Indexed elementary functions in Maple
(David Jeffrey, University of Western Ontario, Canada)

Prime Pairs & Goldbach's Conjecture

by

Rob Gough

Goldbach's Conjecture requires that every even number, E, is the sum of at least one pair of prime numbers. This paper develops an accurate technique for estimating the number of prime pairs. This is based on the prime content of E and a pairing probability called the generalized prime pair measure. It is shown that the estimates are based on simple ratios of the prime content of E combined with the generalized, probabilistic pairing measure.

1. The pairing measures

1.1 Symbols & relationships

E	Even number ($E \geq 6$)
N	Number of pairs of odd numbers adding up to E
A	Lower group of odd integers
B	Upper group of odd integers
a_i	i^{th} integer element of lower group A ($a_1 = 3$)
b_i	i^{th} integer element of upper group B

$$a_i + b_i = E \text{ where } 1 \leq i \leq N$$

Given that $a_1 = 3$ and all a, b are odd then $a_i = 2i + 1$, $b_i = E - 2i - 1$

The relationship between E and N is given by

$$N = \left\lceil \frac{E}{4} \right\rceil - 1$$

The number of primes and composites in A and B, for any value of E, is given by:

$n(A)$	Number of primes in A
$n(B)$	Number of primes in B
$n(\overline{A})$	Number of composites in A
$n(\overline{B})$	Number of composites in B

$$n(A) + n(\overline{A}) = n(B) + n(\overline{B}) = N$$

Finally, there are the four pairing measures for any value of E:

$n(A, B)$	Number of prime-prime pairs
$n(A, \overline{B})$	Number of A-primes pairing with B-composites
$n(\overline{A}, B)$	Number of A-composites pairing with B-primes
$n(\overline{A}, \overline{B})$	Number of composite-composite pairs

$$n(A, B) + n(A, \overline{B}) + n(\overline{A}, B) + n(\overline{A}, \overline{B}) = N$$

$$n(A, B) + n(A, \overline{B}) = n(A), \quad n(A, B) + n(\overline{A}, B) = n(B)$$

1.2 Simple Examples

$E = 18$, $N = 4$

A	3	5	7	9
B	15	13	11	9

$E = 20$, $N = 4$

A	3	5	7	9
B	17	15	13	11

$E = 22$, $N = 5$

A	3	5	7	9	11
B	19	17	15	13	11

$E = 24$, $N = 5$

A	3	5	7	9	11
B	21	19	17	15	13

Note:

1. The primes are shown in red.
2. New odd numbers enter to the left at *B*. As *E* increases these numbers move to the right through *B* until they become fixed, sequentially in *A*.
3. There are two values of *E* for every *N*.
4. The N^{th} element of *A* and *B* has the same value for every alternate value of *E*. This means that *n(A)* and *n(B)* may include one common prime (as in $E = 22$).

From the simple examples above we have the following values:

E	**N**	$n(A)$	$n(B)$	$n(A,B)$	$n(A,\bar{B})$	$n(\bar{A},B)$	$n(\bar{A},\bar{B})$
18	4	3	2	2	1	0	1
20	4	3	3	2	1	1	0
22	5	4	4	3	1	1	0
24	5	4	3	3	1	0	1

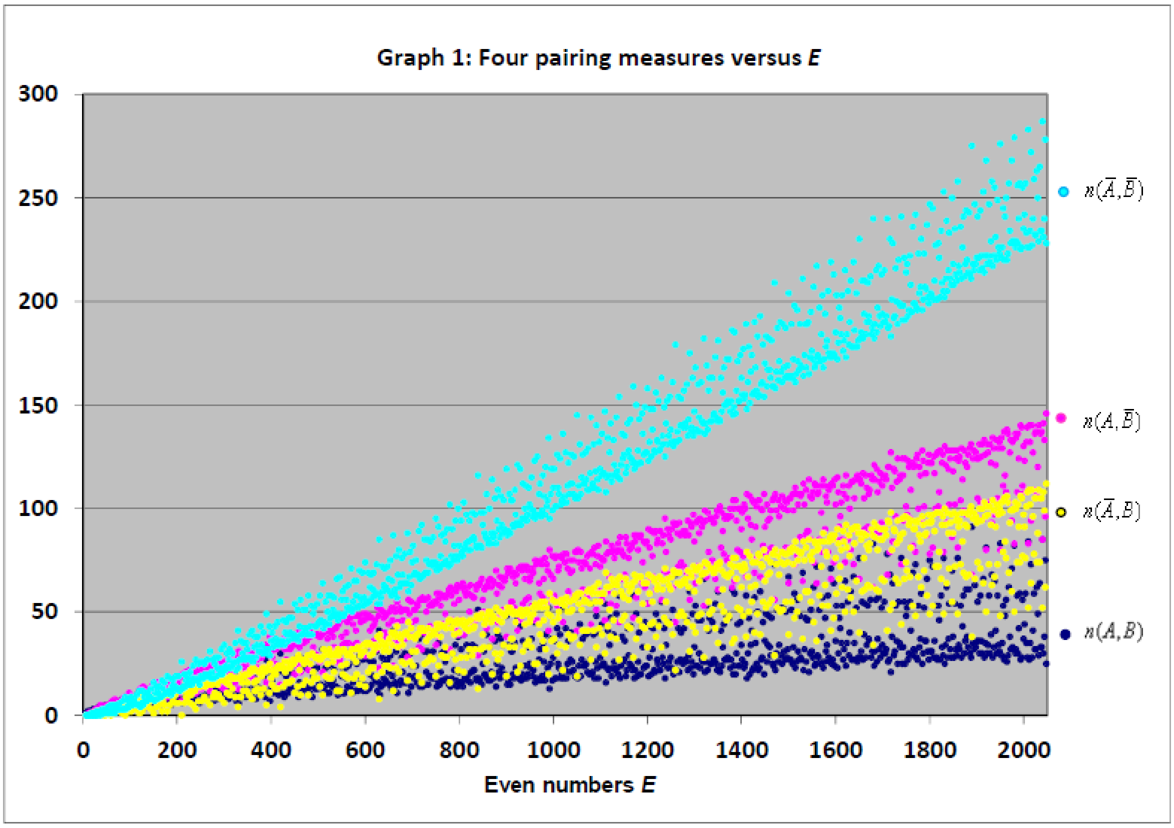
1.3 General pairing maxima and minima

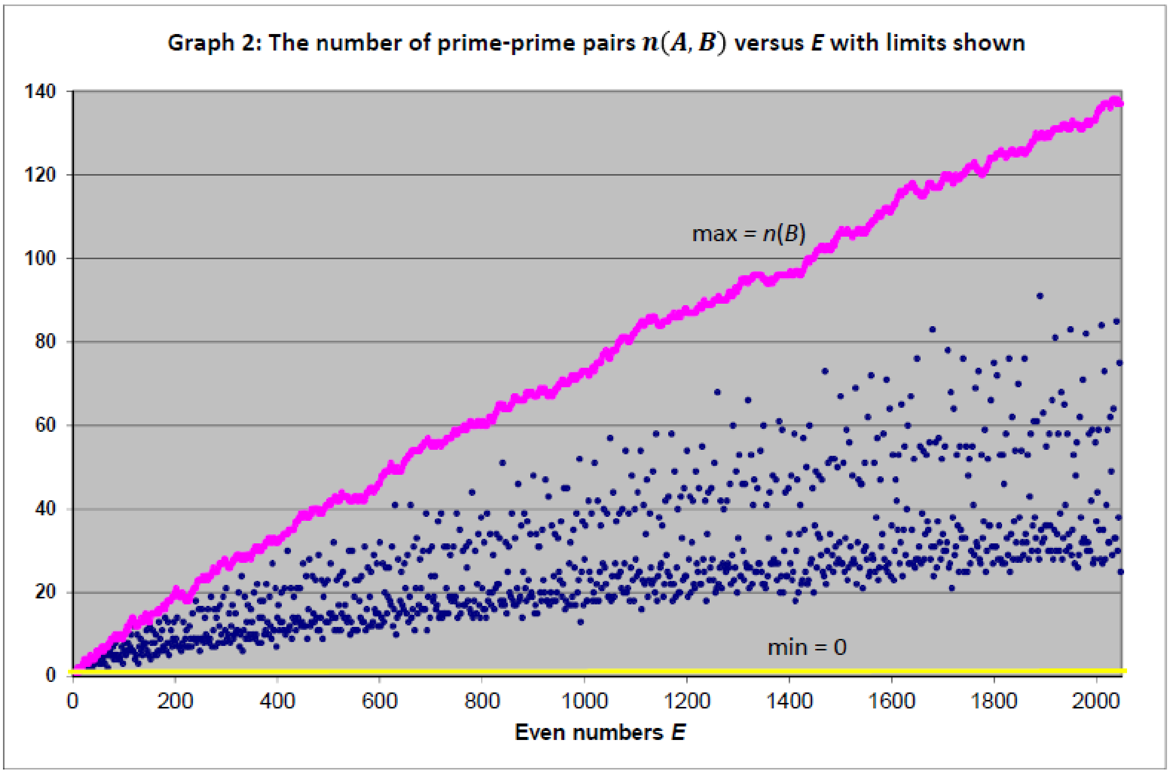
It follows from **Section 1.1** that there are certain maxima and minima in the four pairing measures. These relate to the conditions when Goldbach's Conjecture is false (*GCF* – when there are no prime pairs) and when it is true maximally (*GCTmax* – when all the *B*-primes are paired with *A*-primes). The table below sets out the limits within which our measures must operate.

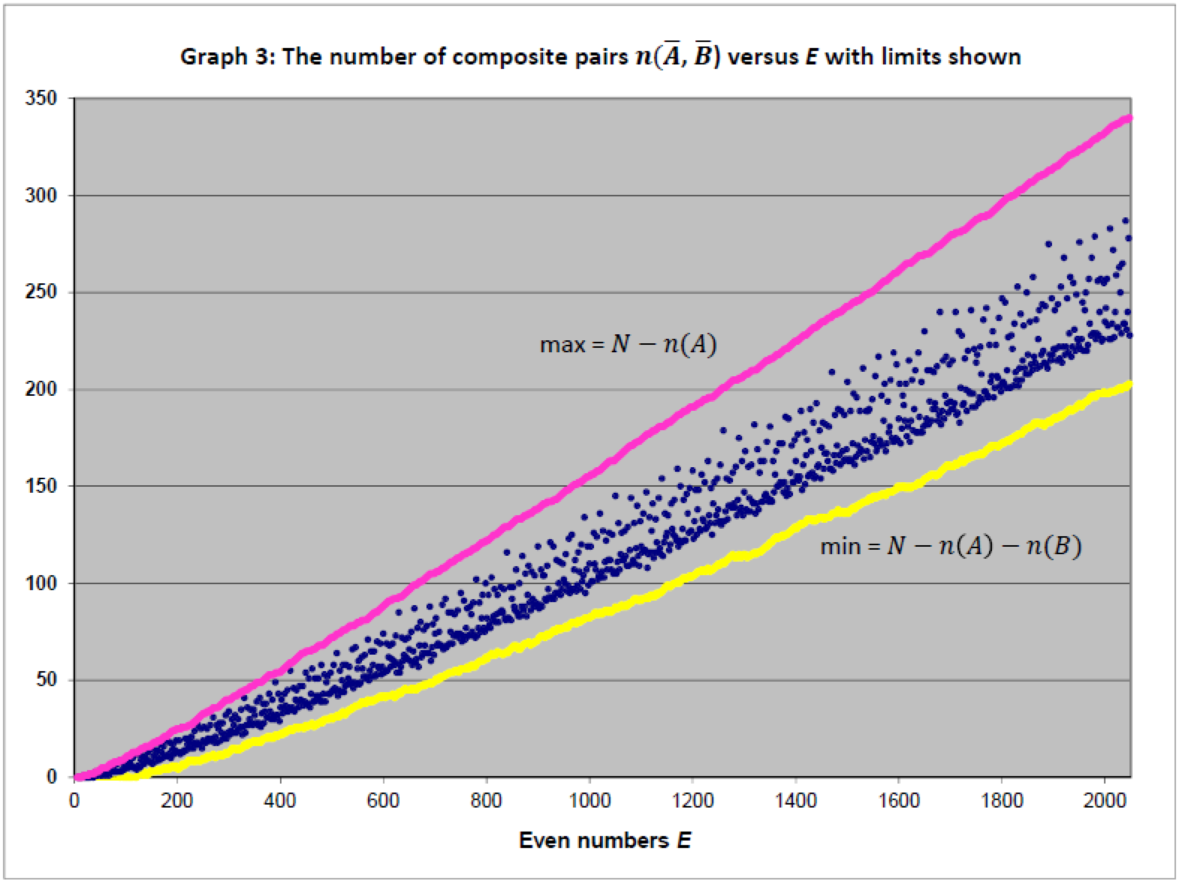
	GCF	**GCTmax**
$n(A,B)$	0 [min]	$n(B)$ [max]
$n(A,\bar{B})$	$n(A)$ [max]	$n(A)-n(B)$ [min]
$n(\bar{A},B)$	$n(B)$ [max]	0 [min]
$n(\bar{A},\bar{B})$	$N-n(A)-n(B)$ [min]	$N-n(A)$ [max]

One other thing stands out from the above analysis: the difference between the maximum and minimum values of *all* four measures is exactly *n*(*B*).

Graph 1 shows all four pairing measures, $n(A,B)$, $n(A,\overline{B})$, $n(\overline{A},B)$ and $n(\overline{A},\overline{B})$ against the even numbers, E, up to 2048. **Graph 2** shows the prime measure, $n(A,B)$, and **Graph 3** shows the composite measure, $n(\overline{A},\overline{B})$. This initial data was created using *Excel*. All subsequent analysis is based on the values of E, N, $n(A)$, and the four pairing measures.







1.4 Initial analysis of Excel data

The following points can be made about the *Excel* graphs and data.

- The individual data points of each pairing measure seem to be random but with clusterings into two major populations
- Concentrating on $n(A,B)$ and $n(\bar{A},\bar{B})$ these clusters are more dense at the lower values, suggesting that these two measure are sympathetic
- Closer analysis shows that the lower cluster consist more of E-numbers of either pure power-2 numbers or with single large primes. The less dense upper cluster contains the prime 3 and other small prime mixtures.
- Generally the pure 2 E-numbers form the lowest strata of the denser main cluster in $n(A,B)$ and $n(\bar{A},\bar{B})$
- E-numbers containing 3-multiples make up 1/3 of the composites in $n(\bar{A},\bar{B})$, pushing this measure up into the less dense upper cluster. The 2/3 non-3 multiples form the more dense lower cluster
- The reason the 3-multiple composites form the upper cluster of $n(\bar{A},\bar{B})$ is because these multiples boost the number of composite pairs generally
- And because of the sympathetic relationship of $n(A,B)$ to $n(\bar{A},\bar{B})$, the 3-multiple composites also boost the number of prime pairs (see **Section 1.5** below for proof).

1.5 The sympathetic prime and composite pairing relationships

Using the identities of **Section 1.1** it is possible to formulate the sympathetic relationship of the prime and composite measures (this will be called the delta rule) as follows:

$$n(\bar{A},\bar{B}) - n(A,B) = N - n(A) - n(B) = \Delta$$

As Δ is fixed by N, $n(A)$ and $n(B)$ then for any particular E if $n(\bar{A},\bar{B})$ is large so too will $n(A,B)$ be, and the converse. This new measure, Δ, will prove useful later on.

1.6 Normalised measures

As the various measures increase with E, it is useful to construct normalized measures as follows:

$$\hat{n}(A) = \frac{n(A)}{N} \quad \hat{n}(B) = \frac{n(B)}{N} \quad \hat{n}(\bar{A}) = \frac{n(\bar{A})}{N} \quad \hat{n}(\bar{B}) = \frac{n(\bar{B})}{N}$$

The prime and composite pairing measures are:

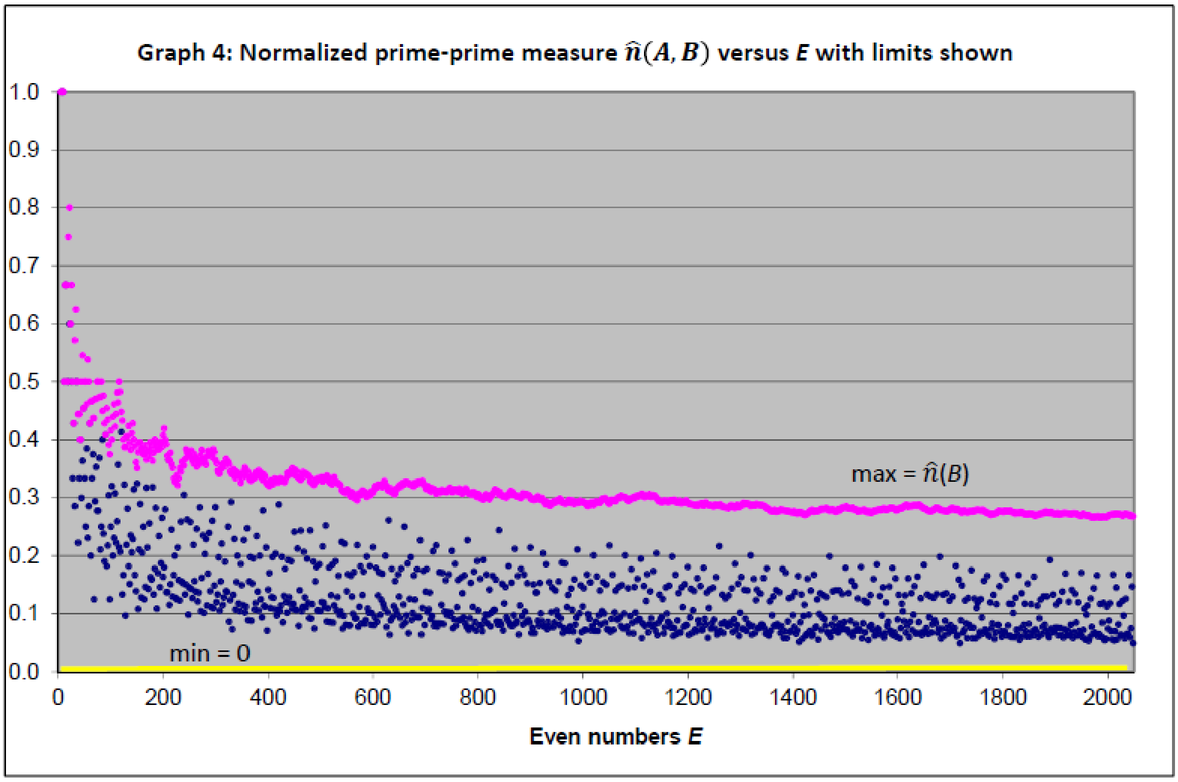
$$\hat{n}(A,B) = \frac{n(A,B)}{N} \text{ and } \hat{n}(\bar{A},\bar{B}) = \frac{n(\bar{A},\bar{B})}{N}$$

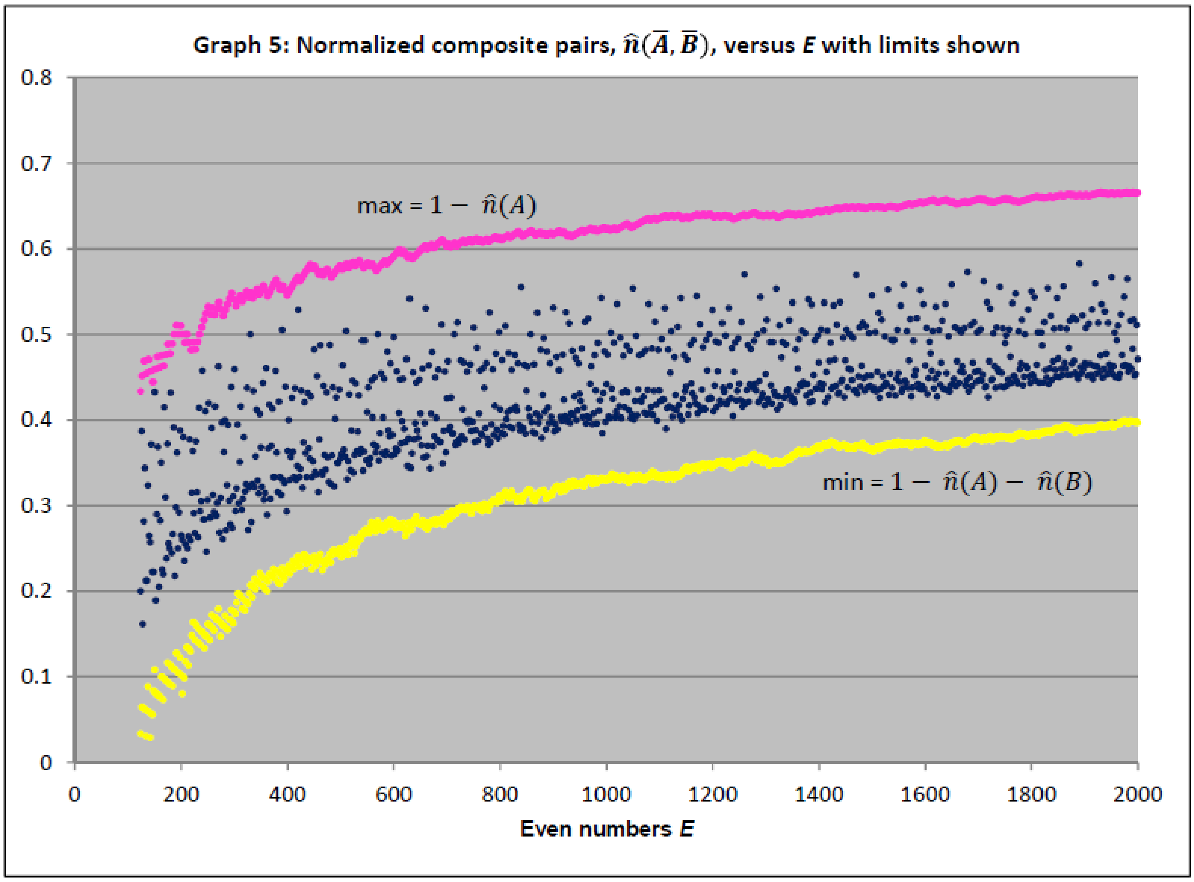
and the normalized delta rule is:

$$\hat{n}(\bar{A},\bar{B}) - \hat{n}(A,B) = 1 - \hat{n}(A) - \hat{n}(B) = \frac{\Delta}{N} = \delta$$

1.7 Normalized graphs

Graph 4 and **5** shows the normalized prime and composite measures respectively.





2. Generalized prime & composite measures

The normalized measures mentioned in **Section 1** can be considered as probabilities. For example, $\hat{n}(A)$ is the probability of finding a prime in A and so on. This leads to the generalized prime-prime measure denoted $\hat{n}_o(A,B)$ that indicates the joint probability that any pair (a_i,b_i) will be prime-prime pairs and the generalized composite-composite measure $\hat{n}_o(\bar{A},\bar{B})$ where:

$$\hat{n}_o(A,B) = \hat{n}(A)\hat{n}(B)$$

$$\hat{n}_o(\bar{A},\bar{B}) = \hat{n}(\bar{A})\bar{n}(\bar{B}) = (1-\hat{n}(A))(1-\hat{n}(B))$$

These are called generalized measures because although they depend on the number of primes in A and B and therefore on E, they do not depend on the specific prime content of E.

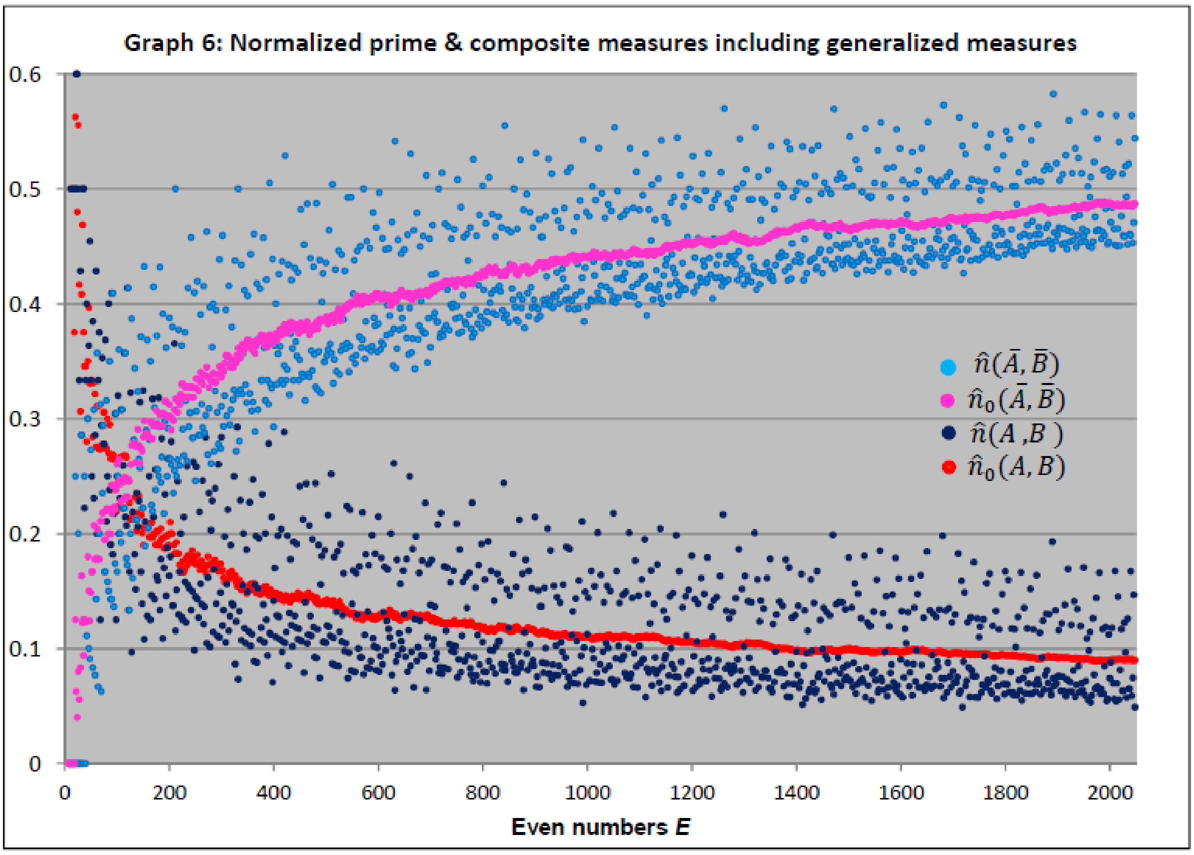
These two measures also obey the delta rule, namely:

$$\hat{n}_o(\bar{A},\bar{B}) - \hat{n}_o(A,B) = 1-\hat{n}(A)-\hat{n}(B) = \delta$$

Graph 6 shows these generalized measures against the background of $\hat{n}(A,B)$ and $\hat{n}(\bar{A},\bar{B})$. The fact that these generalized measures fit into the empty region between the two clusters should not be a surprise:

- The generalized measures do not depend on the prime factors of E
- Statistically, the composite measure lies between the one-third of numbers that contain 3-multiples and the two-thirds of numbers that do not contain 3-multiples.

- The generalized measures behaves in a similar way because of the delta rule
- The generalized measures, however, fit that same developing pattern seen in $\hat{n}(A,B)$ and $\hat{n}(\bar{A},\bar{B})$ - a feature that becomes more apparent in the *DERIVE* analysis that follows.



The generalized measures, however, fit that same developing pattern seen in $\hat{n}(A,B)$ and $\hat{n}(\bar{A},\bar{B})$.

3. Prime-pair manipulation using generalized measures

A lot has been learnt from the *Excel* analysis, but it is now necessary to extend this to larger values of *E* (in the region of 100 million) and in particular look for trends and patterns in our various pairing measures.

3.1 The *Derive* nomenclature

The setup is basically the same as **Section 1**, but now *E* is defined by:

$$E(\alpha,\Omega) = \Omega \times 2^{\alpha}$$

This way we can look at the influence of individual primes in *E* on our pair measures. *Derive* requires a new nomenclature and some examples of this are shown in the table below.

Original	Derive	Generalized measures	
		Original	**Derive**
E	$E(\alpha,\Omega)$	$\hat{n}_0(A,B)$	$\pi 0(\alpha,\Omega)$
N	$N(\alpha,\Omega)$	$\hat{n}_0(\overline{A},\overline{B})$	$\kappa 0(\alpha,\Omega)$
$n(A)$	$nA(\alpha,\Omega)$	**Functions modelled on generalized measures**	
$n(B)$	$nB(\alpha,\Omega)$	$\hat{n}_{03}(\overline{A},\overline{B})$	$\kappa 03(\alpha,\Omega)$
$n(\overline{A})$	$N(\alpha,\Omega)-nA(\alpha,\Omega)$	$\hat{n}_{03}(A,B)$	$\pi 03(\alpha,\Omega)$
$n(\overline{B})$	$N(\alpha,\Omega)-nB(\alpha,\Omega)$	$\hat{n}_{0p}(\overline{A},\overline{B})$	$\kappa 0p(\alpha,\Omega)$
$\hat{n}(A,B)$	$\pi\Omega(\alpha,\Omega)$	$\hat{n}_{0p}(A,B)$	$\pi 0p(\alpha,\Omega)$
$\hat{n}(\overline{A},\overline{B})$	$\kappa\Omega(\alpha,\Omega)$		

General string nomenclature for prime and composite measures in Derive		
1st character	**2nd character**	**3rd group**
π (for prime) κ (for composites)	Ω (for real measures based on odd factors in E)	Followed by Ω-number (eg 1, 3 or 35)
	0 (for generalized measure)	Followed by Ω-number or specific prime p indicating theoretical adaptation
	1 (for theoretical adaptations of $\pi\Omega 1$)	

3.2 Prime & composite measures extended

Graphs 7 and **8** show how particular Ω-numbers affect the prime and composite measures as α increases. From now on we can refer to these curves by their Ω-numbers:

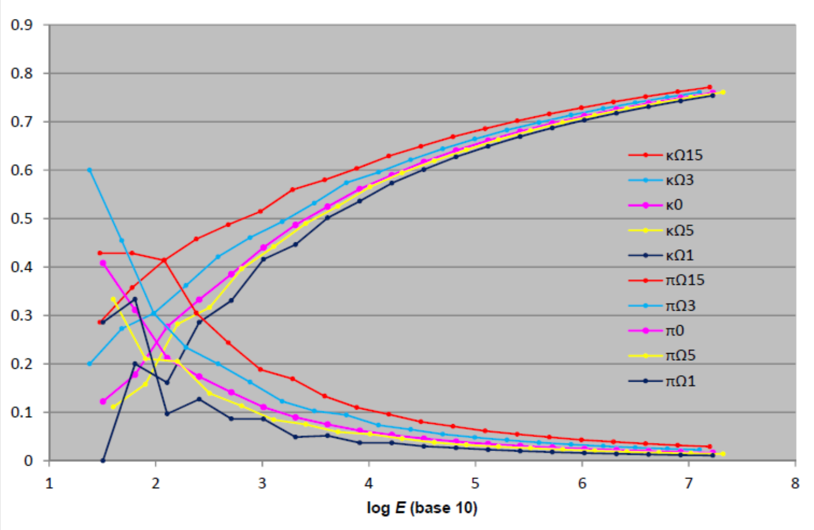
- $\Omega 1$ refers to E with $\Omega = 1$ and thus to numbers that are pure powers of 2
- $\Omega 3$ refers to E with $\Omega = 3$ and therefore 3 is a factor of E, and so on

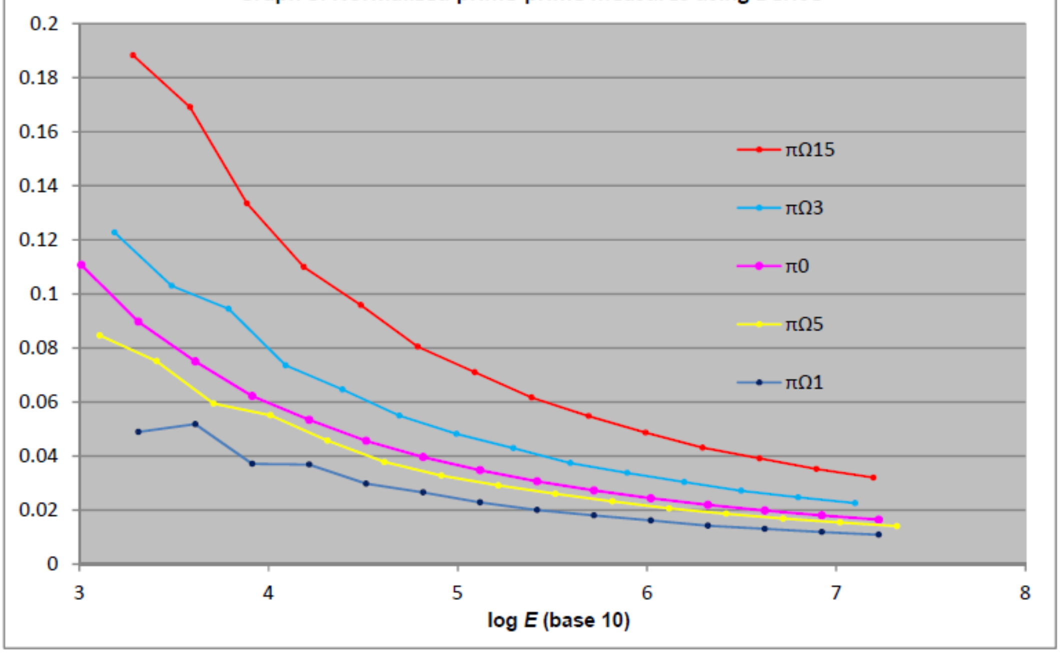
Also included are the curves for the generalized prime and composite measures:

- $\pi 0\ (= \hat{n}_0(A,B))$
- $\kappa 0\ (= \hat{n}_0(\overline{A},\overline{B}))$

Note how well behaved these measures become as E increases (as α increases). The specific calculation points have been connected to show both the general trend in each curve and to make it easier to follow that trend between the curves. For reasons of computation time (generally around 2 hours for each measure), these and all future calculations in *Derive* are based on Ω1 and up to α24. For higher Ω, α is reduced accordingly.

Graph 7: Normalized prime & composite measures using Derive





Graph 8: Normalized prime-prime measures using Derive

3.3 Modelling composite & prime measures with Ω3

We can use the $\pi 0$ and $\kappa 0$ functions and their interdependence with the δ function to create crude approximations to the real $\pi\Omega$ measures. We can demonstrate this using the $\Omega 3$ numbers. The composite-composite measure is easily approached as they are more amenable to analysis than the primes. The plan is to calculate the number of composite pairs with $\Omega 3$ numbers, called $\hat{n}_{03}(\bar{A},\bar{B})$, and then use the δ function to determine $\hat{n}_{03}(A,B)$. In *Derive* these will be $\kappa 03$ and $\pi 03$.

If E contains 3 then all 3-multiples pair up, and so there are at least $N/3$ composite pairs, plus others in the remaining $2N/3$ pairs. The arrangement can be shown diagrammatically as follows where the pair elements in the A and B groups have been rearranged.

$N/3$ 3-multiples in A	$2N/3$ remaining numbers in A
$N/3$ 3-multiples in B	$2N/3$ remaining numbers in B
$N/3$ 3- multiple composite pairs	$\dfrac{2N/3-n(A)}{2N/3}=1-\dfrac{3}{2}\hat{n}(A)$ = the fraction of composites in this section of A $\dfrac{2N/3-n(B)}{2N/3}=1-\dfrac{3}{2}\hat{n}(B)$ = the fraction of composites in this section of B $\left(1-\dfrac{3}{2}\hat{n}(A)\right)\left(1-\dfrac{3}{2}\hat{n}(B)\right)$ = the probability of any pair being composite in this section $\left(\dfrac{2N}{3}\right)\left(1-\dfrac{3}{2}\hat{n}(A)\right)\left(1-\dfrac{3}{2}\hat{n}(B)\right)$ = the number of composite pairs in this section

The theoretical composite measure for Ω3 is therefore:

$$n_{03}(\overline{A},\overline{B}) = \frac{N}{3} + \left(\frac{2N}{3}\right)\left(1-\frac{3}{2}\hat{n}(A)\right)\left(1-\frac{3}{2}\hat{n}(B)\right)$$

$$\hat{n}_{03}(\overline{A},\overline{B}) = \frac{1}{3} + \left(\frac{2}{3}\right)\left(1-\frac{3}{2}\hat{n}(A)\right)\left(1-\frac{3}{2}\hat{n}(B)\right)$$

This can be simplified to:

$$\hat{n}_{03}(\overline{A},\overline{B}) = 1-\hat{n}(A)-\hat{n}(B)+\frac{3}{2}\hat{n}(A)\hat{n}(B) = \delta + \frac{3}{2}\hat{n}_0(A,B)$$

In which case the prime measure is:

$$\hat{n}_{03}(A,B) = \frac{3}{2}\hat{n}_0(A,B) = \frac{3}{2}\pi 0$$

So by isolating a particular group of composite pairs based on a prime present in *E*, this translates as a fractional increase on the generalized prime-prime measure. Note that this is not particularly accurate, but it hints at the fact that *E*-numbers with $\Omega > 1$ create larger prime pairs measures, $n(A,B)$, than *E*-numbers based on pure powers of 2 ($\Omega = 1$).

3.4 Modelling composite & prime measures with Ω*p* (*p*-multiples)

The analysis of **Section 3.3** can be extended to any prime *p* present in *E* ($\Omega = p$).

N/p *p*-multiples in *A*	$N\left(\frac{p-1}{p}\right)$ remaining numbers in *A*
N/p *p*-multiples in *B*	$N\left(\frac{p-1}{p}\right)$ remaining numbers in *B*
N/p *p*-composite pairs	$\frac{N-\frac{N}{p}-n(A)}{N-\frac{N}{p}} = 1-\left(\frac{p}{p-1}\right)\hat{n}(A)$ = the fraction of composites in this section of *A* $\frac{N-\frac{N}{p}-n(B)}{N-\frac{N}{p}} = 1-\left(\frac{p}{p-1}\right)\hat{n}(B)$ = the fraction of composites in this section of *B* $\left(1-\left(\frac{p}{p-1}\right)\hat{n}(A)\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(B)\right)$ = the probability of any pair being composite in this section $N\left(\frac{p-1}{p}\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(A)\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(B)\right)$ = the number of composite pairs in this section

The theoretical composite measure for Ωp is therefore:

$$n_{0p}(\bar{A},\bar{B}) = \frac{N}{p} + N\left(\frac{p-1}{p}\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(A)\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(B)\right)$$

$$\hat{n}_{0p}(\bar{A},\bar{B}) = \frac{1}{p} + \left(\frac{p-1}{p}\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(A)\right)\left(1-\left(\frac{p}{p-1}\right)\hat{n}(B)\right)$$

This can be simplified to:

$$\hat{n}_{0p}(\bar{A},\bar{B}) = 1-\hat{n}(A)-\hat{n}(B)+\left(\frac{p}{p-1}\right)\hat{n}(A)\hat{n}(B) = \delta + \left(\frac{p}{p-1}\right)\hat{n}(A)\hat{n}(B)$$

In which case:

$$\hat{n}_{0p}(A,B) = \left(\frac{p}{p-1}\right)\hat{n}_0(A,B) = \left(\frac{p}{p-1}\right)\pi 0$$

If we introduce a new measure, called the β^* function (where the star denotes that it is a theoretical function, relating how the *p*-prime boosts the prime-prime pairing) then:

$$\beta^*_{p0} = \frac{\hat{n}_{0p}(A,B)}{\hat{n}_0(A,B)} = \frac{p}{p-1} \text{ and in the } \textit{DERIVE} \text{ notation: } \pi 0p = \beta^*_{p0} \times \pi 0.$$

3.5 Modelling composite & prime measures with Ωpq (pq-multiples)

N/p *p*-multiples in *A* | N/q *q*-multiples in *A*

N/p *p*-multiples in *B* | N/q *q*-multiples in *B*

Both groups, however, contain *pq* multiples. To compensate for this, the combined number of *p* and *q* multiples is:

$\frac{N}{p}+\frac{N}{q}-\frac{N}{pq} = N\left(\frac{p+q-1}{pq}\right)$ in *A* and in *B*

$N\left(\frac{p+q-1}{pq}\right)$ composite *p* and *q* pairs in this section

$N - N\left(\frac{p+q-1}{pq}\right) = N\left(\frac{pq-p-q+1}{pq}\right)$ remaining numbers in *A* and in *B*

$N\left(\frac{pq-p-q+1}{pq}\right) - n(A)$ = the number of composites in this section of *A*

$N\left(\frac{pq-p-q+1}{pq}\right) - n(B)$ = the number of composites in this section of *B*

$\left(1-\frac{pq}{pq-p-q+1}\hat{n}(A)\right)$ = the fraction of composites in this section of *A*

$\left(1-\frac{pq}{pq-p-q+1}\hat{n}(B)\right)$ = the fraction of composites in this section of *B*

From this is can be shown that the total number of composite pairs is:

$$n_{0pq}(\bar{A},\bar{B}) = N\left(\frac{p+q-1}{pq}\right) + N\left(\frac{pq-p-q+1}{pq}\right)\left(1-\left(\frac{pq}{pq-p-q+1}\right)\hat{n}(A)\right)\left(1-\left(\frac{pq}{pq-p-q+1}\right)\hat{n}(B)\right)$$

which simplifies to $n_{0pq}(\bar{A},\bar{B}) = \Delta + N\left(\frac{pq}{pq-p-q+1}\right)\hat{n}(A)\hat{n}(B)$

Therefore the normalized composite pair measure is

$$\hat{n}_{0pq}(\overline{A},\overline{B}) = \delta + \left(\frac{pq}{pq-p-q+1}\right)\hat{n}(A)\hat{n}(B)$$

The normalized prime pair measure is therefore:

$$\hat{n}_{0pq}(A,B) = \left(\frac{pq}{pq-p-q+1}\right)\hat{n}(A)\hat{n}(B) = \left(\frac{pq}{pq-p-q+1}\right)\hat{n}_0(A,B)$$

In which case it can be shown that:

$$\beta^*_{pq0} = \frac{\hat{n}_{0pq}(A,B)}{\hat{n}_0(A,B)} = \left(\frac{pq}{pq-p-q+1}\right) = \left(\frac{p}{p-1}\right)\left(\frac{q}{q-1}\right)$$

From this it follows that: $\beta^*_{pq0} = \beta^*_{p0} \times \beta^*_{q0}$

and these measures are multiplicative. The theoretical prime pair measure is therefore:

$$\pi 0pq = \beta^*_{p0} \times \pi 0.$$

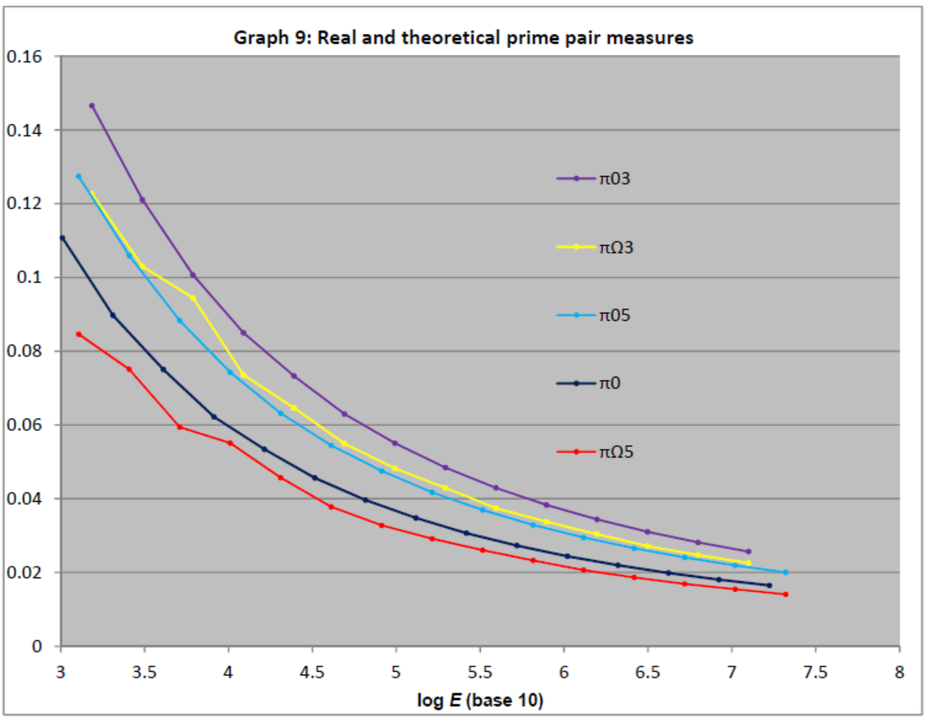
3.6 Comments on calculated composite & prime pairs based on $\pi 0$

In *Derive* the following measures have been calculated:

$$\pi 03 = \beta^*_{30} \times \pi 0 \quad \text{and} \quad \pi 05 = \beta^*_{50} \times \pi 0$$

Graph 9 shows some of these calculated functions based on $\pi 0$ alongside two real prime pair measures, $\pi\Omega 3$ and $\pi\Omega 5$. They show that:

- They are all well behaved
- Their spacings are of the right order
- The theoretical measures are too large compared to the real measures, but this is to be expected as $\pi 0$ is rather large – being larger than all the single prime pair measures except $\pi\Omega 3$.



There are two approaches we must take to overcome the excessive size of the $\pi 0p$ functions compared to the real measures, but they are all based on the idea that $\Omega 1$ numbers form the lowest real prime pair measures. This is a reasonable assumption based on the evidence of **Graphs 1** to **5** but with some reservations to be mentioned later (**Section 4**). These approaches are:

- Calculate the ratio of $\pi\Omega 1$ to $\pi 0$. This will be called the β_{10} function.
- Find more accurate methods to mimic $\pi\Omega p$ measures from the new base-line of $\pi\Omega 1$

From these three approaches we will try to forge a synthesis.

3.7 The beta function applied to $\pi\Omega 1$

The β_{10} function (this has no star because it is calculated on real data) is defined as:

$$\beta_{10}(\alpha,\Omega) = \frac{\pi\Omega 1}{\pi 0}.$$

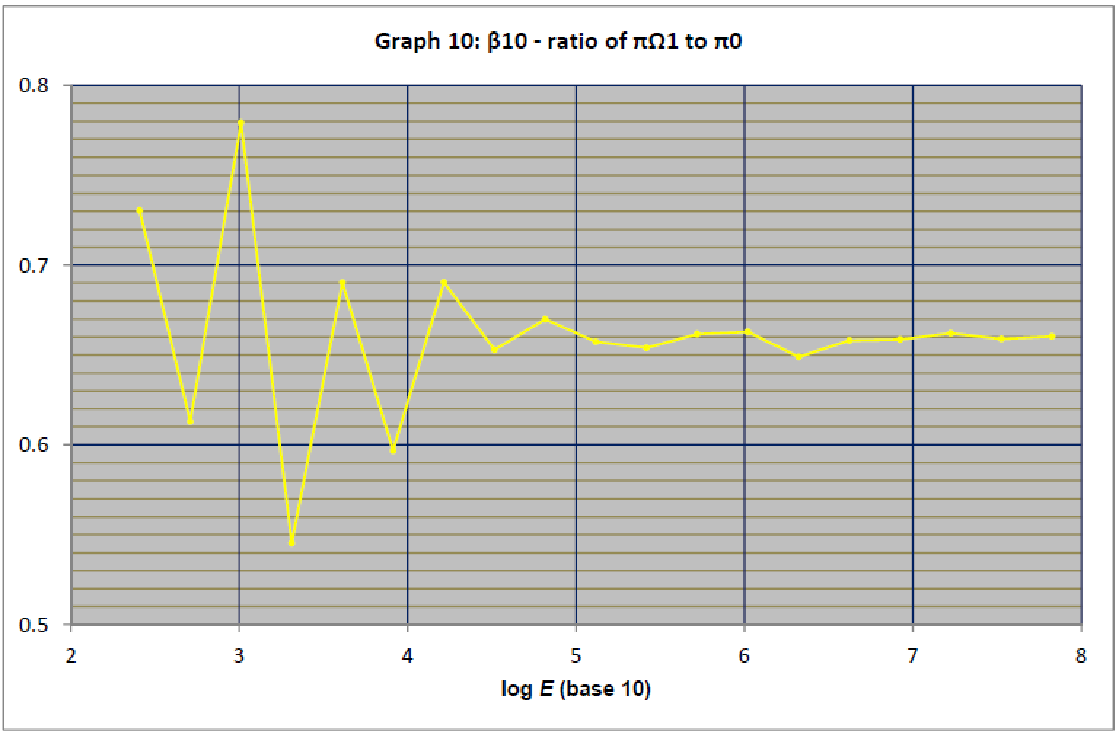
The *Derive* data gives a final value of β_{10} to 9 decimal places as:

$$\beta_{10} = 0.660337319$$

Graph 10 shows how this function develops with increasing α. After initial fluctuations this seems to settle down until at $E \approx 100{,}000{,}000$ the value is about 0.660 to 3dp. For more of a matter of convenience I will adopt this as a fixed value of β_{10} where:

$$\beta_{10} = 0.660$$

These calculations were performed in *Derive* with the function called β10 and took 28 hours of computer time.



At this point there is no clear reason why β_{10} should have this particular value. This awaits later developments.

(Part 2 will follow in the next DNL.)

I had a very intense communication with Rob concerning his paper and the respective DERIVE and Excel files. I was surprised that all his figures were done using MS Excel. Finally he sent `psi modified.dfw` in order to demonstrate how Graph 6 (page 11) can be created using DERIVE. Rob wrote:

One of my big problems was presenting the data in a simple and neat form. I decided to literally manually transfer all data from the DERIVE files to Excel. Painfully slow, but ultimately rewarding because of the superior graph quality available in Excel. If you know a simple way of transferring DERIVE data into Excel spreadsheets I would like to know it.

I will come back to his problem later. Let's first show `psi modified`.dfw and some of its results.

```
#1:   E(α, Ω) := 2^α·Ω

The 2 groups, A & B, contain N elements, related to E by:

#2:   N(α, Ω) := CEILING(E(α, Ω)/4) - 1

The number of primes in A and in B, called nA and nB respectively are:

#3:   nA(α, Ω) := PRIMEPI(2·N(α, Ω) + 1) - 1

#4:   nB(α, Ω) := PRIMEPI(E(α, Ω) - 3) - PRIMEPI(E(α, Ω) - 2·N(α, Ω) - 2)

Also later on another important number called Δ = N - nA - nB
and δ = Δ/N will be used where:

#5:   Δ(α, Ω) := N(α, Ω) - nA(α, Ω) - nB(α, Ω)

#6:   δ(α, Ω) := 1 - nA(α, Ω)/N(α, Ω) - nB(α, Ω)/N(α, Ω)

1. Prime-prime pairs

To compute n(A,B) we need to count the number of prime-prime pairs.
This is achieved via PAB(i) which is true if PA(i) is true and PB(i) is true.
Hence:

#7:   PA(i) := PRIME(2·i + 1)

#8:   PB(α, Ω, i) := PRIME(E(α, Ω) - 2·i - 1)

#9:   PAB(α, Ω, i) := PA(i) ∧ PB(α, Ω, i)

The following program calculates the number of prime-prime pairs, n.
This number however grows large with E and so it is normalized as π = n/N.

      π(α, Ω) :=
        Prog
          i := 1
          n := 0
          Loop
#10:        If PAB(α, Ω, i)
               n := n + 1
            If i = N(α, Ω)
               RETURN n/N(α, Ω)
               i := i + 1
```

Finally, π is computed for a range of α at one value of Ω, along with log E, producing a set of x-y coordinates in Π(Ω,α1,α2)

```
#11:  Π(Ω, α1, α2) ≔ VECTOR([LOG(E(α, Ω), 10), π(α, Ω)], α, α1, α2)
```

For example, the set of prime-prime pairs for Ω1 is:

```
#12:  SetΠΩ1 ≔ Π(1, 5, 24)
```

2. Composite-composite pairs

Using the negation that composites are not-primes then we have:

```
#13:  CA(i) ≔ ¬ PRIME(2·i + 1)

#14:  CB(α, Ω, i) ≔ ¬ PRIME(E(α, Ω) - 2·i - 1)

#15:  CAB(α, Ω, i) ≔ CA(i) ∧ CB(α, Ω, i)
```

The following calculates the number of composite-composite pairs, m, which is then normalized as κ = m/N

```
        κ(α, Ω) ≔
          Prog
            i ≔ 1
            m ≔ 0
            Loop
#16:          If CAB(α, Ω, i)
                 m ≔ m + 1
              If i = N(α, Ω)
                 RETURN m/N(α, Ω)
                 i ≔ i + 1
```

Finally κ is computed for a range of α at one value of Ω, along with log E, as a set of coordinates as K(Ω,α1,α2)

```
#17:  K(Ω, α1, α2) ≔ VECTOR([LOG(E(α, Ω), 10), κ(α, Ω)], α, α1, α2)
```

For example, the set of composite-composite pairs for Ω3 is:

```
#18:  SetKΩ3 ≔ K(3, 3, 22)
```

NOTE: For reasons of computation time the upper limit has been set at Ω1 and α24, which sets the time at about 2 hours.
For higher Ω, α is reduced accordingly.
This reasoning applies to all future calculations.

The following test, T, shows that π and κ conform to the rule that κ − π = δ as it produces a unitary output for all E

#19: $T(\Omega, \alpha1, \alpha2) := \mathrm{VECTOR}\left(\left[\mathrm{LOG}(E(\alpha, \Omega), 10), \dfrac{\kappa(\alpha, \Omega) - \pi(\alpha, \Omega)}{\delta(\alpha, \Omega)}\right], \alpha, \alpha1, \alpha2\right)$

```
#20:  T(1, 6, 16)
```

3. Generalized measures
The following two measures are general normalized prime-prime and composite-composite numbers respectively. Because they do not relate to any specific E-number, these are called π0 and κ0 and can be calculated for any value of Ω because they always yeild the same curve
(but different points on that curve):

3.1 The generalized prime-prime measure

#21: $\pi 0(\alpha, \Omega) := \dfrac{nA(\alpha, \Omega) \cdot nB(\alpha, \Omega)}{N(\alpha, \Omega)^2}$

#22: Π0(Ω, α1, α2) := VECTOR([LOG(E(α, Ω), 10), π0(α, Ω)], α, α1, α2)

#23: SetΠ0 := Π0(Ω, α1, α2)

3.2 The generalized composite-composite measure

#24: $\kappa 0(\alpha, \Omega) := \dfrac{(N(\alpha, \Omega) - nA(\alpha, \Omega)) \cdot (N(\alpha, \Omega) - nB(\alpha, \Omega))}{N(\alpha, \Omega)^2}$

#25: K0(Ω, α1, α2) := VECTOR([LOG(E(α, Ω), 10), κ0(α, Ω)], α, α1, α2)

#26: SetK0 := K0(Ω, α1, α2)

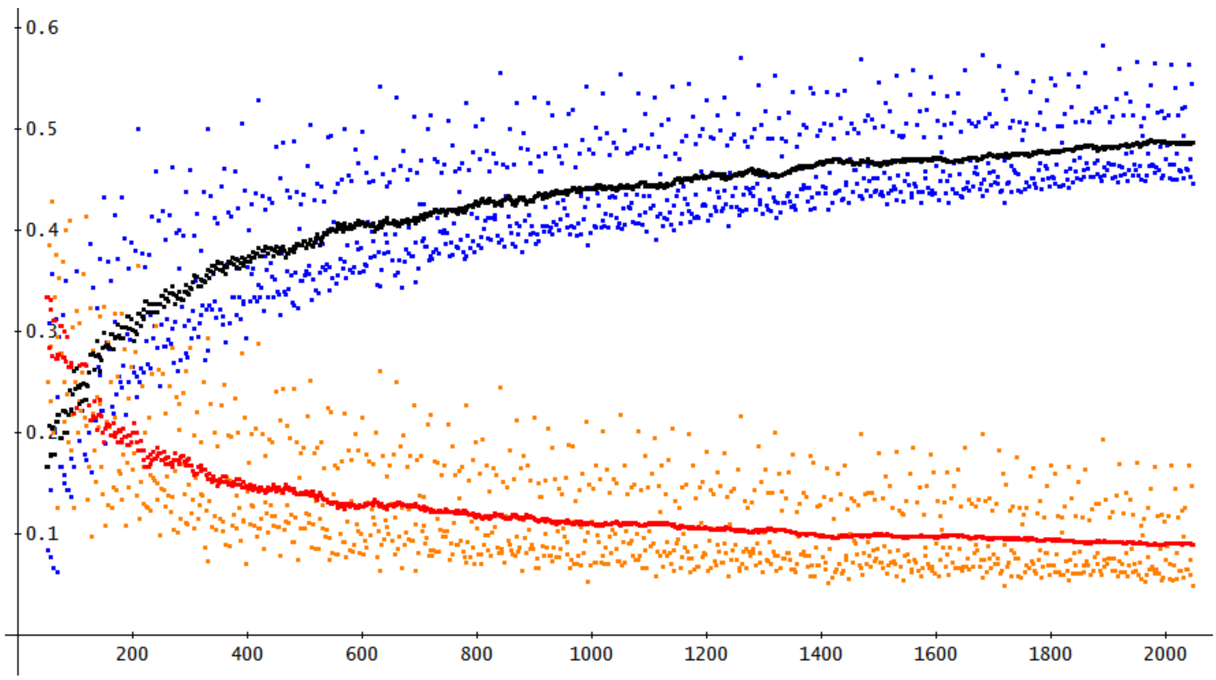
#41: VECTOR([E(1, ψ), κ(1, ψ)], ψ, 25, 1024)

#42: VECTOR([E(1, ψ), π(1, ψ)], ψ, 25, 1024)

#43: VECTOR([E(1, ψ), π0(1, ψ)], ψ, 25, 1024)

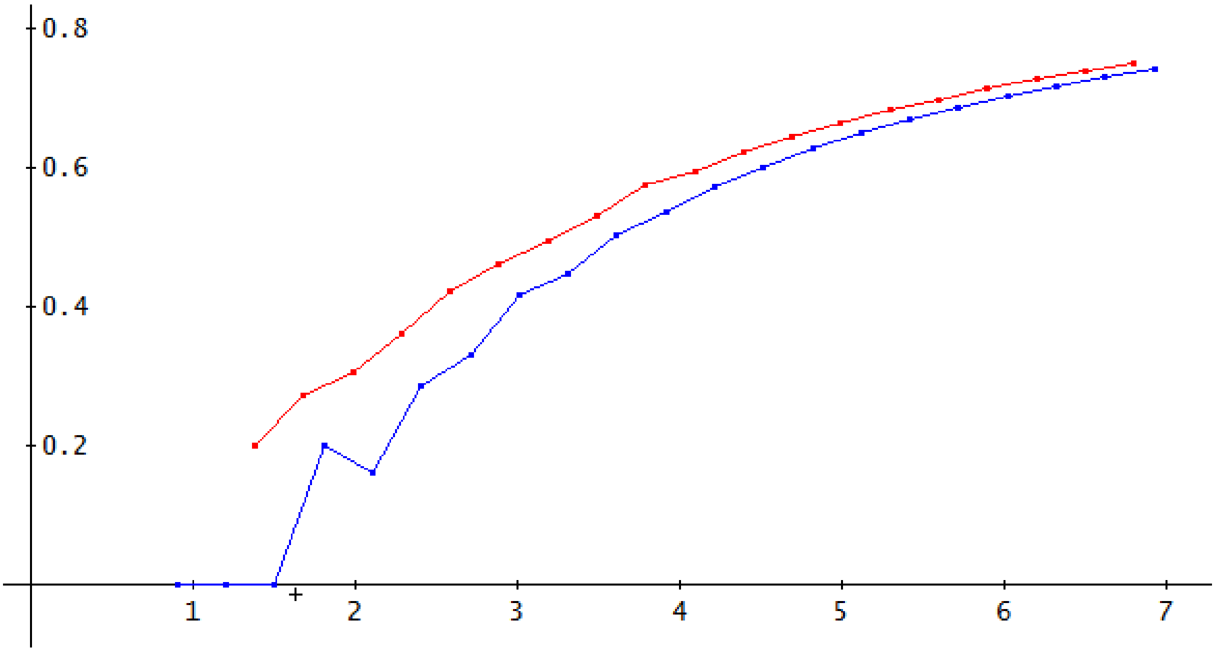
#44: VECTOR([E(1, ψ), κ0(1, ψ)], ψ, 25, 1024)

In contrary to Rob I did not evaluate #41 - #44 before plotting but plotted them directly in the 2D-plot window.



Finally I wanted to reproduce one or two graphs of Rob's Graph 7 (page 12). I approximated K(3,3,n) with n = 15 to 21 which needed 10, 20, 42, …, 898, 2548 seconds. After some 1000 seconds calculation time I gave up approximating K(3,3,22) which is expression # 18 in the above DERIVE file. K(3,3,21) is the red graph.

K(1,3,n) with n = 15 to 23 approximates a bit faster with 8.6, 18, …, 1600, 4850 seconds. The blue graph is the final result. You may imagine how much time was spent by Rob to write his paper!!



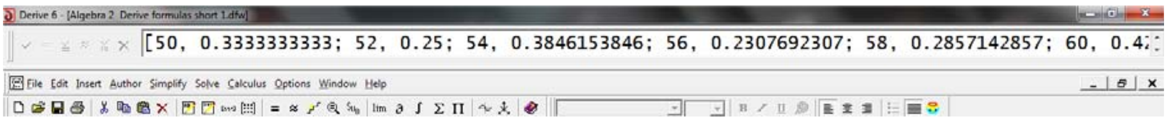
In the following I will describe how to transfer the data (coordinates of the points) to MS Excel.

1st step: Calculate and approximate in DERIVE.

```
VECTOR([E(1, ψ), π(1, ψ)], ψ, 25, 1024)

[[50, 0.3333333333], [52, 0.25], [54, 0.3846153846], [56, 0.2307692307], [58, 0.2857142857],
  [60, 0.4285714285], [62, 0.2], [64, 0.3333333333], [66, 0.375], [68, 0.125], [70,
  0.2941176470], [72, 0.3529411764], [74, 0.2777777777], [76, 0.2777777777], [78,
  0.3684210526], [80, 0.2105263157], [82, 0.25], [84, 0.4], [86, 0.2380952380], [88,
  0.1904761904], [90, 0.4090909090], [92, 0.1818181818], [94, 0.2173913043], [96,
```

2nd step: Copy the result into the Edit Line.



3rd step: Copy and paste the contents of the Edit Line to your text processing program. The first lines will look like:

[50, 0.3333333333; 52, 0.25; 54, 0.3846153846; 56, 0.2307692307; 58, 0.2857142857; 60, 0.4285714285; 62, 0.2; 64, 0.3333333333; 66, 0.375; 68, 0.125; 70, 0.2941176470; 72, 0.3529411764; 74, 0.2777777777; 76, 0.2777777777; 78, 0.3684210526; 80, 0.2105263157; 82, 0.25; 84, 0.4; 86, 0.2380952380; 88, 0.1904761904; 90, 0.4090909090; 92, 0.1818181818; 94, 0.2173913043; 96, 0.3043478260; 98, 0.125; 100, 0.25; 102, 0.32; 104, 0.2; 106, 0.2307692307; 108, 0.3076923076; 110, 0.2222222222; 112, 0.2592592592; 114, 0.3571428571; 116, 0.2142857142; 118, 0.2068965517; 120, 0.4137931034; 122, 4th step: Remove the brackets, then replace the semicolons by the end of paragraph character. Now it should look like shown on the right.

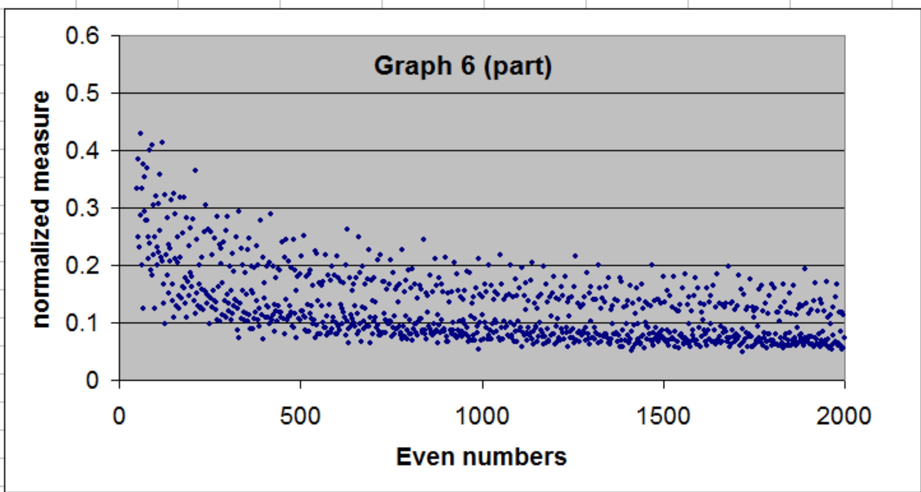
50, 0.3333333333¶
52, 0.25¶
54, 0.3846153846¶
56, 0.2307692307¶
58, 0.2857142857¶
60, 0.4285714285¶
62, 0.2¶
64, 0.3333333333¶
66, 0.375¶
68, 0.125¶
70, 0.2941176470¶
72, 0.3529411764¶

5th step: Convert the text into a table (2 columns) with a comma as separation mark. This should result in a 2 columns table:

50¤	0.3333333333¤	¤
52¤	0.25¤	¤
54¤	0.3846153846¤	¤
56¤	0.2307692307¤	¤
58¤	0.2857142857¤	¤
60¤	0.4285714285¤	¤
62¤	0.2¤	¤
64¤	0.3333333333¤	¤
66¤	0.375¤	¤
68¤	0.125¤	¤
70¤	0.2941176470¤	¤
72¤	0.3529411764¤	¤
74¤	0.2777777777¤	¤
76¤	0.2777777777¤	¤

6th step: Copy the table and paste it into the first cell of your spreadsheet program. (Important for all decimal comma users: You have to change to decimal point input now or you have to replace the decimal point by a comma in the prior step.)

	A	B
1	50	0.3333333
2	52	0.25
3	54	0.3846154
4	56	0.2307692
5	58	0.2857143
6	60	0.4285714
7	62	0.2
8	64	0.3333333
9	66	0.375
10	68	0.125
11	70	0.2941176
12	72	0.3529412
13	74	0.2777778
14	76	0.2777778
15	78	0.3684211
16	80	0.2105263
17	82	0.25
18	84	0.4
19	86	0.2380952



This gives a table with 2 columns and 1000 rows. Then you can easily produce the diagram according to your ideas.

Maxima® for Physics Examples

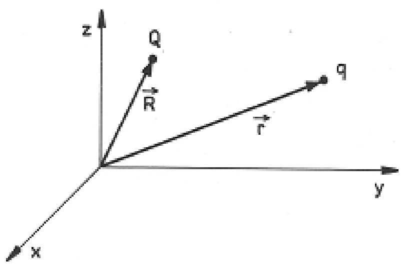
Electric Field

Introduction

The most elementary part of physics dealing with electromagnetic interactions is called electrostatics. Electrostatics describes the interaction between stationary electric charges. The force of interaction between two point charges obeys Coulomb's law. The force $\vec{F}$ acting on a charge q, whose position is $\vec{r}$, due to a charge Q at $\vec{R}$, is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{\left|\vec{r} - \vec{R}\right|^3} (\vec{r} - \vec{R}) \quad ,$$

where ε_o denotes the permittivity of free space.



Electrostatic interactions may also be described in a different way, by means of the concept of an electric field. Within this formalism, the force $\vec{F}$ acting on a charge q is the product of the charge q and the electric field $\vec{E}$ caused by the charge Q

$$\vec{F} = q\vec{E},$$

hence

$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\left|\vec{r} - \vec{R}\right|^3} (\vec{r} - \vec{R}) \ .$$

The modern formulation of electrostatics is based on Gauss's law, which may be written in the form

$$\oint_S \vec{E}\ \overrightarrow{\mathrm{dS}} = \frac{Q}{\varepsilon_o},$$

where Q is the total charge giving rise to the electrostatic field $\vec{E}$, confined by a closed surface *S*. In differential notation Gauss's law is of the form

$$div\vec{E} = \frac{\rho}{\varepsilon_o},$$

where ρ is the charge density.

One very important property of an electrostatic field is the fact that it can be described by a scalar function called the potential. This is a consequence of conservative nature of electrostatic force. The field intensity $\vec{E}$ and the potential Φ are related to each other by the equation

$$\vec{E} = -grad\Phi.$$

Thus, Gauss's law, expressed in terms of the potential, reads

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_o},$$

which is known as Poisson's law (∇^2 denotes the Laplace operator).

The electric field created by a system of electric charges is the vector sum of the fields created by the individual charges (the principle of superposition).

Using the equations given above, we can solve standard exercises as presented below. Some of them will deal with discrete charges, the other ones with the continuously distributed charges.

PROBLEMS

I.1 Split the point Charge *Q* into two point charges *q* and *(Q-q),* separated by a distance of *d,* so that the force of interaction between them is maximum.

Solution: According to Coulomb's law, the magnitude of the force of repulsion between the charges is given by

```
(%i1) F:1/(4*%pi*epsilon[0])*q*(Q-q)/d^2;
```
(%o1) $\frac{q\,(Q-q)}{4\,\pi\,\varepsilon_0\,d^2}$

and

```
(%i2) solve(diff(F,q)=0,q);
```
(%o2) $[\,q=\frac{Q}{2}\,]$

The problem can also be directly solved by finding the maximum value of the function *F* (even its nominator only) i.e. by solving the equation

$$\frac{d}{dq}q(Q-q)=0.$$

Let's find the solution of the above equation

```
(%i3) solve(diff(q*(Q-q),q)=0,q);
```
(%o3) $[\,q=\frac{Q}{2}\,]$

and afterwards sign evaluation of the second derivative at the obtained solution

```
(%i5) sign(diff(q*(Q-q),q,2));
(%o5) neg
```

From %o3 and %o5 we see that at $q=Q/2$ the interactive force is maximum.

Note In the above exercise evaluation of sign function is not necessary. See below

```
(%i6) diff(q*(Q-q),q,2);
(%o6) -2
```

■

I will skip the DERIVE and the TI-Nspire treatment as well because we recognize the quadratic function with zeros at $q_1 = 0$ and $q_2 = Q$. As the graph is a downwards open parabola its vertex is at $q_0 = Q/2$.

I.2 We consider a system of an infinite number of point charges Q, which are distributed on half-line in such a way that the first charge is put at a distance of d from the beginning of the half-line, A, and each successive distance is chosen to be as α big as the previous one. Find the force exerted on a point charge q placed at A.

Solution: The resultant force acting on q is the algebraic sum of Coulomb forces originating from the successive charges Q.

```
(%i1) F:q*Q/(4*%pi*epsilon[0])*sum(1/(alpha^n*d)^2,n,0,inf);
```

(%o1) $$\frac{\left(\sum_{n=0}^{\infty}\frac{1}{\alpha^{2n}}\right) q\,Q}{4\,\pi\,\varepsilon_0\,d^2}$$

Let's try to evaluate the above expression

```
(%i2) assume(alpha>0)$
(%i3) 'F=F,simpsum;
Is  α-1  positive, negative, or zero?p;
```

(%o3) $$\frac{\left(\sum_{n=0}^{\infty}\frac{1}{\alpha^{2n}}\right) q\,Q}{4\,\pi\,\varepsilon_0\,d^2}=\frac{q\,Q}{4\,\pi\,\varepsilon_0\left(1-\frac{1}{\alpha^2}\right)d^2}$$

The above sum was calculated $\alpha - 1 > 0$ i.e. the force takes the simple form for $\alpha > 1$.

For $\alpha < 1$ we get

```
(%i4) 'F=F,simpsum;
Is  α-1  positive, negative, or zero?n;
  sum: sum is divergent.
   -- an error. To debug this try: debugmode(true
```

Let us notice that in our series (geometrical series) the ratio $\frac{1}{\alpha^2}$ appears

```
(%i5) a[n]:=1/alpha^(2*n)$
(%i6) expand(
          a[n+1]/a[n]
                  );
```

(%o6) $\frac{1}{\alpha^2}$

Hence the solution of our problem $(\alpha > 1)$

This is the DERIVE solution:

#1: [CaseMode := Sensitive, InputMode := Word]

#2: $F := \frac{q \cdot Q}{4 \cdot \pi \cdot \varepsilon 0} \cdot \sum_{n=0}^{\infty} \frac{1}{(\alpha^n \cdot d)^2}$

#3: $F := \frac{Q \cdot q \cdot \sum_{n=0}^{\infty} \alpha^{-2 \cdot n}}{4 \cdot \pi \cdot d^2 \cdot \varepsilon 0}$

We see that there is a geometric series with quotient 1/α^2.

#4: $\frac{\alpha^{-2 \cdot (n+1)}}{\alpha^{-2 \cdot n}} = \frac{1}{\alpha^2}$

#5: α :∊ Real (1, ∞)

F from above is recalculated and is given in a simpler form:

#6: $F := \frac{Q \cdot q \cdot \alpha^2}{4 \cdot \pi \cdot d^2 \cdot \varepsilon 0 \cdot (\alpha^2 - 1)}$

Set α ≤ 1:

#7: α :∊ Real (−∞, 1]

#8: $F := \frac{Q \cdot q \cdot \sum_{n=0}^{\infty} \alpha^{-2 \cdot n}}{4 \cdot \pi \cdot d^2 \cdot \varepsilon 0}$

On the TI-Nspire we cannot distinguish between lower and uppercase characters, so we denote *Q* as *qq*:

$$\frac{q \cdot qq}{4 \cdot \pi \cdot \varepsilon 0} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{(\alpha^n \cdot d)^2} \right) \blacktriangleright \frac{\sum_{n=0}^{\infty} \left(\frac{\alpha^{-2 \cdot n}}{d^2} \right) \cdot q \cdot qq}{4 \cdot \varepsilon 0 \cdot \pi}$$

$$\frac{q \cdot qq}{4 \cdot \pi \cdot \varepsilon 0} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{(\alpha^n \cdot d)^2} \right) | \alpha > 1 \blacktriangleright \frac{q \cdot qq \cdot \alpha^2}{4 \cdot d^2 \cdot (\alpha^2 - 1) \cdot \varepsilon 0 \cdot \pi}$$

The third example is a bit more complex:

I.3 Three stationary charges Q_1, Q_2 and Q_3 are placed at the following positions $\vec{R}_1(x_1, y_1, z_1), \vec{R}_2(x_2, y_2, z_2)$ and $\vec{R}_3(x_3, y_3, z_3)$, respectively.

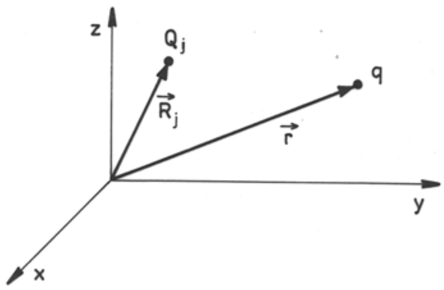
(a) Write down expressions describing the field potential at the point $\vec{r}(x, y, z)$.

(b) Compute the resultant force acting on an electron of charge q for the data:

x= 6m, y= -7m, z= 9m, $Q_1 = 4C,\ Q_2 = -3C,\ Q_3 = (1/2)C$,
$\vec{R}_1(0,\ 2,\ 0),\ \vec{R}_2(-3, 2, 0)$ and $\vec{R}_3(0,0,0)$.

Solution: The potential at the point $\vec{r}$, due to a single charge Q_j placed at the point given by the position vector $\vec{R}_j$ can be expressed as

$$\Phi_j = \frac{1}{4\pi\varepsilon_0} \frac{Q_j}{\left|\vec{r} - \vec{R}_j\right|}$$



According to the principle of superposition, the resultant potential is

$$\Phi = \sum_j \Phi_j ,$$

a) To make the notation more concise the components of the position vectors $\vec{R}_j$ of the charges $\bar{Q}_j$ are entered in the form of the matrix **R_**.

```
(%i1) R_:matrix([0,2,0],[-3,2,0],[0,0,0])*m;
       [  0    2 m  0 ]
(%o1)  [ -3 m  2 m  0 ]
       [  0    0    0 ]
```

while the charges as components of the vector **Q_**.

```
(%i2) Q_:[4,-3,1/2]*C;
                    C
(%o2) [4 C, -3 C, -]
                    2
```

Then the resultant potential at $\vec{r}(x, y, z)$ can be written as follows

```
(%i3) Phi(x,y,x):=1/(4*%pi*%epsilon[0])*
                sum(Q_[j]/sqrt((r_-R_[j]).(r_-R_[j])),j,1,length(Q_));
```

(%o3) $\Phi(x,y,x) := \frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{length(Q\_)} \frac{Q\_{j}}{\sqrt{(r\_ - R\_{j}) \cdot (r\_ - R\_{j})}}$

with

```
(%i4) r_:[x,y,x];
(%o4) [x,y,x]
```

b) Desired force has the form

$$F = \sqrt{\vec{F}.\vec{F}}$$

where

$$\vec{F} = q \cdot \vec{E}, \quad \vec{E} = -grad(\Phi)$$

We load the file “**vect**” and enter the above relations.

```
(%i5) load(vect)$
(%i6) E_(x,y,x):=-ev(express(grad(Phi(x,y,z))),diff)$
(%i7) F_(x,y,x):=q*E_(x,y,z)$
```

The magnitude of the resultant force is

```
(%i8) F(x,y,z):=sqrt(F_(x,y,z).F_(x,y,z))$
```

We import the value of the electron charge and electric constant from the utility file “**physical_constants**”.

```
(%i9) load (physical_constants)$
      propvars (physical_constant);
(%o10) [%c, %mu_0, %e_0, %Z_0, %G, %h, %h_bar, %m_P, %T_P, %l_P, %t_P, %%e, %Phi_0, %G_0, %K_J, %R_K,
%mu_B, %mu_N, α, %R_inf, %a_0, %E_h, %ratio_h_me, %m_e, %N_A, %m_u, %F, %R, %%k, %V_m, %n_0, %ratio_S0
, σ, %c_1, %c_1L, %c_2, %b, %b_prime]
(%i11) get(%%e,description);
(%o11) elementary charge
(%i12) get(%e_0,description);
(%o12) electric constant $1/(\mu_0 c^2)$
```

In the next step we evaluate the magnitude of the resultant force at the point $\vec{r}[6,-7,9]\,m$.

```
(%i13) float(subst([q=constvalue(%%e),
       epsilon[0]=constvalue(%e_0),
       x=6*m,y=-7*m,z=9*m],F(x,y,z)));
```

(%o13) $\frac{1.615850173775426\;10^{-11}\,|c|}{m^2}\;\;\frac{m^2\;c\;N}{s^2\;A^2}$

In the final step we replace symbol "." by "*".

```
(%i14) (1.615850173775426*10^-11*C)/m^2 * (m^2*C*N)/(s^2*A^2);
```

(%o14) $\dfrac{1.615850173775426\,10^{-11}\,C^2\,N}{s^2\,A^2}$

Simplifying units returns

```
(%i15) subst(A=C/s,%);
```

(%o15) $1.615850173775426\,10^{-11}\,N$

This is the DERIVE version:

#1: [CaseMode := Sensitive, InputMode := Word]

#2: LOAD(C:\Program Files\TI Education\Derive 6\Math\PhysicalConstants.mth)

#3: $\left[q := e\_,\ \varepsilon 0 := \dfrac{\frac{1}{36\cdot\pi}\cdot 10^{-9}\cdot \text{coulomb}^2}{\text{newton}\cdot\text{meter}^2}\right]$

#4: $\left[Q\_ := \left[4,\ -3,\ \frac{1}{2}\right]\cdot\text{coulomb},\ R\_ := \begin{bmatrix} 0 & 2 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\cdot\text{meter}\right]$

#5: r_ := [x, y, z]

#6: $\phi := \dfrac{1}{4\cdot\pi\cdot\varepsilon 0}\cdot\sum_{j=1}^{\text{DIM}(Q\_)} \dfrac{Q\_{}_j}{\left|r\_ - R\_{}_j\right|}$

#7: [E_ := − GRAD(φ), F_ := q·E_]

#8: |SUBST(F_, r_, [6, −7, 9]·meter)|

#9: $1.615890105\cdot 10^{-11}\cdot|\text{newton}|$

The next page shows how to work with TI-Nspire CX CAS. The gradient function is not provided, so we use Michel Beaudin's library (presented in DNL#98) in order to apply his grad function.

Elementary charge q and electric constant (permittivity in a vacuum) ε_0 are provided among the "Constants" in the "Unit Conversions" which can be found in the Documents Toolbox.

© electron charge q and permittivity ε0 are implemented as _q and _ε0 (meter = _m).

$q:=\_q:\varepsilon 0:=\_\varepsilon 0:m:=\_m$ 1.000000· _m

$qq\_:=[4\ \ {-3}\ \ 0.5]\cdot \_coul$ $[4.000000\cdot \_coul\ \ {-3.000000}\cdot \_coul\ \ 0.500000\cdot \_coul]$

$$rr\_:=\begin{bmatrix}0 & 2 & 0\\ -3 & 2 & 0\\ 0 & 0 & 0\end{bmatrix}\cdot m \qquad \begin{bmatrix}0 & 2.000000\cdot \_m & 0\\ -3.000000\cdot \_m & 2.000000\cdot \_m & 0\\ 0 & 0 & 0\end{bmatrix}$$

$r\_:=[x\ \ y\ \ z]$ $[x\ \ y\ \ z]$

$$\varphi:=\frac{1}{4\cdot\pi\cdot\varepsilon 0}\cdot\sum_{j=1}^{\dim(qq\_)[2]}\left(\frac{qq\_[1,j]}{\mathrm{norm}(r\_-rr\_[j])}\right)$$

$$\frac{-26962655362.1}{\sqrt{x^2+6.000000\cdot x\cdot \_m+y^2-4.000000\cdot y\cdot \_m+z^2+13.000000\cdot \_m^2}}\cdot \_m\cdot \_V+\frac{}{\sqrt{x^2+y^2\cdot}}$$

$ee\_:=\text{-}kit\_ets\_mb\backslash grad(\varphi,[x\ \ y\ \ z])$

$$\frac{1}{4\cdot\pi\cdot\varepsilon 0}\ \ \sum_{j=1}\ \ \left(\frac{}{\mathrm{norm}(r\_-rr\_[j])}\right)$$

$$\frac{-26962655362.1}{\sqrt{x^2+6.000000\cdot x\cdot \_m+y^2-4.000000\cdot y\cdot \_m+z^2+13.000000\cdot \_m^2}}\cdot \_m\cdot \_V+\frac{}{\sqrt{x^2+y^2\cdot}}$$

$ee\_:=\text{-}kit\_ets\_mb\backslash grad(\varphi,[x\ \ y\ \ z])$

⚠
$$\left[\frac{-26962655362.1\cdot(x+3.000000\cdot \_m)}{\left(x^2+6.000000\cdot x\cdot \_m+y^2-4.000000\cdot y\cdot \_m+z^2+13.000000\cdot \_m^2\right)^{1.500000}}\cdot \_m,\right.$$

$ee\_:=ee\_|x=6\cdot m \text{ and } y=-7\cdot m \text{ and } z=9\cdot m$

$$\left[25965674.5977\cdot\frac{\_N}{\_coul}\ \ {-66776938.0585}\cdot\frac{\_N}{\_coul}\ \ 70979162.7842\cdot\frac{\_N}{\_coul}\right]$$

$ff\_:=q\cdot ee\_$ $[4.160160\text{E-}12\cdot \_N\ \ {-1.069884}\text{E-}11\cdot \_N\ \ 1.137212\text{E-}11\cdot \_N]$

$\mathrm{norm}(ff\_)$ 1.615850E-11· _N

KIT_ETS_MB for NspireCAS 2 (and for DERIVE, of course)

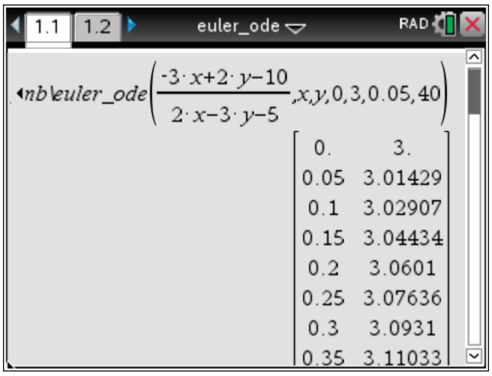
Michel Beaudin (Montréal, CAN) and Josef Böhm (Würmla, AUT)

I present the first three functions and give applications after introducing them.

euler_ode(f,x,y,x0,y0,h,n)

Application of Euler's method on the differential equation $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ with n steps *h* to produce a matrix of *n*+1 rows showing *n*+1 coordinate pairs of points that approximates the solution curve of the equation.

Note: The functions *euler* and *rk23* implemented in TI-Nspire CAS are programs for solving systems of DEs.



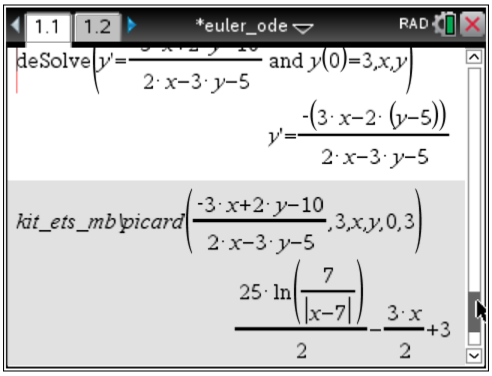
picard(f,p,x,y,x0,y0)

The iterative method of Picard is applied for finding an approximation for the DE $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$.

s is the chosen integration variable (function internally).

eu_ca_ode(a,b,x,y,r)

Gives a solution for $x^2 y'' + a\,x\,y' + b\,y = r(x)$, which cannot be solved by deSolve.



Comment on *picard* (from the DERIVE Online Help):

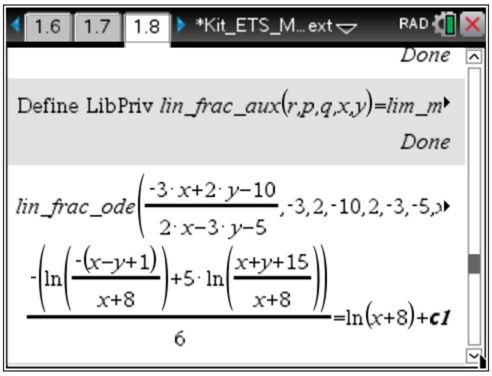
PICARD(r, p, x, y, x0, y0) expands to an improved approximate series solution of the equation y' = r(x, y), given the approximate series solution p(x). Expanding with respect to x often gives a more useful form than just simplifying the expression.

The Picard method involves integrating r(x, p(x)). If no integrals remain in the simplified result, you can try another iteration using the improved approximation for p, and so on. If you have no better first approximation, use the constant y0.

lin_frac_ode(r,a,b,d,p,q,k,x,y)

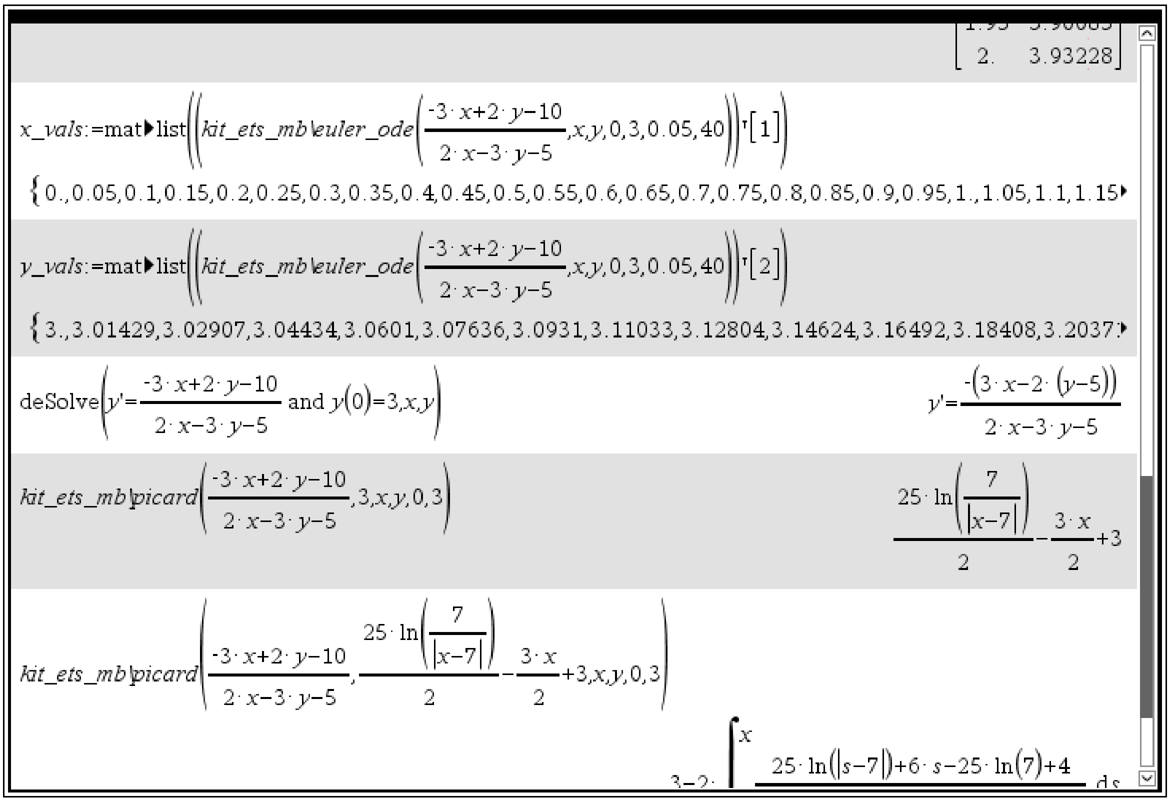
finds a solution for the first order differential equation $\frac{dy}{dx} = r \equiv \frac{ax+by+d}{px+qy+k}$.

This DE cannot be solved with TI-Nspire by the deSolve command.

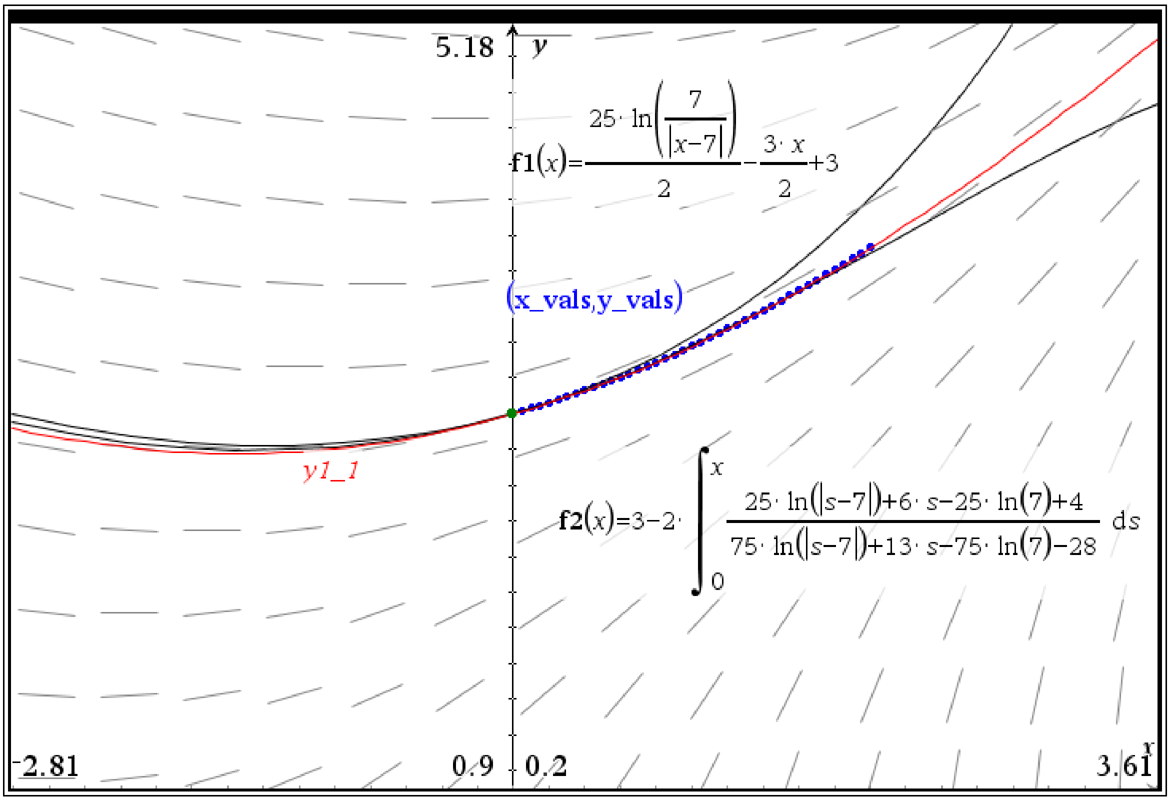


Examples:

Solve $y' = \dfrac{-3x+2y-10}{2x-3y-5}$; $y(0)=3$



As we cannot apply deSolve, so we use Euler's method, convert the two columns of the resulting matrix to lists and then plot the scatter diagram (in blue see below). Another approximating method is Picard iteration which is giving a closed result only for the first iteration.



The following two screen shots show two more examples applying Picard's iteration method.

y' = sin(x)−y, y(0)=1

y1:=**kit_ets_mb\picard**(sin(x)−y,1,x,y,0,1) ▸ $-\cos(x)-x+2$

y2:=**kit_ets_mb\picard**(sin(x)−**y1**,1,x,y,0,1) ▸ $-\sqrt{2}\cdot\cos\left(x+\frac{\pi}{4}\right)+\frac{x^2}{2}-2\cdot x+2$

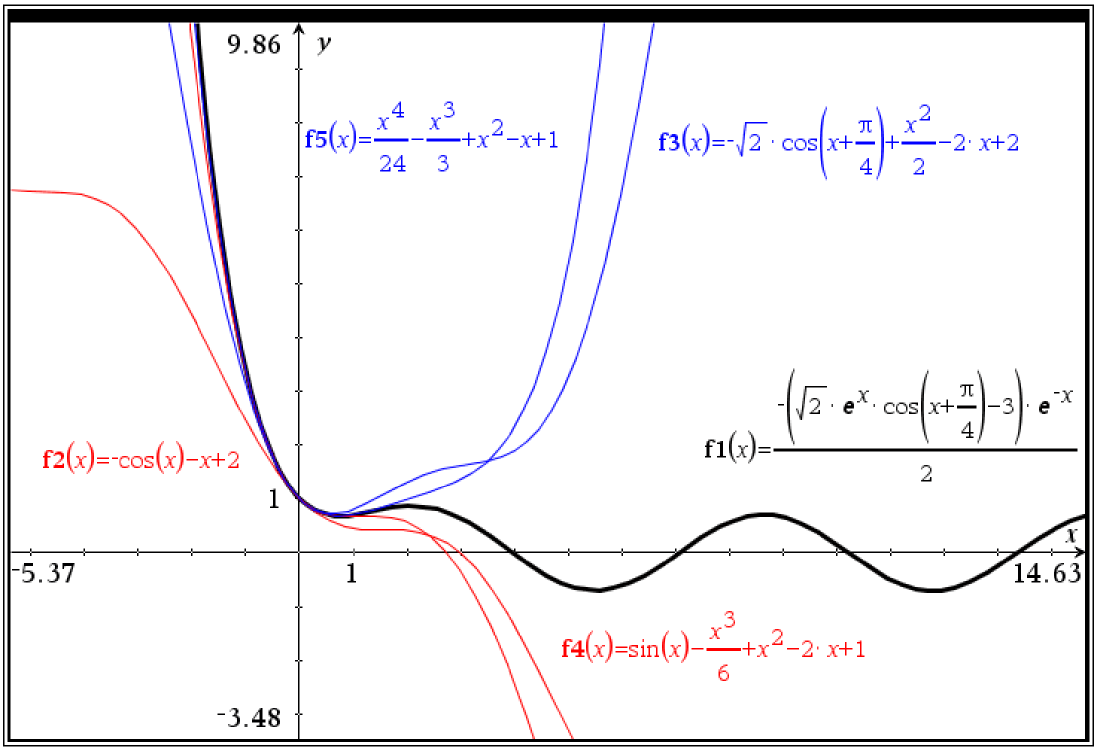
y3:=**kit_ets_mb\picard**(sin(x)−**y2**,1,x,y,0,1) ▸ $\sin(x)-\frac{x^3}{6}+x^2-2\cdot x+1$

y4:=**kit_ets_mb\picard**(sin(x)−y,**y3**,x,y,0,1) ▸ $\frac{x^4}{24}-\frac{x^3}{3}+x^2-x+1$

deSolve(y'=sin(x)−y and y(0)=1,x,y) ▸ $y=\frac{-\left(\sqrt{2}\cdot e^{x}\cdot\cos\left(x+\frac{\pi}{4}\right)-3\right)\cdot e^{-x}}{2}$

f2(x):=−cos(x)−x+2 ▸ *Done* **f3**(x):= $-\sqrt{2}\cdot\cos\left(x+\frac{\pi}{4}\right)+\frac{x^2}{2}-2\cdot x+2$ ▸ *Done*

f4(x):= $\sin(x)-\frac{x^3}{6}+x^2-2\cdot x+1$ ▸ *Done* **f5**(x):= $\frac{x^4}{24}-\frac{x^3}{3}+x^2-x+1$ ▸ *Done*



Another note from the DERIVE Online Help: Note that the highest order terms generated by the Picard method are often incorrect. For example, the coefficient of the above x^3 term should be 1. Distrust terms whose coefficients are not the same for two successive iterates. For this reason and for efficiency, it is wise to discard all but the next higher order term after each iteration. For example, you should discard the above x^3 and x^2 terms for the next iteration.

If any iterate yields a result containing an integral, you can try approximating r(x, p(x)) to make it integrable.

I must admit that I was not very familiar with Picard's iteration method, so I wrote to Michel:

> Dear Michel,
>
> I have a question concerning the Picard Iteration:
>
> I can apply your function picard() without any problems, but I am not quite sure about the function of parameter p (= y0?).
>
> Best regards
> Josef

This is what Michel answered:

Josef, here is a good example. Consider the ODE y' = 3*y^(2/3), y(2) = 0. This ODE does not satisfy the uniqueness theorem in any neighborhood of the point (2, 0) : so we can't expect a unique solution and, in fact, 2 different solutions can be found, namely : y1(x) = 0 for all x and y2(x) = (x-2)^3 (the « desolve » yields the latest one).

Picard(3*y^(2/3),0,x,y,2,0) will simplify in 0 and we will never find the solution (x-2)^3. But if you apply successive Picard iteration (and add « |x>2 »), you will find at the end –using approximate arithmetic for convenience – (x-2)^3!!! See the file.

Regards,

Michel

deSolve$\left(y'=3\cdot y^{\frac{2}{3}} \text{ and } y(2)=0,x,y\right)$ $\quad y^{\frac{1}{3}}=x-2$

solve$\left(y^{\frac{1}{3}}=x-2,y\right)$ $\quad y=(x-2)^3$

kit_ets_mb\picard$\left(3\cdot y^{\frac{2}{3}},0,x,y,2,0\right)$ $\quad 0$

©So if the starting point is p(x)= yo = 0, we will never find the solution $(x-2)^3$ with Picard!

©Note that y(x)= 0 is also a solution to the DE
(this DE does not satisfy the uniqueness theorem in any neighborghood of the point (2, 0)).

©So let's take a different starting, say x−2 and apply successive Picard iterations:

kit_ets_mb\picard$\left(3\cdot y^{\frac{2}{3}},x-2,x,y,2,0\right)|x>2$ $\quad 1.8\cdot (x-2.)^{1.66667}$

kit_ets_mb\picard$\left(3\cdot y^{\frac{2}{3}},1.8\cdot (x-2.)^{1.6666666666667}|x=s,x,y,2,0\right)|x>2$ $\quad 2.10277\cdot (x-2.)^{2.11111}$

$$\text{kit\_ets\_mb\\picard}\left(3\cdot y^{\frac{2}{3}},1.0000239211375\cdot(x-2.)^{2.9999979397155}|x=s,x,y,2,0\right)|x>2 \quad 1.00002\cdot(x-2.)^{3.}$$

$$\text{kit\_ets\_mb\\picard}\left(3\cdot y^{\frac{2}{3}},1.0000164052101\cdot(x-2.)^{2.999998626477}|x=s,x,y,2,0\right)|x>2 \quad 1.00001\cdot(x-2.)^{3.}$$

$$\text{kit\_ets\_mb\\picard}\left(3\cdot y^{\frac{2}{3}},1.0000112420077\cdot(x-2.)^{2.999999084318}|x=s,x,y,2,0\right)|x>2 \quad 1.00001\cdot(x-2.)^{3.}$$

$$\text{kit\_ets\_mb\\picard}\left(3\cdot y^{\frac{2}{3}},1.0000076981443\cdot(x-2.)^{2.9999993895453}|x=s,x,y,2,0\right)|x>2 \quad 1.00001\cdot(x-2.)^{3.}$$

$$\text{kit\_ets\_mb\\picard}\left(3\cdot y^{\frac{2}{3}},1.0000052677471\cdot(x-2.)^{2.9999995930302}|x=s,x,y,2,0\right)|x>2 \quad 1.\cdot(x-2.)^{3.}$$

Comparing my Picard iteration and Michel's one you can see that there are two ways for performing the procedure: one can replace the *y* in the function *r* by the next iteration or one can replace *p* by the next iteration.

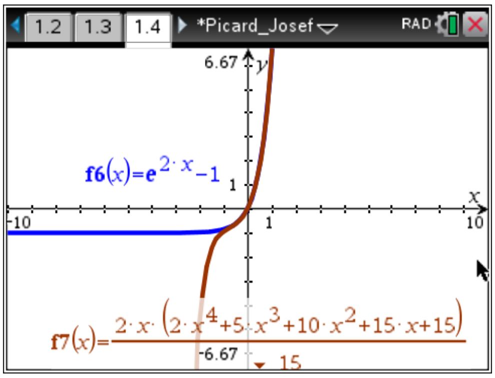
I had the idea to find a function which performs *n* iterations and returns the result:

```
Define picard_its(f,p,x,y,x0,y0,n)=
Func
Local i,yn
i:=1
While i≤n
  yn:=picard(f,p,x,y,x0,y0)
  p:=yn
  i:=i+1
EndWhile
yn
EndFunc
```

I was successful in applying my function on the first DE from above and then another one: $y' = 2y + 2; y(0) = 0$.

$$\text{picard\_its}(\sin(x)-y,1,x,y,0,1,4) \quad \frac{x^4}{24}-\frac{x^3}{3}+x^2-x+1$$

$$\text{picard\_its}(2\cdot y+2,0,x,y,0,0,5) \quad \frac{2\cdot x\cdot(2\cdot x^4+5\cdot x^3+10\cdot x^2+15\cdot x+15)}{15}$$



f6(x) is the exact solution and f7(x) the result of the iteration procedure.

Interesting: this fifth iteration is identical with the Taylor expansion of f6!!

The next screen shows Michel's example treated with *picard_its* followed by an example from the DERIVE Online Help.

$picard\_its(3\cdot y^{3}, x-2, x, y, 2, 0, 1)$ $\qquad \frac{9\cdot(x-2)^3}{5}$

$picard\_its\left(3\cdot y^{\frac{2}{3}}, x-2, x, y, 2, 0, 13\right)$

$\frac{150094635296999121\cdot 1586131^{\frac{1}{3}}\cdot 2059^{\frac{2059}{2187}}\cdot 1009^{\frac{211}{243}}\cdot 331^{\frac{19}{27}}\cdot 211^{\frac{15283}{19683}}\cdot 97^{\frac{665}{729}}\cdot 61^{\frac{5}{9}}\cdot 23^{\frac{11}{27}}\cdot 19^{\cdot}}{141968320691576064581767424926}$

$\frac{150094635296999121\cdot 1586131^{\frac{1}{3}}\cdot 2059^{\frac{2059}{2187}}\cdot 1009^{\frac{211}{243}}\cdot 331^{\frac{19}{27}}\cdot 211^{\frac{15283}{19683}}\cdot 97^{\frac{665}{729}}\cdot 61^{\frac{5}{9}}\cdot 23^{\frac{11}{27}}\cdot 19^{\cdot}}{141968320691576064581767424926}$ $\qquad 1.04851\cdot(x-2.)^{\frac{4766585}{1594323}}$

$1.0485072426292\cdot(x-2.)^{\frac{4766585.}{1594323}}$ $\qquad 1.04851\cdot(x-2.)^{2.98972}$

© more than 13 iterations are impossible to perform

© compare with your results!! pretty the same!!

$picard\_its(y^2+\sqrt{x}, 1, x, y, 0, 1, 1)$ $\qquad \frac{2\cdot x^{\frac{3}{2}}}{3}+x+1$

© the example from the DERIVE Online Help:

$picard\_its(y^2+\sqrt{x}, 1, x, y, 0, 1, 1)$ $\qquad \frac{2\cdot x^{\frac{3}{2}}}{3}+x+1$

$picard\_its(y^2+\sqrt{x}, 1, x, y, 0, 1, 2)$ $\qquad \frac{x^4}{9}+\frac{8\cdot x^{\frac{7}{2}}}{21}+\frac{x^3}{3}+\frac{8\cdot x^{\frac{5}{2}}}{15}+x^2+\frac{2\cdot x^{\frac{3}{2}}}{3}+x+1$

© according to Albert's advice we have to discard the higher degree powers!

© next iteration is:

$x^2+\frac{2\cdot x^{\frac{3}{2}}}{3}+x+1$ $\qquad x^2+\frac{2\cdot x^{\frac{3}{2}}}{3}+x+1$

Now let's have a look on *lin_frac_ode* taking the example from above:

Solve $y' = \dfrac{-3x+2y-10}{2x-3y-5};\; y(0)=3$.

Silly question: Do you know how to solve this DE without Michel's *lin_frac_ode* or DERIVE's lin_frac? I must admit I didn't know. Then I decoded the DERIVE function (Michel's Nspire tool is a reproduction of lin_frac) following the auxliary functions.

Follow me, if you like.

First of all we have to transform this DE with linear numerator and denominator to a homogeneous DE. For this reason we solve the system

-3*m* + 2*n* – 10 = 0 and 2*m* – 3*n* – 5 = 0 → *m* = –8, *n* = –7.

(If the solution is not unique then one has to use another function with DERIVE (fun_lin_ccf). Michel's *lin_frac_ode* recognizes this case and splits internally to another solution method.)

We can proceed by substitute $x = u + m = u - 8$ and $y = v + n = v - 7$ giving

$$v' = \frac{-3(u-8)+2(v-7)-10}{2(u-8)-3(v-7)-5} = \frac{-3u+2v}{2u-3v} = \frac{2\frac{v}{u}-3}{-3\frac{v}{u}+2}.$$ This is now a homogeneuous DE:

We set $s = \dfrac{v}{u} \to v = s\cdot u \to v' = s + s'\cdot u = \dfrac{2s-3}{-3s+2}$

$$\frac{ds}{du}\cdot u = \frac{2s-3}{2-3s} - s = \frac{2s-3-2s+3s^2}{2-3s}$$ The variables can be separated.

$$\frac{(2-3s)\,ds}{3s^2-3} = \frac{dx}{x}$$ which is easy to integrate on both sides.

$$-\frac{\ln(s-1)}{6} - \frac{5\ln(s+1)}{6} = \ln(u)+c \;\to -\frac{\ln\left(\frac{v}{u}-1\right)}{6} - \frac{5\ln\left(\frac{v}{u}+1\right)}{6} = \ln(u)+c$$

Resubstitution and setting $\dfrac{\ln c}{6}$ 6 for *c*: $$-\frac{\ln\left(\frac{y+7}{x+8}-1\right)}{6} - \frac{5\ln\left(\frac{y+7}{x+8}+1\right)}{6} = \ln(x+8)+\frac{\ln c}{6}$$

$$\ln\frac{y-x-1}{x+8} + 5\ln\frac{y+x+15}{x+8} = -6\ln(x+8) - \ln c$$

$$\frac{(y-x-1)(y+x+15)^5}{(x+8)^6} = \frac{1}{c\,(x+8)^6}$$ oder $$(y-x-1)(y+x+15)^5 = c$$

This is the general solution.

Now for the particular solution with $y(0) = 3$: $(3 - 0 - 1)(3 + 0 + 15)^5 = 3779136 = c$.

Finally the solution is: $(y-x-1)(y+x+15)^5 = 3779136$.

Working with *lin_frac_ode* I wondered if it is really necessary to enter the coefficients of numerator and denominator twice. It should be possible to pick them out and then entering the function would be more comfortable. The result of my considerations was *lin_frac_ode2*(*r*,*x*,*y*).

$$lin\_frac\_ode\left(\frac{-3\cdot x+2\cdot y-10}{2\cdot x-3\cdot y-5},-3,2,-10,2,-3,-5,x,y\right)$$

$$\frac{-\left(\ln\left(\frac{-(x-y+1)}{x+8}\right)+5\cdot\ln\left(\frac{x+y+15}{x+8}\right)\right)}{6}=\ln(x+8)+\boldsymbol{c1}$$

$$lin\_frac\_ode2\left(\frac{-3\cdot x+2\cdot y-10}{2\cdot x-3\cdot y-5},x,y\right)$$

$$\frac{-\left(\ln\left(\frac{-(x-y+1)}{x+8}\right)+5\cdot\ln\left(\frac{x+y+15}{x+8}\right)\right)}{6}=\ln(x+8)+\boldsymbol{c1}$$

$$lin\_frac\_ode\left(\frac{-3\cdot x+2\cdot y-10}{6\cdot x-4\cdot y},-3,2,-10,6,-4,-0,x,y\right)$$

$$\frac{-(5\cdot\ln(6\cdot x-4\cdot y+5)-2\cdot(3\cdot x-2\cdot y))}{8}=x+\boldsymbol{c1}$$

$$lin\_frac\_ode2\left(\frac{-3\cdot x+2\cdot y-10}{6\cdot x-4\cdot y},x,y\right)$$

$$\frac{-(5\cdot\ln(6\cdot x-4\cdot y+5)-2\cdot(3\cdot x-2\cdot y))}{8}=x+\boldsymbol{c1}$$

```
lin_frac_ode2                                                    1/7
Define LibPub lin_frac_ode2(r,x,y)=
Func
Local a,b,d,p,q,k
a:=d/dx(getNum(r)):b:=d/dy(getNum(r)):d:=getNum(r)
d:=d|x=0 and y=0
p:=d/dx(getDenom(r)):q:=d/dy(getDenom(r)):k:=getDe
k:=k|x=0 and y=0

{fun_lin(r,p,q,k,x,y),                                  a·q-b·p=0
{lin_frac_aux(r,(b·k-d·q)/(a·q-b·p),(d·p-a·k)/(a·q-b·p),x,y),a·q-b·p≠0
EndFunc
```

The next idea was to provide a function for finding a particular solution. Unfortunately it is not possible to replace the system variable ***c1*** within the function. See how I solved the problem:

$$lin\_frac\_spec\left(\frac{-3\cdot x+2\cdot y-10}{2\cdot x-3\cdot y-5},x,y,0,3\right)$$

$$\frac{-\left(\ln\left(\frac{-(x-y+1)}{x+8}\right)+5\cdot\ln\left(\frac{x+y+15}{x+8}\right)\right)}{6}=\ln(x+8)+\boldsymbol{c1}$$

with c1 =

$$\frac{-\ln(1944)}{3}$$

Done

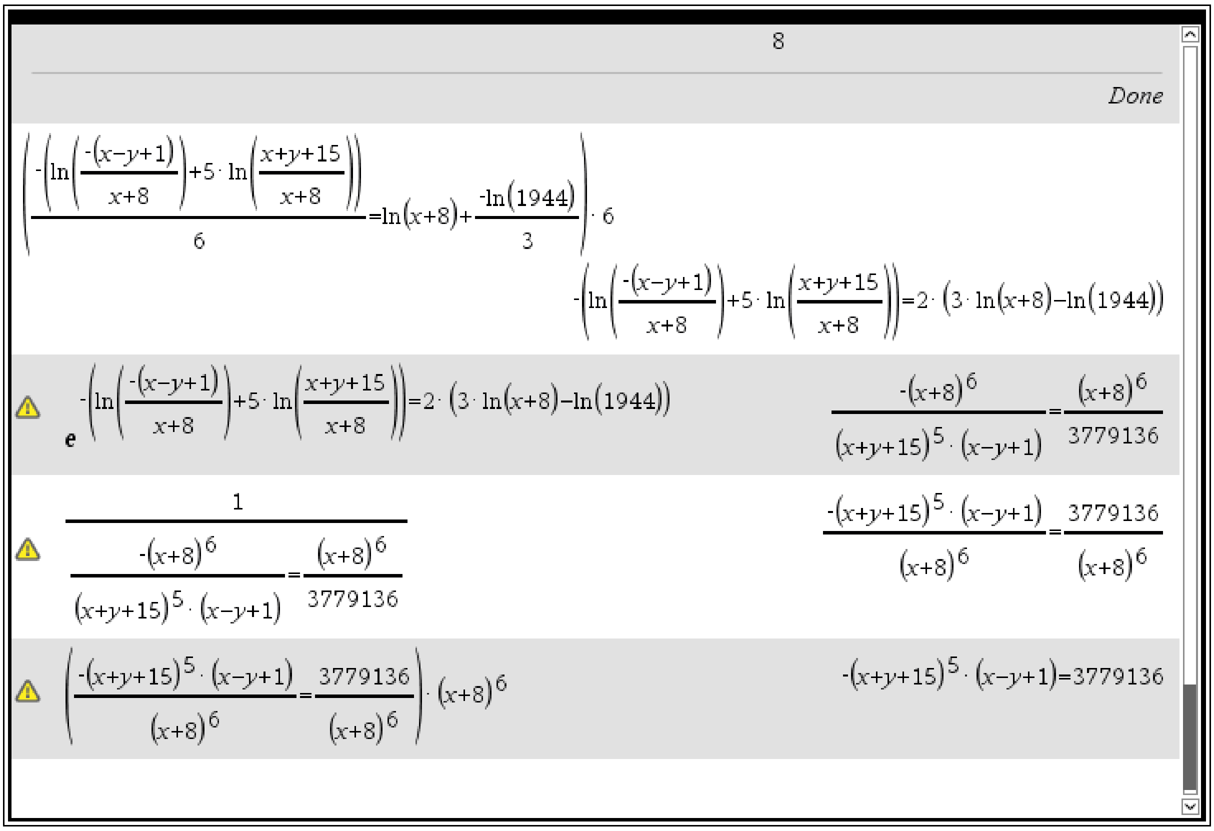
$$lin\_frac\_spec\left(\frac{-3\cdot x+2\cdot y-10}{6\cdot x-4\cdot y},x,y,2,3\right)$$

$$\frac{-(5\cdot\ln(6\cdot x-4\cdot y+5)-2\cdot(3\cdot x-2\cdot y))}{8}=x+\boldsymbol{c1}$$

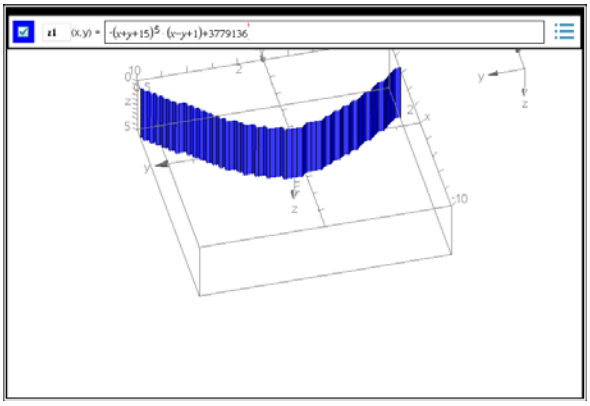
with c1 =

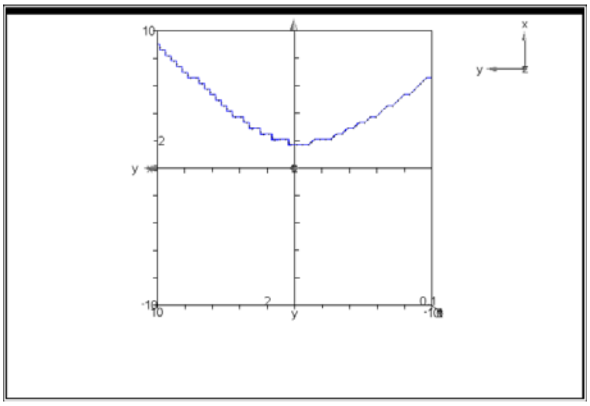
$$\frac{-5\cdot\ln(5)}{8}-2$$

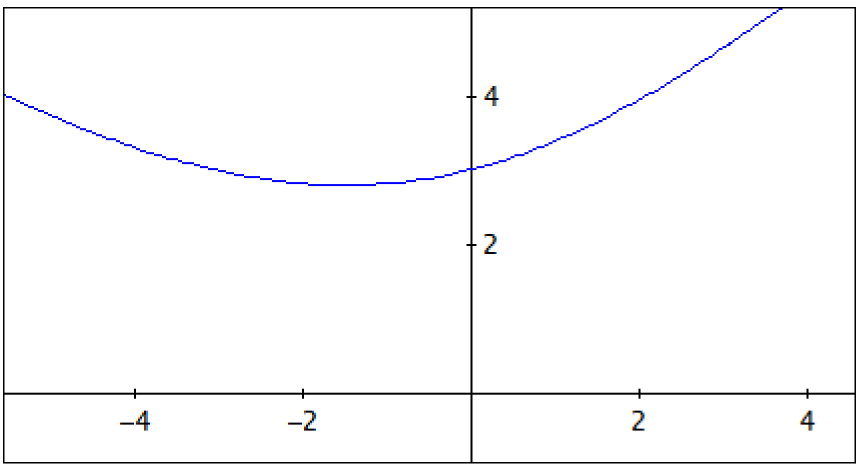
Done



As we are not able producing an implicit plot with TI-Nspire, we can apply a little trick and plot a this slice of the solid $z(x,y) = 3779136 + (x + y + 15)^5 \cdot (x - y + 1)$ in the xy-plane and rotate until we see the top view. The DERIVE made implicit plot is presented below.







taylor_ode1(*f*,*x*,*y*,*x*0,*y*0,*n*)

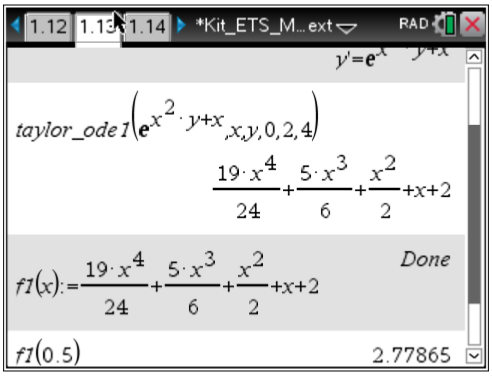
gives the Taylor polynomial of order *n* as approximation for the solution of the differential equation

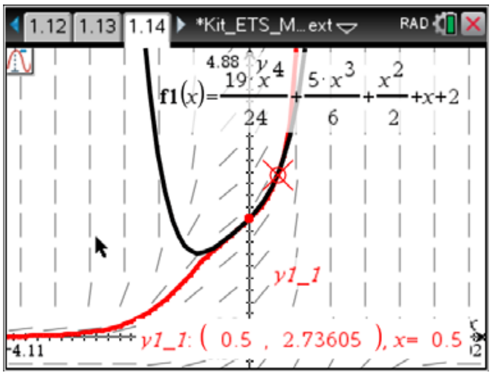
$$\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0 .$$

Solve:

$$y' = e^{x^2 y + x},\ y(0) = 2$$
$$y(0.5) = ?$$

The Graph & Geometry screen shows the Taylor polynomial (black) and the numerical solution produced by TI-NspireCAS using the respective Graph Entry option.





taylor_ode2(*f*,*x*,*y*,*v*,*x*0,*y*0,*v*0,*n*)

gives the Taylor polynomial of order *n* as approximation for the solution of the differential equation

$$\frac{d^2y}{dx^2} = f(x,y,v),\ y(x_0) = y_0, v(x_0) = v_0 .$$

y' is replaced by *v*.

Solve:

$$y'' = x^2 y' + x\,y - 2,\ \ y(1) = 1,\ y'(1) = -\frac{1}{2}$$
$$y(0.5) = ?$$

$$y(0.5) \approx 1.074$$

Entering the DE is a bit tricky:

y1' = y2 with (x_0 = 1, $y1_0$ = 1)
y2' = x^2 y2 + x y1 – 2 with (x_0 = 1, $y2_0$ = -0.5)

The DERIVE Online Help provides a recipe how to verify the correctness of the result.