Thick Lines and Painting in DERIVE

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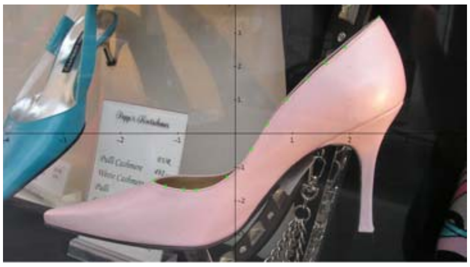
My colleague Tania gave a great presentation in Dresden (on the Proceedings CD). Among many other “motivativing activities” for students she showed her “Cinderalla´s Shoe” modelled by cubic splines. I have seen this earlier, but look at her “DERIVE picture” of the shoe> thick lines and a pink, which is not among the DERIVE colours. I asked how she produced thick lines in DERIVE which has been a long requested feature of DERIVE, and how she brought the colour on the shoe. Here is her answer with some additional comments. Josef



$$\#1: \quad f(x) := \frac{x^4 - 13\cdot x^2 + 30}{30} - 2$$

#2: TABLE(f(x), x, -5, 5, 0.01)

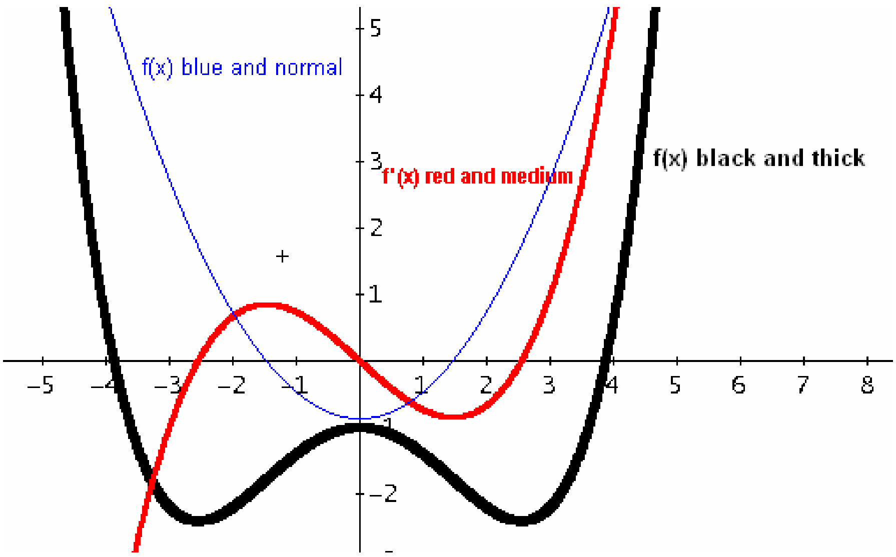
#3: TABLE(f'(x), x, -5, 5, 0.01)

#4: f''(x)

Set Option > Display > Points Connected and Large and Plot #2 (don´t forget to set Options > Approximate before plotting.

Then Set Option > Display > Points connected and Medium and Plot #3.

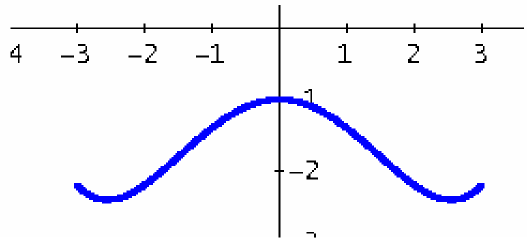
Finally plot #4. Choose colours as appropriate.



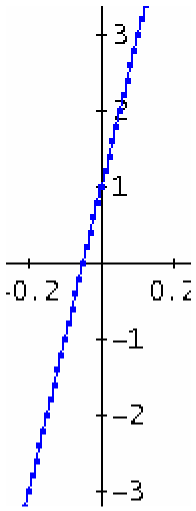
If you intend to use this nice tool more often then you might use a function with fixed parameters which can be changed if needed.

In function **thick()** are the variable and the increment of the table preset with x and 0.01. In most cases this will be fine:

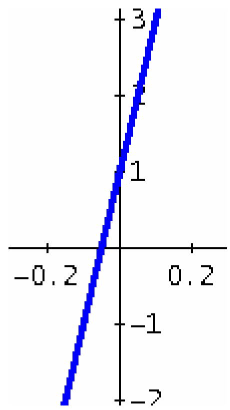
```
#5:    thick(u, mi, ma, v_ := x, inc := 0.01) := TABLE(u, v_, mi, ma, inc)

#6:    thick(f(x), -3, 3)
```



```
#7:    thick(20·t + 1, -0.5, 0.5, t)
```



```
#8:    thick(20·t + 1, -0.5, 0.5, t, 0.001)
```

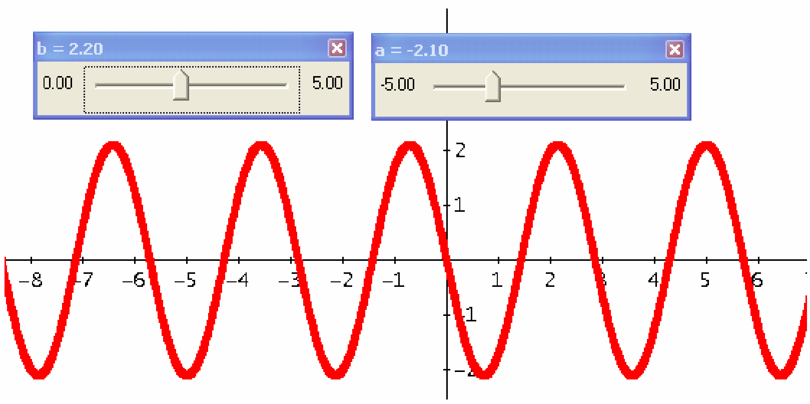


Now it looks fine.

I enter variable *t* and see that the steepness of the graph needs a smaller increment.

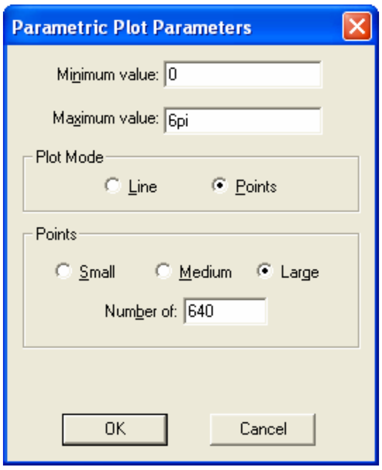
The slider bar is one of my favourite features of DERIVE 6 (and as I know, it is also very often used by Tania in classroom). So I am very happy that the "Thick- and Medium-Line-Trick" is wonderful supported by the slider bars. Sometimes it needs some moments to create the graph, but then the slider bars work without any delay.

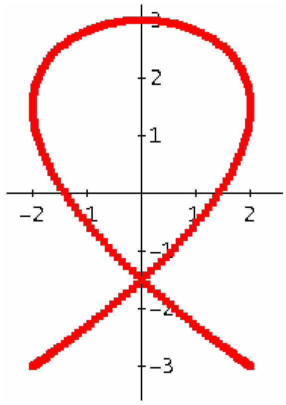
```
#9:    thick(a·SIN(b·x), - 4·π, 4·π)
```



In parameter form we do not need this trick, because we can enter the size of points:

```
#10:   [2·SIN(3·t), 3·COS(2·t)]
```

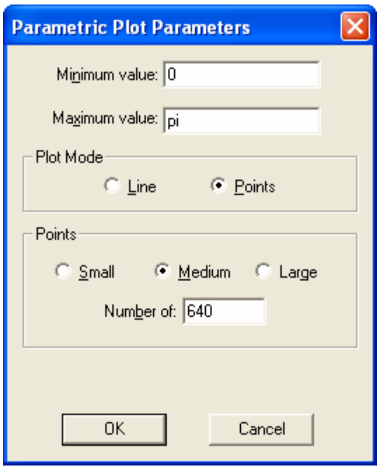


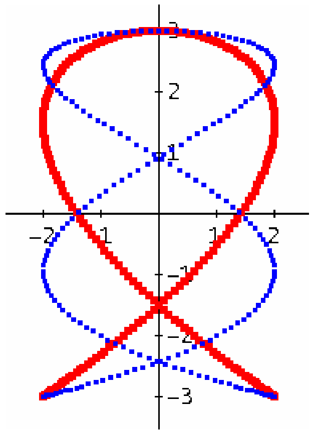


Plotting in Size Medium we face problems because of the restriction to plot a maximum number of 640 points. To obtain a smooth line one has to plot in intersections 0 - π, π-2π, etc.

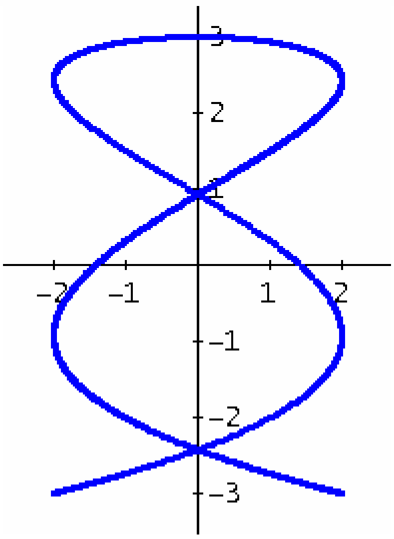
```
#11:   [2·SIN(5·t), 3·COS(2·t)]
```

Or we try to overcome this restriction by defining a function applying the more or less unrestricting VECTOR-command:





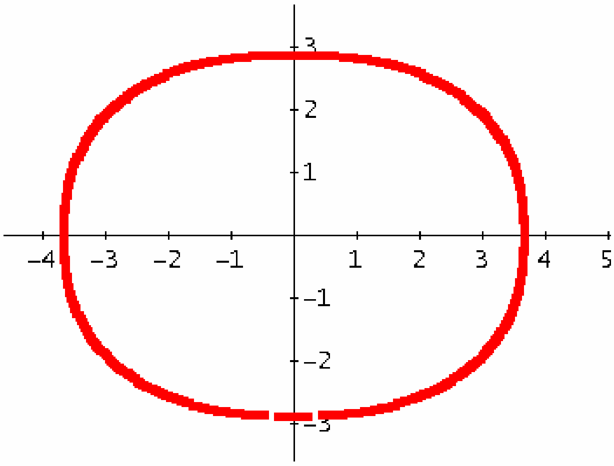
```
#12:   thickpar(u, mi, ma, v_ := x, inc := 0.01) := VECTOR(u, v_, mi, ma, inc)

#13:   thickpar([2·SIN(5·t), 3·COS(2·t)], 0, 3·π, t)
```



Incited and inspired by Tania´s method I wanted to do implicit plots in three line thicknesses, but this is not so easy to perform. One idea is to adapt the utility for plotting level curves (see another contribution in this DNL). I tried another way: Scanning the area in small x- and y-increments and plotting the points which fullfill the implicitly given equation of the curve:

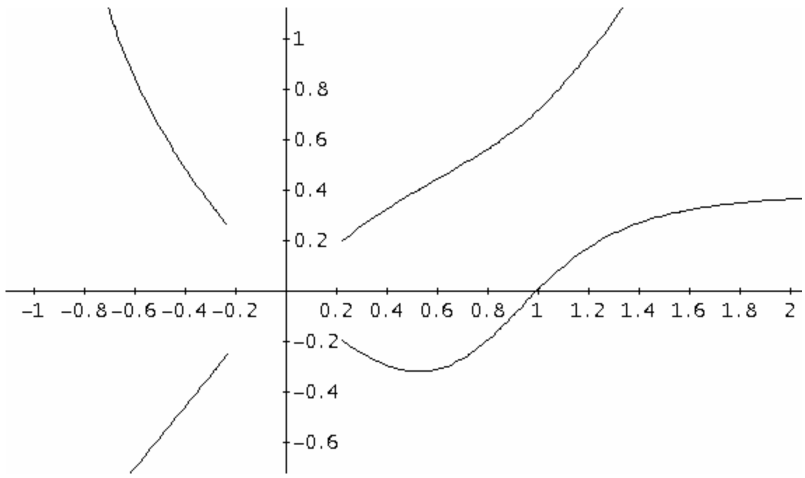
```
        impl(u, xst, xend, yst, yend, inc := 0.01, acc := 0.01, pts, i, j, val) :=
          Prog
            pts := []
            u := LHS(u) - RHS(u)
            i := xst
            Loop
              If i > xend
#15:             RETURN pts
              j := yst
              Loop
                If j > yend exit
                If ABS(SUBST(u, [x, y], [i, j])) < acc
                   pts := APPEND(pts, [[i, j]])
                j :+ inc
              i :+ inc
```

#17: $\mathrm{impl}\left(\left|\frac{x}{3.67}\right|^{2.37} + \left|\frac{y}{2.88}\right|^{2.37} = 1,\ -4,\ 4,\ -4,\ 4,\ 0.03\right)$



I wanted to test my function and invented a strange implicit form and DERIVE gave immediately its graph:

#18: $\frac{x^3 \cdot e^{-x \cdot y}}{x^2 - y^2} = 1$

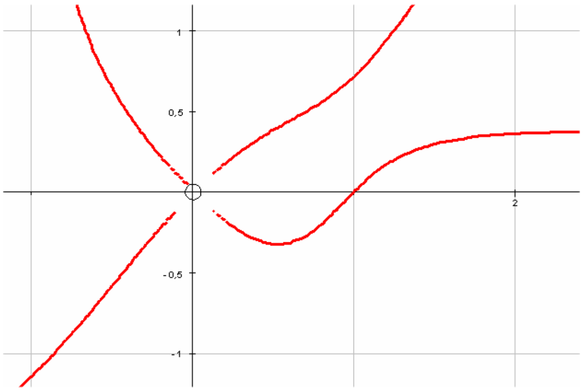


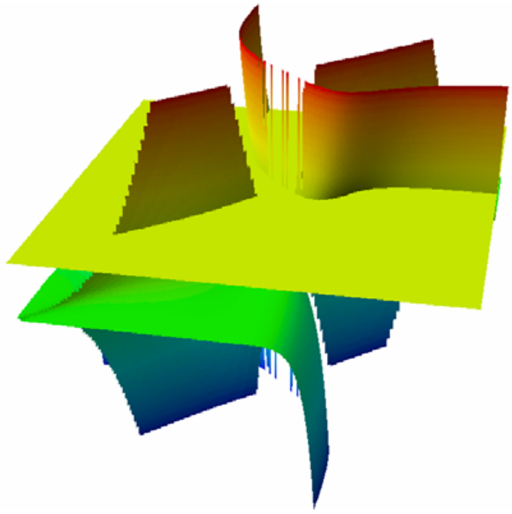
But we want the graph as a medium or as a thick line.

Autograph is not a CAS but has a lot of great features, with a very powerful graphic.

I plotted the expression with a thickness of 3 ½ pts:

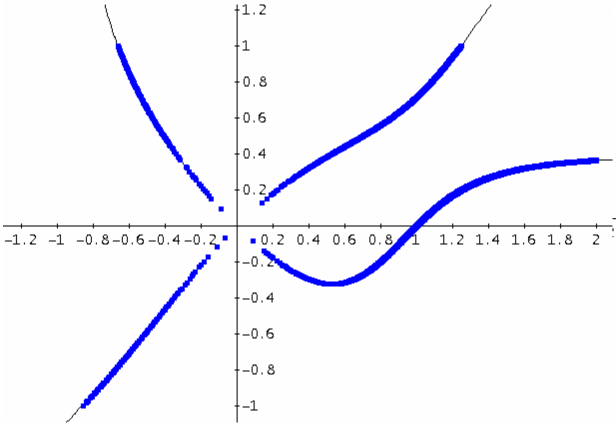
And I took DPGRaph to plot the left hand side surface together with the plane z = 1 for better illustration:

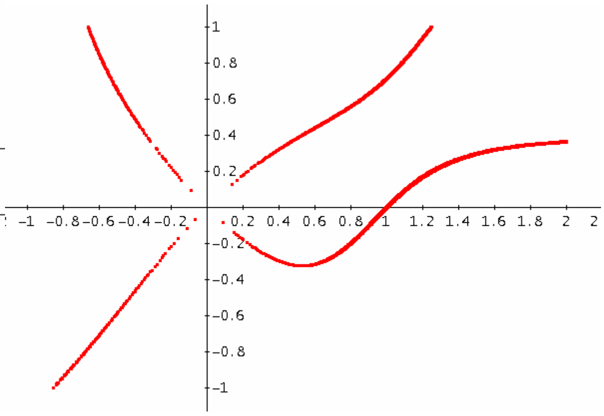




Autograph above and DPGraph at the right hand side…
And the DERIVE – Plots below!

$$\#19:\quad \mathrm{impl}\left(\frac{x^3\cdot e^{-x\cdot y}}{x^2-y^2}=1,\ -1,\ 2,\ -1,\ 1,\ 0.002,\ 0.005\right)$$

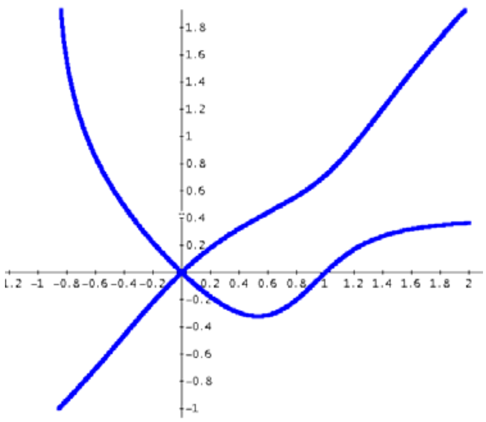


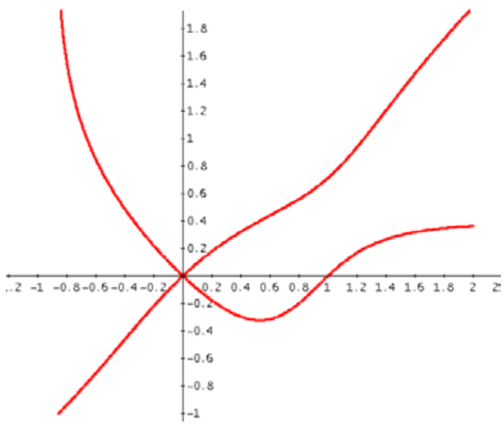


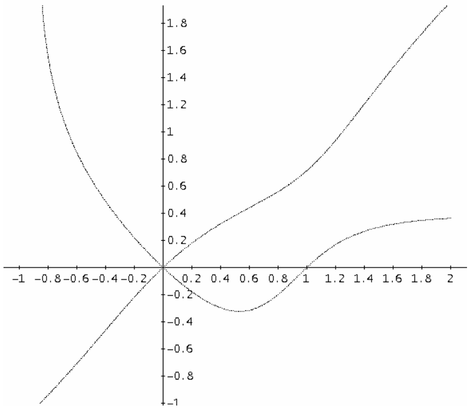
Point Size Large Point Size Medium

Another possibility is offered by adapting one or the other function of producing contour- or level plots (see the respective contribution in this DNL). If you don't mind calculation time then try the following and plot in different point sizes:

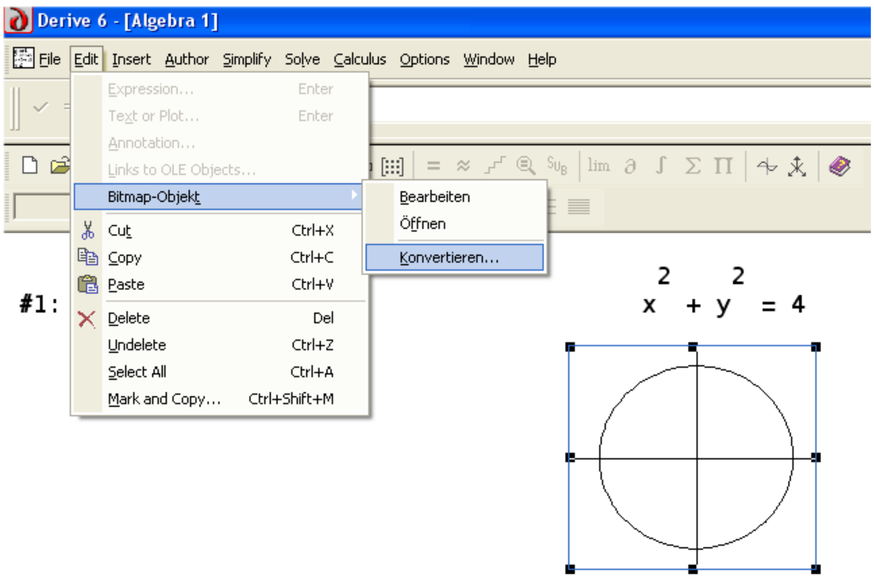
$$\mathrm{IMPPLOT}\left(\frac{x^3\cdot e^{-x\cdot y}}{x^2-y^2}=1,\ -1,\ 2,\ -1,\ 2,\ 0.1,\ 0.1,\ 0.1\right)$$

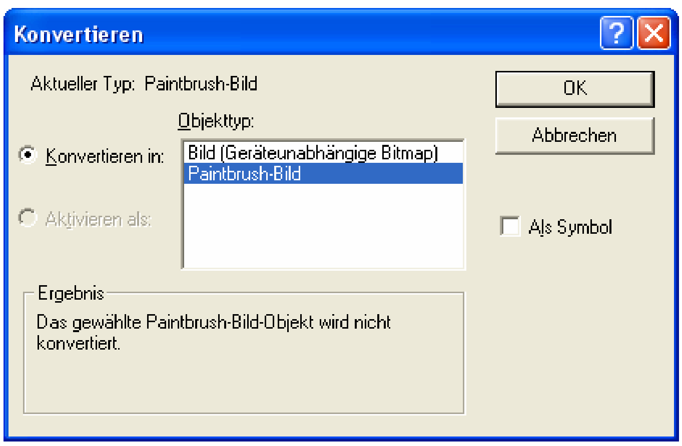




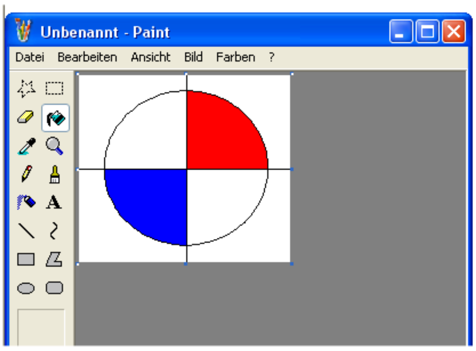


Now let´s turn to Painting in DERIVE. Tania told me that one of her students (female, 16 years) came across an Option in the Edit-Menu which was never mentioned in class: Edit > Bitmap-Object. (Have you ever noticed this?). Just for fun she wanted to know what would happen. She converted the graph (eg a circle) into a Paintbrush Picture. And then with a double click on this picture she found herself in the Windows Tool Painbrush.

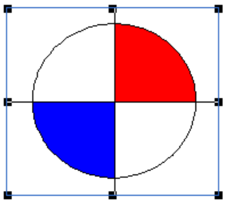




In the Graphics Program you can manipulate the graph – remaining in DERIVE – as you like and clicking in the DERIVE screen (eg on the expression) you see the coloured graph. Any double click on it leads back to Paintbrush for further treatment.



$$x^2 + y^2 = 4$$



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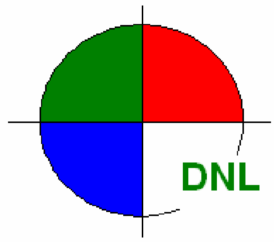
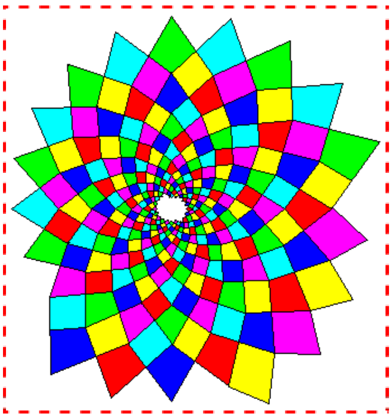


Tania´s students created a whole gallery of pictures using various functions. She sent a selection of pictures and I am very happy to present a part of this gallery in the DNL.



I found the graph below in a mathtematics book and it is a nice task for students to reproduce this pattern.

$$\text{spir} := \left[-0.5\cdot e^{0.5\cdot t}\cdot \text{SIN}(t),\ -0.5\cdot e^{0.5\cdot t}\cdot \text{COS}(t)\right]$$

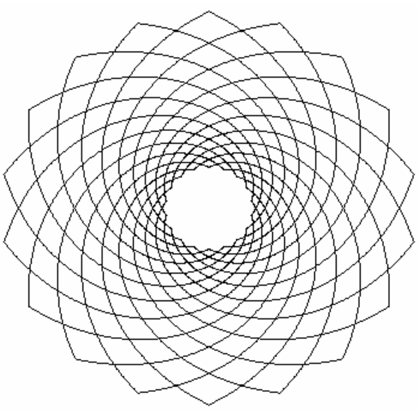
$$\text{spir2} := \left[\frac{e^{t/2}\cdot \text{SIN}(t)}{2},\ -\frac{e^{t/2}\cdot \text{COS}(t)}{2}\right]$$

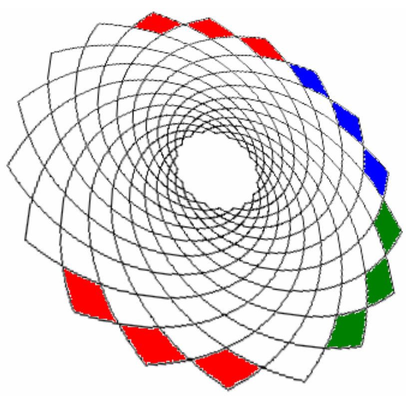
$$\text{rot}(s) := \begin{bmatrix} \text{COS}(s) & -\text{SIN}(s) \\ \text{SIN}(s) & \text{COS}(s) \end{bmatrix}$$

$$\left[\text{VECTOR}\left(\text{spir}\cdot \text{rot}\left(\frac{k\cdot \pi}{9}\right),\ k,\ 0,\ 17\right),\ \text{VECTOR}\left(\text{spir2}\cdot \text{rot}\left(\frac{k\cdot \pi}{9}\right),\ k,\ 0,\ 17\right)\right]$$

Plot with $0 \le t \le \pi$.

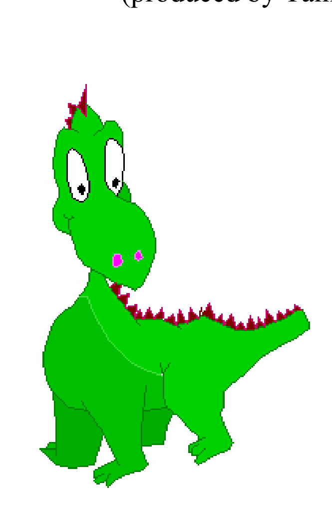
It is not so easy to find the mathematics behind. This is one way to produce a similar pattern by two families of curves. In Paintbrush the graph was distorted and then the tiles can be filled.

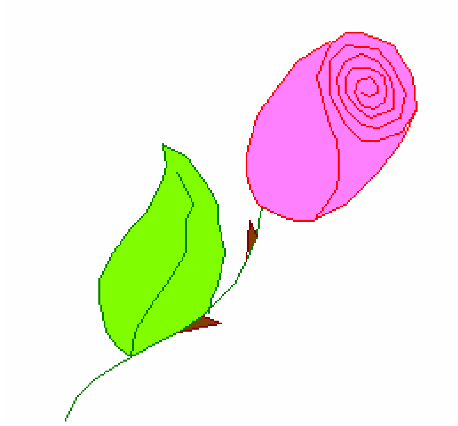


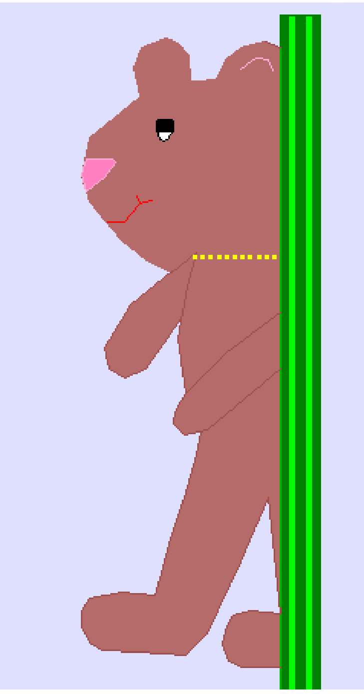


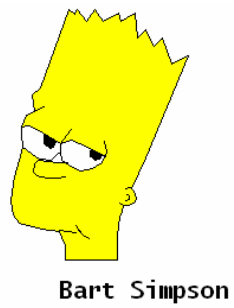
Gallery of DERIVE-Graphs treated as Paintbrush-Pictures

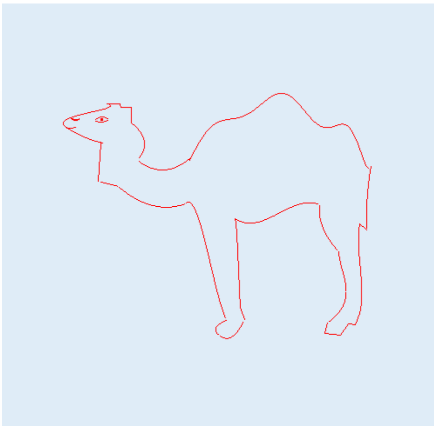
(produced by Tania~s students of the Handelsakademie St. Pölten, Lower Austria)











The dromedary (by artist Daniel Wailzer) is made up by a list of polynomials up to degree 10!

You can find a lot of other students´ products in Tania´s great presentation given at DES-TIME 2006 in Dresden on the Conference CD.

Unfortunately Tania did not unhide her secret how to bring colour on the screen of her Voyage 200!! Josef