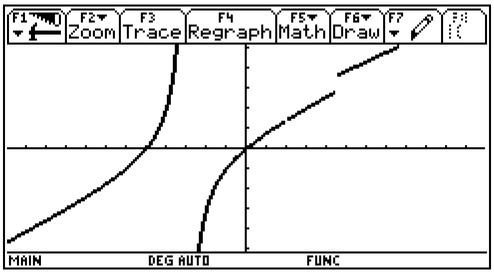
Continuity and Differentiability explored on the TI-89/92

(Josef Böhm)

The DATA-Matrix editor is an excellent tool to support concepts of accumulation points, limits, continuity, average and instantaneous rates of change.

I´ll demonstrate the possibilities using an "exotic" function.

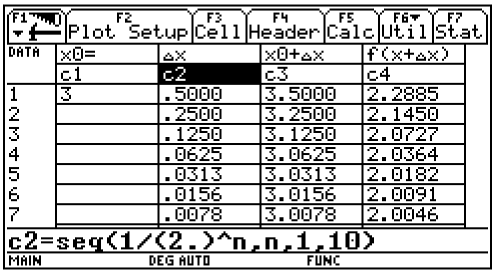
$$f(x) = \frac{x^2 - 4}{2x - 4} + \frac{3\,\mathrm{sign}\,(2x - 9)}{x + 3}$$



Which values for x are excluded from the domain?

To keep the exploration as general as possible I recommend to store the function as f(x) in the Home Screen.

1 Continuity Behaviour at $x_0 = 3$



DATA	x0=	∆x	x0+∆x	f(x+∆x)
	c1	c2	c3	c4
1	3	.5000	3.5000	2.2885
2		.2500	3.2500	2.1450
3		.1250	3.1250	2.0727
4		.0625	3.0625	2.0364
5		.0313	3.0313	2.0182
6		.0156	3.0156	2.0091
7		.0078	3.0078	2.0046

c2=seq(1/(2.)^n,n,1,10)

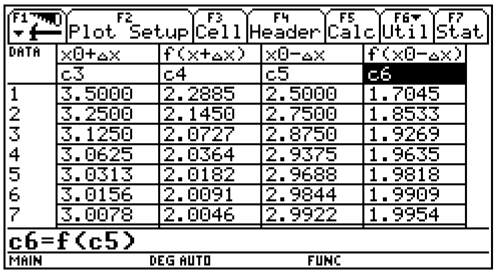
In cell c1 we enter the value fo x_0. In row c2 we define a 0-sequence to obtain in c3 a sequence of x-values with rightside limit 3. Subsequently in c4 we can see the sequence of function values which should tend to $f(x_0)$.

(Experiment changing the sequence in c2).

For c3 enter: c1[1]+c2
For c4 enter: f(c3)

In columns c5 and c6 we generate the left sided approach.

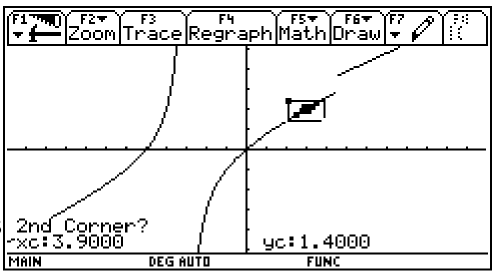
How should we change c2 to accelerate the convergence?

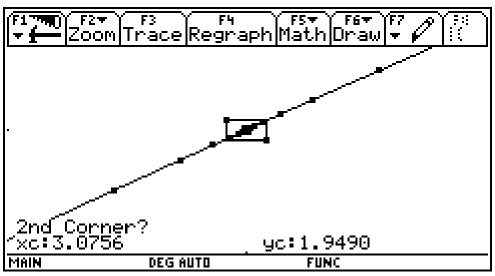


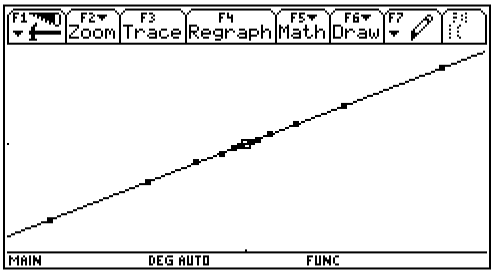
DATA	x0+∆x	f(x+∆x)	x0−∆x	f(x0−∆x)
	c3	c4	c5	c6
1	3.5000	2.2885	2.5000	1.7045
2	3.2500	2.1450	2.7500	1.8533
3	3.1250	2.0727	2.8750	1.9269
4	3.0625	2.0364	2.9375	1.9635
5	3.0313	2.0182	2.9688	1.9818
6	3.0156	2.0091	2.9844	1.9909
7	3.0078	2.0046	2.9922	1.9954

c6=f(c5)

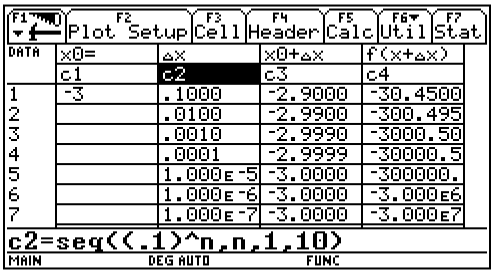
Using the plot-facilities we can visualize the convergence process:





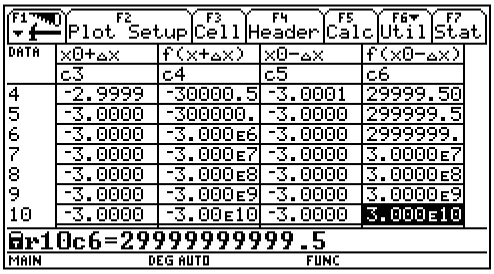


2 Continuity Behaviour at $x_0 = -3$



DATA	x0=	Δx	x0+Δx	f(x+Δx)
	c1	c2	c3	c4
1	-3	.1000	-2.9000	-30.4500
2		.0100	-2.9900	-300.495
3		.0010	-2.9990	-3000.50
4		.0001	-2.9999	-30000.5
5		1.000E-5	-3.0000	-300000.
6		1.000E-6	-3.0000	-3.000E6
7		1.000E-7	-3.0000	-3.000E7

c2=seq((.1)^n,n,1,10)

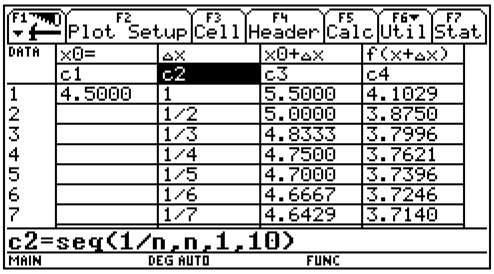


DATA	x0+Δx	f(x+Δx)	x0-Δx	f(x0-Δx)
	c3	c4	c5	c6
4	-2.9999	-30000.5	-3.0001	29999.50
5	-3.0000	-300000.	-3.0000	299999.5
6	-3.0000	-3.000E6	-3.0000	2999999.
7	-3.0000	-3.000E7	-3.0000	3.0000E7
8	-3.0000	-3.000E8	-3.0000	3.0000E8
9	-3.0000	-3.000E9	-3.0000	3.0000E9
10	-3.0000	-3.00E10	-3.0000	3.000E10

r10c6=29999999999.5

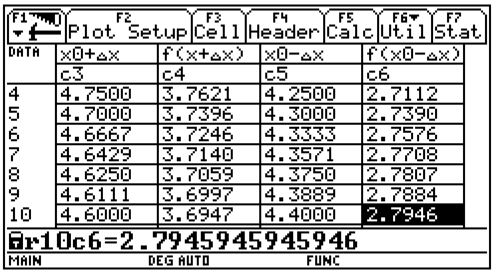
The results in columns c4 and c6 are very informative!!

3 Continuity Behaviour at $x_0 = 4.5$



DATA	x0=	Δx	x0+Δx	f(x+Δx)
	c1	c2	c3	c4
1	4.5000	1	5.5000	4.1029
2		1/2	5.0000	3.8750
3		1/3	4.8333	3.7996
4		1/4	4.7500	3.7621
5		1/5	4.7000	3.7396
6		1/6	4.6667	3.7246
7		1/7	4.6429	3.7140

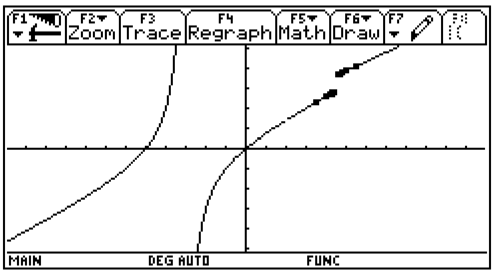
c2=seq(1/n,n,1,10)

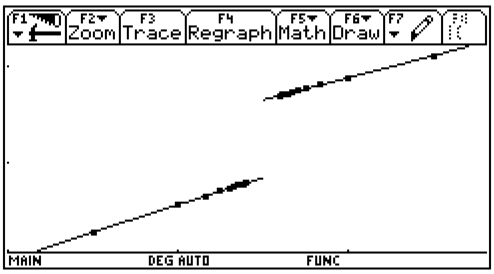


DATA	x0+Δx	f(x+Δx)	x0-Δx	f(x0-Δx)
	c3	c4	c5	c6
4	4.7500	3.7621	4.2500	2.7112
5	4.7000	3.7396	4.3000	2.7390
6	4.6667	3.7246	4.3333	2.7576
7	4.6429	3.7140	4.3571	2.7708
8	4.6250	3.7059	4.3750	2.7807
9	4.6111	3.6997	4.3889	2.7884
10	4.6000	3.6947	4.4000	2.7946

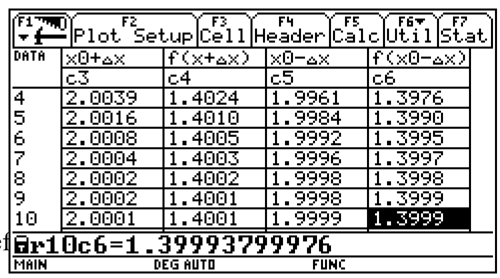
r10c6=2.7945945945946

Discuss the outcomes in columns c4 and c6. Change the sequence in c2. Discuss possible consequences. Produce a graphic representation.



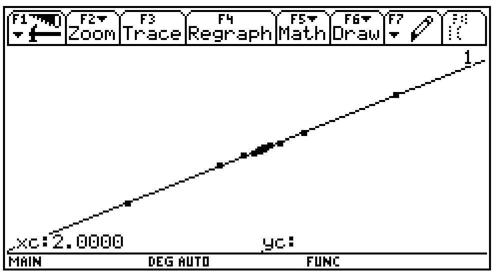


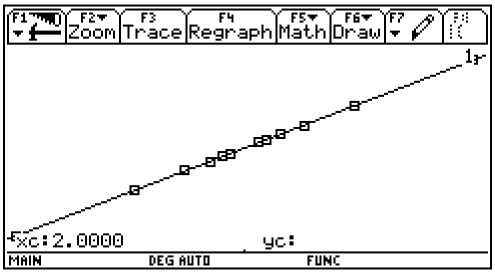
4 The last interesting position is $x_0 = 2$



DATA	x0+Δx	f(x+Δx)	x0-Δx	f(x0-Δx)
	c3	c4	c5	c6
4	2.0039	1.4024	1.9961	1.3976
5	2.0016	1.4010	1.9984	1.3990
6	2.0008	1.4005	1.9992	1.3995
7	2.0004	1.4003	1.9996	1.3997
8	2.0002	1.4002	1.9998	1.3998
9	2.0002	1.4001	1.9998	1.3999
10	2.0001	1.4001	1.9999	1.3999

r10c6=1.39993799976





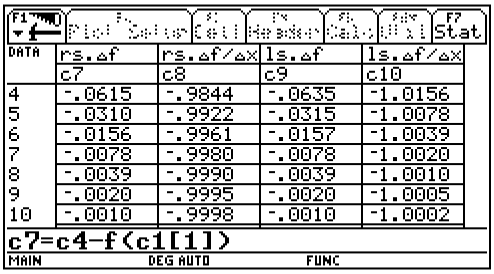
Discuss **all** the messages given on the screen.

Now it is easy to extend this table for further use in calculus teaching. We only have to add some rows for the absolute changes and then for the rates of change leading to the average rate of change and to its limit.

Let´s have another "artificial" function. Do you know the "Marilyn Monroe Curve"?

$$f(x) = \left(\frac{|x|}{2} - 2\right)^2$$

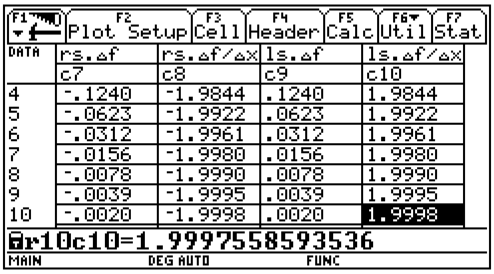
$x_0 = 2$:



DATA	rs.Δf	rs.Δf/Δx	ls.Δf	ls.Δf/Δx
	c7	c8	c9	c10
4	-.0615	-.9844	-.0635	-1.0156
5	-.0310	-.9922	-.0315	-1.0078
6	-.0156	-.9961	-.0157	-1.0039
7	-.0078	-.9980	-.0078	-1.0020
8	-.0039	-.9990	-.0039	-1.0010
9	-.0020	-.9995	-.0020	-1.0005
10	-.0010	-.9998	-.0010	-1.0002

c7=c4-f(c1[1])

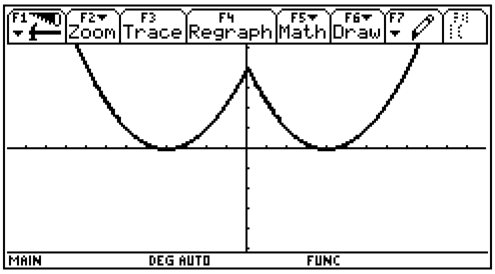
$x_0 = 0$:



DATA	rs.Δf	rs.Δf/Δx	ls.Δf	ls.Δf/Δx
	c7	c8	c9	c10
4	-.1240	-1.9844	.1240	1.9844
5	-.0623	-1.9922	.0623	1.9922
6	-.0312	-1.9961	.0312	1.9961
7	-.0156	-1.9980	.0156	1.9980
8	-.0078	-1.9990	.0078	1.9990
9	-.0039	-1.9995	.0039	1.9995
10	-.0020	-1.9998	.0020	1.9998

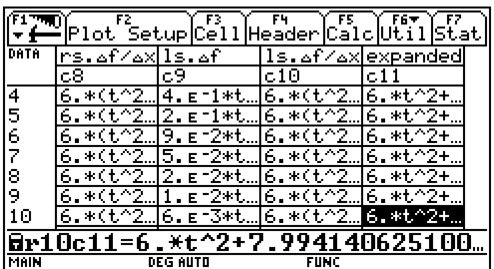
r10c10=1.9997558593536

Do see the difference in the behaviour of the rate of changes progress?
Use various sequences for approaching $x = 0$. Is the function continuous for $x = 0$?



Further explorings:

Try another function, eg $f(x) = 3x\hat{}2 - 4x + 5$. Perform the same investigation like above, but finally enter *x0* for the location:

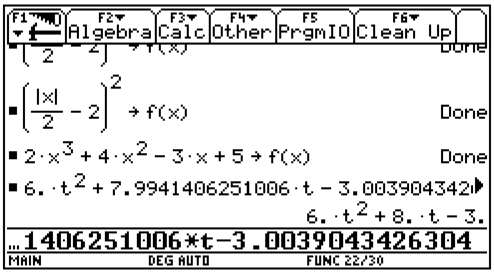


DATA	rs.Δf/Δx	ls.Δf	ls.Δf/Δx	expanded
	c8	c9	c10	c11
4	6.*(t^2…	4.E-1*t…	6.*(t^2…	6.*t^2+…
5	6.*(t^2…	2.E-1*t…	6.*(t^2…	6.*t^2+…
6	6.*(t^2…	9.E-2*t…	6.*(t^2…	6.*t^2+…
7	6.*(t^2…	5.E-2*t…	6.*(t^2…	6.*t^2+…
8	6.*(t^2…	2.E-2*t…	6.*(t^2…	6.*t^2+…
9	6.*(t^2…	1.E-2*t…	6.*(t^2…	6.*t^2+…
10	6.*(t^2…	6.E-3*t…	6.*(t^2…	6.*t^2+…

r10c11=6.*t^2+7.994140625100…

Attach one more column to expand the results from c10 or c8.

Copy and paste content of cell r10c11 into the Home Screen and try to make conclusions for a general rule to find a formula for the instantaneous rate of change.



Overall I´d recommend to start the investigations presented above in connection with a problem from applied mathematics. It is my experience that students then like to follow very "inner mathematical" reasoning.

I want to give full credit to David Bowers who gave a marvellous workshop in San Francisco and in Liverpool as well showing so many possibilities how to use the TI´s DATA-table in a very meaningful way.

I also want to give credit to attendants of the 1st T^3 Winter Academy in Austria (1 - 6 January 2001), who gave the idea of "absolute adressing" a cell in the DATA-table. (K.H.-Keunecke, D.Kirmse, M.Grote).

I´d like to dedicate this paper to Detlev Kirmse who was attacked by an apoplexy of the brain after coming home. All attendants wish Detlev the best for his recovering and hope to discuss and to go skiing with him again next winter.