THE DERIVE - NEWSLETTER #4

THE BULLETIN OF THE

DERIVE

USER GROUP

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revised reprint 2004 **December 1991**

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Mitgliedsbeitrag für 1992

Die DUG-Mitglieder aus Österreich werden gebeten, den Mitgliedsbeitrag von S 300.- für das Jahr 1992 mit beiliegendem Zahlschein zu begleichen!

Die Mitglieder von außerhalb Österreichs bitten wir ihren Beitrag für 1992 über DM 45.- nach Möglichkeit in Form eines Euroschecks an den Herausgeber zu schicken. Alle anderen Überweisungsformen verursachen zum Teil recht hohe (bis zu DM 20.-) Bankspesen.

Herzlichen Dank

Membership Dues for 1992

The Austrian DUG Members are asked for settling the memership dues for 1992 using the enclosed paying-in form!

We invite the members from outside Austria to pay their dues of GM 45.- for 1992 using an euro-cheque as far as possible and sending it to the editor. Each other kind of payment causes considerable banking charges (sometimes GM 20.-).

Thank you

Lieber Derive Anwender!

Mit der 4. Ausgabe des D-N-L beendet die DUG das erste Jahr ihres Bestehens. Die weiterhin steigende Mitgliederzahl - derzeit 205 - ist ebenso erfreulich wie die Tatsache, dass uns immer wieder interessante Artikel erreichen. Trotzdem bitten wir um weitere Beiträge aus allen Gebieten der Lehre und Anwendung. Ich möchte besonders die technischen Anwender auffordern, uns über ihre Einsatzmöglichkeiten von *DERIVE* zu berichten. Schreiben Sie auch, wenn Sie Ideen haben, die Sie gerne behandelt wissen oder zur Diskussion gestellt haben wollen.

Auf Seite 37 finden Sie die Ausschreibung für eine *DERIVE*-Frühjahrs-Schule in Krems, einer netten Stadt in Niederösterreich. Wie Sie vielleicht wissen, hat das dafür zuständige Ministerium für einen Großteil der Höheren Schulen eine Generallizenz für *DERIVE* erworben. Die damit befassten Herren und da ganz besonders LSI Dr. Heugl und MR Dr. Szirucsek erkennen die Herausforderung, die die Computeralgebra für die Schulmathematik darstellt und wollen ihr offensiv begegnen. Man darf entscheidende Impulse von dieser Tagung erwarten.

Wie schon im letzten D-N-L angekündigt, überschreitet die DUG mit dem Jahr 1992 offiziell die Grenzen Europas und lädt auch die US-*DERIVE*-User zum Beitritt ein. Wir können uns schon jetzt auf den Ozean überspannende Kontakte freuen.

Zum Schluß will ich Ihnen noch ein fröhliches Weihnachtsfest sowie ein glückliches Jahr 1992 in Gesundheit und Frieden wünschen. Nehmen Sie die beigelegte Diskette mit allen .MTH–Dateien aus D-N-L#1 bis D-N-L#4 als kleine Aufmerksamkeit für Ihr Interesse an der DUG, als Dank für Ihre Mitarbeit und zugleich als Bitte, auch weiterhin am Ausbau der DUG mitzuwirken.

Mit den besten Grüßen

Dear Derive User,

the DUG ends its first year of existence with the first issue of its bulletin, the D-N-L. The steadily increasing number of members is as pleasing -–205 till now – as the fact that interesting contributions are reaching the editor's desk again and again. Nevertheless we ask for further papers from all fields of teaching and application mathematics. I'd like to invite especially the technical users to inform us about their use of DERIVE. Please also write if you have any ideas which you want to be dealt with or you want to have discussed.

On page 37 you will find the announcement for a *DERIVE* Spring School in Krems, a nice town in Lower Aus-tria. Perhaps you know, that the Ministry of Education has bought the general DERIVE licence for a great part of Austria's Secondary Schools. The people being occupied with this task and especially LSI Dr. Heugl and MR Dr. Szirucsek recognize the big challenge which Computer Algebra performs for maths in school and they will face this challenge bold-ly. We may expect a lot of important impulses from that meeting.

As announced in the last D-N-L, the DUG will go beyond Europe's borders in 1992 and will invite the US *DERIVE* Users for membership. We are looking forward to having contacts reaching over the ocean in the future.

At last I wish you a Merry Christmas and a Happy New Year 1992 in health and peace. Take the disk enclosed containing all the .MTH-files from D-N-L#1 to D-N-L#4 as a little present for your interest and cooperation and at the same time as an invitation to participate in the further development of the DUG

With my best regards

The *Derive-News-Letter* is the Bulletin of the *Derive-User-Group*. It is published at least three times a year with a contents of 30 pages minimum.

The goals of the D-N-L are to enable the exchange of experiences made with Derive as well as to create a group to discuss the possibilities of new methodical and didactic manners in teaching Mathematics.

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Contributions:

Please send all contributions to the above address. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of *D-N-L*. It must be said, though, that non-English articles will be warmly welcomed none-the-less. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in *D-N-L*. The more contributions you will send to the Editor, the more lively and richer in contents the *Derive-News-Letter* will be.

Preview: **Contributions for the next issues:**

Finding a gradient;

Zur Problematik des Einsatzes von DERIVE;

Cyclomania (Hypotrochoids & Epitrochoids);

Two Underground Tunnels (applied Geometry);

Module on sequences and series with Derive (Part 2)

(will be published March 92)

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DERIVE – User - Forum

Waclaw Sadowski, Bukowiec

Dear Sir,
My name is Waclaw Sadowski and i am thirty.three years of age. I live in a small village called Male Konskie in the central region of Poland. I am a higher school graduate and I am a lecturer of mathematics in Secondary School.
I am very much interested in getting know more on the Derive User Group and its bulletin and that's why I've made up my mind to write to you. Recently I have been successful in entering into contacts with some scientists who involve in research on new developments in mathematics and mathematics teaching.
The secondary schools in Poland are secondary schools which provide comprehensive secondary education. The graduates of this school, after having finished the last (fourth) grade, obtain certificates of secondary education and after having passed the final examination (so-called 'maturity') are entitled to apply for admission to the institutions of higher education. The four-year secondary general schools realize plans and curricula of teaching approved by the Ministry of National Education.
We Polish scientists need help from specialists from western countries to help Poland in its new form to integrate into the western part of Europe. I shall much appreciate if you could kindly take action to provide me with information on Derive and one free of charge your bulletin. I look forward to hearing from you before long.

Yours sincerely

Matija Lokar, Kranj

Dear Editor!
Few days ago I got the Derive news letter#1. I found it very interesting and I would like to join the Derive user group. I would do that immediately as I manage the way to pay Annual Membership Dues, as it is now almost impossible to do, because of the situation in our country.
[..some comments to the equation in D-N-L#1, which leads to x7 - 1 = 0]
Although I mostly work with Mathematica, I am very interested in Derive and found it very good. In the last half a year, me and a couple of friends tried to put Derive on the list of software, specially convenient for use in secondary schools. We hope we manage that and to the end of year it is expected that all of 135 secondary schools will receive a copy of Derive for inspection. We also plan to establish special courses about Derive. Therefor I ask you for possible help. Any suggestions, experience with training for use of Derive would be appeciated.

With best regards

D-N-L: We are very glad that the DUG might reach the young democratic states in Eastern Europe. If there are any DUG members who want to establish contact with Mr. Sadowski or Mr./Mrs. Lokar, then please write or call for their addresses.

Herbert Appel, Schweinfurt

[...] Zusätzlich erhalten Sie von mir mit der Bitte um Veröffentlichung einen weiteren Artikel zur Problematik des Einsatzes von DERIVE im Mathematikunterricht. Der Artikel kann, falls notwendig, sinnvoll gekürzt werden; ein vollständiger Abdruck (vielleicht schon im DNL#4) ist mir jedoch am liebsten.

Verbesserungsvorschläge zum DNL:

- könnte dicker sein. 180 Mitglieder müßten eigentlich eine ganze Menge Artikel produzieren. Ich wäre unter Umständen auch bereit, einen höheren Mitgliedsbeitrag zu bezahlen.

- Bitte an die Mitglieder, bei Einverständnis der anderen Mitglieder, Namen und Adressen weitergeben, um miteinander in Kontakt treten zu können.

- Ich benutze Derive auf meinem Atari ST mit ATonce 16Mhz. Derive spricht hier im Gegensatz zu "Cabri-Geometrie", die VGA-Graphikkarte nicht an. Außerdem kann ich trotz GRAPHICS aus DOS 4.0 mit PRTSCR keine Hardcopy auf meinem NEC P6 machen.

- Bitte nicht mit Klammern beim *DNL* sparen. *DNL*#2 war noch mit 3 Klammern am Auseinanderfallen gehindert. (DNL#3 nur noch eine Klammer).

With all - autre chauses - sono contento - weiter so – do swidanje
Im Rahmen der Völkerverständigung.

D-N-L: What is the opinion of the DUG-members? In the next issue we will publish the complete list of the DUG, exept those who have informed me not to be mentioned by name.

I hope that the 202 members of the DUG are as productive as Mr. Appel thinks them to be.

The bad finishing of *DNL*#3 was caused by lack of time. Sorry, hope to do better now!

Mr. Appel has sent an article concerning the problems of using *DERIVE* when teaching mathematics. It will be published in *DNL*#5.

Robert Setif, Annemasse

1. Pourquoi le chronometrage de certains calculs dans un environnement pratiquement identique donne-t-il parfois des résultats trés differents? Voir la fonction mod.
2. La fonction mod de DNL# 2, p 15 m'a surpris. Elle ne fonctionne pas avec par exemple

   `vector(mod(k,2),k,0,24)` Simplify or Approx
3. Voudriez-vous des fonctions aussi simples que `MOD_IT (k,n):=iterate(a-n,a,k,k/n)` ou des fonctions ADD MUL DIV recursives trés peu performantes, mais interessantes pour apprentis-programmeurs?
4. Si cela peut etre utile a quelqu'un, je peux poursuivre le calcul (a la main!) des fractions continues au -delá de √700, mais je préférerais qu'on me trouve une fonction de Derive (ou de MuMATH) qui trouve automatiquement les résultats.
5. Pourriez-vous approfondir le sujet des erreurs pour les fractions continues successives? C'est ma partie faible !
6. Auriez-vous d'autres sujets a me proposer qui ne demandent pas un trop grand quotient intellectuel, car je commence a ètre .. usé..(et je travaille tout en gardant mon petit fils de 8 mois!)?

7. Je ne suis pas equipé pour le bulletin electronique for Derive users. Pourriez-vous publier dans DNL les réponses les plus interessantes ?

8. Y a-t-il une nouvelle version de Derive en chantier ? Avec quelles ameliorations ?

9. Etes-vous utilisateur (ou l avez vous été) de Mumath ? Des fichiers Mumath vous interessent-ils ?
   A mon avis, Mumath est plus interessant que Derive pour les calculs sur N et Q.

Souhaits pour nouvelles versions de Derive

1. **Option notation:** Affichagé, par tranches de 3 chiffres.

   Par exemple 2^{10} = 1 048 576
   $\sqrt{2}$ = 1.414 213 562...

2. **Option notation:** affichagé dans tout l'. écran.

3. **Option mute:**Signal sonoré a la fin d'un calcul, car c'est penible d'attendre (meme si pendant ce temps je peus calculer des fractions continues!) des résultats qui viennent après plus de 1 heure ou même plusieurs minutes.

4. **Option break:** (comme dans MuMATH) qui permette d'interroger certaines variables, de comprendre des erreurs, de connaitre la situation quand on est "dans le noir".

5. Plus de possibilités de programmation , bien qu'il y ait d'ja un progres considerable depuis Derive 1., et ceci avec très peu de mots nouveaux (Vector, Iterate, if).

6. **Passage** Mumath <--> Derive comme il y a Fortran <--> Derive, au moins pour les résultats numeriques et les polynômes.

7. Introduction de **LISTES** : Plus souples que les vecteurs, avec pop et push et liste de listes qui n'auraient pas forcement la même dimension.

8. Possibilité d'une nouvelle sorte de fonction qui afficherait les résultats a mesuré a chaque appui sur une touché speciale (par exemple "space" ASCII 32) au lieu d'attendre que par exemple le vecteur soit tout calculé avec les risques de "memory full".

Avec mes meilleurs sentiments

Bon vacances

Merci pour les améliorisations de mon fichier SYRACUSE.

Here you can find some improvements to the SYRACUS.MTH file from D-N-L#3.

In 1991 it was necessary to produce one´s own mod-function `mod(x,y)`, *(see DNL#3).*

```
       sy(n) :=
         If MOD(n, 2)
#1:         n/2
            3·n + 1

#2:    auxsyr(x0) := ITERATES(IF(x > 1, sy(x), x), x, x0)

#3:    auxsyr(3)

#4:    [3, 10, 5, 16, 8, 4, 2, 1, 1]

#5:    lsyr(n) := DIM(auxsyr(n)) - 1

#6:    lsyr(3)
```

Length of the sequence

```
#7:    8

#8:    syr(x0) := VECTOR((auxsyr(x0))_i, i, lsyr(x0))

#9:    syr(3)
```

The complete sequence

```
#10:   [3, 10, 5, 16, 8, 4, 2, 1]
```

A very long sequence with an ordinary start:

```
#11:  syr(27) = [27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121,
        364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790,
        395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283,
        850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288,
        3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732,
        866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160,
        80, 40, 20, 10, 5, 16, 8, 4, 2, 1]
```

```
#12:  maxsy(n) := ITERATES([1 + v_1, sy(v_2), MAX(v_2, v_3)], v, [1, n, 1], lsyr(n) - 1)
```

#13: maxsy(3) =

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 10 & 3 \\ 3 & 5 & 10 \\ 4 & 16 & 10 \\ 5 & 8 & 16 \\ 6 & 4 & 16 \\ 7 & 2 & 16 \\ 8 & 1 & 16 \end{bmatrix}$$

#14: $\text{max\_el}(n) := (\text{maxsy}(n))_{\text{lsyr}(n),3}$

#15: max_el(3) = 16

Today it is easier than in 1991!

#16: MAX(syr(3)) = 16

#17: maxel(27) = 9232

9232 is the maximum element of the sequence starting with 27

#18: syracus(u, v) := TABLE([lsyr(x), max_el(x)], x, u, v)

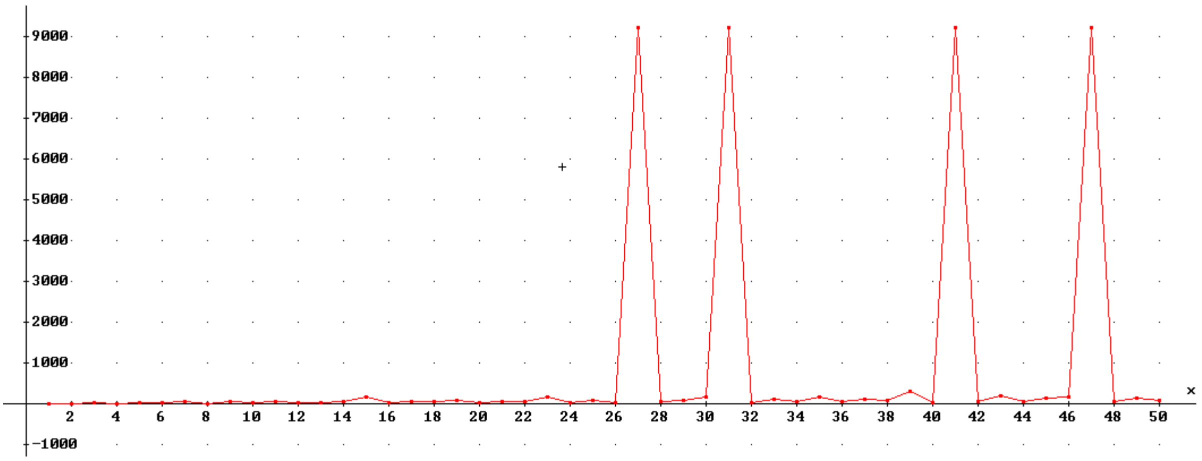
#19: syracus(1, 50)

#20:

...	...	...
24	11	24
25	24	88
26	11	40
27	112	9232
...	...	...

#21: syrgr(u, v) := TABLE(max_el(x), x, u, v)

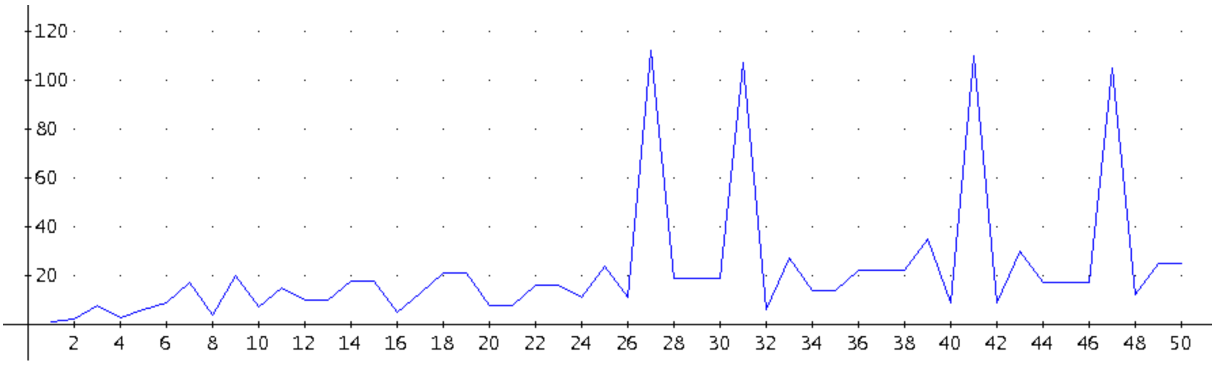
#22: syrgr(1, 50)



Plot of expression #22

LSYRGR(u, v) := VECTOR([x, LSYR(x)], x, u, v)

LSYRGR(1, 50)



This time monsieur Setif opens his treasure box to show us some spirals and polygons. Until expression #5 work with rectangular coordinates, then switch to polar coordinates. (More graphs on p 38)

#1: $\text{SPIRR}(r, k, a, a\_, n) := \text{ITERATES}\left(k \cdot v \cdot \begin{bmatrix} \text{COS}(a\_) & \text{SIN}(a\_) \\ -\text{SIN}(a\_) & \text{COS}(a\_) \end{bmatrix}, v, r \cdot [\text{COS}(a), \text{SIN}(a)], n\right)$

#2: $\text{SPIRR}\left(3, 0.95, 0, \frac{\pi}{3}, 12\right)$

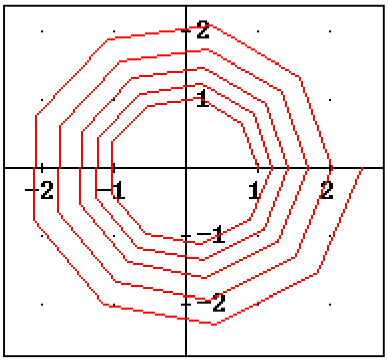
#3: $\text{SPIRR}\left(3, 0.95, 0, \frac{\pi}{3}, 24\right)$

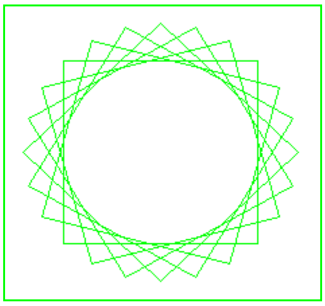
#4: $\text{SPIRR}\left(1, 1.02, 0, \frac{\pi}{5}, 50\right)$

#5: $\text{SPIRR}\left(1, 1.05, 0, \frac{2 \cdot \pi}{3}, 24\right)$

#6: $\text{SPIRR}\left(1, 1.02, 0, \frac{2 \cdot \pi}{5}, 30\right)$

#7: $\text{SPIRR}\left(1, 1.02, 0, \frac{2 \cdot \pi}{9}, 45\right)$





#8: $\text{SQUARE}(r, a) := \text{ITERATES}\left(v + \left[0, \frac{\pi}{2}\right], v, [r, a], 4\right)$

#9: $\text{SQUARE}\left(1, \frac{\pi}{4}\right)$

#10: $\text{TURNSQ}(r, a, t, n) := \text{VECTOR}(\text{SQUARE}(r, a + t \cdot k), k, n)$

#11: $\text{TURNSQ}\left(1.41, \frac{\pi}{4}, \frac{\pi}{12}, 6\right)$

#12: $\text{POLYGON}(r, a, n) := \text{ITERATES}\left(v + \left[0, \frac{2 \cdot \pi}{n}\right], v, [r, a], n\right)$

#13: $\text{POLYGON}(1, 0, 5)$

#14: $\text{POLYGON}(2, 0, 3)$

#15: $\text{STAR5}(r) := \text{ITERATES}\left(v + \left[0, \frac{4 \cdot \pi}{5}\right], v, [r, 0], 5\right)$

Try to find the N-Star!

#16: $\text{STAR5}(2)$

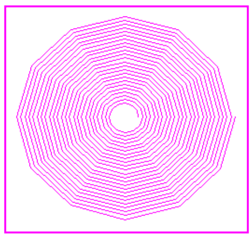
#17: $\text{SPIR}(x0, y0, x\_, y\_, n) := \text{ITERATES}(v + [x\_, y\_], v, [x0, y0], n)$

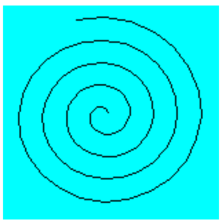
#18: $\text{SPIR}(1, 0, 0.05, 90 \cdot °, 32)$

#19: $\text{SPIR}\left(3, 0, -0.05, \frac{\pi}{2}, 60\right)$

#20: $\text{SPIR}(3, 0, -0.1, 120 \cdot °, 30)$

#21: $\text{SPIR}(0.2, 0, 0.005, 30 \cdot °, 288)$





#26:
```
SEGPOLREC(r, r_, a, a_, n) :=
  If n = 0
     [[r, a]]
     APPEND([[r + n·r_, a + n·a_]], SEGPOLREC(r, r_, a, a_, n - 1))
```

#27: $\text{SEGPOLREC}\left(1, 0.1, 0, \frac{\pi}{8}, 15\right)$

#28: $\text{SEGPOLREC}\left(0.5, 0.1, \frac{\pi}{4}, \frac{\pi}{12}, 11\right)$

#29: $\text{VECTOR}(\text{POLYGON}(a, 0, 5), a, 1, 2.4, 0.2)$

#30: $\text{VECTOR}(\text{STAR5}(a), a, 1.5, 2.5, 0.5)$

With ITERATES to CHAOS

(Part 2)

Dr. Felix Schumm, Stuttgart

3. Gekoppelte Iteration von zwei Variablen x_1 und x_2

3.1 Allgemeine Betrachtungen

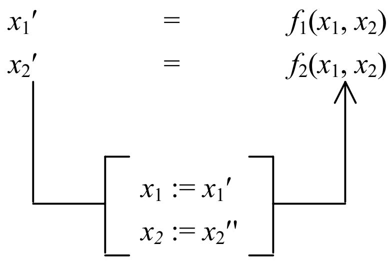
Wir wollen einen Urbildpunkt P(x_1, x_2) abbilden auf einen Punkt P'(x_1', x_2'). Die dazu nötigen Abbildungsvorschriften seien durch zwei Funktionen f_1 und f_2 in folgender Weise gegeben:

$$x_1' = f_1(x_1, x_2)$$
$$x_2' = f_2(x_1, x_2)$$

Wiederholen wir die gleiche Abbildungsvorschrift auf P″, so ergibt sich:

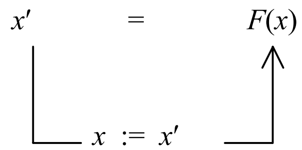
$$x_1'' = f_1(f_1(x_1, x_2), f_2(x_1, x_2))$$
$$x_2'' = f_2(f_1(x_1, x_2), f_2(x_1, x_2))$$

Diesen Prozess können wir nun *n*-mal wiederholen:



n mal wiederholen.

Knapper lässt sich das Verfahren in Vektorform darstellen:



n mal wiederholen.

worin x ein Vektor mit zwei Komponenten [x_1, x_2] und ebenso F ein Vektor mit zwei Komponenten [f_1, f_2] sein muss.

3.2 Umsetzung in DERIVE

Im Befehl `ITERATES(F,x,x0,n)` muss nun `x0` als Startvektor, zB `[0,1]` eingegeben werden und `F` muss einen Vektor der Form [$f1$, $f2$] mit den skalaren Funktionen $f1$ und $f2$ darstellen.

Das Ergebnis sollte dann in Form einer Matrix ausgegeben werden, die je nach Iterationszahl folgende Gestalt hat:

$$[[x_1, x_2],[x_1', x_2'],[x_1'', x_2''],......], \text{ bzw } \begin{bmatrix} x_1 & x_2 \\ x_1' & x_2' \\ x_1'' & x_2'' \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

Die Wirkung des `ITERATES`-Befehles machen wir uns am besten an immer komplizierteren Beispielen klar:

1. Beispiel:

```
[x1 :=, x2 :=]
ITERATES(f, x, [x1, x2], 3) = [[x1, x2], f, f, f]
```

2. Beispiel: ITERATES(3·x, x, [x1, x2], 3) = $\begin{bmatrix} x1 & x2 \\ 3\cdot x1 & 3\cdot x2 \\ 9\cdot x1 & 9\cdot x2 \\ 27\cdot x1 & 27\cdot x2 \end{bmatrix}$ weil $f = 3x$ ein Vektor ist.

3. Beispiel: Setzt man für $f = a \cdot x + c$ ein,worin a eine Matrix $[a_{ij}]$ und c ein Translationsvektor ist, so kann man bequem eine affine Abbildung iterieren. Wir geben zB ein:

```
[c1 :=, c2 :=, a11 :=, a12 :=, a21 :=, a22 :=]
c := [c1, c2]
```

a := $\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$

```
ITERATES(a·x + c, x, [x1, x2], 3)
```

Nach `SIMPLIFY` ergibt dies:

$$\begin{bmatrix} x1 & x2 \\ a11\cdot x1 + a12\cdot x2 + c1 & a21\cdot x1 + a22\cdot x2 + c2 \\ ... & ... \\ ... & ... \end{bmatrix}$$

Wählt man spezielle Zahlenwerte für a, c und für $[x_1, x_2]$, dann sieht das so aus:

$\left[a := \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, c := [0, 0]\right]$

ITERATES(a·x + c, x, [5, 5], 3) = $\begin{bmatrix} 5 & 5 \\ 5 & 10 \\ 5 & 20 \\ 5 & 40 \end{bmatrix}$

Viele Abbildungen sind aber nicht durch affine Transformationen ausdrückbar. Für diesen Fall muss man ein komplizierteres Verfahren einschlagen. Zunächst definieren wir allgemein die Funktionen, die unsere Abbildung liefern sollen.

```
[F1(x1, x2) :=, F2(x1, x2) :=]
```

Dann definieren wir zwei Hilfsfunktionen `G1(x)` und `G2(x)`, die in die Funktionen `F1` und `F2` die Komponenten x_1 und x_2 des Vektors $x = [x_1, x_2]$ einlesen.

$$G1(x) := F1(x_1, x_2)$$

$$G2(x) := F2(x_1, x_2)$$

Der `ITERATES`-Befehl lautet mit diesen Hilfsfunktionen

```
ITERATES([G1(x), G2(x)], x, [x1, x2], 2).
```

und liefert erwartungsgemäß

$$\begin{bmatrix} x1 & x2 \\ F1(x1, x2) & F2(x1, x2) \\ F1(F1(x1, x2), F2(x1, x2)) & F2(F1(x1, x2), F2(x1, x2)) \end{bmatrix}$$

Die Hilfsfunktionen können wir auch gleich in den ITERATES-Befehl mit aufnehmen:

$$\text{ITERATES}\left(\left[F1(x_1, x_2), F2(x_1, x_2)\right], x, \left[x_1, x_2\right], n\right)$$

Es ist gegebenenfalls zweckmäßig, sich dies nun in einem Hilfsfile "ITERATIO.MTH" abzuspeichern, das etwa folgendermaßen aussehen könnte:
(In einer MTH-Datei können keine Textboxes gespeichert werden, Kommentare wie in #1, #2 usw. müssen unter " geschrieben werden.)

It makes sense to save this in a utility file ITERATIO.MTH, which could look like as follows:
(In a MTH-file textboxes cannot be saved, so include comments (#1, #2, etc) under quotes ".)

```
#1:   file ITERATIO.MTH
#2:   The follwing system will be iterated:
#3:   x1_new=F1(x1_old,x2_old)
#4:   x2_new=F2(x1_old,x2_old)
#5:   Define functions F1(x1,x2) and F2(x1,x2)
#6:   Call result_matrix([3,4],5), with [3,4] is the initial vector x
#7:   and you want to get the first 5 iterations
#8:   ============================================================
#9:   [F1(x1, x2) :=, F2(x1, x2) :=]
#10:  result_matrix(x0, n) := ITERATES([F1(x₁, x₂), F2(x₁, x₂)], x, x0, n)
#11:  ============================================================
#12:  1. Beispiel / 1. Example
#13:  [F1(x1, x2) := x1 + x2, F2(x1, x2) := x1·x2]
```

#14: $\text{result\_matrix}([1, 2], 3) = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \\ 11 & 30 \end{bmatrix}$

3.3 Beispiel einer Anwendung

Volterra-Zyklus zwischen Räuber- und Beutetieren Predator – Prey – Cycles

Für Räuber – Beute – Simulationen gilt das Gleichungssystem:
Predator-Prey-simulations can be described by:

$$x_1' = x_1 \cdot (1 - d + c \cdot x_2)$$
$$x_2' = x_2 \cdot (1 + a - b \cdot x_1)$$

mit x_1 = Räuberzahl und x_2 = Beutezahl (x_1 = predators, x_2 = prey)

Dabei haben die Parameter die folgende Bedeutung:

a ...... Wachstumsrate der Beutetiere (growth rate of prey)
b ...... Verlustrate der Beutetiere (loss rate of prey)
c ...... Regenerationsrate der Räuber (rate of regeneration of predators)
d ...... Energieverlust der Räuber (loss rate of energy of predators)

Nachdem wir die Datei `ITERATIO.MTH` geladen haben, geben wir ein:

```
#15:  [a := 1, b := 0.002, c := 0.00001, d := 0.08]
#16:  F1(x1, x2) := x1·(1 - d + c·x2)
#17:  F2(x1, x2) := x2·(1 + a - b·x1)
#18:  result_matrix([350, 6000], 4)
```

$$\#19: \begin{bmatrix} 350 & 6000 \\ 343 & 7800 \\ 342.314 & 10249.2 \\ 350.013 & 13481.5 \\ 369.199 & 17525.6 \end{bmatrix}$$

und erhalten nach `SIMPLIFY` die nebenstehende Matrix.

3.4 Graphische Darstellung

Für unser Iterationsproblem haben wir in Abschnitt 3.3 die Lösung in `result_matrix(x0,n)` zusammengefasst. Sie lieferte eine Matrix in der Form

$$e := \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \\ e_{31} & e_{32} \\ e_{41} & e_{42} \\ \dots & \dots \\ \dots & \dots \\ e_{n+1,1} & e_{n+1,2} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_1' & x_2' \\ x_1'' & x_2'' \\ x_1''' & x_2''' \\ \dots & \dots \\ \dots & \dots \end{bmatrix}$$

Wenn man die entwickelten einzelnen Komponenten von x_1 und x_2 in Abhängigkeit von der Iterationstiefe graphisch darstellen will, muss man daraus, wie in Abschnitt 2.4 besprochen, Matrizen in folgender Form erzeugen:

$$pic\_1 := \begin{bmatrix} 0 & e_{11} \\ 1 & e_{21} \\ 2 & e_{31} \\ 3 & e_{41} \\ \dots & \dots \\ \dots & \dots \\ n & e_{n+1,1} \end{bmatrix} \text{ und } pic\_2 := \begin{bmatrix} 0 & e_{12} \\ 1 & e_{22} \\ 2 & e_{32} \\ 3 & e_{42} \\ \dots & \dots \\ \dots & \dots \\ n & e_{n+1,2} \end{bmatrix}$$

Mit `m SUB i SUB j = m↓i↓j` – `m` ist die Matrix `result_matrix(x0,n)` – lassen sich beide Bilder mit dem `VECTOR`-Befehl durch eine Schleife einlesen.

Expression to extract data for plotting the development of predator population (pic_1) and prey population (pic_2).

$$pic\_1 := VECTOR([i - 1, m_{i,1}], i, DIM(m))$$

$$pic\_2 := VECTOR([i - 1, m_{i,2}], i, DIM(m))$$

Wir prüfen dies nach, indem wir eine Matrix `m` eingeben:

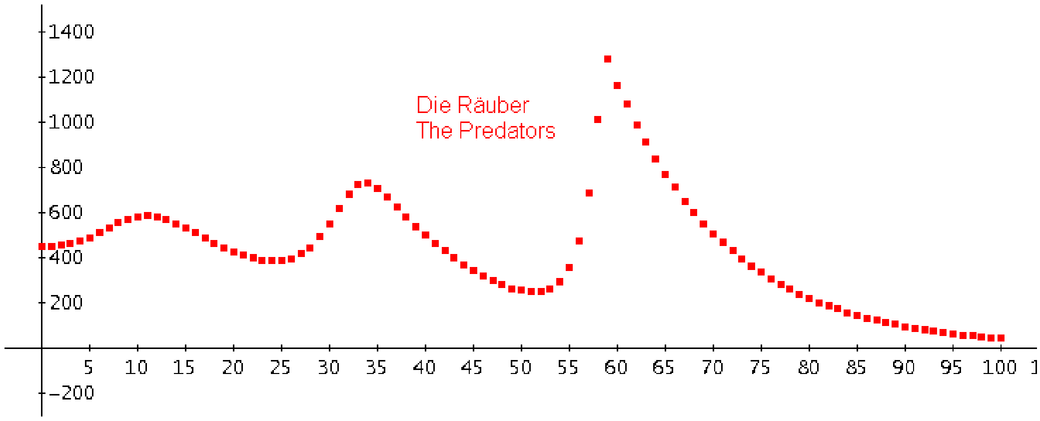
$$m := \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \\ 11 & 30 \end{bmatrix}$$

... und erhalten richtig nach `SIMPLIFY`:

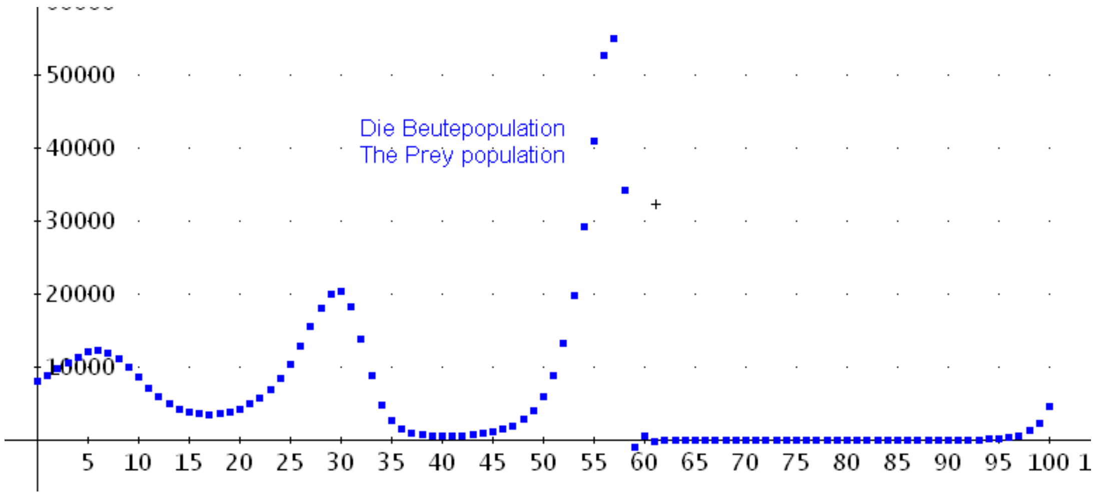
$$[pic\_1, pic\_2] = \left[\begin{bmatrix} 0 & 1 \\ 1 & 3 \\ 2 & 5 \\ 3 & 11 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 2 \\ 2 & 6 \\ 3 & 30 \end{bmatrix} \right]$$

Die Daten für unsere Simulation sind / Data for our simulation are

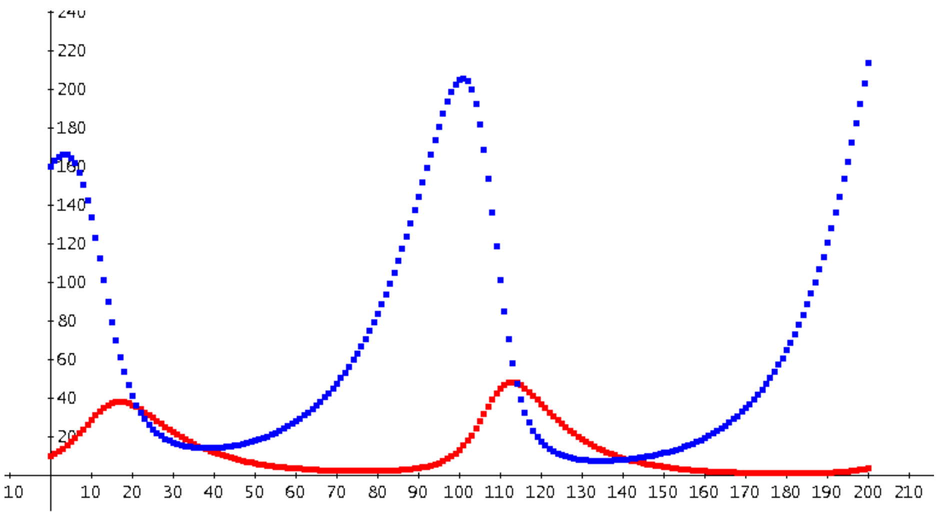
```
[a := 1,b := 0.002,c := 0.00001,d := 0.08,n := 100,x0 := [450, 8050]]
pic_1(x0,n)
```




```
pic_2(x0,n)
```



Eine andere Wahl für die Parameter liefert eine mehr oder weniger zyklischen Verlauf der Populationsentwicklung / Another choice of parameters shows a cyclic up and down of both populations.



Anmerkung des Herausgebers:

In der Zeitschrift "Spektrum der Wissenschaft" (Scientific American) fand ich in 2/84 und 2/85 jeweils einen Artikel über das, von Herrn Schumm bearbeitete Thema. Dabei wurden zwei interessante Graphiken gegenübergestellt:

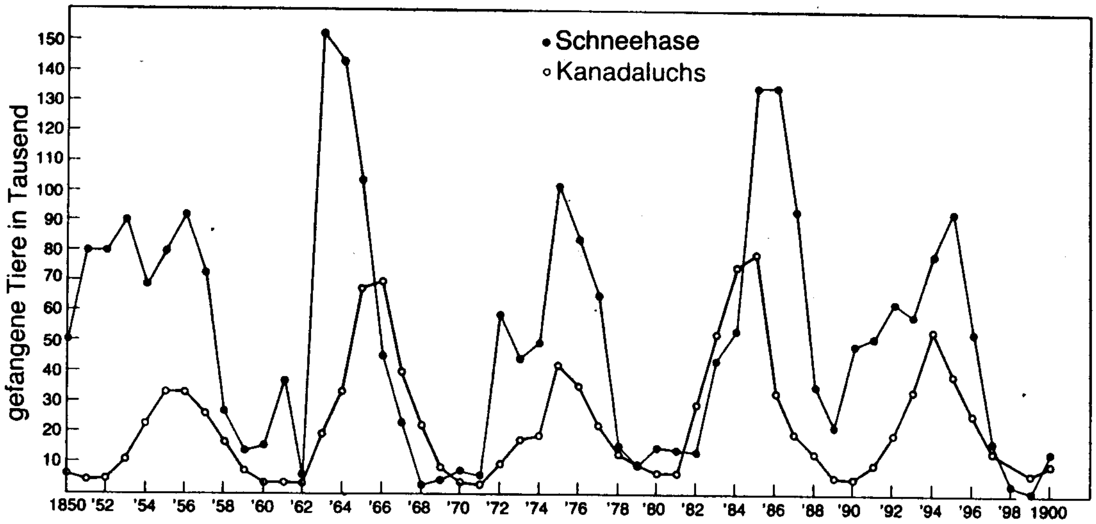


Bild 4: Mengen von Luchsen und Hasen, die von der Hudson's Bay Company zwischen 1850 und 1900 gefangen wurden.

Furs of Snowhare and Lynx caught in Canada between 1850 and 1900 (above) and solution of the associated differential equation (below).

Pictures from "Scientific American", 1984 and 1985.

Dies ist die exakte Lösung der zugehörigen Differentialgleichungen:

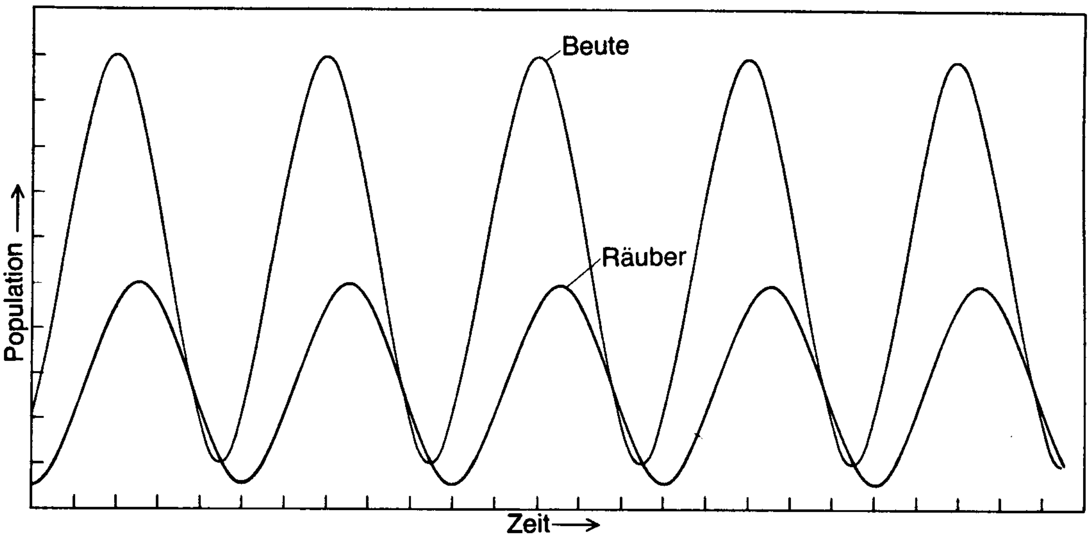


Bild 5: Zyklische Schwankungen in der Population von Räuber und Beute, die sich aus der Lotke-Volterra-Gleichung ergeben.

I enclose a short article on the use of *DERIVE* to find inverse Laplace Transforms which may be of use for the Bulletin of the *DERIVE* User Group. I have deliberately left it open-ended with some suggestions for further investigations.

Yours faithfully
J.V. Royle

Inverse Laplace Transforms using *DERIVE*

J.V. Royle, Leicester

We wish to find the inverse Laplace Transform, $L^{-1}\left(\frac{P(s)}{Q(s)}\right)$, of a rational function $\frac{P(s)}{Q(s)}$, where deg $P(s)$ < deg $Q(s)$. The usual elementary method is to

(i) factorise $Q(s)$ into real linear and quadratic factors,

(ii) express $\frac{P(s)}{Q(s)}$ in terms of Partial Fractions,

(iii) use tables of standard Laplace Transforms to find the inverse Laplace Transform of each of the Partial Fractions. The Expand command in *DERIVE* can be used to find the Partial Fraction expansion required in step (ii), provided the denominator factorises into real linear and quadratic factors with rational coefficients.

Example Find $L^{-1}\left(\frac{s^2+4s-2}{2s^3+11s^2+18s+9}\right)$.

$$\#1: \frac{s^2+4\cdot s-2}{2\cdot s^3+11\cdot s^2+18\cdot s+9}$$

$$\#2: \frac{23}{3\cdot(2\cdot s+3)}-\frac{5}{6\cdot(s+3)}-\frac{5}{2\cdot(s+1)}$$

`Simplify>Expand` delivers the factorisation. There is no need to first factorise the denominator.

Using the standard inverse Laplace Transform $L^{-1}\left(\frac{1}{s-a}\right)=e^{at}$, we get

$$L^{-1}\left(\frac{s^2+4s-2}{2s^3+11s^2+18s+9}\right)=\frac{23\,e^{-\frac{3t}{2}}}{6}-\frac{5\,e^{-3t}}{6}-\frac{5\,e^{-t}}{2}$$

Among the User Contributed files you can find **LaplaceTransforms.dfw** which according to its header computes the inverse Laplace transform of the function f(s) in terms of t. The author of this file is our friend Terence Etchells, who revised it first time in August 2003.
So we load this file as a Utility file and try to find the Inverse Laplace Transform of the above expression to check this calculation – and to check the file, of course.

$$\text{InvLaplace}\left(\frac{s^2+4\cdot s-2}{2\cdot s^3+11\cdot s^2+18\cdot s+9}\right)=-\frac{5\cdot e^{-t}}{2}+\frac{23\cdot e^{-3\cdot t/2}}{6}-\frac{5\cdot e^{-3\cdot t}}{6}$$

Problems with this method arise if

(i) there are Partial Fractions with quadratic denominators,

e.g. for $\dfrac{3s+5}{s^3+6s^2+16s+16}$ *DERIVE* gives the Partial Fraction expansion

$$\text{EXPAND}\left(\frac{3\cdot s+5}{s^3+6\cdot s^2+16\cdot s+16}\right)=\frac{s}{4\cdot(s^2+4\cdot s+8)}+\frac{7}{2\cdot(s^2+4\cdot s+8)}-\frac{1}{4\cdot(s+2)}$$

Using standard inverse Laplace Transforms of the form

$L^{-1}\left(\dfrac{s+a}{(s+a)^2+b^2}\right)=e^{-a\,t}\cos(b\,t)$ still leaves some manipulation to be done which is not easily performed with *DERIVE*.

(ii) the denominator, $Q(s)$, does not factorise into linear and quadratic factors with rational coefficients.

These problems can be surmounted by the following method, which depends on the result if $Q(s)$ has a simple factor of $(s-a)$ then the Partial Fraction expansion of $\dfrac{P(s)}{Q(s)}$ contains a term $\dfrac{A}{s-a}$, where $A=\dfrac{P(a)}{Q'(a)}$.

Hence if $Q(s)$ is an n^{th} degree polynomial with n simple roots a_1, a_2, ..., a_n, real or complex, and deg $P(s) < n$ then:

$\dfrac{P(s)}{Q(s)}=\sum_{i=1}^{n}\dfrac{A_i}{s-a_i}$, where $A_i=\dfrac{P(a_i)}{Q'(a_i)}$, then using $L^{-1}\left(\dfrac{1}{s-a}\right)=e^{at}$ the final result is

$$\boxed{L^{-1}\left(\frac{P(s)}{Q(s)}\right)=\sum_{i=1}^{n}\frac{P(a_i)}{Q'(a_i)}\,e^{a_i\,t}.}$$

Hence to find $L^{-1}\left(\dfrac{P(s)}{Q(s)}\right)$, where $Q(s)$ has only simple roots, the procedure in *DERIVE* is:

(i) define a function `ILT(a):=LIM(p/dif(q,s),s,a)*exp(a·t)`

(ii) author the numerator and denominator as variables p and q

(iii) Solve $q = 0$, best to use Mixed Mode

(iv) assuming there are as many different solutions, a_1, a_2, ..., a_n as the degree of $Q(s)$ then the roots are all simple, find the inverse Laplace Transform by authoring and simplifying

```
ILT(a1) + ILT(a2) + ... + ILT(an).
```

There is no problem with complex roots since, if $\frac{P(s)}{Q(s)}$ is real, complex roots occur in complex conjugate pairs and *DERIVE* automatically expresses complex exponentials, which will occur in the `ILT`-sum as complex conjugate pairs, in terms of real trigonometric functions.

In the following examples it is assumed that the `ILT`-function has been defined as above and that `Precision Mode` has been set to `Mixed`.

Example

Find $L^{-1}\left(\frac{3s+5}{s^3+6s^2+16s+16}\right)$.

#6: $p := 3\cdot s + 5$

#7: $q := s^3 + 6\cdot s^2 + 16\cdot s + 16$

#8: $s^3 + 6\cdot s^2 + 16\cdot s + 16$

#9: SOLVE(q, s)

gives the result

#10: $[s = -2,\ s = -2 - 2\cdot i,\ s = -2 + 2\cdot i]$

so to find the inverse Laplace transform:

#11: $\text{ILT}(-2) + \text{ILT}(-2 - 2\cdot i) + \text{ILT}(-2 + 2\cdot i)$

gives the result

#12: $e^{-2\cdot t}\cdot\left(\frac{\text{COS}(2\cdot t)}{4} + \frac{3\cdot\text{SIN}(2\cdot t)}{2} - \frac{1}{4}\right)$

We check again applying InvLaplace:

$$\text{InvLaplace}\left(\frac{3\cdot s + 5}{s^3 + 6\cdot s^2 + 16\cdot s + 16}\right) = e^{-2\cdot t}\cdot\left(\frac{\text{COS}(2\cdot t)}{4} + \frac{3\cdot\text{SIN}(2\cdot t)}{2} - \frac{1}{4}\right)$$

So in fact this tool seems to be very reliable!

Example Find $L^{-1}\left(\frac{s^2-4s+2}{s^3+3s^2+5s+7}\right)$.

#13: $p := s^2 - 4\cdot s + 2$

#14: $q := s^3 + 3\cdot s^2 + 5\cdot s + 7$

#15: SOLVE(q, s)

#16: $$\left[s = -\left(\frac{2\cdot\sqrt{87}}{9} + 2\right)^{1/3} + \left(\frac{2\cdot\sqrt{87}}{9} - 2\right)^{1/3} - 1,\ s = \left(\frac{\sqrt{87}}{36} + \frac{1}{4}\right)^{1/3} - \left(\frac{\sqrt{87}}{36} - \frac{1}{4}\right)^{1/3} - 1 - i\cdot\left(\left(\frac{3\cdot\sqrt{87}}{8} + \frac{7}{2}\right)^{1/6} + \left(\frac{7}{2} - \frac{3\cdot\sqrt{87}}{8}\right)^{1/6}\right),\ s = \left(\frac{\sqrt{87}}{36} + \frac{1}{4}\right)^{1/3} - \left(\frac{\sqrt{87}}{36} - \frac{1}{4}\right)^{1/3} - 1 + i\cdot\left(\left(\frac{3\cdot\sqrt{87}}{8} + \frac{7}{2}\right)^{1/6} + \left(\frac{7}{2} - \frac{3\cdot\sqrt{87}}{8}\right)^{1/6}\right)\right]$$

#17: $[s = -2.1795,\ s = -0.410245 - 1.74454\cdot i,\ s = -0.410245 + 1.74454\cdot i]$

These roots indicate that q can not be factorised into linear and quadratic factors with rational coefficients.

So to find the inverse Laplace Transform:

ILT(−2.1795) + ILT(−0.410245 − 1.74454·ί) + ILT(−0.410245 + 1.74454·ί)

$$2.50551\cdot e^{-2.1795\cdot t} - e^{-0.410245\cdot t}\cdot(1.50551\cdot \text{COS}(1.74454\cdot t) + 1.23634\cdot \text{SIN}(1.74454\cdot t))$$

Once more I called the new user contributed function

$$\text{INVLaplace}\left(\frac{s^2 - 4\cdot s + 2}{s^3 + 3\cdot s^2 + 5\cdot s + 7}\right)$$

$$e^{-0.4102454876\cdot t}\cdot(6\cdot \text{COS}(1.744543250\cdot t) + 4\cdot \text{SIN}(1.744543250\cdot t))$$

I interrupted simplifying after more than 5 minutes and then achieved a result by approximating the expression – but it was quite different to the result obtained by using J V Royle´s method (compare the two expressions!). I wrote a mail to Terence and asked for advice. His answer was an update of **LaplaceTransform.dfw** from 9 November 2004 which not only simplifies the expression within 0.77 sec to a bulky expression but also approximates to the same result as given above:

$$\text{InvLaplace}\left(\frac{s^2 - 4\cdot s + 2}{s^3 + 3\cdot s^2 + 5\cdot s + 7}\right)$$

$$2.505504641\cdot e^{-2.179509024\cdot t} - e^{-0.4102454876\cdot t}\cdot(1.505504641\cdot \text{COS}(1.744543250\cdot t) + 1.236344531\cdot \text{SIN}(1.744543250\cdot t))$$

$$\text{Laplace}(2.505504641\cdot e^{-2.179509024\cdot t} - e^{-0.4102454876\cdot t}\cdot(1.505504641\cdot \text{COS}(1.74454325\cdot t) +$$

$$1.236344531\cdot \text{SIN}(1.74454325\cdot t)))$$

$$\frac{0.002724795640\cdot(3.628195461\cdot 10^{27}\cdot s^2 - 1.451278183\cdot 10^{28}\cdot s + 7.256390905\cdot 10^{27})}{(6.177695\cdot 10^{6}\cdot s + 1.3464342\cdot 10^{7})\cdot(1.600287999\cdot 10^{18}\cdot s^2 + 1.313021861\cdot 10^{18}\cdot s + 5.139696996\cdot 10^{18})}$$

(Terence says: Note that the inversion has not returned the original expression. However a plot of the original function and the inversion of its Laplace transform show that they are indeed the same function. This latest version of **LaplaceTransform.dfw** is among the files associated to revised DNL#4. Many thanks Terence for your immediate reaction.)

Further extensions of this method are possible:

(i) Find the formula for the coefficients, A and B, in terms of $P(s)$ and $Q(s)$ and their derivatives, for the Partial Fractions $\frac{A}{s-a} + \frac{B}{(s-a)^2}$, corresponding to a double root $s = a$ of $Q(s) = 0$.

Then define a function `ilt2(a):=A exp(a t) + B t exp(a t)` in *DERIVE* using the formulae found for A and B.

(ii) For Laplace Transforms which are not rational functions, finding a Padé approximation valid for large s and inverting this to get an approximation to the inverse Laplace Transform valid for small t.

Joint Venture *DERIVE* und Cabri im Mathematikunterricht

Herbert Appel und Richard Brand, Schweinfurt

1. Einleitung

Das Erscheinen von Programmen für interaktive Geometrie und symbolische Algebra, die eine neue Epoche für den Mathematikunterricht einzuleiten scheinen, erzeugte in letzter Zeit eine Flut sicherlich nötiger und guter Artikel (Schuhmann, Engel, Weigand, Weth u.a.) in didaktischen Zeitschriften. Bücher zu diesem Thema sind momentan noch Mangelware, was sich aber sicher demnächst ändern wird. Ein Mangel haftet jedoch all diesen Betrachtungen an: zum einen beschränken sie sich meist nur auf eine Art von Programm, also entweder auf den Einsatz von z.B. *DERIVE* oder von z.B. Cabri im MU; zum anderen werden darin relativ einfache und grundlegende Beispiele behandelt. Die dadurch mögliche Verquickung von Algebra und Geometrie findet nicht statt.

Mit der folgenden Aufgabe wollen wir speziell dem Lehrer (direkt an der Front) ein Beispiel an die Hand geben, wie er *DERIVE* und Cabri zur Bearbeitung einer etwas anspruchsvolleren Aufgabe kombiniert in seinem Unterricht einsetzen kann.

Appearance of dynamic geometry and computeralgebra programs which seem to open a new era for math education initiated a lot of interesting papers in the last time (Schuhmann, Engel, Weigand, Weth ao). All of these articles are mostly restricted to only one of these programs and tackle very simple examples. There is very little cooperation between algebra and geometry.
By presenting the following problem we will offer an example how to use Derive and Cabri treating a more demanding task in classroom.

2. Methodische Hinweise

Folgende Aufgabe könnte Inhalt einer Übungsstunde in der 9.Klasse der Sekundarstufe I sein. Sie ist beispielhaft für das induktiv-deduktive Vorgehen beim Finden einer Lösung.

Voraussetzung ist, dass ...

... die Schüler bereits im Umgang mit der Bedienung der beiden oben erwähnten Programme versiert sind oder

... eine Systemkombination IBM-kompatibler AT + Panel an der Schule existiert.

Um ein Arbeiten mit Bleistift und Papier kommt der Schüler allerdings schon der Anschaulichkeit wegen nicht herum.

3. Die Aufgabenstellung

Mit den Vektoren AB und AD_n ist eine Schar gleichschenkeliger Trapeze gegeben.

Dabei gilt: $AB = C_nD_n$: $A(2\,|\,1)$; $B(4.5\,|\,0.5)$ und $AD_n = \begin{pmatrix} 4z-4 \\ 2z+2.5 \end{pmatrix}$.

Vectors AB and C_nD_n and define a family of isosceles trapeziums: $AB = C_nD_n$: $A(2|1)$; $B(4.5|0.5)$ and

$AD_n = \begin{pmatrix} 4z - 4 \\ 2z + 2.5 \end{pmatrix}$.

Fragen / Questions:

a) Was lässt sich über den geometrischen Ort der Punkte D_n aussagen?
b) Welche Kurve beschreiben die Punkte C_n?

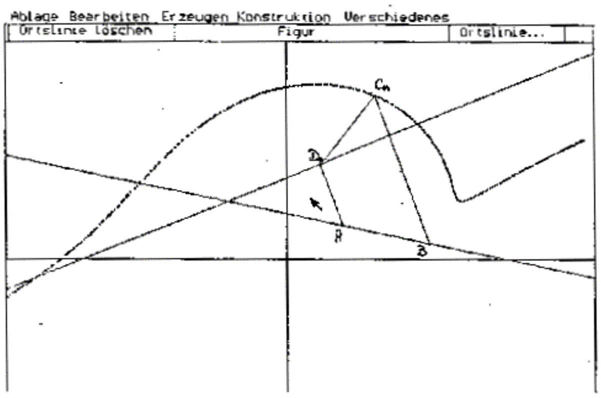
a) Which is the locus of points D_n?
b) Which is the curve described by points C_n?

Lösungsstrategie:

Um den geometrischen Ort der Punkte D_n zu gewinnen ist eine Parameter-Transformation (mit *DERIVE* oder per Hand) durchzuführen. Diese ergibt

$AD_n = \begin{pmatrix} x \\ \dfrac{x+9}{2} \end{pmatrix}$; die Punkte D_n liegen auf einer Geraden (Frage a).

Um sich einen Überblick über den geometrischen Ort der Punkte C_n zu verschaffen wird mit Cabri das (veränderbare) gleichschenkelige Trapez konstruiert. Da in Cabri eine Anfgabe der genauen Punktkoordinaten nicht möglich ist, kann man sich durch Verwendung herkömmlicher Zeichenhilfsmittel (Zirkel, Lineal und Papier) einen groben Überblick über die Lage der Punkte und Geraden im Koordinatensystem verschaffen, eine Idee für die Konstruktion der Trapeze aus den Angaben holen und diese dann mit Cabri durchführen.



Parameter transformation leads to the answer of question a): points D_n are lying on a line.

To obtain an impression about the locus of points C_n we use Cabri to produce the dynamic variable trapezium.

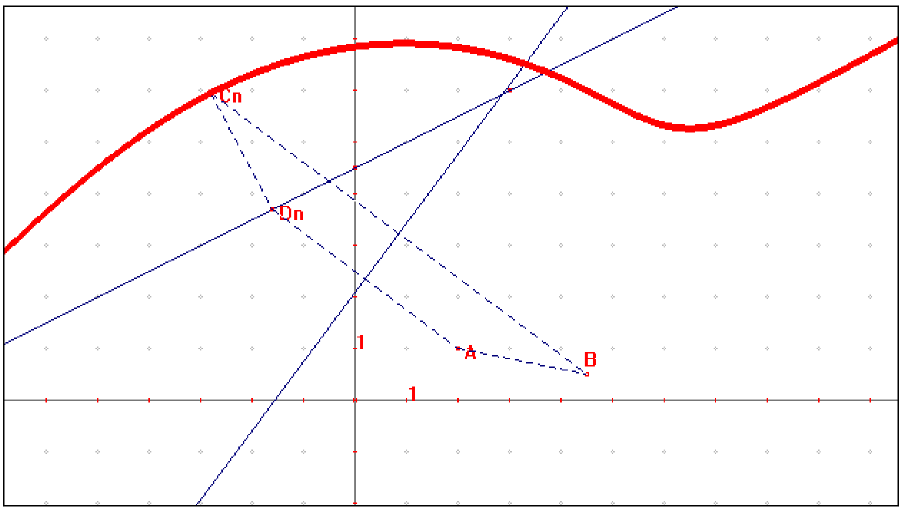
We show the "historic" screenshot from 1991 showing one of the first German Cabri versions.

Recent Cabri versions give a presentation of the locus. We can locate pts *A* and *B* according to the given coordinates.

You can see that we draw the perpendicular bisector of AD_n.

D_n is moveable on the line.

Reflecting point *B* with respect to this bisector gives point C_n.



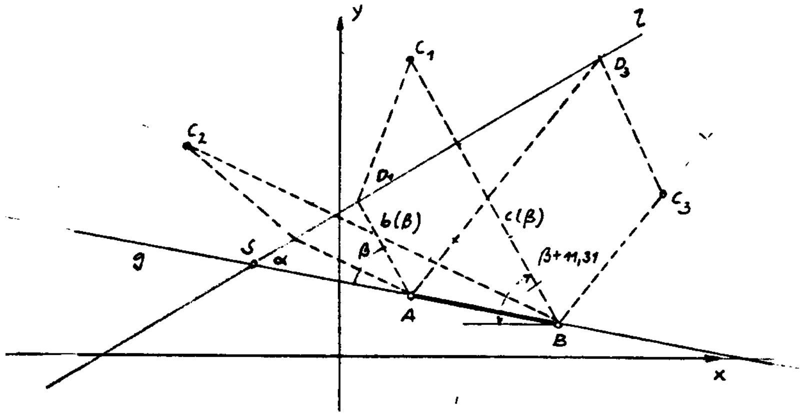
zu b) Konstruktionsbeschreibung

Zeichne Gerade *g* durch *A* und *B*,
zeichne beliebige zweite Gerade, die *g* schneidet (Skizze)
Punkt D_n liegt auf der zweiten Geraden,
zeichne Strecke AD_n,
konstruiere Mittelsenkrechte *h* zu AD_n,
spiegle *B* an *h*. Es ergibt sich C_n,
bewegen des Punktes D_n zum Zeichnen der Ortlinie.

Man erkennt, dass die Kurve eine unbekannte Form (keine Gerade, Parabel, Hyperbel usw.) ist. Eine algebraische Beschreibung der Ortslinie ist sinnvoll. Die gewonnene Ortslinie wird vorsorglich zum späteren Vergleich ausgedruckt.

It is obvious that the curve shows an unknown form (no straight line, parabola, hyperbola, etc). It makes sense to look for an algebraic description of the locus. we print the locus for later comparison.

Zur Veranschaulichung von bekannten und gesuchten Größen dient die Skizze auf dem Blatt. Die Umformungen werden mit *DERIVE* durchgeführt. Follow the sketch.



Nach längerer Rechnung ergibt sich der Schnittpunkt der beiden Geraden g ∩ h *S*(-31/7; 16/7), deren Schnittwinkel $\alpha \approx 37.875°$ und $AS \approx 6.556$ LE. Der Neigungswinkel der Geraden *AB* ist ≈ -11.31°.

Aus dem Sinussatz folgt: $$b(\beta) = \frac{6.556 \sin 37.875°}{\sin(37.875° + \beta)} \quad (1)$$

außerdem $$|AB| = \sqrt{6.5} \quad (2)$$

und $$c(\beta) = b(\beta) + 2|AB|\cos\beta \quad (3)$$

und damit $$C_n = (4.5 - c(\beta)\cdot\cos(11.31° + \beta), 0.5 + c(\beta)\cdot\sin(11.31° + \beta)) \quad (4)$$

We apply the sine rule to express length *b* depending on angle β and find easily an expression for the parallel side *c*(β). Using the coordinates of point *B* we derive the coordinates of C_n which describes the locus in its parameter representation with parameter β.

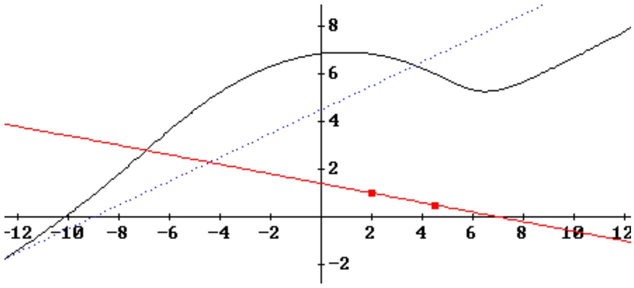
To keep work as easy as possible we use the Substitute command three times, which leads us to a huge and bulky expression. Students are very happy that they didn´t need find it by manual calculation. Derive allows arbitrary long function expressions and is able to plot graphs for functions given in parameter form.

Um den Schreibaufwand gering zu halten, kommt uns *DERIVE* mit dem Befehl `Substitute` sehr entgegen. Dreifaches Anwenden ersetzt (1) und (2) in (3) und (3) in (4).

Heraus kommt ein Riese von einem Term, von dem die Schüler froh sind, dass sie ihn nicht durch schriftliches Umformen herleiten mussten (siehe #34 im Listing).

DERIVE bietet uns nun gegenüber anderen bisher eingesetzten Kurvenplotprogrammen noch zwei Besonderheiten:

- es erlaubt die Eingabe beliebig langer Funktionsterme
- es kann auch Graphen von Funktionen zeichnen, die in Parameterform gegeben sind. Das nutzen wir schamlos aus.



Der Vergleich der von Cabri gezeichneten Punktmenge und dem Graphen, den uns *DERIVE* zeichnet. ergibt eine ausreichende Übereinstimmung und ermöglicht dem Schüler eine tiefere Einsicht in die Zusammenhänge zwischen Algebra und Geometrie.

Comparison of the Cabri- and the Derive results shows an acceptable correspondence and might lead the student to a deeper insight into the connections between algebra and geometry.

Die *DERIVE* Ausarbeitung / The Derive session

#24: $\text{SOLVE}\left[\dfrac{b\_}{SA} = \dfrac{\text{SIN}(\alpha)}{\text{SIN}(\alpha + \beta)},\ b\_\right]$

#25: $b\_ = \dfrac{9\cdot\sqrt{26}}{7\cdot\text{COS}(\beta) + 9\cdot\text{SIN}(\beta)}$

#26: $b(\beta) := \dfrac{9\cdot\sqrt{26}}{7\cdot\text{COS}(\beta) + 9\cdot\text{SIN}(\beta)}$

#27: $c(\beta) := b(\beta) + 2\cdot AB\cdot\text{COS}(\beta)$

#28: $\beta\_ := \text{ATAN}\left[-\dfrac{1}{5}\right]$

#29: $\dfrac{\beta\_}{°}$

#30: -11.30993247

#31: $xc(\beta) := \dfrac{9}{2} - c(\beta)\cdot\text{COS}(\beta - \beta\_)$

#32: $yc(\beta) := \dfrac{1}{2} + c(\beta)\cdot\text{SIN}(\beta - \beta\_)$

#33: $[xc(\beta),\ yc(\beta)]$

#34: $$\left[-\frac{88\cdot\text{COS}(\beta)^3 + 9\cdot\text{COS}(\beta) - \text{SIN}(\beta)\cdot(76\cdot\text{SIN}(\beta)^2 + 23)}{2\cdot(7\cdot\text{COS}(\beta) + 9\cdot\text{SIN}(\beta))},\ \frac{14\cdot\text{COS}(\beta)^3 + 88\cdot\text{SIN}(\beta)\cdot\text{COS}(\beta)^2 + 5\cdot\text{COS}(\beta)\cdot(18\cdot\text{SIN}(\beta)^2 + 5) + 99\cdot\text{SIN}(\beta)}{2\cdot(7\cdot\text{COS}(\beta) + 9\cdot\text{SIN}(\beta))}\right]$$

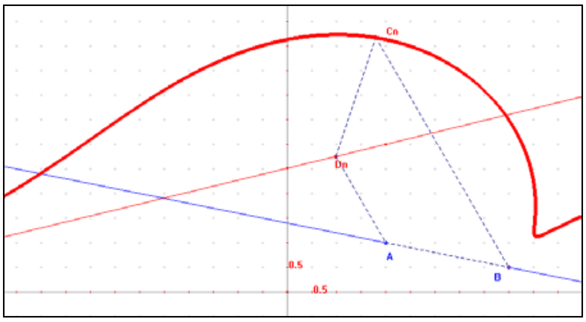
4. Schlussbemerkung

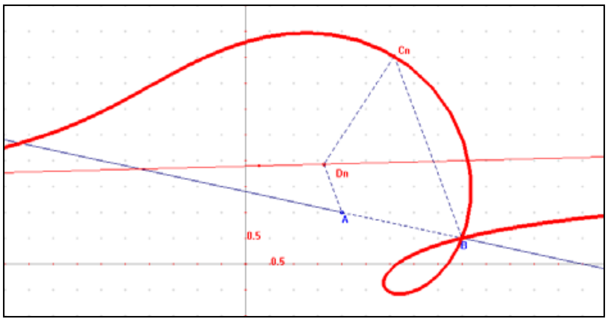
Wir hoffen, dem geneigten Lehrer (Leser) einen Eindruck von der Mächtigkeit, die eine Kombination der beiden Programme darstellt, vermittelt und ihn zum Weiterdenken in dieser Richtung angeregt zu haben.
Ein Programm, das die Fähigkeit von *DERIVE* und Cabri in sich vereinigt ist unseres Wissens nach noch nicht auf dem Markt, aber es wird dem Vernehmen nach daran gearbeitet.

Concluding comment: We hope that we were able to give an impression of the power which is represented by a combination of these two programs and that we inspired some of you to develop more ideas and concepts in this direction. It would be great to have a program which combines the capabilities of both tools.

By rewriting this article from 1991 using the tools of 2004 I found some nice things.
Please follow the next page(s)! Josef

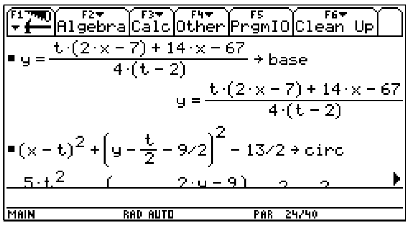
First of all I used Cabri II to find some other loci varying the position of the underlying basic lines.

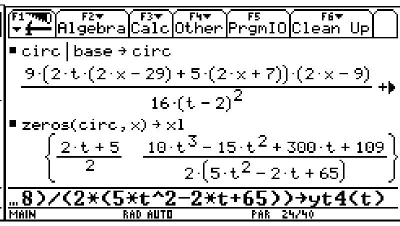


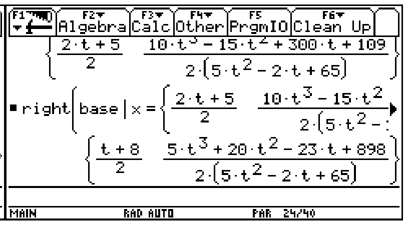


I sat in the train on my way to give a two-days course on the use of CAS-TIs and DERIVE and wanted to tackle Herbert´s problem without using trig functions. The coach was full and I could not open my notebook. So I took my Voyage 200 and started calculating.

The idea was to draw a line parallel to AD_n through point B. Then draw a circle with center D_n and radius AB. The intersection point should give C_n. Finally I tried to find a reasonable algebraic form for the locus.



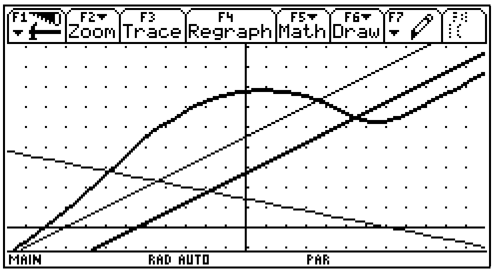




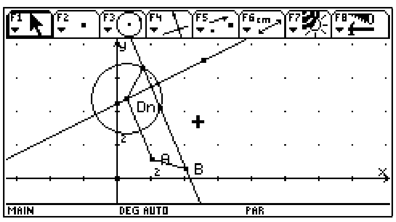
You can see the most important part. Two facts are of special interest:

- The result is a rational expression and more interesting
- We have two loci, one of them appearing as line with the same slope as the locus of D_n!!

I plotted all graphs on the TI using Parametric Mode, which gave a satisfying result.

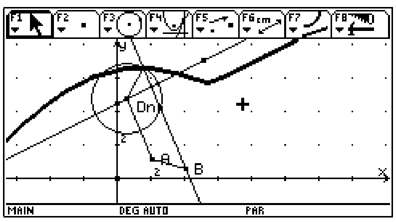


It was a long ride, so started Cabri on the Voyage 200 and reproduced my idea using dynamic geometry in full agreement with Herbert Appel´s idea of combining the two poweful tools and also applying our highly appreciated “Window-Shuttle” principle.

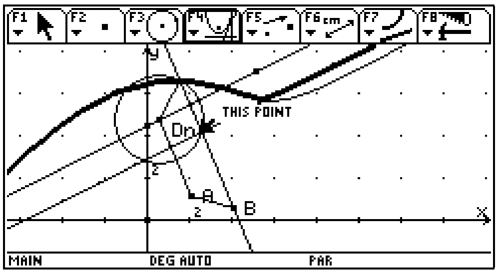


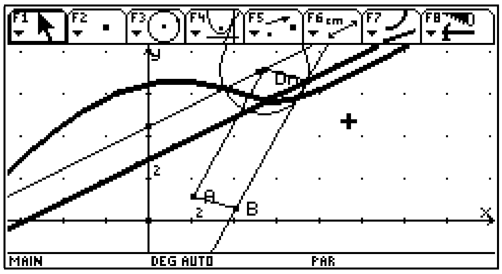
On the left you can follow how to produce the locus. There are two intersection points!

One of them giving the trapezium – the other one giving a ????

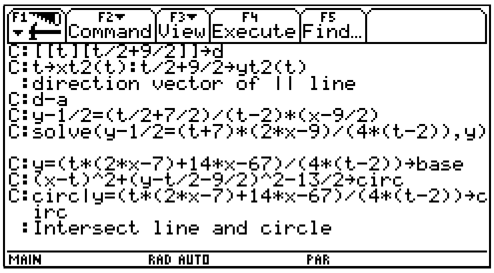


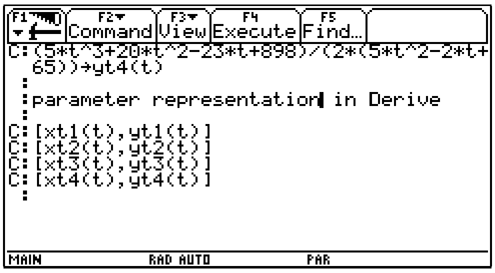
Strange things happen plotting the locus of the trapezium point (see right above): There is a flaw in the locus and it changes from the curve to the line. This is caused by a deficiency of Cabri, because it does not “keep” the right point by passing the position when both quadrangles coincide (in form of a rightangle). Adding the locus of the second intersection point gives the missing parts and now we can see both graphs.





I saved my Voyage 200 session from the Homescreen into a text file `appeltxt` and hoped to be able to tranfer the calculation to Derive 6! I added some comments in the text file and closed with four lines which should give the parameter form in Derive. Plotting them in the 2D-Plot window should immediately give the graphs.





Having arrived in my hotel room in St. Johann/Salzburg I opened the notebook, connected it with the V 200 and launched Derive 6.10 and hoped that InterConnectivity would do it.

I issued `File>TI Handheld>Import from` and typed in `appeltxt`. Transfer started and:

Intersect line and circle

#29: $$x1 := \text{SOLUTIONS}\left(\frac{9\cdot(2\cdot t\cdot(2\cdot x-29)+5\cdot(2\cdot x+7))\cdot(2\cdot x-9)}{16\cdot(t-2)^2}+\frac{5\cdot t^2}{4}-\frac{5\cdot t\cdot(2\cdot x-5)}{4}+\frac{5\cdot x^2}{4}-\frac{43\cdot x}{4}+\frac{845}{16},\ x,\ \text{Real}\right)$$

#30: $$x1 := \left[\frac{18\cdot|t^2-5\cdot t+6|+10\cdot t^3+3\cdot t^2+210\cdot t+217}{2\cdot(5\cdot t^2-2\cdot t+65)},\ -\frac{18\cdot|t^2-5\cdot t+6|-10\cdot t^3-3\cdot t^2-210\cdot t-217}{2\cdot(5\cdot t^2-2\cdot t+65)}\right]$$

#31: $$y1 := \text{RHS}\left(\text{VECTOR}\left(\text{base},\ x,\ \left[\frac{2\cdot t+5}{2},\ \frac{10\cdot t^3-15\cdot t^2+300\cdot t+109}{2\cdot(5\cdot t^2-2\cdot t+65)}\right]\right)\right)$$

It seems to work. I receive the correct – both loci and should be satisfied (next page). Inspecting the Derive procedure more accurately you will find quite another result for the solutions of the important equation which delivers the *x*-values of the intersection points (see expr #30).

parameter representation in Derive

#49: [[xt1(t), yt1(t)]]

#50: $\left[\left[\frac{5\cdot t}{2}+2,\ 1-\frac{t}{2}\right]\right]$

#51: [[xt2(t), yt2(t)]]

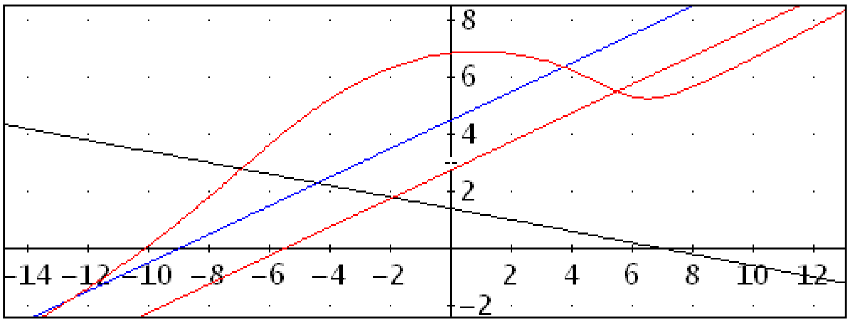
#52: $\left[\left[t,\ \frac{t}{2}+\frac{9}{2}\right]\right]$

#53: [[xt3(t), yt3(t)]]

#54: $\left[\left[\frac{2\cdot t+5}{2},\ \frac{t+8}{2}\right]\right]$

#55: [[xt4(t), yt4(t)]]

#56: $\left[\left[\frac{10\cdot t^3-15\cdot t^2+300\cdot t+109}{2\cdot(5\cdot t^2-2\cdot t+65)},\ \frac{5\cdot t^3+20\cdot t^2-23\cdot t+898}{2\cdot(5\cdot t^2-2\cdot t+65)}\right]\right]$



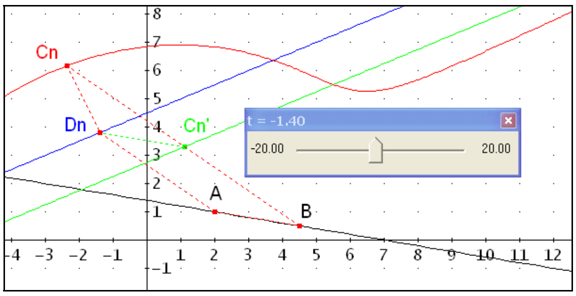
Removing the abs from expression #30 and simplifying once moreI obtain the same expected expressions. (See the end of the page for another way to obtain the expected result.)

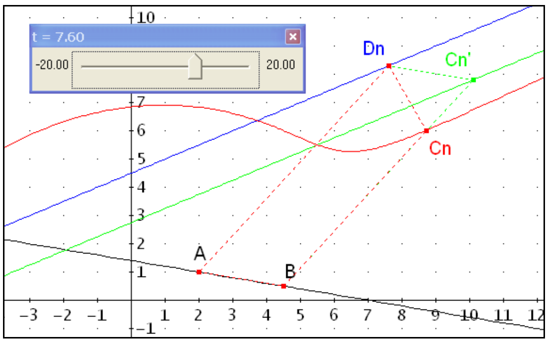
$$\left[\frac{18\cdot(t^2-5\cdot t+6)+10\cdot t^3+3\cdot t^2+210\cdot t+217}{2\cdot(5\cdot t^2-2\cdot t+65)},\ -\frac{18\cdot(t^2-5\cdot t+6)-10\cdot t^3-3\cdot t^2-210\cdot t-217}{2\cdot(5\cdot t^2-2\cdot t+65)}\right]$$

$$\left[\frac{2\cdot t+5}{2},\ \frac{10\cdot t^3-15\cdot t^2+300\cdot t+109}{2\cdot(5\cdot t^2-2\cdot t+65)}\right]$$

In this case InterConnectivity was very helpful for me. However, it must be said that this tool is not perfect at the moment. You have to consider carefully how to use the |-operator on the TI to have a correct translation in Derive. I was said – and from my experience with the developpers I have no reason to doubt – that the programmers are busy to improve InterConnectivity between the CAS-TIs and Derive.

Having used successfully InterConnectivity I wanted to introduce slider bars to have a dynamic Derive which is very close to Dynamic Geometry and forms a useful link between algebraic and graphic representation forms.



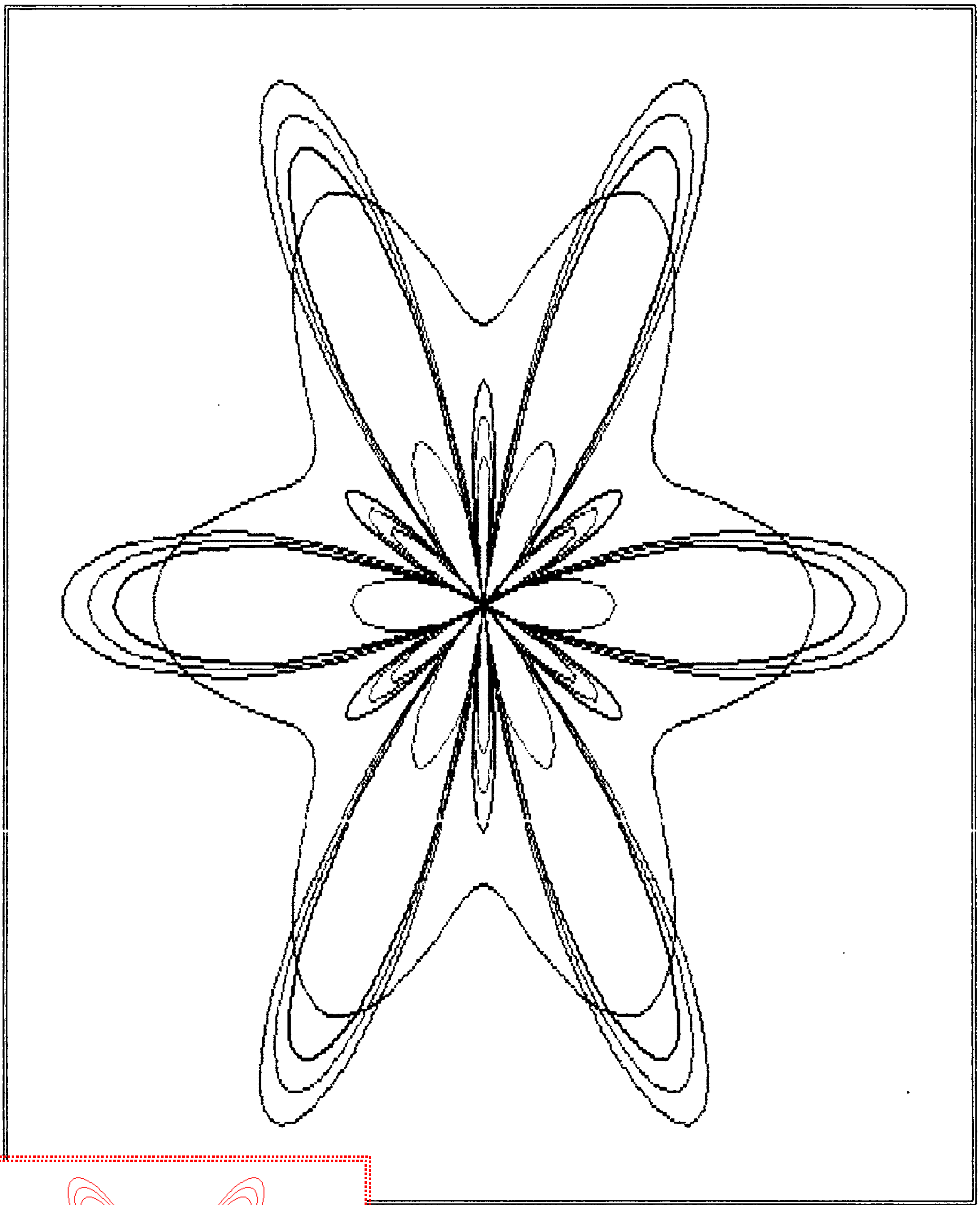


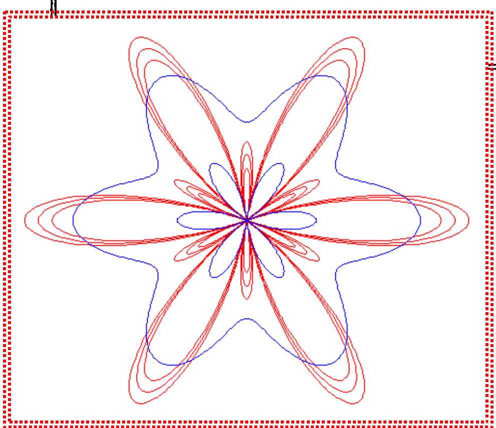
This is a way to overcome Derive´s solution of equation #29 including the absolute values: Factor the expression and then investigate the factors to get the two solutions.

$$\frac{(2\cdot x-2\cdot t-5)\cdot(2\cdot x\cdot(5\cdot t^2-2\cdot t+65)-10\cdot t^3+15\cdot t^2-300\cdot t-109)}{16\cdot(t-2)^2}$$

$$\text{SOLUTIONS}(2\cdot x-2\cdot t-5=0,\ x)=\left[\frac{2\cdot t+5}{2}\right]$$

$$\text{SOLUTIONS}(2\cdot x\cdot(5\cdot t^2-2\cdot t+65)-10\cdot t^3+15\cdot t^2-300\cdot t-109,\ x)=\left[\frac{10\cdot t^3-15\cdot t^2+300\cdot t+109}{2\cdot(5\cdot t^2-2\cdot t+65)}\right]$$





$$\left[2\cdot\cos(6\cdot x)+\frac{4}{5},\ 2\cdot\cos(6\cdot x)+1,\ 2\cdot\cos(6\cdot x)+\frac{6}{5}\right]$$

$$\frac{\cos(6\cdot x)}{2}+2$$

Set Polar Coordinate System

$$\frac{\cos(6\cdot x)}{2}+0.5$$

Module on Sequences and Series with *DERIVE*, Part 1

David E. Hodgkinson, Liverpool

ITERATION

Iteration is a method of repeated calculation that has become very important because computers can iterate quickly and accurately. The process of counting can be represented as an iterative process by starting at 1 and repeatedly adding 1. One mathematical way of writing this is $n := 1$ and $n \leftarrow n + 1$. This means that n is initially assigned to 1 then repeatedly increased by 1. Usually a way of stopping the repetition is included, either by saying how many times the assignment is repeated or providing a condition to stop the repetition.

DERIVE has various functions that provide iteration facilities. To produce a list of the first twelve positive integers use the function `ITERATES` with four arguments. Commas separating the arguments of a function must be typed. Hence typing the following instruction should produce the associated list of integers.

```
#1:    ITERATES(n + 1, n, 1, 11)

#2:    [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
```

is obtained because the second argument of iterates is updated by the assignment to the first argument i.e. $n \leftarrow n + 1$, the third argument is the initial value of n and the fourth argument is the number of iterations required. Hence changing 1 to 0 in iterates gives a list of 13 consecutive integers beginning with 0. It is fairly obvious that as the number of iterations increases the final value in the list will increase, this is an example of a divergent iteration.

Another example is the assigment

$$x \leftarrow \frac{1}{2}\left(x + \frac{2}{x}\right) \qquad \text{starting with } x = 1$$

the first six values of this iteration are found from

$$\#3: \quad \text{ITERATES}\left(\frac{1}{2}\cdot\left(x + \frac{2}{x}\right),\ x,\ 1,\ 6\right)$$

$$\#4: \quad \left[1,\ \frac{3}{2},\ \frac{17}{12},\ \frac{577}{408},\ \frac{665857}{470832},\ \frac{886731088897}{627013566048},\ \frac{1572584048032918633353217}{1111984844349868137938112}\right]$$

However, the list of longer and longer fractions is not very informative. Since iteration is mainly used for numerical calculations, to get decimal approximations use the approx-button (≈):

```
#5:   [1, 1.5, 1.416666666, 1.414215686, 1.414213562, 1.414213562, 1.414213562]
```

i.e. approximating the fractions (or see below for how to repeat the procedure in approximate mode) shows that the iteration is settling to 1.414213. This is recognisable as $\sqrt{2}$ to 6 decimal places. Notice that this iteration is approaching a limiting value as the number of iterations increases, this is an example of a convergent iteration.

There is also a function ITERATE (notice no final 'S') that will just return the final value requested by the value of the fourth argument.

#6: $\text{ITERATE}\left(\frac{1}{2}\cdot\left(x+\frac{2}{x}\right),\ x,\ 1,\ 6\right)$

#7: $\frac{1572584048032918633353217}{1111984844349868137938112}$

To work in generally in Approximate Mode set in the `Options>Mode Settings> Simplification>Precision` to `Approximate` and in `Output>Notation` to `Decimal`.

#8: Precision := Approximate

#9: Notation := Decimal

#10: $\text{ITERATES}\left(\frac{1}{2}\cdot\left(x+\frac{2}{x}\right),\ x,\ 1,\ 6\right)$

#11: [1, 1.5, 1.416666666, 1.414215686, 1.414213562, 1.414213562, 1.414213562]

If you suspect that an iterative sequence is convergent you can use ITERATES without the fourth argument and *DERIVE* will automatically stop when two iterates are the same to the current precision so

#12: PrecisionDigits := 6

#13: NotationDigits := 6

#14: $\text{ITERATES}\left(\frac{1}{2}\cdot\left(x+\frac{2}{x}\right),\ x,\ 1\right)$ = [1, 1.5, 1.41666, 1.41421, 1.41421, 1.41421]

since *DERIVE* stores its numbers in rational form the equivalence of the last three iterates is due to rounding; only the last two have the same fractional form.

If you have been working in approximate mode do not forget to return to exact mode with

#15: Precision := Exact

#16: Notation := Rational

Exercises

1. Repeat the above iteration with $x \leftarrow \frac{1}{2}\left(x+\frac{3}{x}\right)$, starting with $x = 1$,

   (this is usually written $x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right)$ to find the square root of 3, to 9 decimal places.

   Can you modify the command to find the square root of 4, 5, 6?
   What would be the formula for the square root of N, where N is a positive integer?
   Try your formula on $N = 10^7 + 1$.
   Could you think of a way to speed up the convergence?

2. Try the iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n^{\ 2}}\right)$, for N = 27, 8, 4, 2, 3. ($x_0 = 1$).

   What value does this iteration converge to for given N?

3. Try the iteration $x_{n+1} = 2x_n - N \cdot x_n^{\ 2}$

   where N = 2, 3, 4, 5, 6 and the initial value of x lies between 0 and 1. What is the value of this iteration for different N?

4. Find √2 for 10 and 20 decimal places to see the speed with which the convergent answer is reached. If you have been in approximate mode, do not forget to return to exact mode using the above instructions.
   What value does this iteration converge to for given N?

SEQUENCES AND SERIES

In the previous examples one list had the values 1,2,3,4, whilst the other had to 4 decimal places the values 1, 1.5, 1.4166, 1.4142, 1.4142. Lists of numbers separated by commas are called sequences. Mathematically speaking sequences are functions with domain the set of integers, but instead of using the standard $f(n)$ or $g(n)$ notation, the alternative x_n or u_n are preferred, but with *DERIVE* the standard notation will be used.

Closely connected with sequences are series. Series are obtained by adding together the terms of a sequence. The value of this combined addition is called the sum of the series.

e.g. the finite sequence 1,2,3,4,5,6,7,8,9,10,

gives the series 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10,

and the sum af the series is 55.

Using Derive the sum can be obtained iteratively by using an array v in which the first element is the successive numbers of the series and the second element is the sum of the series. The iterative formulae being used are x:=1 and s:=1 initially and x is increased by 1 each time whilst s is increased by x i.e. $x \leftarrow x + 1, s \leftarrow s + x$.

The initial value of v is [1,1]. The first element of v represents the next number in the series, the second element represents the current value of the sum. Derive updates its copy of v using the function `ELEMENT`, which extracts elements of arrays. The next number in the series is found by using

```
ELEMENT(v,1)+1
```

Works still but more comfortable use `v sub 1 + 1` which appears as $v_1 + 1$

The value of the sum is found from

`1+element(v,1)+element(v,2)` or better `1+v sub 1+v sub 2` = $1+v_1+v_2$.

The first two elements represent the new value of the nth term added to the third term representing the previous sum.

The command to automate this is:

#19: $\text{ITERATES}\left(\left[1 + v_1,\ 1 + v_1 + v_2\right],\ v,\ [1,\ 1],\ 6\right)$

#20: $\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 3 & 6 \\ 4 & 10 \\ 5 & 15 \\ 6 & 21 \\ 7 & 28 \end{bmatrix}$

The answer is a two column array in which the first contains the terms of the series and the second column the corresponding sum of the terms so far.

Naturally infinite sequences and series do occur. The idea of convergence can be used with series as well. If the sum to infinity is finite then a series is called convergent. The problems of investigating the convergence of series are many and varied and still the subject of active research. This module will give some ideas of the use and problems with sequences and series.

First note well that a convergent sequence does not mean that the corresponding series will converge.

The infinite sequence 1, 1, 1, 1, 1, 1, 1, ...

is convergent, but the infinite series 1 + 1 + 1 + 1 + 1 + ...

is divergent.

Do not be deceived by the size of the first few terms in a series, it is the long term behaviour of a series that matters.

Use Derive to find the sum of the first few terms of the series

$$\frac{100!}{1}+\frac{100!}{2^2}+\frac{100!}{3^3}+\frac{100!}{4^4}+\frac{100!}{5^5}+\ldots+\frac{100!}{n^n}+\ldots$$

$$100!\cdot\left(1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\frac{1}{5^5}\right)$$

1205087157208740173341246814176706616634074030715166519400874001010255559658506311682762100907010466871597242370877488~
543797418892517493964800000000000000000000

The sum looks very large and impressive. However, this series has a finite sum, since the denominator eventually gets much larger than the numerator. Try

$$\frac{100!}{100^{100}}$$

$$\frac{100!}{200^{200}}$$

$$\frac{100!}{300^{300}}$$

and simplify the expressions.

Whereas, the sum of the series in which each term is the reciprocal of the corresponding one in this series, starts small, but eventually grows without bound. Repeat the trial by looking at the first few sums of $\frac{n^n}{100!}$. Then examine the later terms, these are very large and increasing so that the sum would grow without bounds i.e. the series is divergent.

However, you must not be mislead into thinking that individual terms must be large for the series to be divergent, it is the cumulative effect of adding an unlimited number of terms that is important. In fact the series

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\ldots$$

caused by adding the reciprocals of the integers is divergent, even though the individual terms get very small. It is impossible to show the divergence by just finding the sums to a finite number of terms, the divergence needs a theoretical proof. This series is a famous one in mathematics and is called the **harmonic series**.

ARITHMETIC SERIES

The series

$$1 + 2 + 3 + 4 + 5 + ...$$

is obtained by summing the terms of the sequence

$$u_{n+1} = u_n + 1 \quad \text{with } u_1 = 1.$$

In fact any sequence of the form

$$u_{n+1} = u_n + d$$

with u_1 given an initial is called an **arithmetic sequence** and the sums of the terms of these sequence are called **arithmetic series**.

One of the fascinations of mathematics is finding alternative ways to solve problems. Faced with the problem of adding

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

the obvious way is to do the addition and get 55 either with pencil and paper or using the iteration and calling 9 times. Summing from 1 to 100 is much more arduous and all programming languages have ways of instructing the computer to carry out repetive steps. An alternative way is to find a method of getting the answer from a formula and avoiding the repetetive steps. This is usually not possible, but in most instructive examples the alternative of finding a formula is possible.

In the case of arithmetic series, this alternative of a formula for the sum is possible. The following trials are aimed at producing a formula for the sum of an arithmetic series.

Considering the sequence $u_{n+1} = u_n + 1$ with $u_1 = 1$

it is very easy to find the 10th term, it is 10, the 100th term is 100 etc. so in this case the *n*th term is *n*.

The sum of the terms is less obvious. So try to author a few iterations consulting the previous example if you need to see how. The answer gives two columns: in the first one are the values of *n* and in the second one are the values of the sum to *n* terms. So for the *n* sequence

1, 2, 3, 4, 5, 6, 7

the *s* (i.e. sum) sequence is: 1, 3, 6, 10, 15, 21, 28.

To try and work out a formula connecting the sum to n terms with the value of n, trial and error can be tried, but a systematic approach is best. Assuming $s = a\,n + b$ i.e. a linear relationship, fit this equation to the first two terms to find values of a and b see if the formula predicts the third and higher values. When $n = 1$, $s = 1$; $n = 2$, $s = 3$. *DERIVE* expects you to enter linear equations in the form of a vector, so the [,] brackets must be typed, around the equations.

```
[1 = a + b, 3 = 2·a + b]

SOLVE([1 = a + b, 3 = 2·a + b], [a, b])

[a = 2 ∧ b = -1]
```

Hence author `s = 2n - 1`

To test this conjecture that this is formula is true if $n = 3$ gives $s = 6$. Substitute for n and s:

```
s = 2·n - 1

6 = 2·3 - 1

6 = 5
```

The linear guess is wrong. (Undoubtely you will not need a CAS for this calculation, but it might be useful for later to learn this procedure.)

Trying again with a second degree equation $s = a\,n^2 + b\,n + c$ there are three unknown coefficients a, b and c, so the first three terms will be needed and then the predictions for the fourth and higher terms can be attempted. Author the first three equations as a vector and solve, remember to type the [ ] around the equations.

```
SOLVE([a + b + c = 1, 4·a + 2·b + c = 3, 9·a + 3·b + c = 6], [a, b, c])
```

$$\left[a = \frac{1}{2} \wedge b = \frac{1}{2} \wedge c = 0\right]$$

and these answers make $s = \dfrac{n^2}{2} + \dfrac{n}{2}$. To test for higher terms it is possible to use a *DERIVE* command which allows you to do repeated substitution into a formula. It is called `VECTOR` and has four arguments, the first for the expression, the second for the variable, the third for the starting value and the fourth for the finishing value.

$$\mathrm{VECTOR}\left(\frac{n^2}{2} + \frac{n}{2}, n, 1, 7\right) = [1, 3, 6, 10, 15, 21, 28]$$

giving complete agreement.

Exercises.

1. Consider the iterative sequence

$$u_{n+1} = u_n + 2 \quad \text{with} \quad u_1 = 1.$$

   Generate the first seven terms of this sequence and also the sum for $n = 1, 2, 3, 4, 5, 6, 7$.
   Use your results to produce a formula for the nth term and a formula for the sum to n terms.

2. Use the iterative formula $u_{n+1} = u_n + d$.

   Choose your own u_1 and d and see if you can find the formulae for the nth term and the sum to n terms.

Working with the general arithmetic sequence $u_{n+1} = u_n + d$, $u_1 = a$, you can find its sequence of partial sums by working with iterates and a vector with three elements. This is so that the first element can keep count of the number of terms, the second element can update the current value of the *n*th term and the third element can add the *n*th term to the previous sum.

$$\text{ITERATES}\left(\left[1 + v_1,\ v_2 + d,\ v_3 + v_2 + d\right],\ v,\ [1,\ a,\ a],\ 7\right)$$

The first column sequence represents the number of terms, the second column sequence gives the value of the *n*th term (u) and the third column the sum sequence gives the value of the sum to n terms (s). The problem is to find a formula relating the value of n with the values of the *n*th term u and the sum s.

$$\begin{bmatrix} 1 & a & a \\ 2 & a + d & 2 \cdot a + d \\ 3 & a + 2 \cdot d & 3 \cdot a + 3 \cdot d \\ 4 & a + 3 \cdot d & 4 \cdot a + 6 \cdot d \\ 5 & a + 4 \cdot d & 5 \cdot a + 10 \cdot d \\ 6 & a + 5 \cdot d & 6 \cdot a + 15 \cdot d \\ 7 & a + 6 \cdot d & 7 \cdot a + 21 \cdot d \\ 8 & a + 7 \cdot d & 8 \cdot a + 28 \cdot d \end{bmatrix}$$

Can you spot the relationship between n and u?

In conventional work the first term is usually denoted by a and the commom difference by d, so that the formula for the *n*th term would be $a + (n - 1)\, d$, which should be equivalent to the one you obtained.

The connection between n and s is not as obvious, since both a and d have changing coefficients. Can you spot the formula connecting the *n*th term with the coefficient of a? Write down the sequence of coefficients of d. Have you seen this sequence before? If so can you construct the formula relating the coefficient of d with n?

Finally can you combine your two results to show that your formula is eqivalent to

$$s_n = \frac{n}{2} \cdot \left[2a + (n-1) \cdot d\right]$$

(to be continued)

(David E. Hodgkinson, Department of Applied Mathematics and Theoretical Physics, University of Liverpool)

How to write my own DEMO-file?

Josef Böhm, Würmla

When I want to use *DERIVE* for demonstrating the procedure how to solve a problem I'm glad to have of a Demo-file at my disposal. Students or colleagues start with `File>Load>Demo File>Filename`. They are able to stop the "demonstration" at any position to do their own calculations, change the option settings, plot partial results etc. In addition they can find in the message line a short description, a little hint, an important explanation, ...

I was curious and inspected an original *DERIVE* -.dmo-file. In DOS: `type arith.dmo`. It's pure ASCII-Code. I saw the message to be printed on screen in the first line, starting with a semicolon. The next line was a *DERIVE* -expression, written in its very own syntax. That was all!

Now let me write a Demo-file called `SKETCH.DMO` to demonstrate investigating a function graph. You can use each editor, which creates ASCII-files. I prefer WORD and save my file as ASCII-file with the extension .dmo. Another way to edit this file is in DOS:

`copy con: sketch.dmo` etc.

```
;This is the function to be investigated
f(x):=(x^4-11x^3+29x^2+35x-150)/30
;f(x) = 0 to obtain the zeros
SOLVE(f(x)=0,x)
;f'(x) is the 1st derivative which is defined as f1(x)
f1(x):=DIF(f(x),x)
;zeros of f'(x) give the relative extrema, ESC, Approx, Notation
Decimal, Simplify
SOLVE(f1(x)=0,x)
;These are the turning points! Set back to Exact and Rational
[5,f(5);-0.47038,f(-0.47038);3.72038,f(3.72038)]
;Hurry up for the 2nd derivative
f2(x):=f''(x)
;f''(x) = 0 gives the inflection point(s), ApproX, Notation Decimal
SOLUTIONS(f2(x)=0,x)
;Here are the complete coordinates! Study the Derive command!
VECTOR([u,f(u)],u,SOLUTIONS(f2(x)=0,x))
;We need the slope in the inflection points for a fine sketch
[f1(1.098),f1(4.402)]
;Let's proceed with the indefinite integral in Exact Precision
INT(f(x),x)
;Whatïs the area between graph and x-axis between the zeros
ABS(INT(f(x),x,-2,3))+ABS(INT(f(x),x,3,5))
;ESC, Change to 2D-Plot Window, highlight f(x) and plot the graph
f(x)
;Finally we will shade the area
(-2<=x<=3 AND f(x)<=y<=0) OR (3<=x<=5 AND 0<=y<=f(x))
```

This was from 1991. Nobody will work with copy con .... in 2004??

We have other choices:

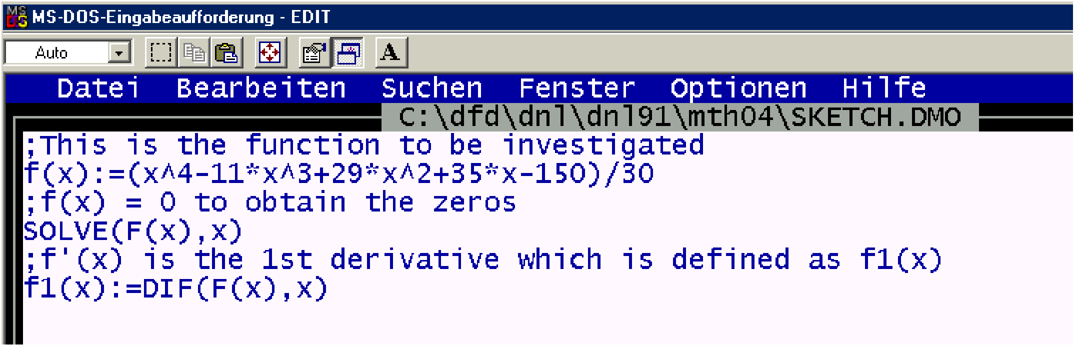
(1) If you like the nostalgic way you might work with the DOS-Editor. So switch to DOS and type `edit sketch.dmo`.

(2) You can do it from the *DERIVE* 6 environment or

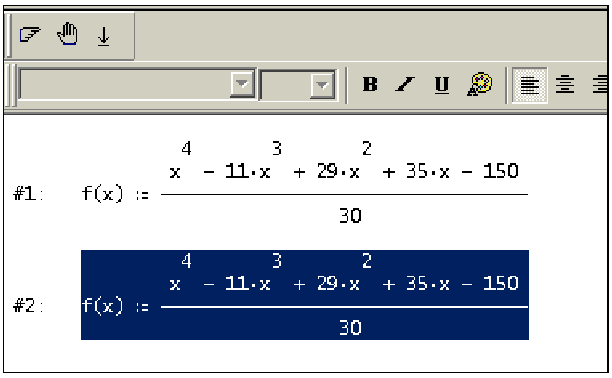
(3) You can use your handheld CAS-Device and apply InterConnectivity.

Don´t use the textprcessor now because of differences between ASCII and ANSI-code!

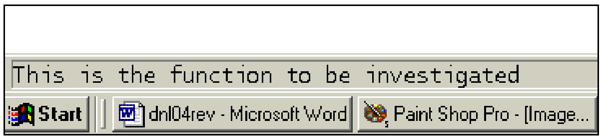
This is the first – nostalgic – way. You must be very careful typing the *DERIVE* commands in the correct syntax and typing them in uppercase. In DOS mode call `edit sketch.dmo` and start editing:



Having finished and saved the file you should be able to load this file as a demo file in *DERIVE*. The first expression appears in the Algebra Window and in the message line on the bottom you will find your comment.



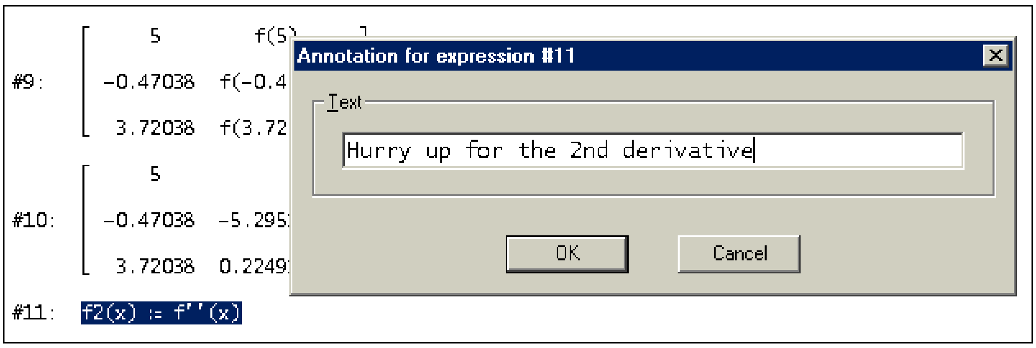
The message line (bottom left):



You can write your complete DEMO-file with the Editor. But possibly you will prefer the *DERIVE* environment, because you want to convert a successful *DERIVE*-session into a Demo file and without typing in all the stuff once more in the editor.

So do the following:

Highlight the expression which should be executed, go for `Edit>Annotation` and write your comment as shown below. Later this comment will appear in the message line.

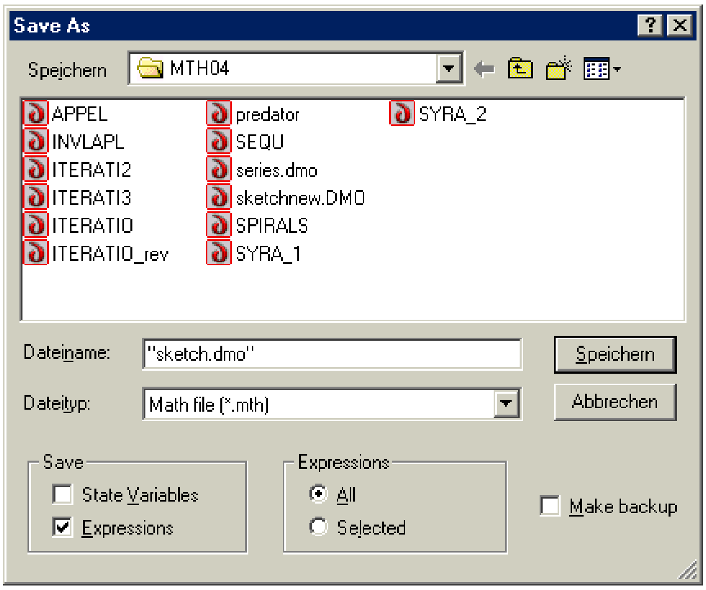


I recommed to delete all simplified or approximated results before saving the file. So I would delete expression `#10`. Otherwise they would appear twice.

When you are ready with the session then you have to save the file in such a way that *DERIVE* will recognize it as a demo-file.

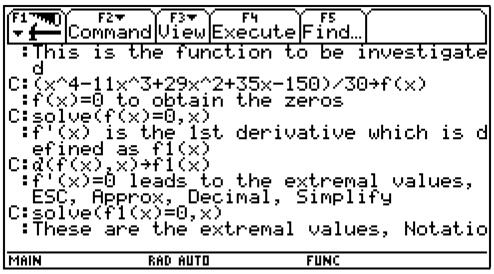
You must take care of two details:

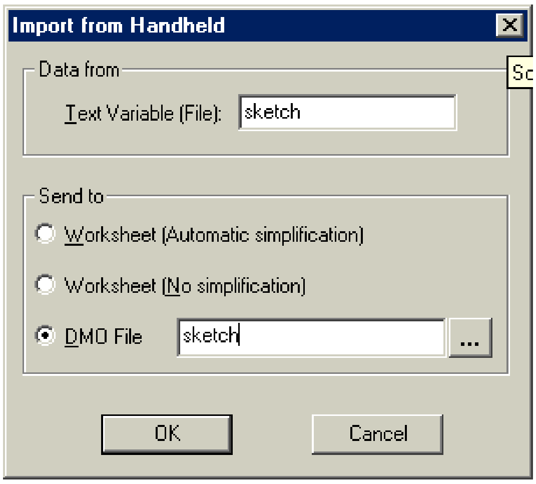
- Save the file as a Math- (mth-) file
- Type the name of the file under quotes with the filename extension `.dmo`.



If you are the happy owner of a TI-92 PLUS, Voyage 200, TI-89 or TITANIUM then you have a third chance to produce a DEMO-file:

At first create your demo as a script on the hand-held device.



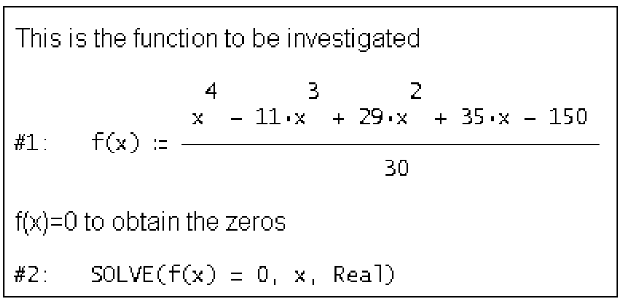


It is not necessary to save the *.dmo-file under the name of the script!!

All your one-line comments in the script which are before an expression will appear as comments in the DEMO-file.

If you would import the same file as a worksheet then you would find your comments in text-boxes.

The picture shows the start of the file.



CALL FOR APPLICATIONS

International Spring School on the Didactics of Computer Algebra

April 27-30, 1992

Krems, Austria

Computer Algebra is a new technology with an immense impact on techniccal sciences. The personal computer algebra system DERIVE has found its way to all levels of education. It has pleased many teachers but also surprised others. The availability of DERIVE on very inexpensive hardware and the new palmtop size Pcs enables more and more students to use the system for their assignments or in school.

How will math teaching need to be changed to cope with thie new development? this spring school aims to bring together people with an interest in developing a didactics of computer algebra for the math education of students aged 12-18.

The program will consist of presentations of classroom experiments, discussions and working groups. The organizers provide full lodging and boarding. The number of participants is strictly limited.

We invite your application to participate. Applications should include the following items:

- full name and address,
- list of activities related to 'computer agebra in education' (own experiments, publications, workshops, seminars, etc.),
- an extended abstract on what you will present at the meeting,

to be sent to the following address:

Research Institute for Symbolic Computation
Univ. of Linz, att. Dr. B. Kutzler
A-4040 Linz, Austria, fax +43-7236-3231-30

to arrive no later than

February 15, 1992.

The Notification of acceptance will be mailed February 29,1992.

This spring school is jointly organized by the following institutions:

✳ Landesschulrat für Niederösterreich, Wipplingerstr. 28,
A-1010 Wien, Austria (LSI Dr. Helmut Heugl)

✳ Bundesministerium für Unterricht und Kunst, Wipplingerstr. 20,
A-1010 Wien, Austria (MR Dr. Eduard Szirucsek)

✳ Research Institute for Symbolic Computation, University of Linz,
A-4040 Linz, Austria (Dr. Bernhard Kutzler)