THE DERIVE - NEWSLETTER #6

THE BULLETIN OF THE

DERIVE®

USER GROUP

Contents:

revised reprint 2005 **June 1992**

For our English DUG-Members:

One of the results of the DUG-Meeting in Nottingham was the wish of some members to have more contacts within England. Two participants of this meeting are willing to build up a regional DERIVE Group to exchange informations and experiences within the UK. I support these efforts and hope to extend the relations to my British friends. I want to thank all participants of the meeting in Nottingham for their co-operation and especially for their cordiality. These days in England were a very special event for my daughter, my wife and me. We will have best memories on this meeting.

DERIVE Group South England, please contact

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DERIVE Group North England, please contact

Terence A. Etchells
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Belmont, Bolton BL7 8AL

Best greetings to Kevin and Terence. I wish you both much success and I'm sure to receive many interesting contributions for the DNL in the future.

Maybe that there are some DUG-Members in other countries who want to found a regional DERIVE-Group. We would be glad to encourage and to support your ideas.

A short report of the International Spring School

The first International Spring School on the Didactics of Computer Algebra took place in Krems, Austria, from April 27 - 30, 1992. It was an inspiring and innovative meeting of 40 experts. But there was a tragic event, too. Dr Dennis Tanner, London, had a bad accident on the first evening from which he couldn't recover. So I'm sorry to tell you, that our friend Dennis died 10 days later in a hospital. I've met Dennis twice for some hours and I will never forget his humour, his spirit and his vitality.

In the next DNL you will find more details about the Krems Meeting. The proceeding is to be published (about 320 pages!!). On the last page you can find a list of the participants, who gave a lecture. If anybody wants to receive some details about one or another item, then please inform the editor.

I must thank all participants for their friendship. I've got a lot of letters since then and I think that this week was a very special week in our life. I can't describe it really, but there was more than a common interest in DERIVE. We all felt, that something more happened in Krems. Best greetings to all of you and I am sure to meet you again somewhere.

My special thank to Dr. Heugl and Dr. Kutzler. Your organization was perfect. I'll enjoy the next Spring- or Summer School.

Lieber Derive Anwender!

Ich möchte mich dafür entschuldigen, dass dieser DNL so spät im Juni erscheint. Aber es gab so viel zu tun in letzter Zeit. Vor allem die Aufarbeitung der DERIVE Spring School in Krems und die Zusammenstellung des Tagungsberichtes hat einige Arbeit gemacht. Wenn ich Ihnen sagen kann, in welcher Form dieser Bericht erscheint, werde ich Sie natürlich hier informieren.

Der angekündigte Artikel über italienische DERIVE-Aktivitäten musste für den nächsten DNL aufgeschoben werden. Erstens war die Übersetzung für mich etwas mühsam: italienisch - deutsch - englisch und zweitens liegt mit H. Appels Wunschliste bereits ein didaktischer Aufsatz vor. Und ich bin der Meinung, dass der DNL keine nur Mathematikdidaktische Schrift werden soll, obwohl didaktische Themen nach wie vor willkommen sind. Daher auch an dieser Stelle meine wiederholte Aufforderung an alle DERIVE-Anwender: bitte berichten Sie von Ihrer ganz speziellen DERIVE-Anwendung!

Unmittelbar nach Fertigstellung des DNL#6 erhielt ich aus Hawai eine Diskette mit den Anfragen und Antworten aus dem DERIVE Electronic Bulletin Board von September 91 bis Mai 92. Zugleich erhielt ich die Zusage, monatlich mit den neuesten Informationen versorgt zu werden. Vielen Dank, mahalo nui loa!

Juni ist Rosenmonat, daher habe ich einige Maurer-Rosen in diesen DNL eingestreut (P.Maurer, "A rose is a rose ...", Amer. Math. Monthly, 94). Im nächsten DNL können Sie die DERIVE Wurzeln und Zweige – und Dornen–- dazu finden.

Ich wünsche allen DERIVE-Freunden einen schönen Sommer und freue mich bereits jetzt auf ein „Wiederlesen“ im Herbst.

Mit den besten Grüßen

Dear Derive User,

I have to apologize for publishing this DNL so late in June. But there was so much to do last time, above all the working up of the DERIVE Spring School held in Krems and then collecting and editing the proceedings. I will let you know in the DNL when and in which form this book will be available.

The announced contribution concerning Italian DERIVE activities from R.M.Castelletti had to be postponed for the next issue because of two reasons: at first it was a little bit difficult for me to translate Italian → German → English and on the other hand there is already a didactic article in this DNL. In my opinion the DNL should not become an only maths didactic magazine, although didactic items are welcomed now as before. Therefore once more my invitation to all DERIVE users: please report about your special DERIVE-application(s)!

...When DNL#6 had just been finished I received a diskette from Hawaii containing requests, answers and comments from the DERIVE Electronic Bulletin Board from Sepember 91 through Mai 92. I got the promise to be informed about the latest news from the BBS monthly from now on. Many thanks Hawaii, mahalo nui loa!

June is the month of roses, therefor I've strewed some Maurer roses for you in this DNL (P.Maurer, "A rose is a rose...", Amer. Math. Monthly, 94). In the next DNL you will find the roots and branches - and thorns.

I wish a fine summer to you and your families and I'm looking forward to a "reading again" in autumn

With my best regards

The Derive-News-Letter is the Bulletin of the Derive User Group. It is published at least three times a year with a contents of 30 pages minimum. The goals of the *D-N-L* are to enable the exchange of experiences made with Derive as well as to create a group to discuss the possibilities of new methodical and didactic manners in teaching Mathematics.

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Contributions

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE - Newsletter* will be.

Preview: Contributions waiting to be published

Logic with DERIVE

Maurer Roses

Probability Theory

DERIVE in Italy

will be published in September 1992

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Prof. Dr. V. Weißpfennig, Passau

During a course on polynomial factorization that I gave at the University of Passau this winter, I tried to use *DERIVE* for computing some examples. In this connection, *DERIVE* turned out to be of very limited use only, since most factorization algorithms work by modular techniques. On the other hand, *DERIVE* does not admit polynomial calculations (such as GCD calculations or various factorizations) over integers modulo a prime p.

The biggest surprise, however, was the polynomial

$$f(x) = (x^3 + 3x^2 - 1)(x^3 - x + 1) = x^6 + 3x^5 - x^4 - 3x^3 + 3x^2 + x - 1$$

that I chose as a convenient example for factorization in Z[x] using Berlekamp's method over Z/2 and Hensel lifting. When I asked *DERIVE* to factor $f(x)$ over the rationals (or even over the complex numbers), *DERIVE* insisted that $f(x)$ is irreducible!

Another example of this kind is

$$g(x)=(x^4 - x^2 + 5x - 1)(x^3 + 4x + 1) = x^7 + 3x^5 + 6x^4 - 5x^3 + 19x^2 + x - 1$$

Has anyone else noticed this bug? Probably, these examples are just the tip of an iceberg.

D-N-L: David Stoutemyer confirmed your question. The algorithms mentioned by you are not implemented in *DERIVE*. Maybe future versions will be able to do better factorization.

As you can see, this problem has been resolved in the meanwhile, Josef 2005:

$$x^6 + 3\cdot x^5 - x^4 - 3\cdot x^3 + 3\cdot x^2 + x - 1$$

$$(x^3 - x + 1)\cdot(x^3 + 3\cdot x^2 - 1)$$

$$x^7 + 3\cdot x^5 + 6\cdot x^4 - 5\cdot x^3 + 19\cdot x^2 + x - 1$$

$$(x^3 + 4\cdot x + 1)\cdot(x^4 - x^2 + 5\cdot x - 1)$$

Volker Neurath, Velbert

... Ich betreibe unter dem Betriebssystem DR-DOS 5.0 die Version 2.07 von *DERIVE*.
Das Problem ist, dass bei mir der Befehl **Transfer Print Printer** nicht funktioniert. DR-DOS bricht den Vorgang mit einer Fehlermeldung ab. Zur Zeit drucke ich daher mit **Transfer Print File** in ein File, dessen Inhalt ich über **Options Execute** mit Hilfe einer Batchdatei drucken lasse.

D-N-L: The reason is an incompatibility of DR-DOS 5.0. Please ask DR for a patch. You will get it and then *DERIVE* will print its files.

Some weeks later I received another mail from Mr. Neurath:
...Danke auch für Ihre Information bezüglich DERIVE/DR-DOS 5.0. Allerdings hat sich dieses Problem inzwischen erledigt. Teilen Sie allen Anwendern mit: DERIVE und DR-DOS 6.0 verstehen sich in jeder Beziehung prima. Ich setze jetzt seit circa 4 Wochen DR-DOS 6.0 ein und bin bisher sehr zufrieden.

Some days ago, when I had nearly finished DNL#6 I received this interesting letter about *DERIVE* & WINDOWS, DavidSjöstrand has promised to send when we met in Krems. Thank you very much for your letter David, and best regards to the north.

David A Sjöstrand, Onsala, Sweden

A little notice about how to copy formulas and plots from *DERIVE* to a document of MS-Word for Windows.

I find it very useful to copy formulas and plots, which I have produced during a *DERIVE*-session, to a document in MS–Word for Windows. This is easily done if you are running *DERIVE* under MS-Windows 3.0 or 3.1. I will give a description of how to do.

1. Start Windows. You must have the possibility to run Windows under so called Enhanced 386-mode. You can do this if you have a computer with 386-processor and at least 2MB memory. I have 4 MB myself and what I am describing below works quickly and comfortably.

2. Open Word.

3. Open *DERIVE*. One way to do this is to do in the following way.

   a) Activate the Program Manager.
   b) Choose Run in the File menu.
   c) Type the pathname and DERIVE.EXE. (E.g. c:\Derive\Derive.exe)

(A fact which has amused me, is that you can have more than one copy of *DERIVE* open at the same time. You just have to follow points a) to c) once more. *DERIVE* is really such a good program that it could sometimes be worth while to have many copies of it open at the same time.)

4. During your *DERIVE*-session you have to run *DERIVE* in **Full Screen** mode It is also possible to run *DERIVE* or any non Windows application in an **Application window**. You can change from one mode to the other by pressing **Alt+Enter**.

5. When time has come to copy your formulas and plots to Word you have to have *DERIVE* in graphics mode. Now press **Alt+Enter** in order to have *DERIVE* in a window. In the left upper corner of the *DERIVE* window you have a bar, the so called **Control-menu box**. Click on this with the mouse. A menu emerges. Select **Edit** and then **Mark**.

6. Now you can mark what you want to copy to **Word** with the mouse or with the arrow keys.

7. When you have marked what you want to copy to Word, press **Enter**. Now the information you want to copy is in the clip-board of Windows.

8. Activate Word. Move the cursor to the place in your Word document, where you want to have the information from *DERIVE*. Choose **Edit Paste** and you will hopefully find that you have successfully copied what you wanted to copy from *DERIVE* to Word.

Arney, *Exploring Calculus with DERIVE*, Addison-Wesley, ISBN 0-201-52839-8

DeMarois, *College Algebra Laboratories Using Derive*, MathWare, 604 East Mumford Drive, Urbana, IL, 61801, USA

Harper, Woof & Hodgkinson, *A Guide to Computer Algebra Systems*, John Wiley, ISBN 0-471-92910-7

Johnson & Evans, *Discovering Calculus with Derive*, John Wiley, ISBN 0-471-55155-4

Salter & Gilligan, *Linear Algebra Experiments Using the Derive Program*, Gilmar Publishing Company, P.O. Box 6376, Cincinatti, OH, 45206, USA

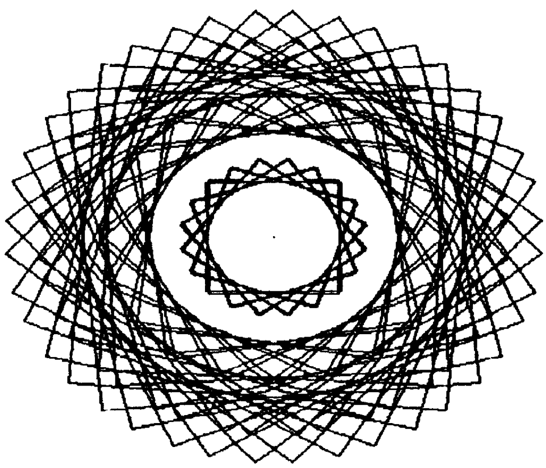
Wasen & Sjöstrand, *Svensk Handledning med Ovningsuppgifter och Introduktion till Algebraisk Modellering och Datoralgebra: Derive*, GraftiTexData, Storgatan 11, 590 40 Kisa, Sweden

Marlewski, *Derive Pomocnik matematyczny wersja 2.0*, Wydawnictwo NAKOM, Poznan, Poland. ISBN 83-85060-49-9;A polish manual, with a lot of examples (355 p.)

Scheu, *Arbeitsbuch Computer-Algebra mit DERIVE*, Dümmler, Bonn, ISBN 3-427-45721-4
Beispiele, Algorithmen und Aufgaben aus der Schulmathematik
aus dem Inhalt: Kurvenscharen, Newtonverfahren, Regula falsi,
Grenzwerte, Programmieren mit *DERIVE*, Experimente mit
Primzahlen, ...

Scheu, *Entdeckungen in der Menge der Primzahlen mit DERIVE*, PM 34, (Heft 3) 1992

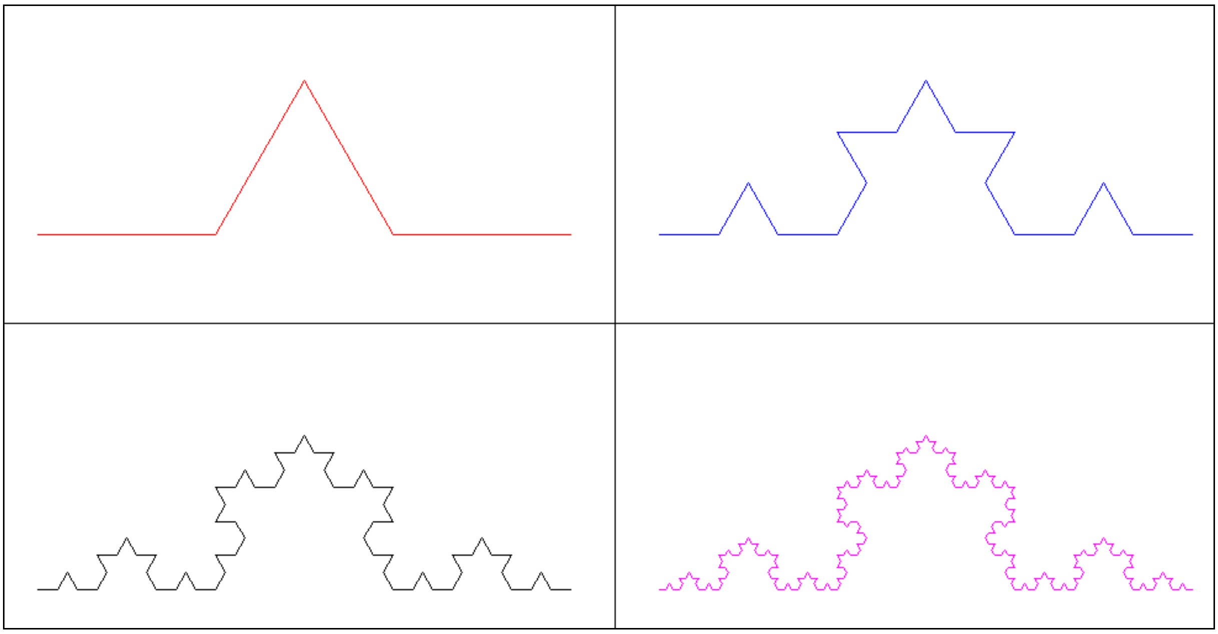
Hischer (Hrsg.), *Mathematikunterricht im Umbruch? Erörterungen zur möglichen „Trivialisierung" von mathematischen Gebieten durch Hardware und Software*; Proceedings, Verlag Franzbecker, Hildesheim 1992.
A lot of lectures concerning *DERIVE* (in German).



This time we open Mr Setif´s Treasure Box and there we can find some jewels glittering like Fractals.

`APV(u,v)` *was Mr Setif´s function to append vectors. Later* `APPEND(u,v)` *was implemented in DERIVE. So one can replace all* `APV`*-appearances* by `APPEND`.

```
APV(u,v):=VECTOR(IF(m_≤DIMENSION(u),ELEMENT(u,m_),
                 ELEMENT(v,m_-DIMENSION(u))),m_,DIMENSION(u)+DIMENSION(v))

FL(x,y,l,a,m,n):=IF(n<0,[],
                    IF(n,[[x+l·COS(m·π/3),y+l·SIN(m·π/3)]],
                       APV(FL(x,y,l/3,m,n-1),
                       APV(FL(x+l·COS(m·π/3)/3,y+l·SIN(m·π/3)/3,l/3,m+1,n-1),
                       APV(FL(x+l·(3·COS(m·π/3)-√3·SIN(m·π/3))/6,
                             y+l·(3·SIN(m·π/3)+√3·COS(m·π/3))/6,l/3,m-1,n-1),
                       FL(x+2·l·COS(m·π/3)/3,y+2·l·SIN(m·π/3)/3,l/3,m,n-1))))))

FLK(x,y,l,m,n):=APV([[x,y]],FL(x,y,l,m,n))

FLA(x,y,l,a,m,n):= IF(n<0,[],
                 IF(n,[[x+l·COS(a+m·π/3),y+l·SIN(a+m·π/3)]],
                 APV(FLA(x,y,l/3,a,m,n-1),
                 APV(FLA(x+l·COS(a+m·π/3)/3,y+l·SIN(a+m·π/3)/3,l/3,a,m+1,n-1),
                 APV(FLA(x+l·(3·COS(a+m·π/3)-√3·SIN(a+m·π/3))/6,
                         y+l·(3·SIN(a+m·π/3)+√3·COS(a+m·π/3))/6,l/3,a,m-1,n-1),
                 LA(x+2·l·COS(a+m·π/3)/3,y+2·l·SIN(a+m·π/3)/3,l/3,a,m,n-1))))))

FLKA(x,y,l,a,m,n):=APV([[x,y]],FLA(x,y,l,a,m,n))

FLAKE_R(x,y,l,n):=[FLKA(x,y,l,0,0,n),FLKA(x+l,y,l,2·π/3,0,n),FLKA(x+l/2,
                   y+l·√3/2,l,4·π/3,0,n)]

FLAKE_S(x,y,l,n):= [FLKA(x,y,l,π/3,0,n),FLKA(x+l/2,y+l·√3/2,l,-π/3,0,n),
                    FLKA(x+l,y,l,π,0,n)]

Picture 1 (for the DOS-version)

FLK(-3,0,6,0,1)
FLK(-3,0,6,0,2)
FLK(-3,0,6,0,3)
FLK(-3,0,6,0,4)

Picture 1 (for DERIVE 6)

FLK(-6.5,1,6,0,1)
FLK(0.5,1,6,0,2)
FLK(-6.5,-3,6,0,3)
FLK(0.5,-3,6,0,4)
```



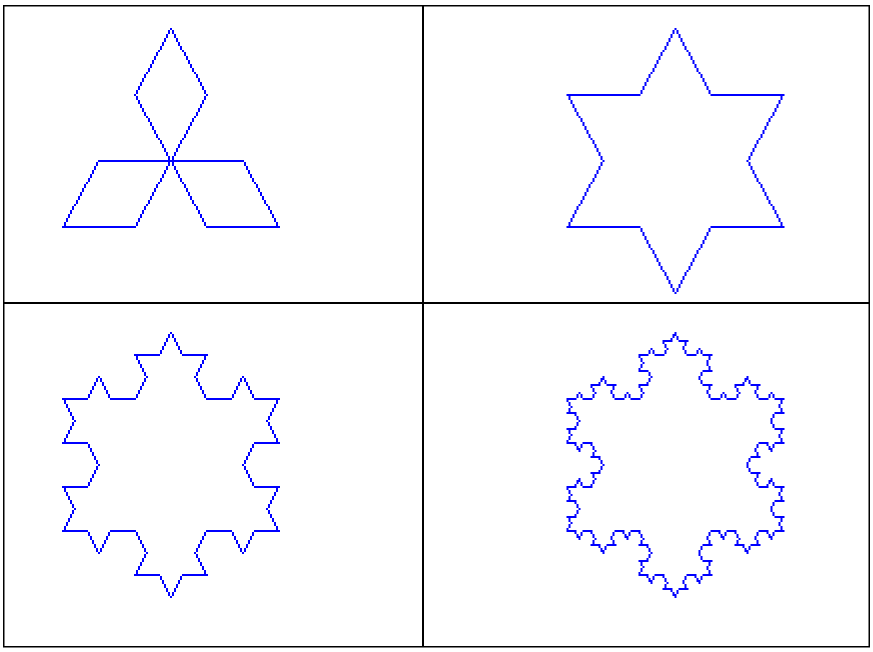
Picture 2

$$\left[\mathrm{FLKA}\left(-1,\ -1,\ 3,\ \frac{\pi}{3},\ 0,\ 3\right),\ \mathrm{FLKA}\left(\frac{1}{2},\ \frac{3\cdot\sqrt{3}}{2}-1,\ 3,\ -\frac{\pi}{3},\ 0,\ 3\right),\ \mathrm{FLKA}(2,\ -1,\ 3,\ \pi,\ 0,\ 3)\right]$$

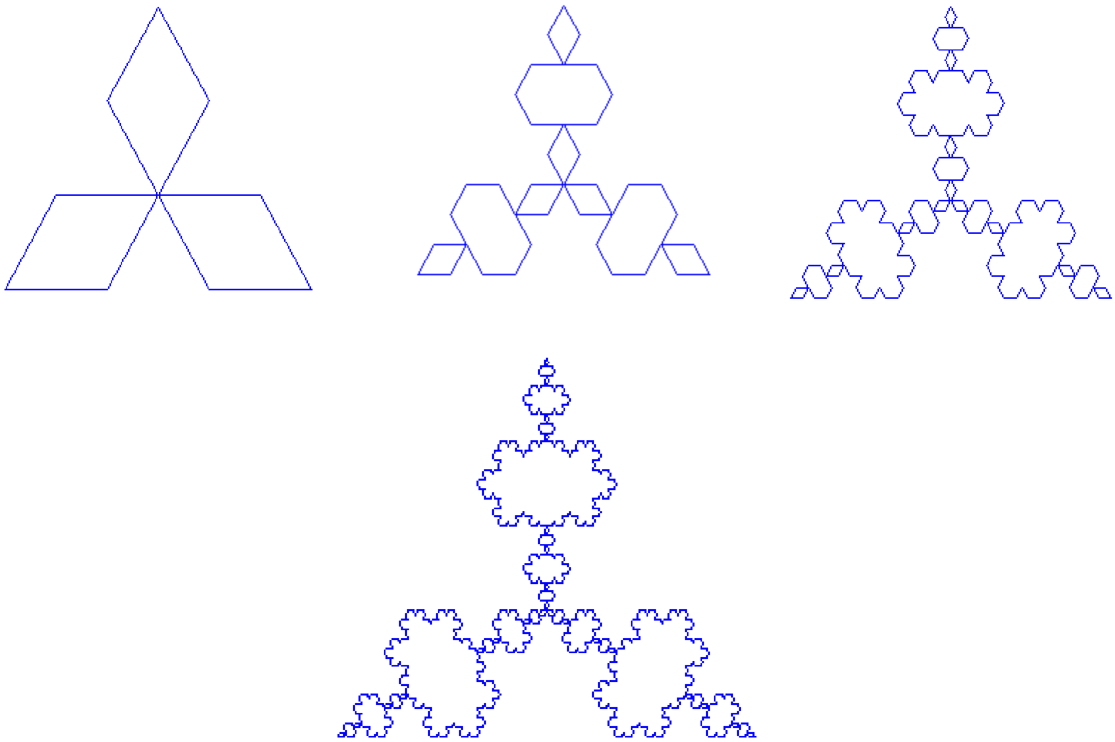
$$\left[\mathrm{FLKA}\left(2,\ +1,\ 3,\ \frac{\pi}{3},\ 0,\ 1\right),\ \mathrm{FLKA}\left(\frac{1}{2}+3,\ \frac{3\cdot\sqrt{3}}{2}+1,\ 3,\ -\frac{\pi}{3},\ 0,\ 1\right),\ \mathrm{FLKA}(5,\ +1,\ 3,\ \pi,\ 0,\ 1)\right]$$

$$\left[\mathrm{FLKA}\left(-5,\ -3,\ 3,\ \frac{\pi}{3},\ 0,\ 2\right),\ \mathrm{FLKA}\left(\frac{1}{2}-4,\ \frac{3\cdot\sqrt{3}}{2}-3,\ 3,\ -\frac{\pi}{3},\ 0,\ 2\right),\ \mathrm{FLKA}(-2,\ -3,\ 3,\ \pi,\ 0,\ 2)\right]$$

$$\left[\mathrm{FLKA}\left(2,\ -3,\ 3,\ \frac{\pi}{3},\ 0,\ 3\right),\ \mathrm{FLKA}\left(\frac{1}{2}+3,\ \frac{3\cdot\sqrt{3}}{2}-3,\ 3,\ -\frac{\pi}{3},\ 0,\ 3\right),\ \mathrm{FLKA}(5,\ -3,\ 3,\ \pi,\ 0,\ 3)\right]$$

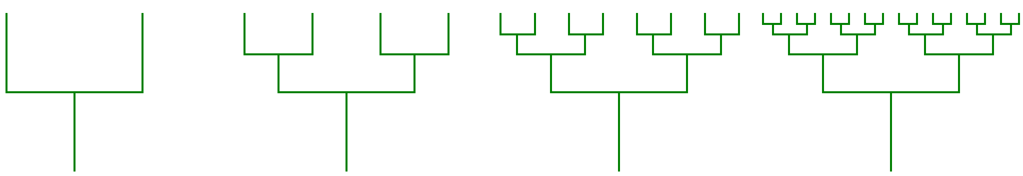


```
Picture 3
FLAKE_R(-1, -1, 3, 1)
FLAKE_R(-1, -1, 3, 2)
FLAKE_R(-1, -1, 3, 3)
FLAKE_R(-1, -1, 3, 4)
```



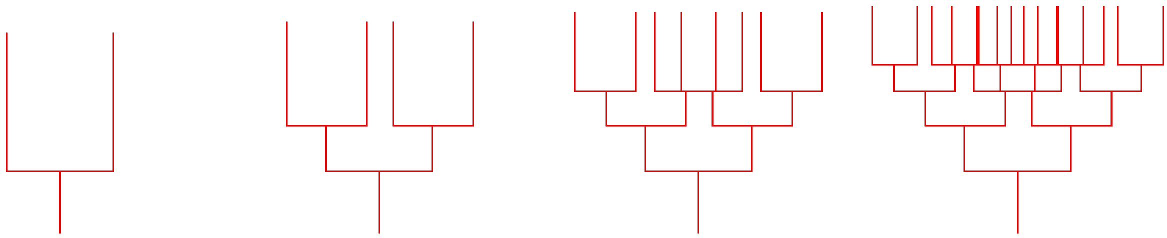
First we calculated some iterations of the Koch-curve. The next file shows fractal candelabers:

```
CAND(x,y,l,n):= IF(n,[x, y + 2·l; x, y],
                   APV([x, y + l; x + l, y + l],
                          CAND(x + l, y + l, l/2, n - 1), [[x - l, y + l]],
                          CAND(x - l, y + l, l/2, n - 1), [x, y + l; x, y]))

ml = multiply length branch
mc = multiply length candle

CANDMM(x,y,l,ml,mc,n):= IF(n,[x, y + mc·l; x, y],
                           APPEND([x, y + l; x + l, y + l],
                                  CANDMM(x + l, y + l, ml·l, ml, mc, n - 1),
                                  [[x - l, y + l]],
                                  CANDMM(x - l, y + l, ml·l, ml, mc, n - 1),
                                  [x, y + l; x, y]))

picture 4

[CAND(-6, -1, 1, 1), CAND(-2, -1, 1, 2), CAND(2, -1, 1, 3), CAND(6, -1, 1, 4)]
```



Picture 5

[CANDMM(-9, -2, 1, 0.75, 3, 1), CANDMM(-3, -2, 1, 0.75, 3, 2),
 CANDMM(3, -2, 1, 0.75, 3, 3), CANDMM(9, -2, 1, 0.75, 3, 4)]

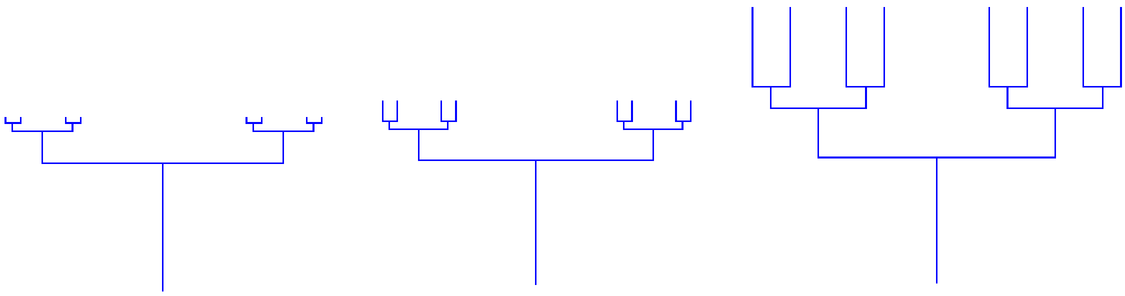


Picture 6

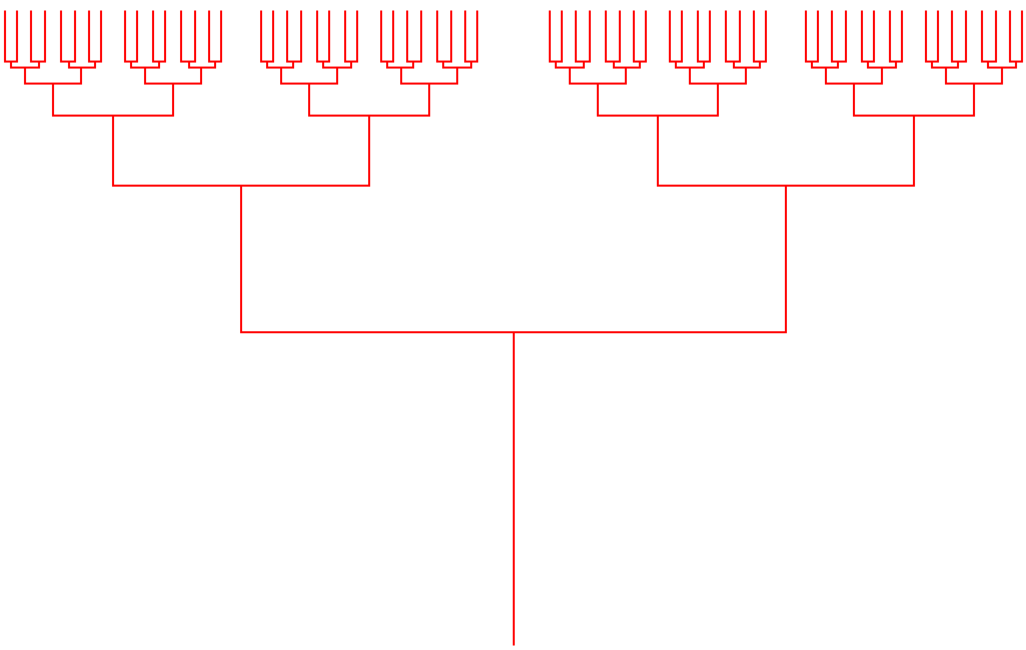
CANDMM(0, -2, 1, 0.25, 3, 3)

CANDMM(0, -2, 1, 0.25, 10, 3)

CANDMM(0, -2, 1, 0.4, 10, 3)



CANDMM(0, -2, 1, 0.47, 15, 6)



Trickfilme mit *DERIVE*
Graphisches Differenzieren

Hans-Jürgen Kayser, Düsseldorf

DERIVE ist hervorragend zur Erstellung veranschaulichender Bildsequenzen geeignet; dies soll hier am Beispiel eines Kurzfilms zum graphischen Differenzieren gezeigt werden. Unter Verwendung der Hilfsfunktionen `LINE`, `TANGENT`, `PUNKT` und `STELLEN` definieren wir die Funktion `GD_SCHAR`, bei der die Parameter *f* (für die zu differenzierende Funktion), *a*, *b* (für die Intervallgrenzen) und *n* (für die Anzahl der Teilintervalle) gewählt werden können, so dass dem Funktionsbenutzer noch genügend Spielraum bleibt.

Die hier abgedruckten Bilder können DERIVEs Leistungen als Filmproduzent naturgemäß nur sehr unvollkommen wiedergeben. Der Leser sollte lieber DERIVE starten, die wesentlichen der im Listing abgedruckten Zeilen (#3, #5, #7, #9 und #14 sowie #19, #24, #25, #26) eingeben und die Filme starten. Das Abschreiben ist bekanntlich schnell erledigt, wenn man die F3- und F4-Taste geschickt nutzt.

Noch ein Tip: Die Ablaufgeschwindigkeit der Filme lässt sich durch den Parameter Accuracy unter Plot-Options beeinflussen.

Und nun viel Vergnügen!

Das Drehbuch:

DERIVE is an excellent tool for producing illustrating sequences of graphs. We will demonstrate this by an example of a short film illustrating graphic differentiation. By using auxiliary functions LINE, TANGENT, PUNKT (point) and STELLEN (places) we define the function GD_SCHAR (gd_family). We can choose parameters *f* (the given function), *a* and *b* (the given interval) and *n* (the number of partial intervals) in order to leave room for investigations.

Much fun, the script follwows:

The other comments are with respect to *DERIVE* for DOS. I´ll come back to them later. Josef

#1: ---------------------- Graphisches Differenzieren --------------------

#2: Wir definieren die Funktionen LINE, TANGENT, PUNKT, STELLEN und GD_SCHAR:

#3: $\mathrm{LINE}(x0,\ y0,\ d1,\ x) := d1\cdot x + y0 - d1\cdot x0$

#4: (Hilfsfunktion für TANGENT)

#5: $\mathrm{TANGENT}(u,\ x,\ x0) := \mathrm{LINE}\left(x0,\ \lim_{x\to x0} u,\ \lim_{x\to x0} \frac{d}{dx} u,\ x\right)$

#6: (Term für die Tangente in B(x0;u(x0)) an den Graphen von u)

#7: $\mathrm{PUNKT}(u,\ x0) := \left[x0,\ \lim_{x\to x0} u\right]$

#8: (Punkt auf dem Graphen von u an der Stelle x0)

#9: $\mathrm{STELLEN}(a,\ b,\ n) := \mathrm{VECTOR}\left(a + \frac{(k - 1)\cdot(b - a)}{n},\ k,\ 1,\ n + 1\right)$

#10: (Stellen auf der x-Achse; a: Start, b: Ziel, n: Anzahl der Teilintervalle)

```
#11:  Beispiel:

#12:  STELLEN(2, 5, 10)

#13:  [2, 2.3, 2.6, 2.9, 3.2, 3.5, 3.8, 4.1, 4.4, 4.7, 5]
```

$$\#14:\ \text{GD\_SCHAR}(f, a, b, n) := \left[f,\ \text{VECTOR}\left(\left[\text{PUNKT}(f, \text{ELEMENT}(\text{STELLEN}(a, b, n), k)),\ \text{TANGENT}(f, x, \text{ELEMENT}(\text{STELLEN}(a, b, n), k)),\ \text{PUNKT}\left(\frac{d}{dx} f, \text{ELEMENT}(\text{STELLEN}(a, b, n), k)\right)\right], k, 1, n + 1\right), \frac{d}{dx} f\right]$$

Zeichnet den Graphen von f, n äquidistante Berührpunkte über [a;b], die Tangenten an den Graphen von f in diesen Punkten, Ableitungspunkte an den Berührstellen und zum Schluß den Ableitungsgraphen.

Plots the graph of f, points on the graph in [a;b], the tangents in this points, the points of the 1st derivative and finally the graph of the first derivative.

```
#15:  Beispiel 1: / Example 1:
```

$$\#16:\ \text{GD\_SCHAR}(x^2, -2, 2, 10)$$

```
#17:  X für approX ergibt:
```

$$\#18:\ \left[x^2,\ \begin{bmatrix} [-2, 4] & -4\cdot x - 4 & [-2, -4] \\ [-1.6, 2.56] & -3.2\cdot x - 2.56 & [-1.6, -3.2] \\ [-1.2, 1.44] & -2.4\cdot x - 1.44 & [-1.2, -2.4] \\ [-0.8, 0.64] & -1.6\cdot x - 0.64 & [-0.8, -1.6] \\ [-0.4, 0.16] & -0.8\cdot x - 0.16 & [-0.4, -0.8] \\ [0, 0] & 0 & [0, 0] \\ [0.4, 0.16] & 0.8\cdot x - 0.16 & [0.4, 0.8] \\ [0.8, 0.64] & 1.6\cdot x - 0.64 & [0.8, 1.6] \\ [1.2, 1.44] & 2.4\cdot x - 1.44 & [1.2, 2.4] \\ [1.6, 2.56] & 3.2\cdot x - 2.56 & [1.6, 3.2] \\ [2, 4] & 4\cdot x - 4 & [2, 4] \end{bmatrix},\ 2\cdot x\right]$$

Fig. 1 - Fig. 3: Drei Momentaufnahmen des Films GD_SCHAR (x^2, -2, 2, 10).

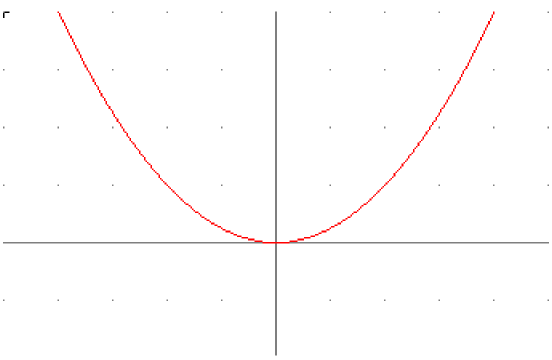


Fig. 1: Kurve / graph

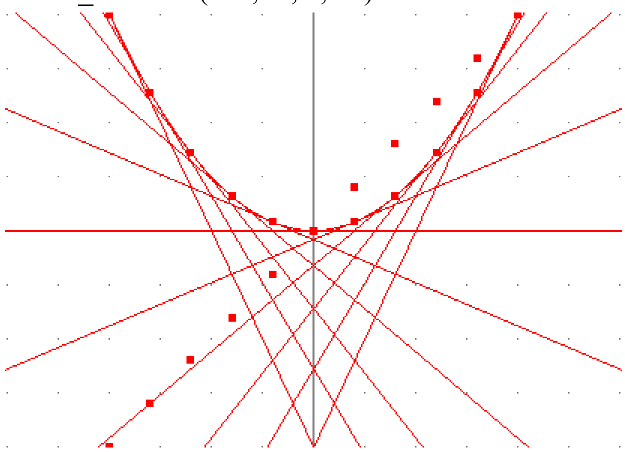
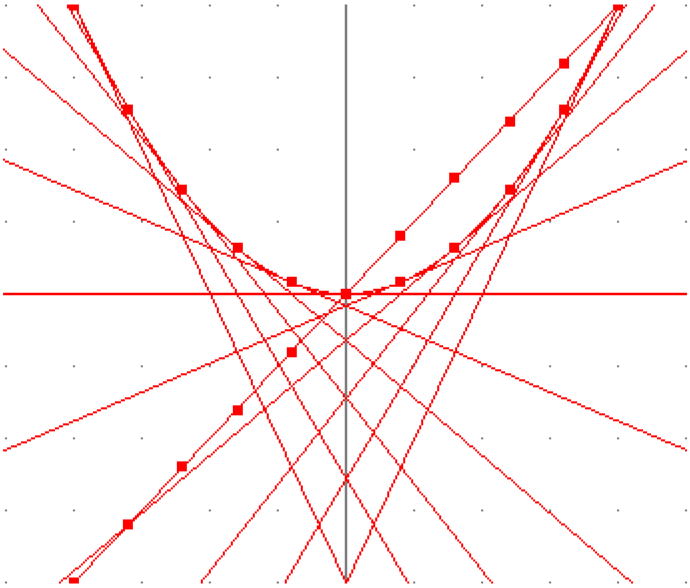


Fig. 2: Kurve mit Tangenten und Ableitungspunkten
Graph, tangents and points of 1st derivative

Fig. 3: Kurve mit Tangenten, Ableitungspunkten und erster Ableitung.

Graph, tangents, points of 1st derivative and derivative.



Drei Schlussszenen .- three "great movie-finals"

aus "GD_Schar(x3, -2.5, 2.5, 50)",
aus "GD_Schar(exp(x), -2.5, 2.5, 20)",
aus "GD_Schar(sin(x), -x, x, 10)"

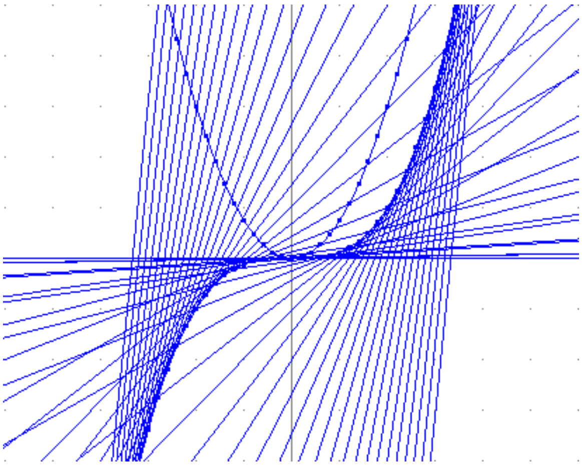


Fig. 4: Ein Hauch von Ästhetik

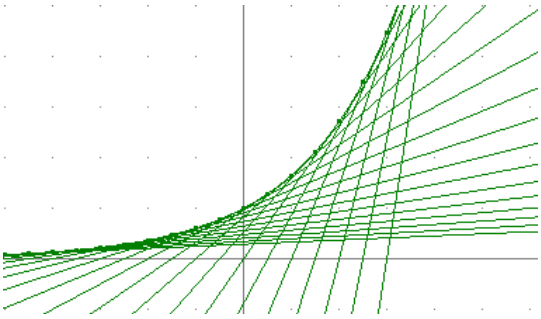


Fig. 5: Was fällt bei der Exponentialfunktion auf?

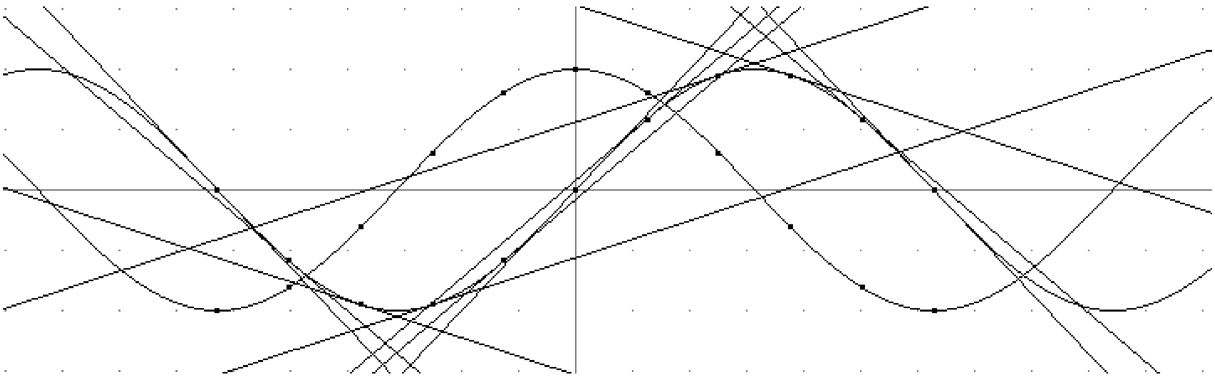


Fig. 6: Vorbereitung einer wichtigen Ableitungsformel / Preparation of an important 1st derivative

This was the original contribution from 1992. At this time we had slow computers and we could control plotting velocity by changing plotting accuracy. So the plotting process displayed as a movie. Now we have very fast computers with fast graphic adapters and we cannot influence plotting velocity. So we have to miss the nice movie?

No, we don´t. See the following trick: You must create your own delay function. Plotting shaded areas takes some time. So let plot one or more shaded circles far outside of the plot region which is presented on the screen.

#24: $\text{delay}(n) := \text{VECTOR}((x - 100)^2 + (y - 100)^2 \le \text{RANDOM}(1) + k,\ k,\ n)$

#25: $\text{GD\_SCHAR\_del}(f,\ a,\ b,\ n,\ t := 0) := \left[f,\ \text{delay}(t),\ \text{VECTOR}\left(\left[\text{PUNKT}(f,\ \text{ELEMENT}(\text{STELLEN}(a,\ b,\ n),\ k)),\ \text{TANGENT}(f,\ x,\ \text{ELEMENT}(\text{STELLEN}(a,\ b,\ n),\ k)),\ \text{PUNKT}\left(\frac{d}{dx} f,\ \text{ELEMENT}(\text{STELLEN}(a,\ b,\ n),\ k)\right),\ \text{delay}(t)\right],\ k,\ 1,\ n + 1\right),\ \frac{d}{dx} f\right]$

`delay(n)` plots *n* shaded circles far away. The last parameter of the function controls the plot velocity. Omitting this parameter results in a picture – without the movie-effect. *t* = 1, 2 leads to nice animations.

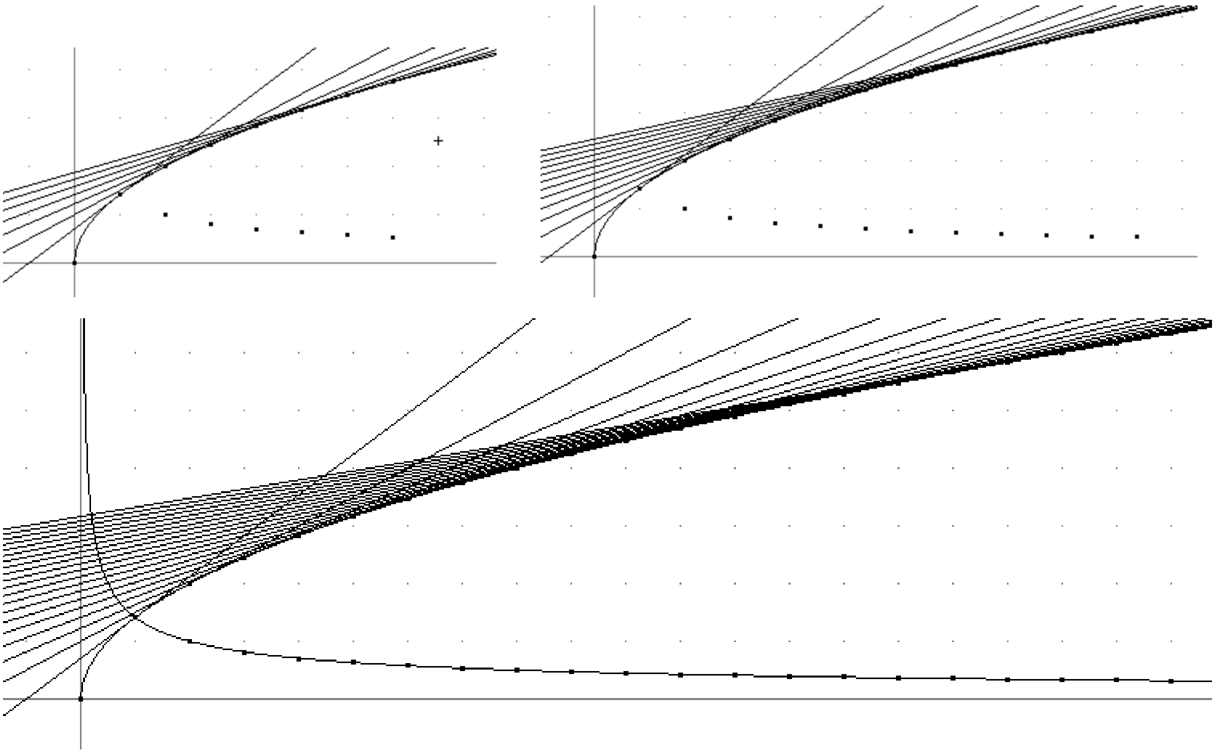
It is worth a try! (Note: Don´t simplify the expressions in the Algebra Window, activate Simplify before plotting in the 2D-Plt Window.)

#26: $\text{GD\_SCHAR\_del}(x^2,\ -2,\ 2,\ 10)$

#27: $\text{GD\_SCHAR\_del}(x^2,\ -2,\ 2,\ 10,\ 1)$

#28: $\text{GD\_SCHAR\_del}(x^2,\ -2,\ 2,\ 10,\ 2)$

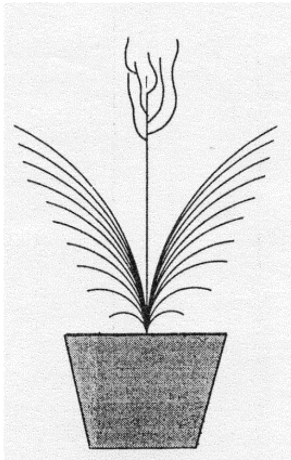
Three stills from the exciting movie `GD_SCHAR_del(√x, 0, 10, 20, 1)`



Nonstandard

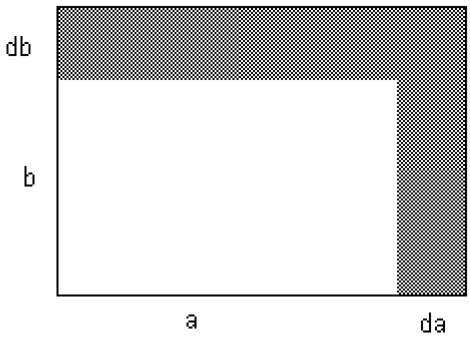
ANALYIS
wieder
ein Versuch

Helmut Wunderling
Berlin



Produktregel:

$$f(x) = g(x) \cdot h(x)$$
$$y = a \cdot b$$
$$\frac{dy}{dx} = ?$$



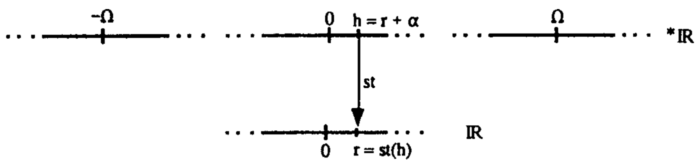
$$dy = da \cdot b + db \cdot a + da \cdot db$$
$$\frac{dy}{dx} = \frac{da}{dx} \cdot b + \frac{db}{dx} \cdot a + \frac{da}{dx} \cdot db$$
$$f'(x) = g'(x) \cdot h(x) + h'(x) \cdot g(x) + g'(x) \cdot db$$

WICHTIG: Differentiale *führen* zum „bewiesenen“ Ergebnis!

Standardanteil:

Jede endliche hyperrrelle Zahl ist eindeutig als Summe einer reellen Zahl und einer infinitesimal kleinen Zahl darstellbar.

Eindeutigkeit:
Gäbe es für eine hyperrelle Zahl h zwei Darstellungen:
$h = r + \alpha$ und $h = s + ß$, wäre $r - s = ß - \alpha$.
Dabei ist $r - s$ reell und $ß - \alpha$ infinitesimal klein, also muss $r - s = 0$ und $ß - \alpha = 0$ sein,
denn 0 ist die einzige infinitesimal kleine Zahl.



Rückschau auf die Produktregel:

$$
\begin{aligned}
f'(x) &= st\left(\frac{dy}{dx}\right) \\
&= st\left(\frac{da}{dx}\cdot b + \frac{db}{dx}\cdot a + \frac{da}{dx}\cdot db\right) \\
&= st\left(\frac{da}{dx}\cdot b\right) + st\left(\frac{db}{dx}\cdot a\right) + st\left(\frac{da}{dx}\cdot db\right) \\
&= st\left(\frac{da}{dx}\right)\cdot st(b) + st\left(\frac{db}{dx}\right)\cdot st(a) + st\left(\frac{da}{dx}\right)\cdot st(db) \\
&= g'(x)\cdot f(x) + f'(x)\cdot g(x) + g'(x)\cdot 0
\end{aligned}
$$

Hyperreelle Zahlen:

Konstruktion der Zahlenmengen aus den natürlichen Zahlen:

a) Äquivalenzklassenbildung
(welches Kriterium ist jeweils brauchbar?)

b) Definition der (erweiteren) Rechenarten
(unabhängig von der Wahl der Repräsentanten möglich?)

c) Einbettung der bisherigen Zahlen
(was wird identifiziert?)

(1, 2, 3, 0, 0, 0, ...) · (0, 0, 0, 4, 4, 4, ...) = (0, 0, 0, 0, 0, 0, ...)
NULLTEILERKATASTROPHE!!

(1, 2, 3, 0, 0, 0, ...) :≡ (0, 0, 0, 0, 0, 0, ...)
(0, 0, 0, 4, 4, 4, ...) :≡ (4, 4, 4, 4, 4, 4, ...)

(0, 1, 0, 1, 0, 1, 0, ...) · (1, 0, 1, 0, 1, 0, 1, ...) = (0, 0, 0, 0, 0, 0, 0, ...)
NULLTEILERKATASTROPHE!!

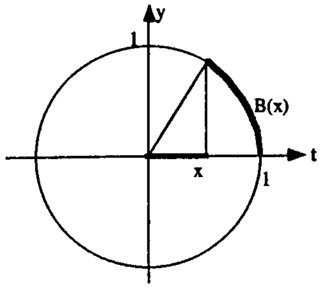
(0, 1, 0, 1, 0, 1, 0, ...) :≡ (0, 0, 0, 0, 0, 0, 0, ...)
(1, 0, 1, 0, 1, 0, 1,...) :≡ (1, 1, 1, 1, 1, 1, 1, ...)

Grundsatz:
Es gibt ein Entscheidungsmaß *m*, das jeder Menge natürlicher Zahlen entweder die Zahl 0 oder 1 zuordnet und folgende Eigenschaften hat:

a) $\boldsymbol{m}(\mathbb{N}) = 1$
b) **T endlich ⇒ *m*(T) = 0**
c) **T ∩ S = ∅ ⇒ *m*(T ∪ S) = *m*(T) + *m*(S)**

Definition:
Zwei Folgen heißen äquivalent genau dann, wenn hinreichend viele Fogenglieder übereinstimmen.
“hinreichend viel”:
Die Menge der natürlichen Zahlen, die übereinstimmende Folgenglieder bezeichnen, hat das Entscheidungsmaß 1.
Jede Äquivalenzklasse reeller Zahlenfolgen ist eine hyperrelle Zahl.

Ableitung der Kreisfunktionen:



$B(x)$ ist die Bogenlänge auf dem Einheitskreis $\{x \in [-1,1]\}$ =
= Summe von Teilbögen.

$B(x) = \int_x^1 db$.

$db = \sqrt{dt^2 + dy^2}$ und $t^2 + y^2 = 1$,

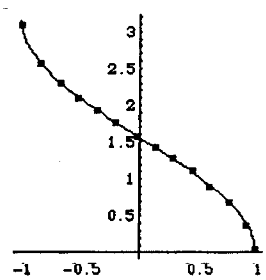
$f(t) := y = \sqrt{1-t^2}, \quad db = \sqrt{1 + \left(\frac{df(t)}{dt}\right)^2} \cdot dt.$

μ Anzahl der Summanden $\Rightarrow$ $dt = \frac{(1-x)}{\mu}$

Bogenlänge = $\sum_{i=1}^{\mu} \sqrt{1 + \left(\frac{df(t)}{dt}\right)^2} \cdot dt$ mit $df(t) = df(t_i) = f(x + i \cdot dt) - f(x + (i-1) \cdot dt)$.

$$B(x) = \int_x^1 \sqrt{1 + \left(\frac{df(t)}{dt}\right)^2} \cdot dt := st\left(\sum_{i=1}^{\mu} \sqrt{1 + \left(\frac{df(t)}{dt}\right)^2} \cdot dt\right).$$

DERIVE



$x \in [-1,1]$
Punkte des Graphen zu

$B(x) = \int_x^1 \sqrt{1 + \left(\frac{df(t)}{dt}\right)^2} \cdot dt$

arccos(x) := $B(x)$
cos (erweiterte) Umkehrfunktion dazu

Ableitung von arccos.

$B(x) = \int_x^1 W(t) \cdot dt = st\left(\sum_{i=1}^{\mu} W(x + (i-1) \cdot dt) \cdot dt\right)$ $\qquad [\mu \cdot dt = 1 - x]$

$\frac{dB}{dx}$? $\qquad dB = B(x + dx) - B(x) \qquad dx := dt.$

$dB = \sum_{i=2}^{\mu} W(x + (i-1) \cdot dt) \cdot dt - \sum_{i=1}^{\mu} W(x + (i-1) \cdot dt) \cdot dt = W(x) \cdot dt$
$= W(x) \cdot dx.$

$$B'(x) = st\left(\frac{dB}{dx}\right) = W(x) = -\frac{1}{\sqrt{1-x^2}}$$

DERIVE lässt sich auch zur (ANALYSIS) Infinitesimalrechnung nutzen.

Der Standardteil lässt sich wie folgt definieren:

#1: FFFFF(x) :=

#2: $ST(fffff, \alpha\alpha, \beta\beta, \delta\delta) := \lim_{\delta\delta \to 0} \lim_{\beta\beta \to 0} \lim_{\alpha\alpha \to 0} fffff$

Definition der 1. Ableitung F1 als Standardanteil des Differentialquotienten

Definition of 1st derivative F1 as standard part of differential quotient

#3: F(x) :=

Differentialquotient

#4: $DQ(f, a, da) := \dfrac{\left(\lim_{x \to a + da} f\right) - \lim_{x \to a} f}{da}$

#5: F1(f, b, db) := ST(DQ(f, b, db), db)

#6: $F1(x^3, u, du)$

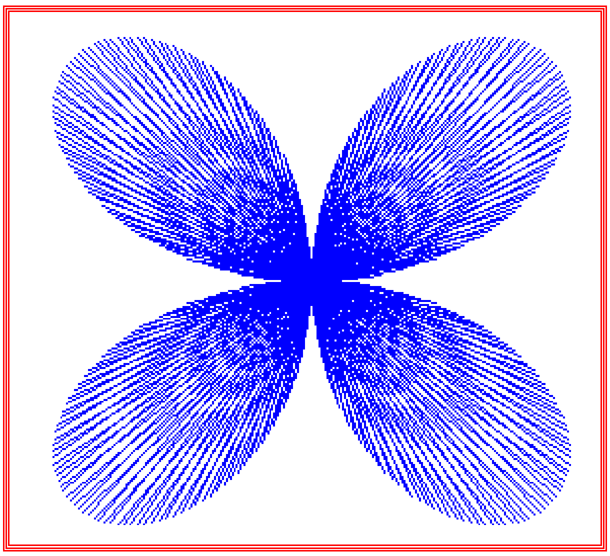
#7: $3 \cdot u^2$

Integration als Standardanteil ebenfalls für nicht zu komplexe Terme möglich.

Integration as standard part for not too complex expressions possible.

Comment: Non-standard analysis was introduced in the early 1960s by the mathematician Abraham Robinson.

Related websites: en.wikipedia.org/wiki/Non-standard_analysis
en.wikipedia.org/wiki/Non-standard_calculus
www.math.wisc.edu/~keisler/calc.html
mathforum.org/dr.math/faq/analysis_hyperreals.html



Wunschliste für Verbesserungen

Wishlist for Improvements

Herbert Appel, Schweinfurt

Wie bereits im DNL#5 berichtet, wurden an der staatlichen Realschule in Forchheim einige Unterrichtssequenzen zum Thema „DERIVE im Mathematikunterricht“ durchgeführt. Meine Beobachtungen während dieser Unterrichtsstunden und die abschließende Befragung der Schüler brachte einige neue Erkenntnisse, die hier aufgezählt werden sollen. Sie sind als Anregungen bei der Weiterentwicklung von DERIVE und als Forderungen an derartige Programme im Allgemeinen zu verstehen, um diese auch für den Einsatz im Unterricht der Realschule bzw. der Sekundarstufe I geeignet zu machen.

Obwohl sich die Schüler durchwegs positiv über das Programm äußerten und sich vorstellen konnten, auch in Zukunft mit dem Programm zu arbeiten, ertönten doch auch kritische Stimmen.

Forderungen bezüglich der Software:

- Am häufigsten wurden die schwer merkbaren englischen Befehle beanstandet. Weiterhin wurde beklagt, dass das Programm nicht über Menüs und Maus bedienbar ist. Die Schüler sind von Computern (APPLE, AMIGA, ATARI, MS-Windows etc.) anderes (besseres?) gewohnt.
(The students want to work with menus and to use the mouse.)

- Schüler tendieren dazu, den Lehrer auf seine Kompetenz hin überprüfen zu wollen. Nicht benötigte Menüpunkte müssen abschaltbar sein!
(The teacher should be able to switch off menu options which are not needed.)

- Die Handhabung des Insert/Overwrite-Modus ist bei einigen Menüpunkten inkonsequent. Die zu ersetzende Variable wird wird trotz "Overwrite" beim Kopieren mit F3 bei Manage Substitute an das Zeilenende verschoben.
(When using the "Overwrite Mode" and copying with F3 to Manage Substitute the expression does not overwrite the variable.)

- Die Handhabung der Befehle Center, Zoom, Scale etc. ist unökonomisch, langwierig und daher unbefriedigend (Einsatz der Maus).
(Working with Center, Zoom, Scale etc. is boring and unsatisfying - could be done better using a mouse.)

Für den Mathematiker stellen viele Eigenheiten des Programms DERIVE keine Schwierigkeiten dar; anders ist es bei unseren Schülern. In theoretischen Überlegungen wird meist vom idealtypischen, d.h. von einem an der Mathematik interessierten, konzentriert arbeitenden, seine Gedanken auf den Problemlöseprozess ausrichtenden Schüler ausgegangen. An den allgemeinbildenden Schulen treffen wir jedoch den „realen“ Schüler an, der häufig bereits bei einfachen Lerninhalten Schwierigkeiten hat. Der Teufel steckt wie so oft im Detail.

Weitere Forderungen und Anregungen:

- Die Sichtbarkeit des Multiplikationspunkts sollte einschaltbar sein. (Bsp.: 3·x·y oder 2·2 anstelle von 3 x y und 2 2.)
(It should be possible to switch on the visibility of the multiplication point - 3·x·y or 2·2 instead of 3 x y und 2 2)

- Die Darstellung der Klammern (runde statt der eckigen und geschweiften, bzw. eckige Klammern statt der runden) weicht von den allgemeinen Konventionen ab.
(The use of parentheses and brackets does not follow the usual conventions.)

- Gleichungen der Form xy/z = wy/v werden nicht automatisch weiter vereinfacht, auch dann nicht, wenn y als „positive" deklariert wurde.
(Equations of the form xy/z = wy/z will not be simplified automatically.)

- Es sollte möglich sein, die Variablen (z.B. für das Verfahren der Parametertransformation $x' = 3x + 2$ und $y' = x^2 - 5$ mit einem " ' " versehen zu können.
(It should be possible to write variables with a dash - x',y'.)

- Die Schreibweise (2x = 4)/2, die beim schrittweisen Umformen der Gleichungen verwendet wird, ist völlig unüblich.
(Writing (2x = 4)/2 used when transforming an equation step by step is not common.)

- Umformungen an trigonometrischen Ausdrücken bleiben für Schüler durch die Verwendung der Signum- und Absolutwertfunktion schwer durchschaubar.
(Transformations of trigonometric expressions are difficult for pupils because of the use of the SIGN- and the ABS-function.)

- Es ist unbedingt nötig vom Bogenmaß ins Gradmaß umschalten zu können, da Schüler kaum mit der Bogenmaßdarstellung arbeiten. [1)]
(It must be possible to switch from rad to degree, because pupils are not accustomed to work with rad.) [1)]

- Der in der Geometrie häufig benutzte griechische Buchstabe γ kann wegen der bereits vorhandenen Γ-Funktion nicht für Winkelbezeichnungen verwendet werden.
(Letter γ cannot be used for naming angles, because of the Γ-function.)

- Die Angabe der Intervallgrenzen beim Zeichnen eines Graphen sollte handlicher gestaltet werden. Insbesondere wäre eine Beschriftbarkeit des Graphen wünschenswert.
(The declaration of an interval for plotting a graph should be more comfortable, annotations of graphs would be great.)

- Werden mit BUILD zwei Terme voneinander subtrahiert, wobei der zweite ein Bruch mit negativem Vorzeichen ist, so folgen in der Darstellung zwei Rechenzeichen aufeinander.

(Subtracting two expressions using the BUILD-command, we find two following signs, in case of the 2nd expression being negative.)

(I tried to translate Mr. Appels wishes for the non-german speaking DUG-members, the editor).

[1)] I think you can use the deg-function (e.g. SIN(60 DEG) will display as SIN (60ø), the editor)

The following article was written by David R. Stoutemyer and was first published by the

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I am very grateful for the permission to reprint this contribution.

Mechanical Engineering Applications of the *DERIVE* PC Symbolic Math Program

David R. Stoutemyer,
Honolulu, Hawaii

ABSTRACT

The Derive® symbolic math program was developed by Soft Warehouse, Inc. of Honolulu Hawaii. The program runs on any IBM-PC or NEC PC-9801 compatible personal computer having at least 512 kilobytes of RAM and MS-DOS version 2.1 or later. Derive features powerful symbolic capabilities, 2D and 3D function plots, and numerical methods. These are integrated in a menu-driven windowing interface with on-line help, so it is easy to learn. The combination of power, low cost, modest hardware requirements, and short learning curve make Derive highly suitable for both engineering education and research. As illustrations, this article shows, some of the ways Derive can be used to assist in the following applications:

a) random vibration stability,

b) computing moments of intertia,

c) robot kinematics,

d) statics,

e) automatic control,

f) computing a radiative heat transfer view factor,

g) deriving the Hamilton-Jacobi equations of motion for a constrained system, and,

h) deriving the Navier-Stokes equation in paraboloidal coordinates.

1. INTRODUCTION

Derive is a menu-driven symbolic math program with on-line help, making it easy to learn (References 1, 2, and 7). Derive includes a windowing system that enables you to split the screen into windows or overlay them. Figure 1 shows a screen dump in which there is an algebra window containing two expressions, a 2D-Plot window showing a parametric plot of the first expression, and a 3D-Plot window showing a surface plot of the second expression.

Version 2.0 described here is programmable. Its built-in symbolic capabilities include integration, differentiation, solution of algebraic and differential equations, vector algebra and calculus, and matrix algebra on matrices having nonnumeric elements. Derive also includes graphic and approximate numeric capabilities to a wide variety of mechanical engineering problems. Some of these applications use functions defined in auxiliary files distributed with Derive. Others also use functions written for this article that are available free on the Derive bulletin board: (217) 337-0926.

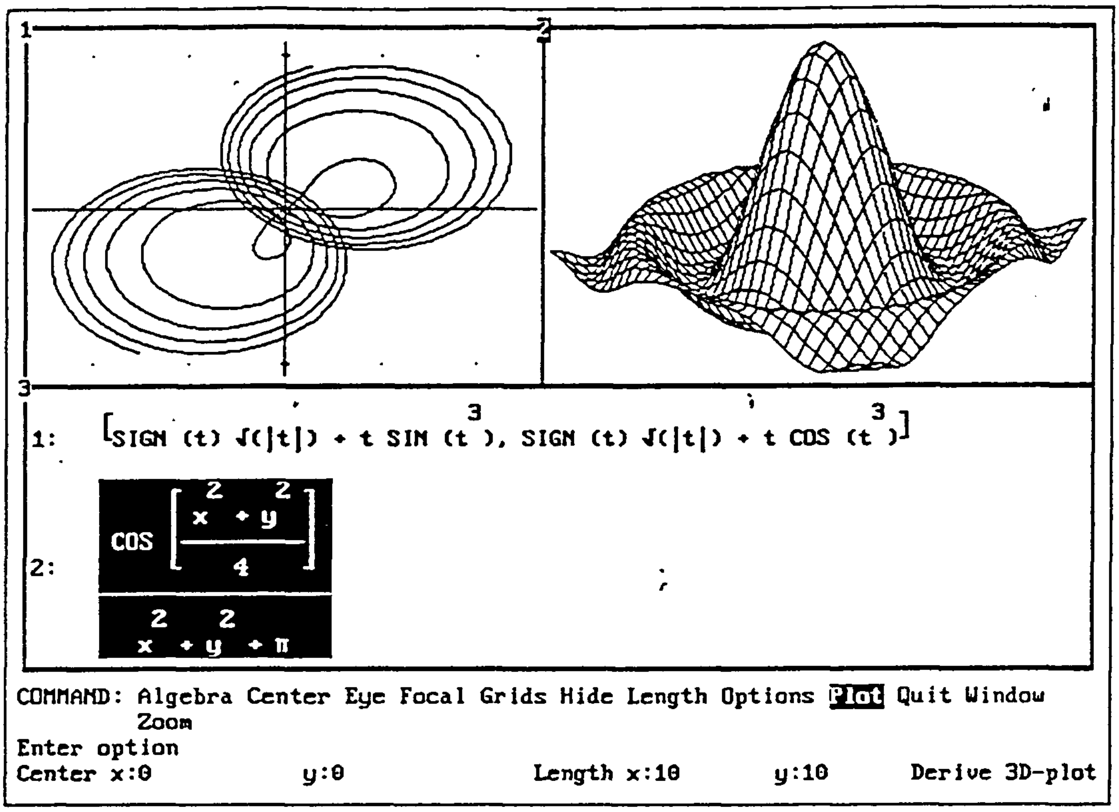


Figure 1: Derive Screen Dump (Derive 2.0)

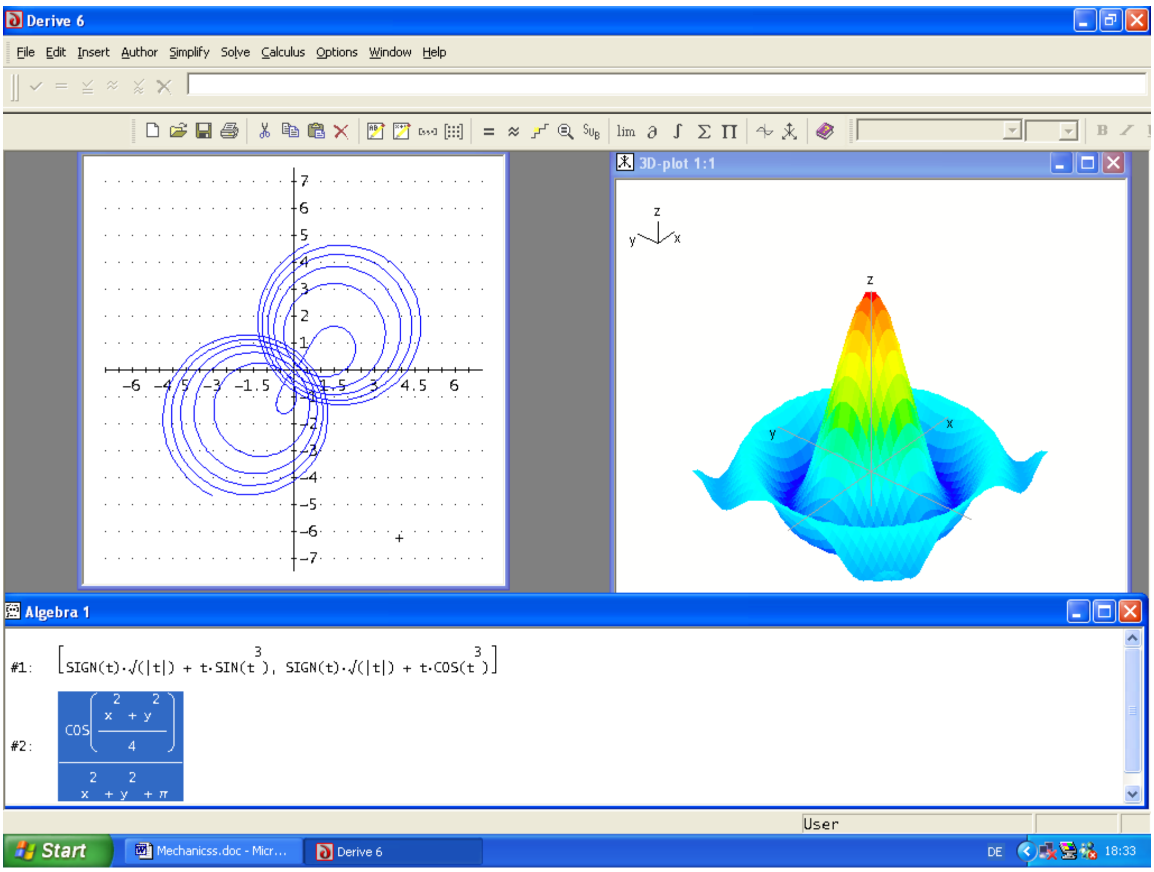


Figure 1a: Derive Screen Dump (Version 6.10)

2. RANDOM VIBRATION STABILITY

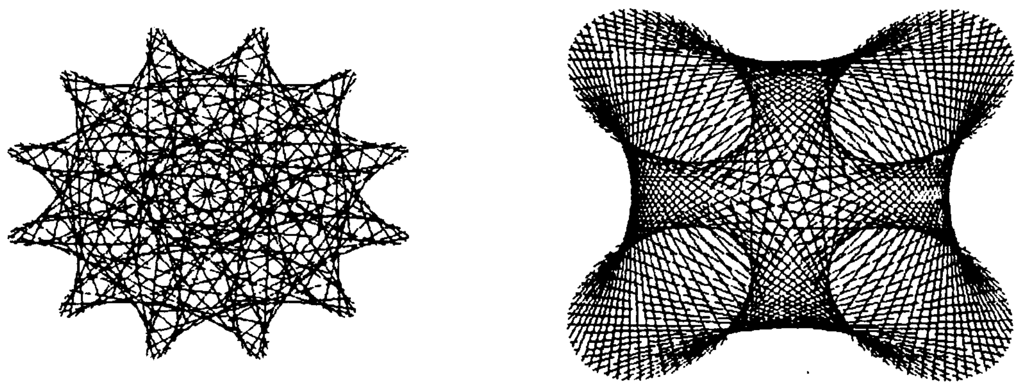
Reference 5 characterizes the stability of a certain system under random vibration according to the zeros of the determinant

$$\mathrm{DET}\begin{bmatrix} -\mu & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\omega & -\beta-\mu & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\cdot\mu & -2\cdot\beta-\mu & 0 & 0 & s & 0 & 0 & 0 & 0 \\ s & 0 & 0 & -2\cdot b-\mu & -2\cdot w & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -b-\mu & -w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -w & -b-\mu & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega & 0 & -\beta-\mu & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & t & -\omega & -w & -\beta-b-\mu \end{bmatrix}$$

We would like the determinant expressed as a polynomial in μ, so we declare that μ orders ahead of any other variables, then simplify this expression. This yields the following product of a fourth and a sixth degree polynomial:

$$(\mu^4 + 2\cdot\mu^3\cdot(b+\beta) + \mu^2\cdot(b^2 + 3\cdot b\cdot\beta + \beta^2 + 2\cdot\omega + 2\cdot w) + \mu\cdot(b^2\cdot\beta + b\cdot(\beta^2 + 2\cdot\omega + 2\cdot w) + 2\cdot\beta\cdot(\omega + w) - 2\cdot t) + b^2\cdot\omega + b\cdot(\beta\cdot(\omega + w) - t) + \beta^2\cdot w - \beta\cdot t + (\omega - w)^2)\cdot(\mu^6 + \mu^5\cdot(3\cdot b + 3\cdot\beta + 2) + \mu^4\cdot(2\cdot b^2 + 3\cdot b\cdot(3\cdot\beta + 2) + 2\cdot(\beta^2 + \omega + 2\cdot w)) + 2\cdot\mu^3\cdot(b^2\cdot(3\cdot\beta + 2) + b\cdot(3\cdot\beta^2 + 3\cdot\omega + 2\cdot w) + 2\cdot(\beta\cdot(\omega + 3\cdot w) + 2\cdot w)) + 4\cdot\mu^2\cdot(b^2\cdot(\beta^2 + \omega) + b\cdot(3\cdot\beta\cdot(\omega + w) + 2\cdot w) + 2\cdot w\cdot(\beta^2 + \omega)) + 8\cdot\mu\cdot(b^2\cdot\beta\cdot\omega + b\cdot w\cdot(\beta^2 + \omega) + 2\cdot\beta\cdot\omega\cdot w) + 16\cdot b\cdot\beta\cdot\omega\cdot w - 4\cdot s^2)$$

Try deriving this result manually!



Two DOS-DERIVE Maurer Roses

3. MOMENT OF INERTIA

Let μ denote a density of some quantity such as mass, which can vary with position. A common application of integration is to determine the integral of μ or of its first or second moments along a curve or over a plane, curved surface, or volume. $\mu = 1$ is used to compute arclength, area, volume, centroid, or the geometric inertia tensor. Applications include center of pressure, center of bouyancy, bending of beams, torsion of bars, and rigid body dynamics. File INERTIA.MTH, distributed with Derive, contains general purpose functions for computing such quantities in Cartesian, spherical or polar cylindrical coordinates. For example, here is one such function definition:

VOLUME_INERTIA_CARTESIAN(μ, x, a, b, y, u, v, z, s, t):=

$$\int_a^b \int_u^v \int_s^t \mu \begin{bmatrix} y^2 + z^2 & -x\,y & -x\,z \\ -x\,y & x^2 + y^2 & -y\,z \\ -x\,z & -y\,z & x^2 + y^2 \end{bmatrix} dz\ dy\ dx$$

Here and throughout this article, displayed expressions are excerpts from files generated by Derive, closely matching the appearance of these expressions on the screen or when printed.

As an example of using this function definition, suppose that we want to compute the volume inertia tensor of the intersection of the two homogeneous cylinders:

$$z^2 = r^2 - x^2$$
$$z^2 = r^2 - y^2$$

Using $\mu = 1$ for the uniform density, the appropriate expression to simplify is then

$$\text{VOLUME\_INERTIA\_CARTESIAN}(1,\ x,\ -r,\ r,\ y,\ -\sqrt{(r^2 - x^2)},\ \sqrt{(r^2 - x^2)},\ z,\ -\sqrt{(r^2 - y^2)},\ \sqrt{(r^2 - y^2)})$$

giving

$$\begin{bmatrix} \frac{112\cdot r^5}{45} & 0 & 0 \\ 0 & \frac{128\cdot r^5}{45} & 0 \\ 0 & 0 & \frac{112\cdot r^5}{45} \end{bmatrix}$$

Note that this is a general answer for any r. Such an exact parameterized symbolic solution can´t be obtained with numerical integration.

Comment: You cannot find INERTIA.MTH among the utility files distributed with higher versions of Derive. It was replaced by ItnegrationApplications.mth. VOLUME_INERTIA seems to be the same or at least very similar:

$$\mathrm{VOLUME\_INERTIA}(x, -r, r, y, -\sqrt{r^2 - x^2}, \sqrt{r^2 - x^2}, z, -\sqrt{r^2 - y^2}, \sqrt{r^2 - y^2})$$

$$\begin{bmatrix} \frac{7 r^2}{15} & 0 & 0 \\ 0 & \frac{8 r^2}{15} & 0 \\ 0 & 0 & \frac{7 r^2}{15} \end{bmatrix}$$

The results are not the same, but if you multiply the result by the volume of the body, then:

$$\mathrm{VOLUME\_INERTIA}(x, -r, r, y, -\sqrt{r^2 - x^2}, \sqrt{r^2 - x^2}, z, -\sqrt{r^2 - y^2}, \sqrt{r^2 - y^2}) \cdot \mathrm{VOLUME}(x, -r, r, y, -\sqrt{r^2 - x^2}, \sqrt{r^2 - x^2}, z, -\sqrt{r^2 - y^2}, \sqrt{r^2 - y^2}, 1)$$

$$\begin{bmatrix} \frac{112 r^5}{45} & 0 & 0 \\ 0 & \frac{128 r^5}{45} & 0 \\ 0 & 0 & \frac{112 r^5}{45} \end{bmatrix}$$

4. ROBOT KINEMATICS

The position and orientation of the tip of a robot arm can be computed by multiplication of matrices representing joint rotations and translations. For example, reference 3 gives the relevant matrices for the Cincinnati T3-756 Robot as

$$m1 := \begin{bmatrix} \mathrm{COS}(\alpha) & 0 & -\mathrm{SIN}(\alpha) & 0 \\ \mathrm{SIN}(\alpha) & 0 & \mathrm{COS}(\alpha) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m2 := \begin{bmatrix} \mathrm{COS}(\beta) & -\mathrm{SIN}(\beta) & 0 & a\ \mathrm{COS}(\beta) \\ \mathrm{SIN}(\beta) & \mathrm{COS}(\beta) & 0 & a\ \mathrm{SIN}(\beta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m3 := \begin{bmatrix} \mathrm{COS}(\theta) & -\mathrm{SIN}(\theta) & 0 & b\ \mathrm{COS}(\theta) \\ \mathrm{SIN}(\theta) & \mathrm{COS}(\theta) & 0 & b\ \mathrm{SIN}(\theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m4 := \begin{bmatrix} \mathrm{COS}(\phi) & 0 & \mathrm{SIN}(\phi) & 0 \\ \mathrm{SIN}(\phi) & 0 & -\mathrm{COS}(\phi) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m5 := \begin{bmatrix} \mathrm{COS}(\sigma) & 0 & -\mathrm{SIN}(\sigma) & 0 \\ \mathrm{SIN}(\sigma) & 0 & \mathrm{COS}(\sigma) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m6 := \begin{bmatrix} \mathrm{COS}(\tau) & -\mathrm{SIN}(\tau) & 0 & 0 \\ \mathrm{SIN}(\tau) & \mathrm{COS}(\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To multiply these matrices we simplify the expression

```
m1 m2 m3 m4 m5 m6
```

giving a matrix having complicated trigonometric expressions as entries. For example, here is the 3-3 element of this result:

```
  COS(β) (SIN(θ) SIN(σ) COS(φ) + COS(θ) SIN(σ) SIN(φ))
+ SIN(β) (COS(θ) SIN(σ) COS(φ) - SIN(θ) SIN(σ) SIN(φ))
```

To attempt further simplification, we can apply the factor command, giving

```
SIN(σ) (COS(β) (SIN(θ) COS(φ) + COS(θ) SIN(φ))
      + SIN(β) (COS(θ) COS(φ) - SIN(θ) SIN(φ)))
```

The parallel structure between the two overall terms in the cofactor of `SIN(σ)` suggests collecting products of sinusoids into angle sums in this cofactor. To do so, we can switch trigono-metric simplification to collection, highlight the cofactor, then resimplify to obtain the more compact equivalent:

```
Manage Trigononometry  Direction: Collect
SIN(σ) SIN(β + φ + θ)
```

Comment: Doing the same in Derive 6 we receive another compact equivalent:

```
Trigonometry := Collect
```

$$\frac{\mathrm{COS}(\beta + \theta - \sigma + \phi)}{2} - \frac{\mathrm{COS}(\beta + \theta + \sigma + \phi)}{2}$$

Again we have a general answer for any angles, rather than a numeric answer for one particular set of angles.

5. STATICS

Reference 6 describes a student static problem in which a slender beam of length u and mass m is suspended from a point by two cables making angle θ which each of the two ends of the beam where they attach. The problem is to determine the tension f in the remaining cable just after the other cable breaks. Let α be the angular acceleration of the beam, and let a be the magnitude of its linear acceleration. The equations for horizontal, vertical and angular acceleration are

$$f\ \mathrm{COS}(\theta) = m\ a\ \mathrm{SIN}(\theta)$$

$$f\ \mathrm{SIN}(\theta) - m\ g = m\left(-\ a\ \mathrm{COS}(\theta) - \frac{u\ \alpha}{2}\right)$$

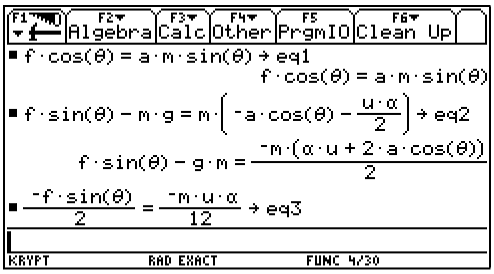
$$-\ \frac{f\ \mathrm{SIN}(\theta)\ u}{2} = -\ \frac{m\ u^2\ \alpha}{12}$$

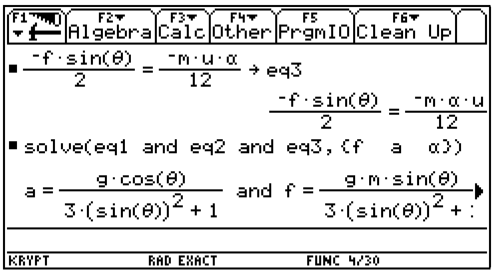
Collecting these equations into a vector, then applying the solve command yields the solution

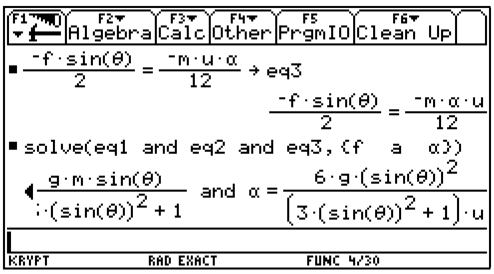
$$\left[a = \frac{g\cdot \mathrm{COS}(\theta)}{4 - 3\cdot \mathrm{COS}(\theta)^2},\ \alpha = \frac{6\cdot g\cdot \mathrm{SIN}(\theta)^2}{u\cdot(4 - 3\cdot \mathrm{COS}(\theta)^2)},\ f = \frac{g\cdot m\cdot \mathrm{SIN}(\theta)}{4 - 3\cdot \mathrm{COS}(\theta)^2}\right]$$

Note that as with many student problems, the parameters are symbolic rather than numeric. This lessens the chance of careless numeric errors masking the instructors' assessment of the students' understanding, and it makes the answer more general. Computer algebra similarly further lessens the chance of careless algebraic errors masking the instructors' assessment of the students' modeling capabilities.

Comment: As many of you might know it was David Stoutemyer who was very deep involved in implementing computer algebra on the TI-92, which has now developed into the Voyage 200. So it is nice to see how this problem is solved by the TI-CAS-devices. The solution does not look the same, students should find out if the outputs are equivalent or not.







6. AUTOMATIC CONTROL

Reference 4 describes an automatic control example, and here is how it can be done using Derive:

Suppose that we have derived the Laplace domain characteristic expression

$$(s + a)\ (s + b) + k\ (s + 2\ a)\ (s + 4\ b)\ s$$

For stability analysis we can use the expand command to expand and collect similar powers of s giving

$$k\ s^3 + s^2\ (2\ a\ k + 4\ b\ k + 1) + s\ (a\ (8\ b\ k + 1) + b) + a\ b$$

Throughout the analysis we want s to order ahead of the other variables, so we declare that. Next we can use the solve command to determine the value of k that makes this expression 0:

$$k = -\frac{(b + s)\cdot(a + s)}{s\cdot(2\cdot a + s)\cdot(4\cdot b + s)}$$

Next to determine the sensitivity of this solution, we can assign the right side to k, then simplify

$$\frac{k}{s\cdot\frac{d}{ds}\ k}$$

to obtain

$$-\frac{(b + s)\cdot(4\cdot b + s)\cdot(a + s)\cdot(2\cdot a + s)}{2\cdot a^2\cdot(4\cdot b^2 + 2\cdot b\cdot s + s^2) + a\cdot s\cdot(8\cdot b^2 + b\cdot s + 2\cdot s^2) + s^2\cdot(4\cdot b^2 + 2\cdot b\cdot s + s^2)}$$

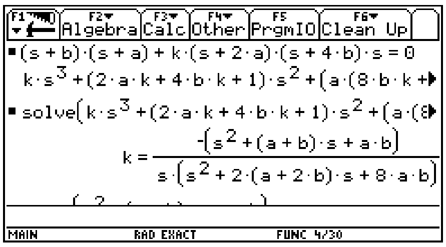
Finally, we can use the solve command to determine the values of s that make the sensitivity 0:

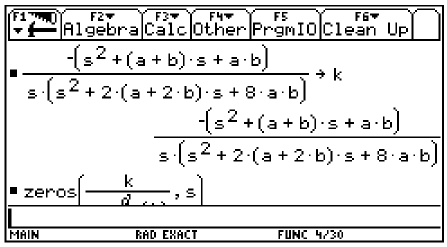
$$s = -\ 4\cdot b$$

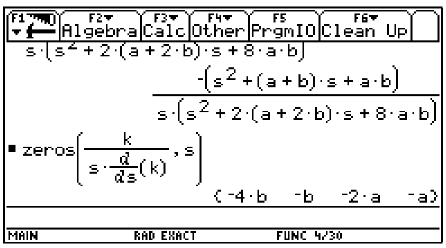
$$s = -b$$

$$s = -\ 2\cdot a$$

$$s = -a$$







7. RADIATIVE HEAT TRANSFER

Radiative heat transfer involves evaluating difficult multiple integrals to compute view factors. For example, the view factor from the rectangle $0 \le x \le a$, $0 \le y \le b$, $z = 0$ of the rectangle $x = 0$, $0 \le y \le b$, $0 \le z \le c$ is

$$\frac{1}{a\cdot c}\cdot\int_0^c\int_0^b\int_0^a\int_0^a \frac{y\cdot z}{\pi\cdot((x-w)^2+y^2+z^2)^2}\,dx\,dw\,dy\,dz$$

Simplifying this expression gives

$$-\frac{\sqrt{b^2+c^2}\cdot\mathrm{ATAN}\left(\frac{a}{\sqrt{b^2+c^2}}\right)}{\pi\cdot c}+\frac{b\cdot\mathrm{ATAN}\left(\frac{a}{b}\right)}{\pi\cdot c}+\frac{\mathrm{ATAN}\left(\frac{a}{c}\right)}{\pi}-$$

$$\frac{(a^2-b^2-c^2)\cdot\mathrm{LN}(a^2+b^2+c^2)}{4\cdot\pi\cdot a\cdot c}+\frac{(a^2-b^2)\cdot\mathrm{LN}(a^2+b^2)}{4\cdot\pi\cdot a\cdot c}+\frac{(a^2-c^2)\cdot\mathrm{LN}(a^2+c^2)}{4\cdot\pi\cdot a\cdot c}-$$

$$\frac{a\cdot\mathrm{LN}(a^2)}{4\cdot\pi\cdot c}-\frac{(b^2+c^2)\cdot\mathrm{LN}(b^2+c^2)}{4\cdot\pi\cdot a\cdot c}+\frac{b^2\cdot\mathrm{LN}(b^2)}{4\cdot\pi\cdot a\cdot c}+\frac{c\cdot\mathrm{LN}(c^2)}{4\cdot\pi\cdot a}$$

Try deriving this result manually, even with the help of integral tables!

Figure 2 shows a 3D Derive plot of the value of this expression for $c = 1$, $-5 \le a \le 5$, $-5 \le b \le 5$.

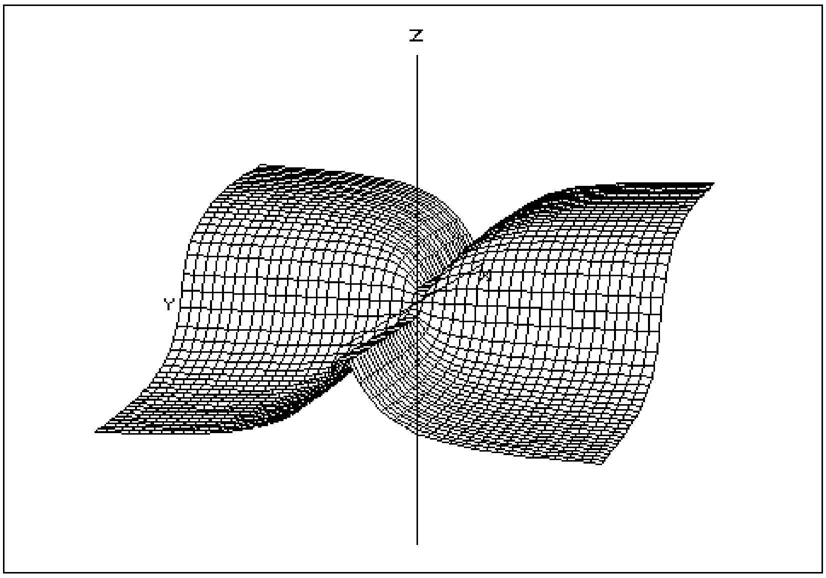


Figure 2

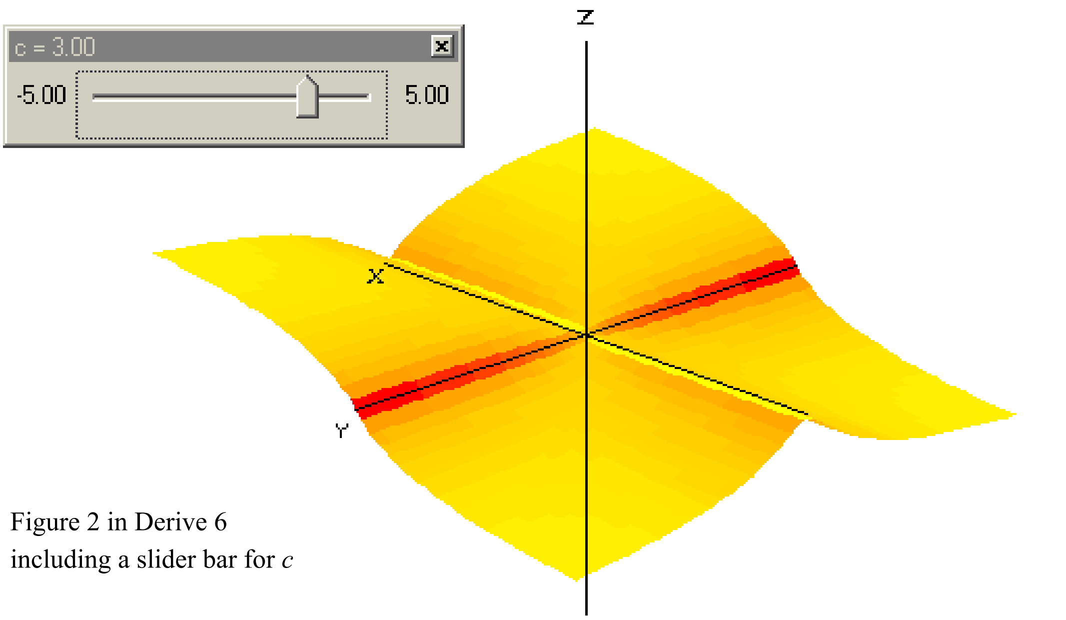


Figure 2 in Derive 6
including a slider bar for *c*

8. HAMILTON-JACOBI EQUATIONS OF MOTION

The Hamiltonian approach is one alternative for determining the equations of motion of a constrained mechanical system. Most numerical ordinary-differential-equation solvers require a system of first-order equations, and the Hamiltonian approach has the advantage of directly yielding first-order rather than second-order differential equations. However, the algebra can be extremely tedious, making it a good candidate for computer algebra. Glossing over details:

a) The potential and kintec energies are expressed in terms of the generalized coordinates and velocities.

b) The Lagrangian is computed as the kinetic minus potential energy.

c) The vector of generalized momenta is defined as the gradient of the Lagrangian with respect to the vector of generalized velocities.

d) These simultaneous algebraic equations are solved for the velocities in terms of the momenta.

e) The Hamiltonian is then expressed in terms of the generalized coordinates and momenta.

f) The derivative of the vector of generalized coordinates is then equated to the gradient of the Hamiltonian with respect to the vector of momenta.

g) The derivative of the vector of momenta is then equated to minus the gradient of the Hamiltonian with respect to the vector of generalized coordinates.

File HAMILTON.MTH on the Derive electronic bulletin board automates the process. First to accomplish steps a) through e), simplify an expression of the form

```
HAMILTONIAN(ke, pe, θdot, p)
```

Here `ke` is the kinetic energy, `pe` is the potential energy, `θdot` is a vector of variables representing the velocities, and `p` is a vector of variables representing the momenta.

Next, to perform step f), simplify an expression of the form

DCOORDINATES_DT(h, p)

where h is the Hamiltonian.

Finally, to perform step g), simplify an expression of the form

DMOMENTUM_DT(h, q)

where q is a vector of variables representing the generalized coordinates.

As an example of this process, consider a planar double pendulum composed of two uniform slender rods of length r and mass m. One rod is attached to the end of the other. Taking α as the angle of the upper rod measured from straight down and β as the angle of the second rod measured from straight down, the potential energy is

$$pe := m\ g\ r \left(\frac{3}{2}\ \mathrm{COS}(\alpha) + \frac{1}{2}\ \cos(\beta) \right)$$

Denoting the corresponding velocities by αdot and ßdot the kinetic energy is

$$ke := \frac{1}{2}\ m\ r^2 \left(\frac{4}{3}\ \alpha dot^2 + \frac{1}{3}\ \beta dot^2 + \alpha dot\ \beta dot\ \mathrm{COS}(\beta - \alpha) \right)$$

Denoting the corresponding generalized momenta by σ and τ, to form the Hamiltonian we then simplify the right side of

h := HAMILTONIAN(ke, pe, [αdot, βdot], [σ, τ])

giving

$$h := \frac{18\cdot\sigma\cdot\tau\cdot\mathrm{COS}(\alpha - \beta)}{m\cdot r^2\cdot(9\cdot\mathrm{COS}(\alpha - \beta)^2 - 16)} + \frac{6\cdot(\sigma^2 + 4\cdot\tau^2)}{m\cdot r^2\cdot(16 - 9\cdot\mathrm{COS}(\alpha - \beta)^2)} + \frac{3\cdot m\cdot r\cdot g\cdot\mathrm{COS}(\alpha)}{2} + \frac{\mathrm{COS}(\beta)\cdot m\cdot r\cdot g}{2}$$

To determine the time derivative of the vector [α, ß] we simplify the right side of the expression

DCOORDINATES_DT(h, [σ, τ])

giving

$$\left[\frac{6\cdot(3\cdot\tau\cdot\mathrm{COS}(\alpha - \beta) - 2\cdot\sigma)}{m\cdot r^2\cdot(9\cdot\mathrm{COS}(\alpha - \beta)^2 - 16)},\ \frac{6\cdot(3\cdot\sigma\cdot\mathrm{COS}(\alpha - \beta) - 8\cdot\tau)}{m\cdot r^2\cdot(9\cdot\mathrm{COS}(\alpha - \beta)^2 - 16)} \right]$$

To form the time derivatives of the vector [σ, τ] we simplify the right side of expression

DMOMENTUM_DT(h, [α, β])

which produces an analogous but larger expression.

$$\left[- \frac{3\cdot(81\cdot g\cdot m^2\cdot r^3\cdot\mathrm{SIN}(\alpha)\cdot\mathrm{COS}(\alpha - \beta)^4 - 36\cdot\mathrm{COS}(\alpha - \beta)^2\cdot(3\cdot\sigma\cdot\tau\cdot\mathrm{SIN}(\alpha - \beta) + 8\cdot g\cdot m^2\cdot r^3\cdot\mathrm{SIN}(\alpha)) + 72\cdot(\sigma^2 + 4\cdot\tau^2)\cdot\mathrm{SIN}(\alpha - \beta)\cdot\mathrm{COS}(\alpha - \beta) + 64\cdot(4\cdot g\cdot m^2\cdot r^3\cdot\mathrm{SIN}(\alpha) - 3\cdot\sigma\cdot\tau\cdot\mathrm{SIN}(\alpha - \beta)))}{2\cdot m\cdot r^2\cdot(9\cdot\mathrm{COS}(\alpha - \beta)^2 - 16)^2},\ - \frac{18\cdot\mathrm{SIN}(\alpha - \beta)\cdot(9\cdot\sigma\cdot\tau\cdot\mathrm{COS}(\alpha - \beta)^2 - 6\cdot(\sigma^2 + 4\cdot\tau^2)\cdot\mathrm{COS}(\alpha - \beta) + 16\cdot\sigma\cdot\tau)}{m\cdot r^2\cdot(9\cdot\mathrm{COS}(\alpha - \beta)^2 - 16)^2} \right]$$

9. NAVIER-STOKES EQUATIONS IN PARABOLOIDAL COORDINATES

Sometimes boundary shapes encourage the use of non-Cartesian coordinates. For example, the airflow around a paraboloidal antenna encourages the use of paraboloidal coordinates, making it useful to derive the governing Navier-Stokes equations in those coordinates.

The coordinate-free vector form of the Navier-Stokes equation is well known, and Derive has built-in functions for determining its differential operators in any orthogonal curvilinear coordinate system. Moreover, file COORD.MTH, distributed with Derive, has built-in functions, that determine the metrical coefficients `h1`, `h2`, and `h3` necessary for computing the differential operators in any coordinate system.

Let the transformation from curvilinear coordinates `[α, β, τ]` to Cartesian coordinates `[x, y, z]` be given by

```
x := e1

y := e2

z := e3
```

where `e1`, `e2` and `e3` are expressions depending on `α, β,` and `τ`. Then simplifying an expression of the form

```
COVARIANT_METRIC_TENSOR([e1, e2, e3; α, β, τ])
```

yields the covariant metric tensor `g`.

If `g` is a diagonal matrix, then simplifying an expression of the form

```
[α, β, τ ; SQRT_DIAGONAL(g)]
```

yields a geometry matrix whose first row is the curvilinear coordinates and whose second row is the corresponding metrical coefficients.

Such a matrix is an optional second argument for the vector differential operators such as `LAPLACIAN`, `GRAD` and `DIV`. If omitted, the default is Cartesian coordinates `[x, y, z]`, with unit metrical coefficients.

As an example of this process, to compute the metric tensor for paraboloidal coordinates, we can simplify the right side of

$$g := \mathrm{COVARIANT\_METRIC\_TENSOR}\left(\mathrm{JACOBIAN}\left(\left[\alpha\cdot\beta\cdot\tau,\ \alpha\cdot\beta\cdot\sqrt{1-\tau^2},\ \frac{\alpha^2-\beta^2}{2}\right],\ [\alpha,\ \beta,\ \tau]\right)\right)$$

giving

$$g := \begin{bmatrix} \alpha^2+\beta^2 & 0 & 0 \\ 0 & \alpha^2+\beta^2 & 0 \\ 0 & 0 & \dfrac{\alpha^2\cdot\beta^2}{1-\tau^2} \end{bmatrix}$$

This is diagonal, so to compute the corresponding geometry matrix, we simplify the right side of

```
m ≔ [[α, β, τ], SQRT_DIAGONAL(g)]
```

or now with DERIVE 5 or 6

```
m ≔ GEOMETRY_MATRIX([α, β, τ], g)
```

giving the 2 by 3 matrix

$$m := \begin{bmatrix} \alpha & \beta & \tau \\ \sqrt{(\alpha^2 + \beta^2)} & \sqrt{(\alpha^2 + \beta^2)} & \dfrac{i\,|\alpha\,\beta|}{\sqrt{(\tau^2 - 1)}} \end{bmatrix}$$

Now for the Navier-Stokes equation: Let v denote the dependent velocity vector, t the time, μ the viscosity, p the pressure, σ the density, and m the above geometry matrix. The Navier-Stokes equation is then given by the function definition

```
NAVIER_STOKES(v, t, μ, p, σ, m) ≔
```

$$\frac{d}{dt} v = \mu\cdot\mathrm{LAPLACIAN}(v, m) - v\cdot\mathrm{GRAD}(v, m) + \frac{\mu\cdot\mathrm{GRAD}(\mathrm{DIV}(v, m), m)}{3} - \frac{\mathrm{GRAD}(p, m)}{\sigma}$$

v generally depends on time and position, and this equation involves time and spatial derivatives of v. To make these dependencies manifest so that these derivatives won't simplify to 0, we can declare each component as an undefined function as follows:

```
Vα(α, β, τ, t) ≔

Vβ(α, β, τ, t) ≔

Vτ(α, β, τ, t) ≔
```

We can then collect these components together as v:

```
v ≔ [Vα(α, β, τ, t), Vβ(α, β, τ, t), Vτ(α, β, τ, t)]
```

P similarly depends on time and position and is differentiated, so we also declare it as an undefined function:

```
P(α, β, τ, t) ≔
```

We can then derive the Navier-Stokes equations in paraboloidal coordinates by simplifying

```
NAVIER_STOKES(v, t, μ, P(α, β, τ), σ, m)
```

The result is a vector partial differential equation. As an indication of its complexity, the first component of the left side is

$$\frac{d}{dt} V\alpha(\alpha, \beta, \tau, t)$$

and when expanded with respect to μ the first component of the right side is

$$
\begin{aligned}
&\frac{\mu\cdot\frac{d}{d\beta}\frac{d}{d\alpha}V\beta(\alpha,\beta,\tau,t)}{3\cdot(\alpha^2+\beta^2)} - \frac{\frac{d}{d\alpha}P(\alpha,\beta,\tau,t)}{\sigma\cdot\sqrt{(\alpha^2+\beta^2)}} + \frac{4\cdot\mu\cdot\left(\frac{d}{d\alpha}\right)^2 V\alpha(\alpha,\beta,\tau,t)}{3\cdot(\alpha^2+\beta^2)} + \left(\frac{d}{d\alpha}V\alpha(\alpha,\beta,\tau,t)\right)\cdot\left(\frac{4\cdot\mu}{3\cdot\alpha\cdot(\alpha^2+\beta^2)} - \right.\\
&\left.\frac{V\alpha(\alpha,\beta,\tau,t)}{\sqrt{(\alpha^2+\beta^2)}}\right) + \frac{\mu\cdot\left(\frac{d}{d\beta}\right)^2 V\alpha(\alpha,\beta,\tau,t)}{\alpha^2+\beta^2} + \left(\frac{d}{d\beta}V\alpha(\alpha,\beta,\tau,t)\right)\cdot\left(\frac{\mu}{\beta\cdot(\alpha^2+\beta^2)} - \frac{V\beta(\alpha,\beta,\tau,t)}{\sqrt{(\alpha^2+\beta^2)}}\right) + \\
&\frac{\mu\cdot(1-\tau^2)\cdot\left(\frac{d}{d\tau}\right)^2 V\alpha(\alpha,\beta,\tau,t)}{\alpha^2\cdot\beta^2} - \frac{\mu\cdot\tau\cdot\frac{d}{d\tau}V\alpha(\alpha,\beta,\tau,t)}{\alpha^2\cdot\beta^2} + \frac{\mu\cdot(\alpha^2+2\cdot\beta^2)\cdot\frac{d}{d\alpha}V\beta(\alpha,\beta,\tau,t)}{3\cdot\beta\cdot(\alpha^2+\beta^2)^2} - \frac{\alpha\cdot\mu\cdot\frac{d}{d\beta}V\beta(\alpha,\beta,\tau,t)}{3\cdot(\alpha^2+\beta^2)^2} - \\
&\frac{\mu\cdot(4\cdot\alpha^4+2\cdot\alpha^2\cdot\beta^2+\beta^4)\cdot V\alpha(\alpha,\beta,\tau,t)}{3\cdot\alpha^2\cdot(\alpha^2+\beta^2)^3} - \frac{\alpha\cdot\mu\cdot(\alpha^2+4\cdot\beta^2)\cdot V\beta(\alpha,\beta,\tau,t)}{3\cdot\beta\cdot(\alpha^2+\beta^2)^3} - \\
&\frac{\imath\cdot\sqrt{(\tau^2-1)}\cdot\mathrm{SIGN}(\alpha)\cdot\left(\alpha\cdot\mu\cdot\frac{d}{d\tau}\frac{d}{d\alpha}V\tau(\alpha,\beta,\tau,t) - 3\cdot\alpha\cdot\sqrt{(\alpha^2+\beta^2)}\cdot V\tau(\alpha,\beta,\tau,t)\cdot\frac{d}{d\tau}V\alpha(\alpha,\beta,\tau,t) - \mu\cdot\frac{d}{d\tau}V\tau(\alpha,\beta,\tau,t)\right)}{3\cdot\alpha^2\cdot\sqrt{(\alpha^2+\beta^2)}\cdot|\beta|}
\end{aligned}
$$

The other two components are analogous.

10. CONCLUSIONS

The above examples range in difficulty from typical student problems through research examples. They also range widely throughout mechanical engineering applications. However, eight examples can only sparsely sample the space of possible applications. Among the relevant Derive capabilities not illustrated here are Fourier series, Taylor series, exact, numeric and series solution of differential equations, the use of Bessel and other special functions, approximate numeric integration and root finding. Nonetheless, I hope that these examples are varied enough to suggest others in your discipline. I also hope that you will send me preprints or reprints of your applications so that I can direct others to your work. In fact, I plan to accumulate similar collections of applications in electrical engineering, civil engineering, physics and other fields.

11. REFERENCES

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