THE DERIVE - NEWSLETTER #12

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THE BULLETIN OF THE

DERIVE®

USER GROUP

Contents:

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The list of new **DERIVE** related books is very extended this time. I want to thank many of you for your allusions to new publications. I specially would recommend two German books **[7]** and **[8]** to the non german speaking friends. I think that these two books are so rich in mathematical and "Derivish" contents, that you need not know the connecting words in most cases. We all can be proud that all the authors – with only one exception - are members of the DUG.

D E R I V E - B O O K - S H E L F

[1] Linear Algebra Experiments Using the Derive Program, M. Salter & L. Gilligan, GILMAR Publ., P.O.Box 3676, Cincinnati, Ohio

[2] Computer Algebra Systems in the Classroom, J.Monaghan & T.Etchells, Centre for Studies in Science and Mathematics Education, The University of Leeds, Leeds LS2 9JT

[3] Discovering Calculus with DERIVE, J. Johnson & B.Evans, John Wiley & Son, Inc. 1992

[4] Learning Mathematics through DERIVE, l.S.Berry, E.Graham & A.l.P.Watkins Ellis Horwood. 1993

[5] Algebraprogramme und Tabellenkalkulation im Unterricht, W.Cyrmon, Hölder-Pichler- Tempsky , Wien. 1993

[6] Mathematik mit DERIVE, W.Koepf, A.Ben-Israel u. B.Gilbert, Vieweg. 1993 (Diskette dazu erhältlich)

[7] DERIVE -ein mathematischer Assistent, Landesinstitut für Schule und Weiterbildung (Kayer, Neveling) Best.Nr .1759 beim Soester Verlagskontor, Gabelsbergerstr. 1, D-59069 Hamm (inkl. Diskette)

[8] Mathematik mit DERIVE, Arbeitsblätter zur experimentellen Mathematik, Mauve und Moos Dümmlerbuch 4588, Dümmler. 1993 (vor kurzem erst erschienen, Diskette erhältlich)

[9] Mathematikstunden in der 11.Jahrgangsstufe, schriftl Hausarbeit von St.Ref. Yasser Kanaan, Hans-Leinsberger-Gymnasium Landshut

[10] Integration mit Derive, W .Koepf und A.Ben-Israel, Aufsatz in Didaktik der Mathematik 1, 1993. Bayerischer Schulbuchverlag

[11] Zur Berechnung der trigonometrischen Funktionen, W.Koepf, Preprint A/16-93, FU Berlin - Fachbereich Mathematik

[12] Ein elementarer Zugang zu Potenzreihen, W .Koepf, Preprint A/22-93, FU Berlin -Fachbereich Mathematik

[13] Usando Derive - Fundamentos da Matematica, A.J.P.Watkins, Editora Grypho, 1993. Rio de Janeiro

DERIVE USER GROUP UK Meeting 1993

On 20th September this meeting took place in Birmingham. 38 participants followed the presentations: Prof. J. Berry: **Derive International Journal & Conference**, Dr. D. Salinger: **Integrating like a computer algebra package**, Dr .B. Kutzler: **Derive in Europe**, Short presentations from meeting attendees, K. Williamson: **Finite fields alive with version 2.5**, Dr. D. Stoutemyer: **The Future of Derive** and others.

I have to thank all members for coming. My very special thanks to Terence Etchells for organizing the meeting. He had a lot of work to do and to overcome a big portion of stress. At last thanks to the staff of the TMT Conference in Birmingham for the wonderful cooperation and the nice days in Birmingham. Thanks Pam, Mrs Vicepresident of the DUG.

Liebes DUG-Mitglied,

mit großem Vergnügen lege ich Ihnen nun die bereits 12. Ausgabe Ihres *DNL* vor. Ja, ich sage ganz bewusst "Ihres" *DNL,* denn diese Zeitschrift kann nur durch Ihre Mithilfe, Ihr Engagement und Ihr Interesse bestehen. Dank Ihrer Zuschriften gelingt es immer wieder ein, wie ich glaube, interessantes Magazin zusammenzustellen.

Bitte bleiben Sie weiterhin so aktiv. In dieser Ausgabe konnte zum ersten Mal ein Beitrag aus Asien aufgenommen werden. Afrika und Australien fehlen noch auf der Autorenliste. Ich möchte vor allem die zahlreichen Mitglieder aus Australien ermutigen, mir zu schreiben. Als Anreiz setze ich für den ersten australischen Beitrag eine Jahresmitgliedschaft bei der DUG als Preis. Das gilt natürlich auch für Afrika, wenn die DUG dort vertreten sein sollte.

Ich danke auf diesem Weg den vielen Freunden, die uns - meiner Frau und mir - immer wieder schreiben. Ich kann unmöglich alle Briefe gleich beantworten, aber Ihre Aufmunterungen und Anregungen machen mir die vielen Stunden hinter dem Computer noch erfreulicher.

Meine Frau war der Meinung, in eine Dezemberausgabe gehört nun einmal ein Christbaum, so erfand ich eine Christbaumfunktion (siehe nächste Seite - das XMAS.MTH-file ist auf der Diskette). Bei derartigen Problemstellungen wundert sie sich dann, dass ich viel Zeit hinter dem Computer verbringe. Ich habe schon Angst, dass ich zu Ostern …, na, ein Osterei ginge ja noch, aber ein Osterhase???

Sie finden als Beilage eine Rechnung für Ihren Mitgliedsbeitrag 1994. Ich hoffe natürlich, dass Sie Ihre Mitgliedschaft erneuern. Schicken Sie bitte wie bisher einen Euroscheck oder geben Sie uns mit der Allonge Ihre Kreditkartennummer bekannt. Für die Mitglieder aus Österreich liegt ein Zahlschein bei.

Im nächsten *DNL* werde ich über die DERIVE- Konferenz in Krems berichten. Bei dieser Gelegenheit möchte ich Sie auf die nächstjährige Konferenz in Plymouth hinweisen. Eine Information liegt diesem *DNL* bei.

Herr Dr. Wiesenbauer von der TU Wien hat versprochen, eine Kolumne für Zahlentheorie und Algebra mit DERIVE einzurichten. Herzlichen Dank dafür. Wir sind gespannt auf seine Ideen.

Noch ein Gedanke meiner Frau: ich werde im nächsten *DNL* die Gelegenheit wahrnehmen, mich persönlich näher vorzustellen. Ich glaube, das gehört auch dazu.

So bleibt mir nur noch, Ihnen ein schönes Weihnachtsfest und ein gutes, gesundes und vor allem friedliches Jahr 1994 zu wünschen.

Mit den besten Griißen Ihr

Dear DUG-Member,

It is a great pleasure for me to present the 12th issue of your *DNL*. Yes, I insist on saying "your" *DNL,* because this paper can only exist with your help, your enthusiasm and your interest. With your help it is possible to make up an interesting journal. Please remain in your activity in the future! In this issue you will find the first time a contribution from Asia. I miss Africa and Australia on the author’s list. I especially want to encourage our many members from Australia to write. As an incitement I grant a one year membership for the first Australian and African contribution.

By the way I have to thank the many of you, who wrote to me. Its not possible for me to answer all the letters immediately, but your friendly words and your ideas embellish my hours behind the PC.

It was my wife’s idea that in a Decembers issue a Christmas tree should not be missing. So I created a Xmas-tree-function (see next page -XMAS-MTH is on the diskette). She sets the problems and then she is wondering at the time I spend at the computer. I fear Easter time, an Easter-egg is would not be too difficult, but an Easter-Bunny. ...?

In this issue you will find an invoice for the dues for 1994. I hope that you will renew your membership. Please send a cheque or use the attached form to allow us to charge your creditcard (VISA or Master/Eurocard only).

I will give a report on the 2nd DERIVE Conference in Krems In the next *DNL*. At this occasion I want to remind you on the DERIVE Conference in Plymouth in July. You can find a respective information in this DNL.

Dr. Wiesenbauer from the Technical University of Vienna has promised to install a special column in the *DNL*: DERIVE – Algebra and Number Theory. Many thanks and we are looking forward to his future contributions.

Another one of my wife's ideas (she is full of them!): in the next DNL I will introduce myself.

At the end of this letter I wish you a merry Christmas and a happy, healthy and peaceful year 1994. I hope to meet you all again next year .

With my best regards as ever

The *DERIVE NEWSLETTER* is the Bulletin of the *DERIVE User Group*. It is published at least three times a year with a contents of 30 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE* as well as to create a group of experts to discuss the possibilities of new methodical and didactic manners in teaching Mathematics.

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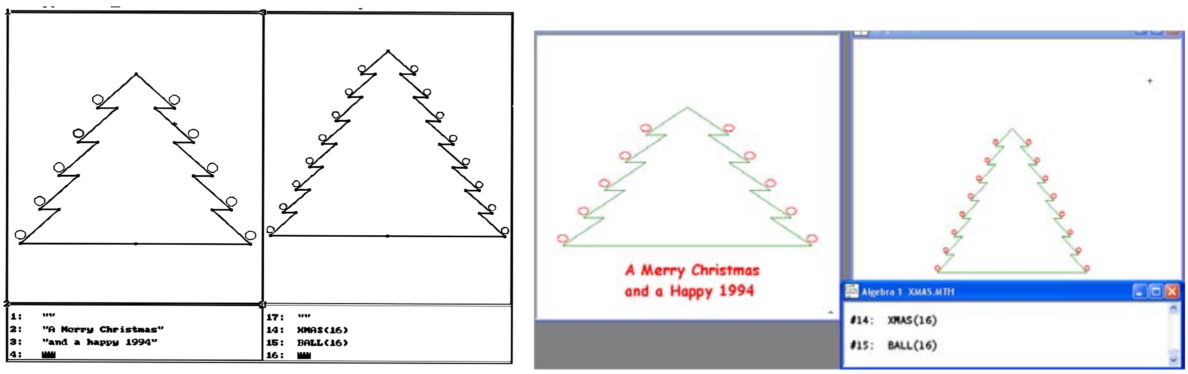
Contributions:

Please send all contributions to the below address. Non-English speakers are encouraged to write their contributions in English to underline the internationality of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send to the Editor, the more lively and richer in contents the Derive-News-Letter will be.

Preview: (Contributions for the next issues):

Fluid flow in DERIVE, Reuther a.o., BRA
Applications in Electrical Engineering, Scheuermann, GER
Minimization of a "Flat function", Lopes, POR
Stability od systems of ODEs, Kozubik, SLO
Los desplazamientos en las funciones elementales, Rarnos, ESP
Computer Algebra and Excel, Sjöstrand, SWE
Algebraic Operations on Polynomials in DERIVE, Roanes, ESP
Prime Iterating Number Generators, Wild, UK
Graphic Integration, Probability Theory, Linear Programming, Böhm, A
DERIVE in Austrian Schools, some examples, Lechner & Eisler, A
Tilgung fremderregter Schwingungen, Klingen, GER, and others



Original DOS version from 1993 versus DERIVE 6.10 version from 2006

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P .Gondrie, Tilburg, Netherlands

... Further more a question of *DERIVE*: Is it possible to declare a variable of type integer. For example: cos (k π) becomes then $(-1)^k$. Now it cannot simplify this answer.

In 1993 it was not possible, but now you can:

```
#1:   k :∈ Integer
#2:   COS(k·π) = (-1)^k
```

Dr.F.Schumm. Stuttgart. German~

Beiliegend schicke ich eine kleine ausgearbeitete Erwiderung zu "Mr. Robins 2nd problem" (*DNL* #11):

Da Herrn Robins Problem allgemeinerer Natur ist und bei DERIVE immer wieder auftaucht, gehe ich ausführlicher darauf ein. *(Because this problem is a general one I will treat it more precisely).*

1. Die "ungeschickte" Definition einer Ableitung

Es sei: $f(x) = x^3$. Für die Ableitung bilden wir `DIF(f(x),x)`, also $DF(x) := \frac{d}{dx} f(x)$

Geben wir nun `DF(u)` ein und wenden **Simplify** an, so erhalten wir wie erwartet: $3u^2$.

Das richtige Ergebnis ist jedoch trügerisch! Versuchen wir es entsprechend mit `DF(2)`, so erhalten wir den vielleicht nicht erwarteten Ausdruck $\frac{d}{d2} f(2)$.

If we define DF(x) := DIF(f(x),x), then we obtain DF(u) = $3u^2$ for f(x):= x^3 but we find also - probably not expected $\frac{d}{d2} f(2)$.

2. Begründung für das "falsche" Ergebnis

Schreiben wir `DF(2)` so meinen wir, daß erst nach x differenziert werden und dann im Resultat *x* durch 2 ersetzt werden soll. *DERIVE* hingegen substituiert erst und versucht dann zu differenzieren! Soweit ich beurteilen kann, werden grundsätzlich bei Funktionen F(*x*, *y*, *z*):= bei Ersetzen der Argumente *x*, *y*, *z* durch andere Ausdrücke z.B. F(2, *b* ,*c* – *d*) überall auf der rechten Seite der Funktionsdefinitionen sofort diese Ersetzungen durchgeführt, gleichgültig wo.
If we write DF(2) we intend to substitute 2 for x after forming the derivative, but DERIVE first substitutes and then does the derivation wrt the "new" variable.

3. Lösung des Problems

Wir müssen die Reihenfolge "erst Differenzieren, dann Substituieren" künstlich erzwingen. Das Erzwingen von Reihenfolgen lässt sich in DERIVE (fast?) immer mit dem Grenzwert `LIM` und einer Hilfsvariablen (hier *x_*) durchführen.
Wir definieren daher die Ableitung DF(x) besser so: wir differenzieren erst f(*x_*) nach *x_* und ersetzen im Resultat *x_* durch *x* mit Hilfe des Grenzübergangs `LIM` *x_* → *x*.

Jetzt ergibt auch DF(2) nach Simplify auch richtig den Wert 12.

$$\#2: \quad DF(x) := \lim_{x\_ \to x} \frac{d}{d\ x\_} f(x\_)$$

$$\#3: \quad [DF(u),\ DF(2)] = [3 \cdot u^2,\ 12]$$

Wenn man sich die UTlLITY-Files von DERIVE ansieht, so findet man an vielen Stellen `LIM`-Symbole, wo man sie zunächst nicht erwartet. Meist handelt es sich dann um das gleiche Problem, eine bestimmte Rechenreihenfolge einzuhalten, bzw. salopp formuliert: "Erst allgemein berechnen, dann substituieren."

("You should first calculate generally and then substitute!" So we *force DERIVE to do so, using LIM.)*

More examples from Mr Robin´s mail – now treated correctly:

$$\#3:\quad \left[\int x\cdot DF(x)\,dx,\ \int (x+u)\cdot DF(x)\,dx\right] = \left[\frac{3\cdot x^4}{4},\ \frac{3\cdot x^4}{4} + u\cdot x^3\right]$$

$$\#4:\quad \left[\int x\cdot DF(x+u)\,dx,\ DF(x+y)\right] = \left[\frac{(x+u)^3\cdot(3\cdot x - u)}{4},\ 3\cdot(x+y)^2\right]$$

Dr. Hubertus Wöhl, Freiburg, Germany

Solar physics with DERIVE on the HP95XL

As an astrophysicist with the actual working field of experimental solar research I am using since more than two years the Hewlett-Packard Palmtop PC (HP95IX) including –since several months – the DERIVE 2.5 program. Here are some of my experiences:

For a mobile PC with a mass of about 300 grams, which is always with you on the road, in the train, and on the plane, the main usages of the HP95LX are of course the time planning and telephone infos. In addition many statistical reductions can be performed using the LOTUS 1-2-3 (vers.2.2) package and the included HP calculator of the standard HP95LX. Since the (upgraded) version has 1 MB of RAM and I use a 2 MB SRAM card in the A-drive there is enough storage for the DERIVE program, its additional subroutines and all user group programs.

My applications of DERIVE are mainly curve fitting with determinations of the errors of the fit parameters, the plotting of data and curves, and applications of Fourier transforms to time series to determine periods. Most of these usages are during travelling, scientific conferences and observing periods at a solar observatory on the Canary Islands. In general no printouts are needed. Since I use at my office MAPLE V on a UNIX based workstation a part of the using of DERIVE is also testing of results from MAPLE applications. Of course this is limited to not very time consuming operations, because the processor speed of the HP95IX is rather slow.

A special feature of my HP95LX is the fact that it operates with solar power: it uses two NiCd rechargeable batteries lasting for several hours of computing time. The NiCd cells are recharged with solar energy powered equipment.

Josef Lechner, Viehdorf, Austria

Beim Befehl `ITERATES` sollte der Iterationsschritt als 5. Parameter dem Benutzer zugänglich sein, damit würde der `ITERATES`-Befehl viel an Flexibilität gewinnen.

Josef would like to have an additional 5th parameter in the ITERATES-command for the iteration step. ITERATES would gain much more flexibility.

Ein Beispiel: Das Abarbeiten einer "Befehlsliste" mit verschiedenen Funktionen wäre möglich.
As a example working through a "command list" containing various functions would become possible.

```
bliste:=[F1(x), F2(x), ..., Fn(x)]
PROGRAM(bliste):=ITERATES(ELEMENT(bliste,k),x,b0,DIMENSION(bliste),k)
```

mit dem Ergebnis (*with the result*):

bisher (*until now*): `[x0, F(x0), F(F(x0)), F(F(F(x0))),...]`
und dann (*and then*): `[x0, F1(x0), F2(F1(x0)), F3(F2(F1(x0))),...]`

D-N-L (from 2006): ITERATES didn´t change since 1993, but now we can write a little program which performs applies to Josef Lechner´s suggestions:

```
#1:    nested(funcs, x0, x, f) :=
         Prog
           Loop
             If DIM(funcs) = 0 exit
             f_ := FIRST(funcs)
             x0 := LIM(f_, x, x0)
             funcs := REST(funcs)
           x0
```

#2: $\text{nested}\left(\left[\text{SIN}(x),\ 1 + x,\ \frac{x^2}{2},\ \frac{1}{x}\right],\ \frac{\pi}{2}\right) = \frac{1}{2}$

#3: $\text{nested}\left(\left[\text{SIN}(x),\ 1 + x,\ \frac{x^2}{2},\ \frac{1}{x}\right],\ t\right) = \frac{2}{\text{SIN}(t)^2 + 2\cdot\text{SIN}(t) + 1}$

The 1st argument `funcs` is the list of functions which should be nested, followed by the initial value `x0` for variable `x` (by default). Compare the two next function calls:

#4: $\text{nested}\left(\left[\text{COS}(y),\ x\cdot y^2,\ e^{x\cdot y}\right],\ \pi\right) = e^{y^3\cdot\text{COS}(y)}$

#5: $\text{nested}\left(\left[\text{COS}(y),\ x\cdot y^2,\ e^{x\cdot y}\right],\ \pi,\ y\right) = e^{x^2}$

Klaus Herdt, Osnabrück, Germany

Wie bereits in der Beantwortung Ihres Fragebogens fehlen m.E. manchmal Hinweise mehr technischer Art. Beispielsweise könnte ich mir vorstellen, dass es eventuell auch anderen Anwendern nicht immer gelingt, die auf der Hardware ihres Rechners prinzipiell verfügbaren (und unter anderen Programmen auch durchaus einsetzbaren) Grafikmöglichkeiten auch unter DERIVE zu nutzen. Ich habe beispielsweise auf meinem PC eine Grafikkarte EIZO VA41 (1 MB) im Einsatz, die theoretisch eine Auflösung von 1024x768 bei 256 Farben ermöglicht. Die vom Hersteller angegebenen Modi 37h, bzw. 38h (16, bzw. 256 Farben bei einer Auflösung von 1024x768) werden von DERIVE zwar syntaktisch akzeptiert, nicht jedoch unterstützt. Neben DERIVE benutze ich bisweilen auch MAPLE V, rev.2 und dieses System bedient sich bei Betrieb unter DOS ganz analog zu DERIVE einer Initialisierungsdatei (MAPLE.INI). Hierin lassen sich die Darstellungsarten 800x600x16 bis 1024x768x256 ansprechen durch die (genormten?) Modusangaben 258 bis 261 (die dann wohl intern in die herstellerspezifischen Kennzahlen umgesetzt werden). Natürlich liegt es nahe, auch bei DERIVE einen Versuch mit diesen Modi zu unternehmen. Eine Anfrage beim Hersteller erbrachte lediglich die wenig befriedigende Antwort, dass dann wohl die Hardware nicht korrekt arbeite, was durch das MAPLE-Beispiel ja wohl widerlegt sein dürfte.
Vielleicht hat der eine oder andere Anwender ähnliche Erfahrungen gemacht und das Problem lösen können?!

D-N-L: I hope that there are any members who are willing to offer their experiences with "trouble shooting" graphic adapters or with trouble shooting DERIVE in connection with graphic cards. I have similar problems not with DERIVE but with GRASP and a Cirrus Logic Card.

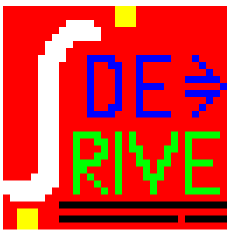
Wolfgang Pröpper, Nürnberg, Germany

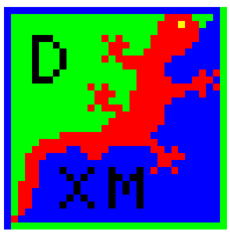
Auf beiliegender Diskette befindet sich ein Icon, das ich mir letzte Woche gestrickt habe. Ich stelle es gerne der Allgemeinheit zur Verfügung, d.h., ich verzichte auf alle, mir aus der Erstellung dieses grossen Werks zustehenden Urheberrechte. Damit müssen DERIVE-Anwender, die von WINDOWS aus starten, nicht mehr auf ein eigenes Icon verzichten und können DERIVE wie andere schöne Programme (MathCad, Mathematica} durch Doppelklick starten.

Die Installierung ist denkbar einfach: DERIVEXM.ICO wird ins DERIVE- bzw. ins WINDOWS-Verzeichnis kopiert. Im ProgMan wird im Menüpunkt *Datei-Eigenschaften* … der Pfad für DERIVE und gegebenenfalls das Arbeitsverzeichnis angelegt und dann mit *Anderes Symbol*. ... und *Durchsuchen* dieses Icon mit DERIVE verbunden. Mitgliedern der DUG können Sie diese Datei auf der 93er DNL-Datei mitschicken.

Noch viel schöner ware es allerdings, wenn möglichst bald eine WINDOWS- und/oder eine OS/2-Version von DERIVE erschiene. Am Icon muss es nicht mehr scheitern.

D-N-L: Many thanks Mr Pröpper for this nice Christmas gift for the User Group. With pleasure I copy your creation DERIVEXM.ICO onto the disk of the year 1993. But that our users who don't work with the XM-version have not to miss their icon I tried - stimulated by your idea - to "paint" DER1.ICO and DER2.ICO. Installation is very simple. Copy the icons into the DERIVE- or WINDOWS-directory and connect DERIVE with one of the icons using the Program Manager. If anyone of you have created other DERIVE-Icons, then please let me know. Let's have a DERIVE-Icon-Exhibition. Merry Christmas, Wolfgang, and many Christmas presents for you.





Hubertus Wöhl. Freiburg, Germany

Zu den Grafiken Erkki Ahonens sollte deutlich darauf hingewiesen werden, dass die Resultate im Wesentlichen auf Grund eines Programmfehlers entstehen und bei vielen DERIVE-Nutzern (so auch bei meinem Rechner mit Version 2.5) gar nicht verifiziert werden konnen.

(Mr Wöhl wants to point to the fact that the results of Erkki Ahonen are caused by a bug in the program and that many users who are working with later versions cannot verify them. ed.)

Günter Scheu, Pfinztal, Germany

Hier sind einige Befehle zum Halbierungsverfahren, wie wir es im Unterricht durchgeführt haben. Die Anregung erhielten wir im DNL#11 (Dyer).

(Some functions to the Bisection Method as we used them in school. We were inspired by Mr Dyer's submission in DNL#11.)

#1: Das Halbierungsverfahren – Bisectionmethod – G.Scheu,HALB_A1.MTH

#2: Beispiel zur Durchführung mit DERIVE – Example

#3: $$H(f, a, b) := \begin{bmatrix} a & m & b \\ a & \frac{a+b}{2} & b \\ f(a) & f(m) & f(b) \\ \lim\limits_{x \to a} f & \lim\limits_{x \to (a+b)/2} f & \lim\limits_{x \to b} f \end{bmatrix}$$

#4: Schrittweise Berechnung – Step by Step Calculation

#5: $H(x^2 - 4, 1, 4)$

#6: $$\begin{bmatrix} a & m & b \\ 1 & \frac{5}{2} & 4 \\ f(a) & f(m) & f(b) \\ -3 & \frac{9}{4} & 12 \end{bmatrix}$$

#7: $H\left(x^2 - 4, 1, \frac{5}{2}\right)$

#8: $$\begin{bmatrix} a & m & b \\ 1 & \frac{7}{4} & \frac{5}{2} \\ f(a) & f(m) & f(b) \\ -3 & -\frac{15}{16} & \frac{9}{4} \end{bmatrix}$$

#10: $$\begin{bmatrix} a & m & b \\ \frac{7}{4} & \frac{17}{8} & \frac{5}{2} \\ f(a) & f(m) & f(b) \\ -\frac{15}{16} & \frac{33}{64} & \frac{9}{4} \end{bmatrix}$$

#11: $H\left(x^2 - 4, \frac{7}{4}, \frac{17}{8}\right)$

#12: $$\begin{bmatrix} a & m & b \\ 1.75 & 1.9375 & 2.125 \\ f(a) & f(m) & f(b) \\ -0.9375 & -0.24609375 & 0.515625 \end{bmatrix}$$

#13: Programmierte Berechnung – Calculation programmed

#14:
```
HI(f, a, b, eps) :=
  If LIM(f, x, (a + b)/2) = 0 ∨ ABS(a - b) < eps
     H(f, a, b)
     If LIM(f, x, a)·LIM(f, x, (a + b)/2) < 0
        HI(f, a, (a + b)/2, eps)
        HI(f, (a + b)/2, b, eps)
```

#15: $HI(x^2 - 4, 1, 3, 0.001)$

#16: $$\begin{bmatrix} a & m & b \\ 1 & 2 & 3 \\ f(a) & f(m) & f(b) \\ -3 & 0 & 5 \end{bmatrix}$$

```
                2
#17:  HI(x  - 4, 1, 5, 0.001)
```

#18:

a	m	b
1	2	3
f(a)	f(m)	f(b)
-3	0	5

```
                2
#19:  HI(x  - 4, 1, 4, 0.001)
```

#20:

a	m	b
1.999755859	2.000122070	2.000488281
f(a)	f(m)	f(b)
$-9.765625\cdot10^{-4}$	$4.8828125\cdot10^{-4}$	$\frac{16}{8191}$

Hellmut Scheuermann, Hofheim/T., Germany

We have found a bug? In version 2.52, when we wanted to show that abs(x) cannot be differentiated at $x_0 = 0$, what to do with the result SIGN(0)?

#1: $F(x) := |x|$

#2: $MS(x) := \frac{F(x + h) - F(x)}{h}$

#3: Limit from the left:

#4: $MTL(x) := \lim_{h\to0-} MS(x)$

#5: MTL(0)

#6: -1

#7: wie erwartet richtig – how expected correct!

#8: $MTR(x) := \lim_{h\to0+} MS(x)$

#9: MTR(0)

#10: 1

#11: auch richtig, aber jetzt – correct, too, but now

#12: $MTB(x) := \lim_{h\to0} MS(x)$

#13: MTB(0)

#14: SIGN(0)

The recent version of DERIVE gives an answer for sign(0):

#16: MTB(0) = ±1

Message 3004 was entered on 5/24/93 at 8:52 AM.
From ROGER FOLSOM to SOFT WAREHOUSE about #2974 / COMMENTS ON DERIVE

I've discovered that one can get around the 512 limit on author line length by using the #number facility. That's worth mentioning in the manual.

Message 3019 was entered on 5/28/93 at 2:10 AM. (Read 10 times)
From ROGER FOLSOM to SOFT WAREHOUSE about DERIVE SUGGESTIONS

BUILD. A brief paragraph listing the advantages of BUILD over Author (bypassing line edit buffer size of 512 bytes, recording the sources of an expression, etc.) would be **well** worth including in the manual.

Message 3020 was entered on 5/28/93 at 2:11 AM. (Read 10 times)
From ROGER FOLSOM to SOFT WAREHOUSE about DERIVE SUGGESTIONS

POLYNOMIAL ORDERING

You write" I think we all agree that the polynomial $a\ x^2 + b\ x + c$ should be displayed in the above order. "I for one most defmitely do **not** agree. In at least some areas of applied mathematics (e.g. my own field of economics) the usual is to display polynomials in ascending powers, e.g. as $a_0 + a_1\ x + a_2\ x^2$ and so forth, simply because the degree is often not known initially, and adding or subtracting a term is a notational pain because it requires relabeling the coefficients. Turning your polynomial into a cubic would make it

$$a\ x^3 + b\ x^2 + c\ x + d \text{ or } a_0\ x^3 + a_1\ x^2 + a_2\ x + a_3;$$

note in your version that coefficient a no longer is for x^2, and in the last version that the coefficient subscripts do not match the term exponents. My apologies for belaboring the obvious, but such issues are what drives some of us to prefer ascending powers. [If the model will be estimated statistically, the degrees of polynomials used may depend on the data set and the degrees of freedom required -- more data, or "tighter" data (around a regression hypersurface), lets one consider higher degree polynomials; a model may go through several iterations in which polynomial degrees are varied depending on the statistical results]. Admittedly, ascending powers makes division of polynomials awkward (which is probably why "real" mathematicians such as yourself prefer descending powers), and in a division situation even I would order a polynomial in descending powers (as you would), but for many of us that case is an exception (the last time I divided any polynomials was in the early 1970s).

You write further: "Naturally, a switch could be added that would reverse the order of all sums, but this radical approach would screw up the display of quadratics like the above." For me, a quadratic or cubic displayed as

$$a + b\ x^2 + c\ x^3 + d\ x \quad \text{or} \quad a_0\ x^2 + a_1\ x^3 + a_2\ x + a_3$$

is **NOT** "screwed up" -- it is displayed normally. That simple switch is what I, and I think others, want.

I won't think anyone is asking DERIVE to remember whether a polynomial came from a Taylor expansion. We just want the option to force Derive to order **all** polynomials in ascending powers, with the constant term listed **first**. If you want to get fancy, you might include an override to cause polynomials in a particular expression (or subexpression) to be ordered opposite to however the global "polynomial ordering" switch is set.

AUTO-SIMPLIFICATION.

Thanks much for inviting our attention to the undocumented feature (entering a Transfer Load or Transfer Merge filename with a Ctrl-Enter, or Ctrl-J) that causes **every** line to be simplified as it is read in. I have discovered this to be helpful, but often one wants a line entered and **then** simplified, to keep a record of the steps in a calculation. Of course one can enter each command twice: once surrounded by quotes so that Derive takes it as a comment, and second as a real command that Derive will simplify automatically (using the Ctrl-Enter option). To have a simplify command that simplified the preceding expression would be somewhat more convenient. Nevertheless, the Ctrl-Enter option will be useful; thanks for un-undocumenting (aka documenting) it. My wife says I should ask: what other undocumented goodies are buried in Derive's depths?

Message 3021 was entered on 5/28/93 at 2:13 AM.
From ROGER FOLSOM to SOFT WAREHOUSE about DERIVE SUGGESTIONS

EXPAND and FACTOR

I realize that Expand can expand a sum to be **not** over a common denominator. However, in doing so it often rearranges variable ordering (of sums and products) in ways not to my liking. I realize also that this is supposed to be fixable by declaring secondary and tertiary etc. expand variables, but I have yet to figure out how those work. The manual (2.08) needs work here. Some way to factor an **expression** out of another expression would be very useful; e.g. if the expression a+b appears in several terms of a sum, it would be nice to be able to factor it out. Also, sometimes it improves readability to be able to factor an expression out to the front, or to the rear; to be able to choose would be a convenience.

Finally, for me the FACTOR command does not always work: I had a variable *dyda* (word mode) that I wanted to pull out of a two-term expression (where *dyda* appeared in each term), but Derive insisted on leaving it where it was. That is, factoring *dyda* out of $dyda*X_1 + dyda*X_2$, where X_1 and X_2 were two expressions, remained as $dyda*X_1 + dyda*X_2$, rather than as $dyda*(X_1 + X_2)$. The error here probably was mine, but I don´t know what my error was.

Message 3022 was entered on 5/28/93 at 6:35 AM. (Read 11 times)
From JERRY GLYNN to PUBLIC about ORDER AND EXPAND

Inspired by R Folsom's questions: If I aske Derive to EXPAND $(a+b)^2$ could I get $a^2 +2ab +b^2$ and if I EXPAND $(b+a)^2$ get $b^2 + 2ba + a^2$?

Message 3032 was entered on 6/2/93 at 10:27 PM.
From SOFT WAREHOUSE to ROGER FOLSOM about #3019 / DERIVE SUGGESTIONS

Thanks for your wide ranging thoughtful and insightful suggestions for improving DERIVE. The following is in response to some of them:

SCREEN PRINTING: When printing screen images, DERIVE actually reads the screen pixels to determine what to send to the printer. It is impossible to read what is not actually on the screen. However, some super VGA graphics adapter cards have video modes that can display 132 character columns, instead of the normal 80 columns. If then such a mode is properly supported by the BIOS and you know the number of the mode, select "Other" in the Adapter field of the Options Display command and enter the video mode number. Then, you can print screen images containing much longer expressions.

DOMAIN DECLARATION SYNTAX: Yes, the Declare Variable Domain command will create an expression declaring a variable' s domain, just like the Declare Variable Value command creates an expression declaring a variable's value.

VARIABLE ORDERING: Using an external text editor, you can make one line INI files that just contain a variable ordering list. Then use the Transfer Load State command to load these truncated INI files. It should work.

MATCHING PARENTHESES: Being a long-time LISPer I can certainly appreciate the advantage of jumping to matching parentheses. Also blinking matching parentheses would be nice.

BUILD COMMAND: In the next printing of the manual, I will discuss the benefits and drawbacks of the Build command. Aloha, Al Rich, Soft Warehouse, Inc.

Message 3033 was entered on 6/2/93 at 10:28 PM.
From SOFT WAREHOUSE to ROGER FOLSOM about #3020 / DERIVE SUGGESTIONS

POLYNOMIAL ORDERING: Your points are weIl taken. I will look into the ramifications of a flag for reversing sums.

VARIABLE ORDERING: Let's see if the above change is sufficient before making additional changes.

AUTO-SIMPLIFICATION: The Transfer Demo command reads in and displays an expression, and then simplifies it. However, it pauses after each simplification. The obvious solution is to have an option to not pause. Aloha, Al Rich, Soft Warehouse, Inc.

Message 3034 was entered on 6/2/93 at 10:29 PM.

From SOFT WAREHOUSE to ROGER FOLSOM about #3021 *I* DERIVE SUGGESTIONS

EXPAND and FACTOR: What is the specific example where *dyda* will not trivially factor out of $dyda*X1 + dyda*X_2$?

Message 3037 was entered on 615193 at 5:56 AM.

From JERRY GL YNN to SOFf W AREHOUSE about #3032 / DERIVE SUGGESTIONS

Hooray for Al Rich.. .just when I think I've wasted my breath some months ago I learn that I didn't. This will inspire new requests!

Message 3045 was entered on 6/6/93 at 4:41 PM. (Read 17 times)

From ROGER FOLSOM to SOFT WAREHOUSE about DERIVE SUGGESTIONS

Al Rich: a few responses to your much appreciated responses to my suggestions:

VARIABLE ORDERING: you note that to get a variable ordering list appropriate for a particular paper one is working on, one can load an *.ini file that contains only a one line ordering list. Thanks much for the suggestion. The manual might usefully mention that loading a second *.ini file does **not** dump all defaults previously loaded by an earlier *.ini file or while within Derive; only the new file's explicit **changes** are loaded.

Incidentally, my experience (Derive 2.08) is that the ordering list cannot be more than 128 characters; I bumped this limit the other day with a bunch of multi-character variables. Derive's error message was something akin to "can't fmd the end of flle; contact Soft-Warehouse, " but pruning the ordering list below 128 characters solved the problem. A 256 or even 512 character limit would be useful, but I realize that there are severe memory constraints and tradeoffs (at least in "ordinary" rather than XM Derive).

I will continue to observe carefully how Derive responds (in sums and products) to my ordering list, and I hope I can do better at predicting and forcing the patterns I want. You are right that an ascending powers switch will do much to make me even happier than I now am with Derive.

Nevertheless, I tentatively still believe that an order list (maybe call it a sequence list) going from least to most main, together with a "Derive, don't get too clever" switch, telling Derive to order all factors (and maybe equal power sums) in that sequence regardless of what Derive thinks is a parameter and what is a variable, would be a useful option.

EXPAND AND FACTOR: I should have explained more fully. The expression $dyda*X1 + dyda*X2$ initially involved a common denominator, and was written as

1) $$\frac{dyda\,(X1 \ + \ N * X2)}{N}$$

I didn't want it over a common denominator, so I expanded it around *N*, and got

2) $$\frac{dyda * X1}{N} + dyda * X2$$

Then when I **simplified** (2), I got (1) back again. When I **factored** *dyda* out of (2), I got (1) back again. So I gave up! I wanted, but was unable to get,

3) $$dyda\left(\frac{X1}{N} + X2\right)$$

As noted earlier, also useful would be the ability to factor an expression out of another expression; perhaps when the factoring command asks for the variable to factor, an expression (rather than a simple variable) could be typed, or selected by highlighting.

AUTO-SIMPLIFICATION. The "control-enter" option for loading a file, so that the file simplifies as it loads is useful, but it does not substitute fully for a statement within the file to simplify a particular statement. Sometimes one does **not** want all statements simplified.

Example: when I used "control-enter" (or Load Demo) to load a file containing the definitions y(Nt,LNt,t,a): = and Y: =y(Nt,LNt,t,a), Derive simplifies these to y(Nt,LNt,t,a) and y(Nt,LNt,t,a), and then doesn't know what to do with later expressions such as DIF(Y,x) because it doesn't know what Y is. I don't see how to use Transfer Demo (with or without a pause, especially since the pause comes **after** the simplification) to deal with this problem. Thus a "simplify previous (or next) line" command still appears useful. With it, Derive would **add** some of the advantages of a "compiled" program, that one could "run" on a slow machine while one does something else (e.g. eat dinner, or go to bed). But simplifying everything is too much.

Message 3056 was entered on 6/12/93 at 5:52 AM.
From HARALD LANG to JEFF I about #3049 / TO: Y' ALL

In addition to Jerry Glynn ' s suggestion, you need to tell the printer to use the IBM character set. Furthermore, the output gets much nicer, in my opinion at least, if you choose a line spacing of 12 lines per inch (or maybe even 16). I send the following sequence of control codes when I print to laser printers from Derive:

< ESC > E < ESC > (10U < ESC > &112D

In addition, I send control sequences to set margins and so on, but since my European paper format is different from yours, I suppose you will find them useless. Note: < ESC > stands for the ASCII character 27. Regards -Harald Lang

Ebene Algebraische und Transzendente Kurven (2)

Thomas Weth, Würzburg, Germany

Die Strophoide

In einem Briefwechsel zwischen F. de Verdus und Torricelli vom 15. März 1645 und dem 19.Mai 1645 wird eine "Linie" beschrieben. die" ... in Frankreich Ala oder Pteroide genannt wird ...". Die Erfindung der *Strophoide*, wie die Kurve heute allgemein genannt wird, geht somit auf einen französischen Mathematiker, wahrscheinlich Roberval zurück. Die ursprüngliche etwas komplizierte räumliche Konstruktion geschieht mit Hilfe von Brennpunkten von Kegelschnitten. In der mathematischen Literatur fand die Strophoide dann 1757 im
IV. Band der *lnstituti Bononiensis Comentarii* in den zwei Denkschriften von Gregor Casali *De conicarum sectionum focis* (Über die Brennpunkte von Kegelschnitten) Beachtung, geriet aber zunächst in Vergessenheit. Casali schlug den Namen *Pteroides torricellanea* vor, um die Verdienste Torricellis (einem Schüler Galileis) um die Kurve zu würdigen. Ab Mitte des 18. Jhdts beschäftigten sich u.a. de Moivre, der eine Konstruktion in der Ebene fand und die Quadratur der Kurve angab, Maria Gaetana Agnesi (1748), Cramer (1750), Ricati und Saladini (1765) mit dieser Kurve. 1819 wurden die Kurven von A. Quetelet in der *Dissertatio de quibusdam locis geometricis* (Abhandlung über einige geometrische Örter) als *Fokale* bezeichnet. Durch eine Preisaufgabe aus dem Jahre 1840, die den französischen Lyceen und 1861 im Examen für die Zulassung zur Ecole normale in Paris vorgelegt wurde, entstanden zahlreiche Untersuchungen der Kurve. Zum erstenmal tritt der Name *Strophoide* in den Nouv. Ann. V., 1846 in der Arbeit *La strophoide* von Montucci auf; abgeleitet ist er vom griechischen στροφοζ (= Band, Seil). Bis zum Ende des 19. Jhdts trat die Kurve unter den Namen *Kukumaeide* (Bezeichnung durch Uhlhorn – 1809 – und Lehmus – 1842), *Logocyklika* (Booth – 1858) und *harmonische Kurve* (Rummer – 1868) auf.

Unter den vielen interessanten Eigenschaften der Strophoide seien hier genannt: Die Strophoide ergibt sich als Fußpunktkurve einer Parabel, als Bild einer gleichseitigen Hyperbel bei der Inversion am Kreis oder als sog. "Gegenkurve" einer Geraden. Außerdem ist sie eine *anallagmatische* Kurve, also eine Kurve, die unter der Inversion am Kreis auf sich selbst abgebildet wird.

The Strophoid

In a correspondence between F. de Verdus and Torricelli (1645) a "line" is described which " ... in France is called Ala or Pteroide ...". The *strophoid* was discovered out in France, probably by Roberval. The first construction used the foci of conics in space. Later we can find the curve in two papers of Casali. He proposed the name *Pteroides torricellanea.* In the 18th century Moivre, M. G. Agnesi, Cramer, Ricati and Saladini dealt with this curve. In the 19th century especially french mathematicians investigated the curve. In Montucci's publication *La strophoide* from 1846 you can find the curve's name the first time. The name is derived from the greek word στροφοζ (= band, rope). Till the end of the 19th century there were some other names used as *Kukumaeide, Logocyklika* and *harmonic Curve*.

Among the many interesting properties of the strophoid you can find: it is pedal curve of a parabola, image of an equilateral hyperbola using the inversion on a circle or the so called "anticurve" of a line. The strophoid is an anallagmatic curve, i.e. it is mapped onto itself using the inversion on a circle
.

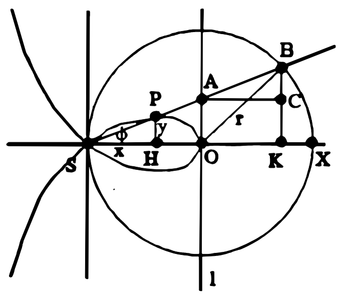
Die Konstruktion der Kurve

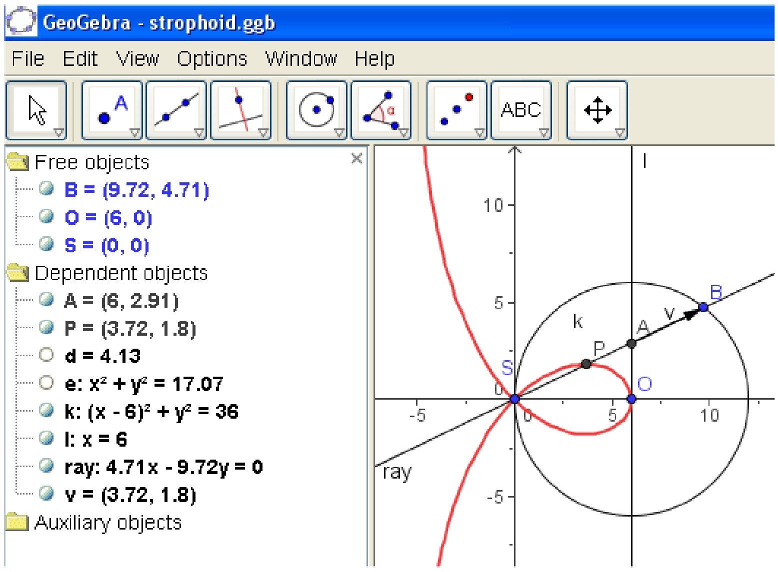
Gegeben ist ein Kreis k(O,r), sowie zwei senkrechte Durchmesser SOX and l. Durch S zeichnet man einen Strahl, der l in A und den Kreis in B schneidet. Auf dem Strahl trägt man von S aus die Strecke AB ab und erhält den Endpunkt P. Dreht man nun den Strahl um S, so beschreibt P eine Strophoide.

Construction of the Curve

Given is the circle k(O,r) and two orthogonal diameters SOX and l. Draw a ray through S giving the intersections A with l and B with k. Using the distance AB = SP you will obtain point P. Rotating the ray round S point P will describe a strophoid.

Compare the sketch from 1993 with the pretty GeoGebra construction from 2006.





Herleitung der Kurvengleichung **(Derivation of the equation of the curve)**

Gesucht sind die Koordinaten des Strophoidenpunktes P (x,y). Aus dem Strahlensatz folgt

$\overline{AO} : \overline{SO} = y : x$, daher $\overline{AO} = \frac{y \cdot r}{x}$. Da nach der Konstruktion $\overline{SP} = \overline{AB}$, sind die Dreiecke ΔSHP und ΔACB kongruent. Damit gilt auch $\overline{SH}(= x) = \overline{AC} = \overline{OK}$.

Für die Koordinaten von B ergibt sich somit $B\left(x + r, \frac{y \cdot r}{x} + y\right)$. Da B ein Kreispunkt ist, gilt:

$(x + r - r)^2 + \left(\frac{y \cdot r}{x} + y\right)^2 = r^2$. Für den Strophoidenpunkt gilt dann (nach Umformung mit *DERIVE*):

$$y^2(x + r) + x^2(x - r) = 0 \quad \text{bzw.} \quad y = \pm x\sqrt{\frac{r - x}{r + x}}.$$

Damit sind die Strophoiden Kurven dritter Ordnung. (The strophoids are algebraic curves of order 3.)

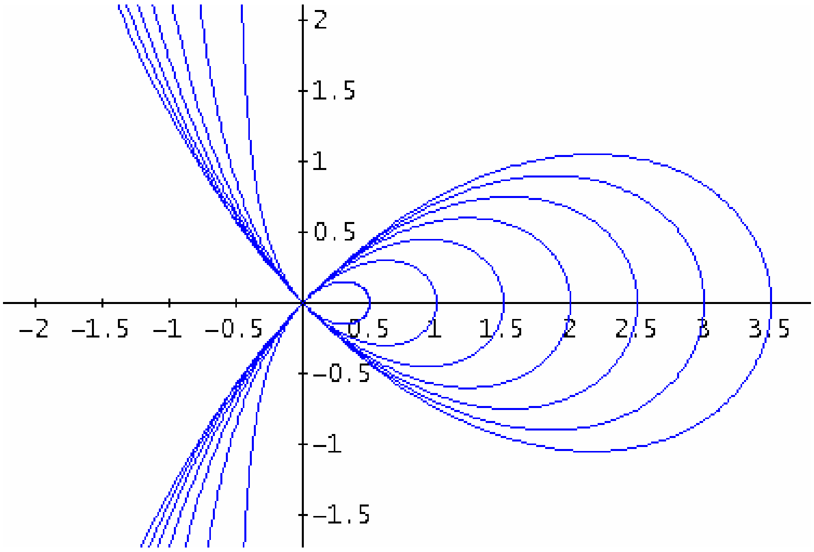
Mit $x = \rho \cos \varphi$ und $y = \rho \sin \varphi$ erfolgt mit einiger Umformung (*DERIVE*) die Polardarstellung der Strophoide und eine mögliche Parameterdarstellung (Polar form and one possible Paramter form):

$$\rho = r\frac{\cos 2\varphi}{\cos \varphi} \quad \text{bzw.} \quad \begin{pmatrix} x = r\dfrac{t^2 - 1}{t^2 + 1} \\ y = r\,t\dfrac{t^2 - 1}{t^2 + 1} \end{pmatrix}.$$

(See also the following DERIVE file.)

Kurzreferenz für DERIVE User:

Die Strophoide / The Strophoid



VECTOR(r·COS(2·φ)/COS(φ), r, 0.5, 3.5, 0.5)

Polarform:	ρ = r·COS(2·φ)/COS(φ)
Parameterform:	[r·(t^2 - 1)/(t^2 + 1), r·t·(t^2 - 1)/(t^2 + 1)]
Algebraische Kurvengleichung:	y^2 (x + r) + x^2·(x - r) = 0

The area of the loop is $2r^2 - \frac{r^2\pi}{2}$; the area between the curve and the asymptote $x = -r$ is $2r^2 + \frac{r^2\pi}{2}$.

#1: CaseMode := Sensitive

#2: $k := (x - r)^2 + y^2 = r^2$

#3: l := x = r

#4: ray := y = TAN(f)·x

#5: A := [r, TAN(f)·r]

#6: SOLUTIONS(k ∧ ray, [x, y])

#7: $\begin{bmatrix} 0 & 0 \\ 2\cdot r\cdot COS(f)^2 & 2\cdot r\cdot SIN(f)\cdot COS(f) \end{bmatrix}$

#8: $B := \left[2\cdot r\cdot COS(f)^2,\ 2\cdot r\cdot SIN(f)\cdot COS(f)\right]$

#9: $d := |A - B|$

#10: $d := \left|2\cdot r\cdot COS(f) - \frac{r}{COS(f)}\right|$

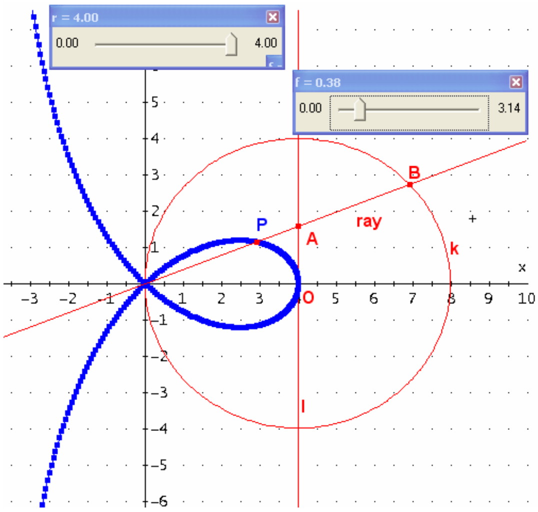
#11: $\text{SOLUTIONS}(\text{ray} \wedge x^2 + y^2 = d^2, [x, y])$

#12: $$\begin{bmatrix} 2\cdot r\cdot \text{COS}(f)^2 - r & 2\cdot r\cdot \text{SIN}(f)\cdot \text{COS}(f) - r\cdot \text{TAN}(f) \\ r - 2\cdot r\cdot \text{COS}(f)^2 & r\cdot \text{TAN}(f) - 2\cdot r\cdot \text{SIN}(f)\cdot \text{COS}(f) \end{bmatrix}$$

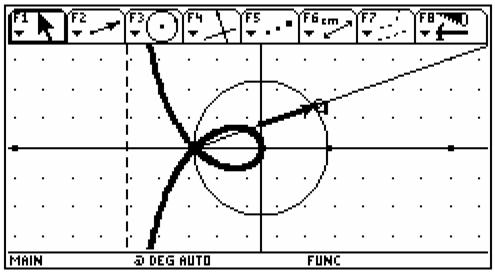
#13: $P := [2\cdot r\cdot \text{COS}(f)^2 - r, 2\cdot r\cdot \text{SIN}(f)\cdot \text{COS}(f) - r\cdot \text{TAN}(f)]$

#14: $[2\cdot r\cdot \text{COS}(t)^2 - r, 2\cdot r\cdot \text{SIN}(t)\cdot \text{COS}(t) - r\cdot \text{TAN}(t)]$

#15: $\left(\text{TABLE}\left([2\cdot r\cdot \text{COS}(t)^2 - r, 2\cdot r\cdot \text{SIN}(t)\cdot \text{COS}(t) - r\cdot \text{TAN}(t)], t, 0, \pi, \frac{\pi}{400}\right)\right)\downarrow\downarrow[2, 3]$



The strophoid on the Voyage 200



It is important to work with the vector AB to translate the origin on the ray OB for obtaining point P. Then we can either trace point P or generate the locus. (I didn´t label the points on the TI-screen)

#16: $[2\cdot r\cdot \text{COS}(\phi)^2 - r, 2\cdot r\cdot \text{SIN}(\phi)\cdot \text{COS}(\phi) - r\cdot \text{TAN}(\phi)]$

#17: Trigonometry := Expand

#18: $\left[2\cdot r\cdot \text{COS}(\phi)^2 - r, 2\cdot r\cdot \text{SIN}(\phi)\cdot \text{COS}(\phi) - \frac{r\cdot \text{SIN}(\phi)}{\text{COS}(\phi)}\right]$

#19: Trigonometry := Auto

#20: $\text{SOLUTIONS}\left(\rho\cdot \text{COS}(\phi) = 2\cdot r\cdot \text{COS}(\phi)^2 - r \wedge \rho\cdot \text{SIN}(\phi) = 2\cdot r\cdot \text{SIN}(\phi)\cdot \text{COS}(\phi) - \frac{r\cdot \text{SIN}(\phi)}{\text{COS}(\phi)}, [\rho, x]\right)$

#21: $\left[\left[2\cdot r\cdot \text{COS}(\phi) - \frac{r}{\text{COS}(\phi)}, @1\right]\right]$

This is the polar form; plot it changing the system of coordinates

#22: $2\cdot r\cdot \text{COS}(\phi) - \frac{r}{\text{COS}(\phi)}$

Bring over one common denominator and then Trig Collect

#23: $\frac{2\cdot r\cdot \text{COS}(\phi)^2 - r}{\text{COS}(\phi)}$

#24: Trigonometry := Collect

#25: $\frac{r\cdot \text{COS}(2\cdot \phi)}{\text{COS}(\phi)}$

#26: $\left[x = 2\cdot r\cdot \text{COS}(\phi)^2 - r, y = 2\cdot r\cdot \text{SIN}(\phi)\cdot \text{COS}(\phi) - \frac{r\cdot \text{SIN}(\phi)}{\text{COS}(\phi)}\right]$

#27: $\left[x = 2\cdot r\cdot \mathrm{COS}(\mathrm{ACOS}(t))^2 - r,\ y = 2\cdot r\cdot \mathrm{SIN}(\mathrm{ACOS}(t))\cdot \mathrm{COS}(\mathrm{ACOS}(t)) - \frac{r\cdot \mathrm{SIN}(\mathrm{ACOS}(t))}{\mathrm{COS}(\mathrm{ACOS}(t))}\right]$

#28: $\left[x = r\cdot(2\cdot t^2 - 1),\ y = r\cdot\sqrt{1 - t^2}\cdot\left(2\cdot t - \frac{1}{t}\right)\right]$

#29: $\mathrm{SOLUTIONS}(x = r\cdot(2\cdot t^2 - 1),\ t)$

#30: $\left[\frac{\sqrt{2}\cdot\sqrt{\left(\frac{x + r}{r}\right)}}{2},\ -\frac{\sqrt{2}\cdot\sqrt{\left(\frac{x + r}{r}\right)}}{2}\right]$

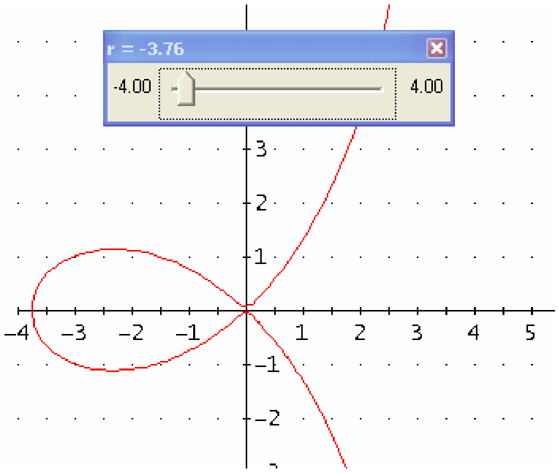
#31: $y = r\cdot\sqrt{1 - t^2}\cdot\left(2\cdot t - \frac{1}{t}\right)$

#32: $y = r\cdot\sqrt{\left(1 - \left(\frac{\sqrt{2}\cdot\sqrt{\left(\frac{x + r}{r}\right)}}{2}\right)^2\right)}\cdot\left(2\cdot\frac{\sqrt{2}\cdot\sqrt{\left(\frac{x + r}{r}\right)}}{2} - \frac{1}{\frac{\sqrt{2}\cdot\sqrt{\left(\frac{x + r}{r}\right)}}{2}}\right)$

#33: $y = \frac{x\cdot\sqrt{\left(\frac{r - x}{r}\right)}}{\sqrt{\left(\frac{x + r}{r}\right)}}$

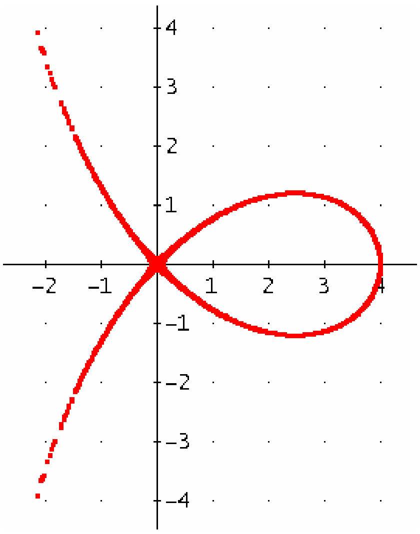
#34: $\left(y = \frac{x\cdot\sqrt{\left(\frac{r - x}{r}\right)}}{\sqrt{\left(\frac{x + r}{r}\right)}}\right)^2$

#35: $y^2 = \frac{x^2\cdot(r - x)}{x + r}$



This is the plot of the strophoid using a tool from from DNL#63 (2006).

$\mathrm{impl}\left(y^2 = \frac{x^2\cdot(4 - x)}{x + 4},\ -4,\ 4,\ -4,\ 4\right)$



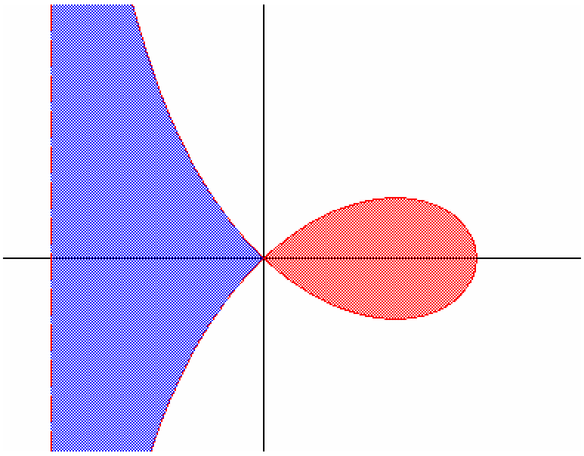
The area included by the bend and the area included by the curve and the asymptote $x = -r$ can easily be computed by the CAS.

The red coloured area:

$$\#37:\quad 2\cdot\left|\int_{0}^{r}\frac{x\cdot\sqrt{\left(\frac{r-x}{r}\right)}}{\sqrt{\left(\frac{x+r}{r}\right)}}\,dx\right| = \frac{r^2\cdot(4-\pi)}{2}$$

The blue coloured area

$$\#38:\quad 2\cdot\left|\int_{-r}^{0}\frac{x\cdot\sqrt{\left(\frac{r-x}{r}\right)}}{\sqrt{\left(\frac{x+r}{r}\right)}}\,dx\right| = \frac{r^2\cdot(\pi+4)}{2}$$



As a appendix (2006) I´ll show how to create the strophoid as pedal curve of a parabola. The pedal point is the origin of the directrix (=P). To make calculation easier we set the parabola into the plane that the *y*-axis is the directrix, i.e. the origin is the pedal point: $y^2 = 4r(x-r)$. The pedal curve is the locus of all intersection points between the tangents of the parabola and the perpendicular lines through the origin wrt the tangents.

This is a nice – and not too difficult – task for students. Introducing slider bars adds some animation.

$$\#1:\quad y^2 = 4\cdot r\cdot(x-r)$$

$$\#2:\quad [0,\ 0]$$

$$\#3:\quad \text{tang} := y = \frac{r}{\sqrt{(r\cdot t - r^2)}}\cdot(x-t) + 2\cdot\sqrt{(r\cdot t - r^2)}$$

$$\#4:\quad \text{perp} := y = -\frac{x\cdot\sqrt{(r\cdot(t-r))}}{r}$$

$$\#5:\quad (\text{SOLUTIONS}(\text{tang} \wedge \text{perp},\ [x,\ y]))_1$$

$$\#6:\quad \left[\frac{r\cdot(2\cdot r-t)}{t},\ \frac{(t-2\cdot r)\cdot\sqrt{(r\cdot(t-r))}}{t}\right]$$

Insert slider bars for *r* and *t* and see the tangents, perpendicular lines and their intersection points moving around.

#6 is – one possible – parameter form of the strophoid. For plotting purposes one has to substitute for *t*, because *t* is used as slider bar variable. Elimination of the parameter leads to the implicit form of the strophoid.

$$\#7:\quad \left[x = \frac{r\cdot(2\cdot r-u)}{u},\ y = \frac{(u-2\cdot r)\cdot\sqrt{(r\cdot(u-r))}}{u}\right]$$

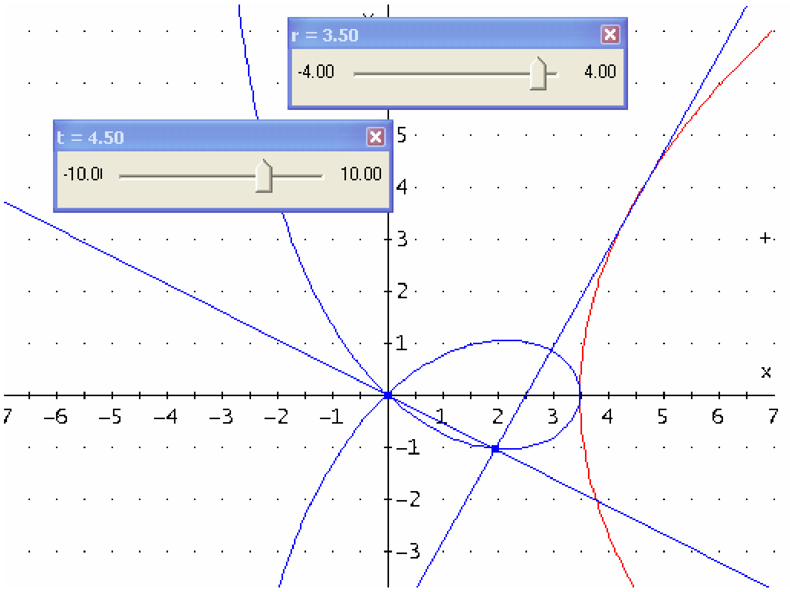
$$\#8:\quad \left(\text{SOLUTIONS}\left(x = \frac{r\cdot(2\cdot r-u)}{u},\ u\right)\right)_1 = \frac{2\cdot r^2}{x+r}$$

$$\#9:\quad \text{SUBST}\left(y = \frac{(u-2\cdot r)\cdot\sqrt{(r\cdot(u-r))}}{u},\ u,\ \frac{2\cdot r^2}{x+r}\right)$$

$$\#10:\quad y = -x\cdot\text{SIGN}(r)\cdot\sqrt{\left(\frac{r-x}{x+r}\right)}$$

$$\#11:\quad \left(y = -x\cdot\text{SIGN}(r)\cdot\sqrt{\left(\frac{r-x}{x+r}\right)}\right)^2$$

$$\#12:\quad y^2 = \frac{x^2\cdot(r-x)}{x+r}$$



Explore Chaos with NEWTON and RAPHSON

Josef Böhm, Würmla, Austria

I am sure that in a very near future many classical items of school mathematics will be obsolete if we are able and willing to use Computer Algebra tools. They will not be obsolete at all, but the way we teach and use them will have to change. Lets have a glance to some of teachers' most beloved "children": discussion of a function - calculating the zeros, differentiating and solving equations to find maximum and minimum values and inflection points and using the results for sketching the graph. Now a simple keypress on two buttons gives the graph within less than a second and DIF in connection with SOLVE will do the work for most of the examples which can be found now in our students' textbooks. I really don t believe that we will not have to teach this any longer, but there are new and fascinating possibilities and we can give traditional problems some new qualities. I remember with great pleasure Paul Drijvers' lecture held in Krems this year. He showed examples how the investigation of a function's properties can be changed in a thrilling way for students – and teachers as well.

Another problem has a fixed place in our school mathematics. We are glad to demonstrate the Newton-Raphson algorithm as a first application of calculus. But now, which need is for this algorithm any longer, when pressing L in approximate mode solves the equation (DOS-DERIVE). We should use the occasion to focus upon the algorithm and specially on the importance of the initial value or initial guess of this iterative process in order to understand some "strange" results given by CAS-programs.

Using DERIVE's graphic features it is easy to create a function for plotting the family of tangents emerging from any given initial value. So you can observe the iteration and convergence process. Zooming into the neighbourhood of the zero gives more insight.

We will have a very special view on this method:

First of all load the utility file RAPHSON.MTH, edit the function `u:=f(x)` and plot `TAN_FAM(u,x0,n)`, which gives a family of *n* tangents (linear approximations) with initial value *x0*. Set Points Connected (and Small Size) and you will see the first *n* steps of the linear approximation process – with hopefully visualization of the convergence towards a zero of the function.

```
#1:  NEW_ROW(u, x, x0, n) := ITERATES(x - u/∂(u, x), x, x0, n)

#2:  NEW_COL(u, x, x0, n) := NEW_ROW(u, x, x0, n)`

#3:  NEWTON(u, x, x0, n) := ITERATE(x - u/∂(u, x), x, x0, n)

#4:  LINE(x0, y0, d1, x) := d1·x + y0 - d1·x0

#5:  TANGENT(u, x, x0) := LINE(x0, LIM(u, x, x0), LIM(∂(u, x), x, x0), x)

#6:  VERT(u, x0) := [x0, 0; x0, LIM(u, x, x0)]

#7:  TAN_FAM(u, x0, n) := [VECTOR([VERT(u, (NEW_ROW(u, x, x0, n))↓k), TANGENT(u,
     x, (NEW_ROW(u, x, x0, n))↓k)], k, n)]

#8:  NEW_ATTR(u, start, end, inc) := VECTOR(NEWTON(u, x, start + k), k, 0,
     end - start, inc)

#9:  NEW_ATTR_GR(u, start, end, inc, n) := VECTOR([start + k, NEWTON(u, x,
     start + k, n)], k, 0, end - start, inc)
```

Functions #1 - #7 are from an excellent new DERIVE publication (Book Shelf [7]). Expression #2 returns the values of the algorithm as a column vector.

Example 1: Take the simple cubic equation $x^3 - x = 0$. What can be done? It is clear that there are three zeros: -1, 0 and +1. There is no need to apply a numeric process to find the zeros. But it might be interesting for students that three initial values which are very close like $x_0 = 0.44720$, $x_{0'} = 0.44725$ and $x_0 = 0.44730$ will converge towards quite different zeros. Furthermore they will recognize that they will need more than 10 iteration steps in case of the third initial value for noticing any convergence. 15 steps ore more will be enough, depending on the requested precision.

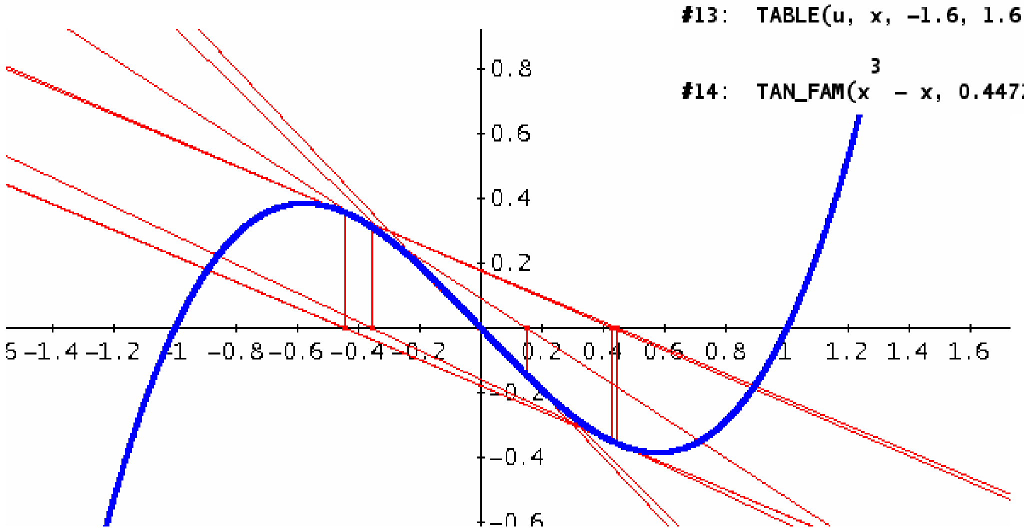
#10: $u := x^3 - x$

#11: NEW_ROW(u, x, 0.4472, 10)

#12: [0.4472, -0.4471320317, 0.4467245477, -0.4442912969, 0.4300999982, -0.3575504623, 0.148295498, -0.006983212209, $6.811758641\cdot 10^{-7}$, 0, 0]

#13: TABLE(u, x, -1.6, 1.6, 0.001)

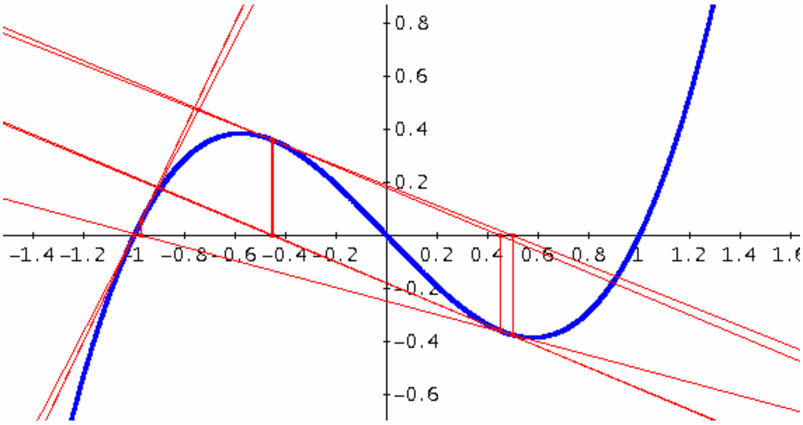
#14: TAN_FAM(x^3 - x, 0.4472, 10)



#15: NEW_ROW(u, x, 0.44725, 10)

#16: [0.44725, -0.4474320891, 0.4485269635, -0.4551814749, 0.4984238057, -0.972212844, -1.001238546, -1.000002294, -1, -1, -1]

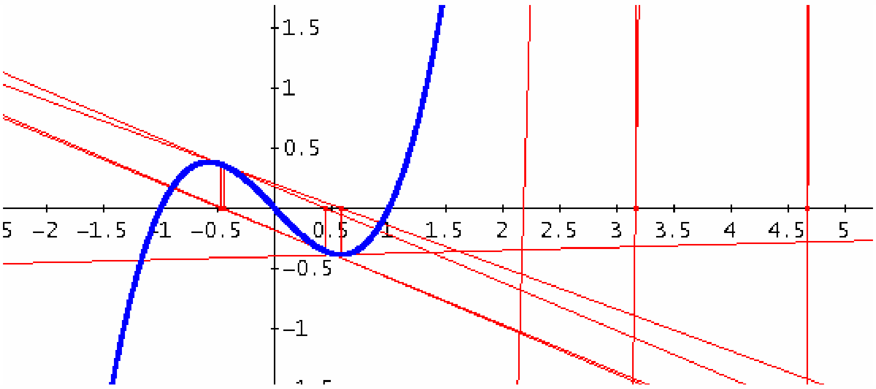
#17: TAN_FAM(x^3 - x, 0.44725, 10)



#18: NEW_ROW(u, x, 0.4473, 10)

#19: [0.4473, -0.4477323983, 0.4503400086, -0.4664759754, 0.5847059419, 15.59091934, 10.40821911, 6.960229289, 4.672301492, 3.16316677, 2.181452043]

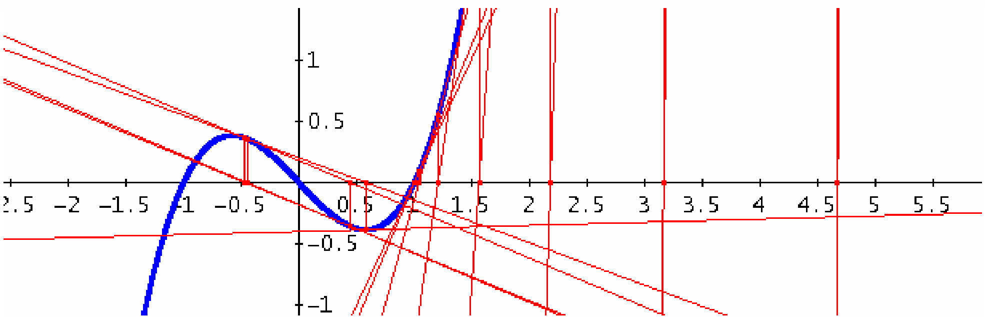
#20: TAN_FAM(x^3 - x, 0.4473, 10)



#21: TAN_FAM(x^3 - x, 0.4473, 15)

#22: NEW_ROW(u, x, 0.4473, 15)

#23: [0.4473, -0.4477323983, 0.4503400086, -0.4664759754, 0.5847059419, 15.59091934, 10.40821911, 6.960229289, 4.672301492, 3.16316677, 2.181452043, 1.563843359, 1.207086802, 1.043431923, 1.002569376, 1.000009843]

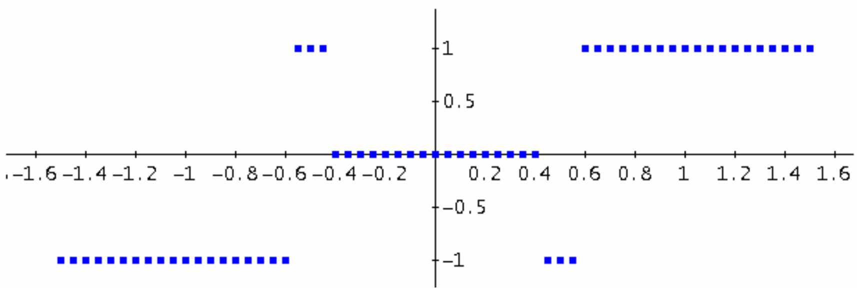


Expression #8 `NEW_ATTR(u,start,end,inc)` returns a list containing all "results" of the Newton-Raphson algorithm with initial values x_0 = *start* to x_0 = *end* with increment *inc*. So let´s try:

```
#24:  NEW_ATTR(u, -1.5, 1.5, 0.05)

#25:  [-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, 1, 1, 1, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1]
```

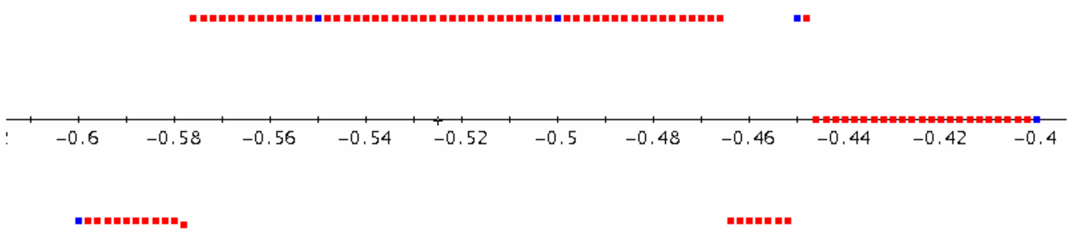
In #9 I define `NEW_ATTR_GR(u,start,end,inc,n)` to return a list of points [x_0, value of the process after *n* iterations] with *start* ≤ x_0 ≤ *end*. I plot `NEW_ATTR_GR(u,-1.5,1.5,0.05,10)`. (Don´t forget to activate *Approximate before plotting* in the Plot Options!)

```
#26:  NEW_ATTR_GR(u, -1.5, 1.5, 0.05, 15)
```

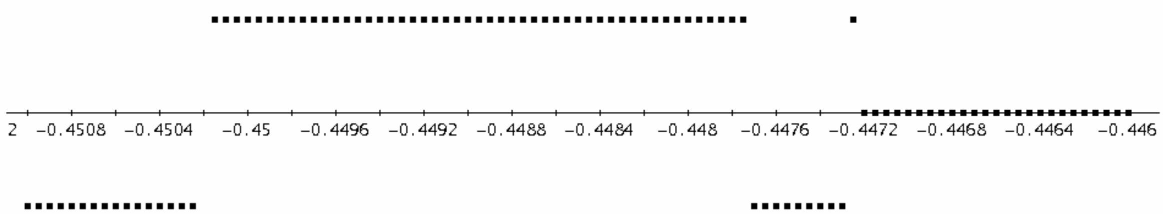


Then we get closer – zooming into the interval [-0.6, -0.4] using an increment of 0.02 – and then into the interval [-0.451, -0.446] with *inc* = 0.00005.

```
#27:  NEW_ATTR_GR(u, -0.6, -0.4, 0.002, 15)
```




```
#28:  NEW_ATTR_GR(u, -0.451, -0.446, 0.00005, 15)
```



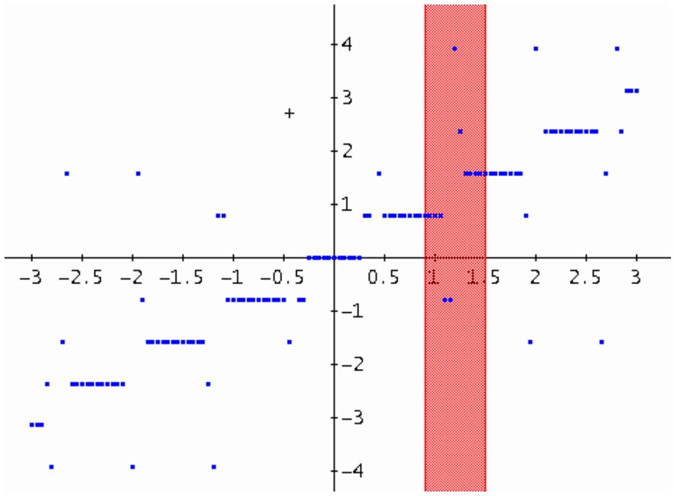
By zooming in we can observe – unexpected – similar patterns. A relationship to "CHAOS" is obvious. Selfsimilarity is a significant characteristic of some chaotic evolutions!!

It is clear that functions with many zeros will show much more interesting patterns. We should ask the students to define functions, make investigations and then report about their findings.

I add one much more striking example:

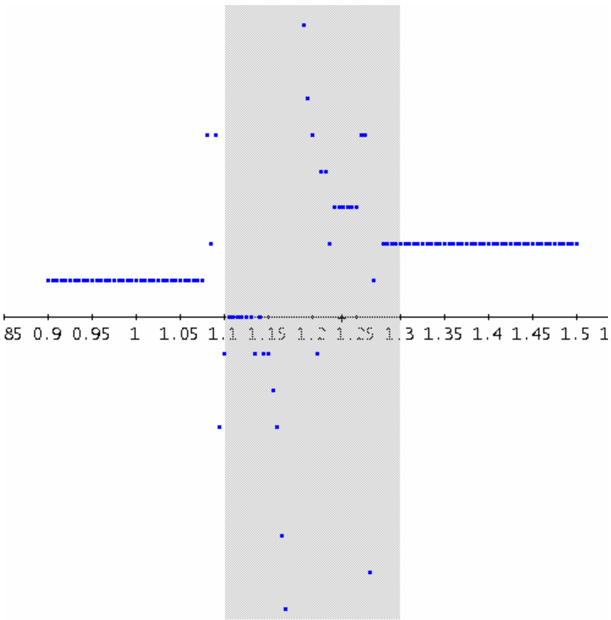
Example 2: I want to investigate the behaviour of this algorithm with another function. I am taking $u = \sin 4x$, having in mind that there will appear "many" zeros.

```
#29:   u := SIN(4·x)

#30:   NEW_ATTR_GR(u, -3, 3, 0.05, 15)

#31:   0.9 < x < 1.5
```



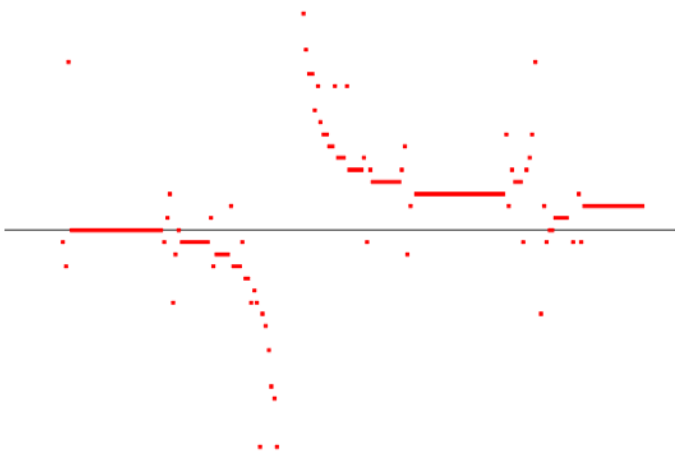
I zoom into the interval [0.9, 1.5] and discover an unexpected pattern:

```
#32:   PrecisionDigits := 20

#33:   NotationDigits := 20

#34:   NEW_ATTR_GR(u, 0.9, 1.5, 0.005, 15)

#35:   1.1 < x < 1.3
```



Next zoom in into [1.1,1.3] showing another pattern. **CHAOS IS PERFECT.**

```
#36:   NEW_ATTR_GR(u, 1.1, 1.3, 0.001, 15)
```



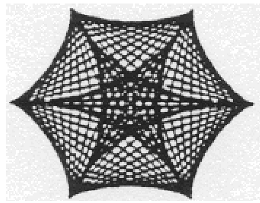
This is an example how I try using the computer and DERIVE teaching mathematics: doing things which I was not able to do ever before.

It is my strong belief that the concepts and the ideas are within the capabilities of our students and I hope to transfer another – a new – view what mathematics can be.

[1] DERIVE – ein mathematischer Assistant, Dr. H.J. Kayser u.a., Landesinstitut für Schule und Weiterbildung,
Soester Verlagskontor

[2] Dynamische Systeme und Fraktale, K. H. Becker & M. Dörfler, 1989, Vieweg, Braunschweig

Extending Algebra Concepts with Technology {DERIVE}



Edward Y. Sawada, Mililani, Hawaii

I recently attended the **NCTM** conference in Seattle, Washington. I was fortunate to hear Tan Teague speak on some of his favourite problems using technology. He illustrated some interesting patterns occurring when the coefficients **A**, **B**, **C** vary in the linear equation **A*x* + B*y* + C = 0**. Dan showed what happens when **A**, **B** and **C** vary in an arithmetic progression and in a geometric progression. I will add to these patterns by showing what happens when **A + B = C** and **A – B = C**, **B – A = C**, and **√(A^2 + B^2) = C**. ***Other patterns will be found by adjusting the coefficients of a Linear Equation***.

Arithmetic Progression

Linear equation whose coefficients are in an arithmetic progression

a∗x + (a+d)∗y + (a+2d) = 0

Solving for y:

y = – (a∗x + a + 2d)/(a + d)

Creating a Family of Lines in an Arithmetic Progression:

VECTOR(VECTOR(- (a·x + a + 2·d)/(a + d), a, -3, 3), d, -3, 3)

See Fig. 1

Geometric Progression

Linear equation whose coefficients are in a geometric progression

x + r∗y + r^2 = 0

Solving for y:

y = – (x + r^2)/r

Creating a Family of Lines in a Geometric Progression:

VECTOR(-(x + r^2)/r, r, -3, 3, 0.25)

See Fig. 2

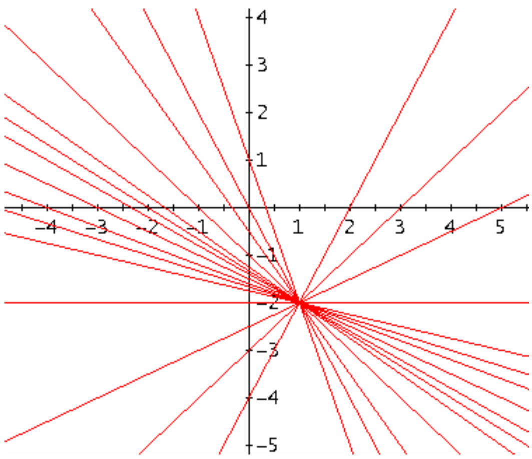


Fig. 1

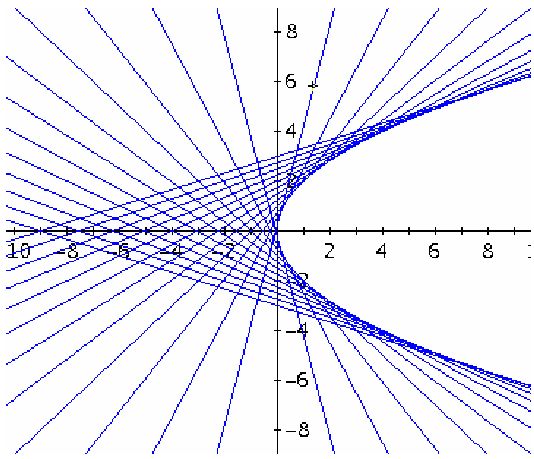


Fig. 2

Proof that all *linear equations* having **arithmetic progessions** for their coefficients will intersect at (1, –2).

We can do the proof with PAP (Paper & Pencil) or we leave the work for DERIVE:

```
SOLVE(a·x + (a + d)·y + a + 2·d = 0 ∧ e·x + (e + f)·y + e + 2·f = 0, [x, y])

                         x = 1 ∧ y = -2
```

Proof that all *linear equations* having coefficients **Ax + By + (A + B) = 0** will intersect at (–1, –1).

We can do again the proof with PAP (Paper & Pencil) or we leave the work for DERIVE:

```
SOLVE(a·x + b·y + (a + b) = 0 ∧ c·x + d·y + (c + d) = 0, [x, y])

                         x = -1 ∧ y = -1
```

Other family of equations such as **Ax + By + (A – B) = 0** *will intersect at* (–1, 1) *and* **Ax + By + (B – A) = 0** *will intersect at* (1, –1). *These facts can be proven in a similar way.*

Additional Questions from 2006:

(1) Is there any similar family of linear equations which intersect at (1, 1)?

(2) Is there an easier way to prove the conjectures?

```
SUBST(a·x + (a + d)·y + a + 2·d = 0, [x, y], [1, -2]) = true

SUBST(a·x + b·y + (a + b) = 0, [x, y], [-1, -1]) = true
```

ARITHMETIC PROGRESSION

Examples: 1, 2, 3, 4, …
6, 9, 12, 15, …
5, 13, 21, 29, …
4, 14, 24, 34, …

Examples of equations having arithmetic progression for coefficients:

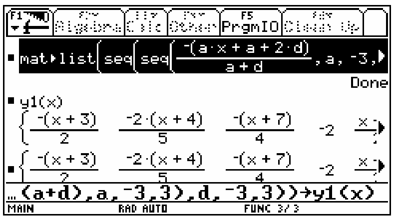
1. $x + 2y + 3 = 0$
2. $6x + 9y + 12 = 0$
3. $5x + 13y + 21 = 0$
4. $4x + 14y + 24 = 0$

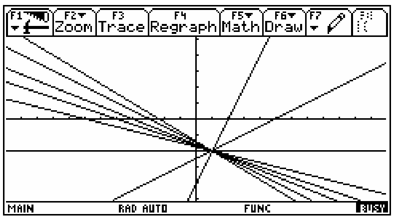
Solving any pair of equations for x and y:

Eq 2 & Eq 4 $\Rightarrow$ $(x = 1, y = -2)$
Eq 1 & Eq 3 $\Rightarrow$ $(x = 1, y = -2)$

All *linear equations* having arithmetic progressions for their coefficients will intersect at (1, –2).

Plotting the family of equations on the TI is possible but it is very time consuming. Notice the function for converting the matrix of expressions into a plotable list!





Systems of Equations {Set 1}
Try solving any two of the following *linear equations*:

1. $x + 2y + 3 = 0$
2. $6x + 9y + 12 = 0$
3. $5x + 13y + 21 = 0$
4. $4x + 14y + 24 = 0$
5. $-3x + 5y + 13 = 0$
6. $10x - y - 12 = 0$
7. $x + 3y/2 + 2 = 0$
8. $-3x + y + 5 = 0$
9. $x - 2y - 5 = 0$
10. $0.1x + 0.8y + 1.5 = 0$

They all will intersect at **(1, –2)**.

Systems of Equations {Set 2}
Try solving any two of the following *linear equations*:

1. $x + 2y + 3 = 0$
2. $-2x + 3y + 1 = 0$
3. $2x - 5y - 3 = 0$
4. $-4x - 2y - 6 = 0$
5. $3x - 7y - 4 = 0$
6. $5x + 3y + 8 = 0$
7. $x + 3y/2 + 5/2 = 0$
8. $-3x - 7y - 10 = 0$
9. $x - y = 0$
10. $0.1x + 0.8y + 0.9 = 0$

All *linear equations* in the form
$\mathbf{A}x + \mathbf{B}y + (\mathbf{A} + \mathbf{B}) = 0$ will intersect at **(–1, –1)**.

Systems of Equations {Set 3}
Try solving any two of the following *linear equations*:

1. $x + 2y - 1 = 0$
2. $6x + 9y - 3 = 0$
3. $-x - 2y + 1 = 0$
4. $2x - y + 3 = 0$
5. $3x + 2y + 1 = 0$
6. $4x + 5y - 1 = 0$
7. $x + 3y/2 - 1/2 = 0$
8. $-3x + y - 4 = 0$
9. $x - 2y + 3 = 0$
10. $0.1x + 0.8y - 0.7 = 0$

All *linear equations* in the form
$\mathbf{A}x + \mathbf{B}y + (\mathbf{A} - \mathbf{B}) = 0$ will intersect at **(–1, 1)**.

Systems of Equations {Set 3}
Try solving any two of the following *linear equations*:

1. $x - y - 2 = 0$
2. $2x + y - 1 = 0$
3. $-2x - 3y - 1 = 0$
4. $3x + 2y - 1 = 0$
5. $4x - 3y - 7 = 0$
6. $7x + 2y - 5 = 0$
7. $x + 3y/2 + 1/2 = 0$
8. $9x - 7y - 16 = 0$
9. $9x + 7y - 2 = 0$
10. $0.1x + 0.8y + 0.7 = 0$

All *linear equations* in the form
$\mathbf{A}x + \mathbf{B}y + (\mathbf{B} - \mathbf{A}) = 0$ will intersect at **(1, –1)**.

Linear Equation of the form

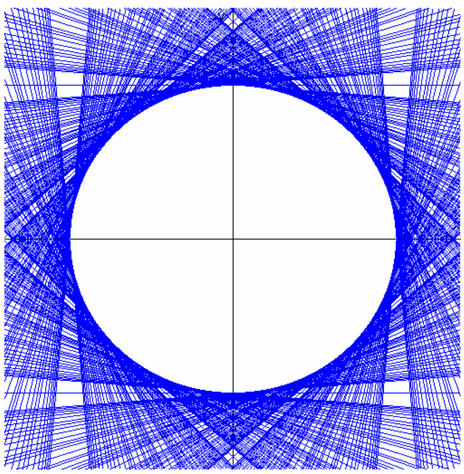
$$\mathbf{A}x + \mathbf{B}y + \sqrt{\mathbf{A}^2 + \mathbf{B}^2} = 0$$

will all be tangent to a circle $x^2 + y^2 = 1$

Creating a Family of Lines of this form

$$y = -(\mathbf{a}\ \mathbf{x} + \mathrm{SQRT}(\mathbf{a}^2 + \mathbf{b}^2))/\mathbf{b}$$

VECTOR(VECTOR(- (a·x + √(a^2 + b^2))/b, a, -3, 3, 1/4), b, -3, 3, 1/4)



FAMILY OF EQUATIONS HAVING (a, b) AS ITS SOLUTION

We have discussed patterns in the linear equation **A*x* + B*y* + C = 0**, and have had various solutions to the system of familiy members. I would now like to show you how to generate a family of equations having the solution **(a, b)**, where you determine the values of **a** and **b**. Let the solution dictate the pattern of the equation.

If a system of linear equations are to have a solution **(a, b)**, then the linear equations must pass through the point **(a, b)**. Since two points determine a line, let **(u, v)** be the other point the linear equation passes through. Using the point-slope form for the equation line, we can generate a pattern of family members having **(a, b)** as its solution.

E. Sawada´s Proof:

#12: $y - b = \frac{v - b}{u - a}\cdot(x - a)$

#13: $\text{EXPAND}\left(\left(y - b = \frac{v - b}{u - a}\cdot(x - a)\right)\cdot(u - a)\right)$

#14: $a\cdot y + b\cdot u - u\cdot y = a\cdot v + b\cdot x - v\cdot x$

#15: $(a\cdot y + b\cdot u - u\cdot y = a\cdot v + b\cdot x - v\cdot x) - (a\cdot y + b\cdot u - u\cdot y)$

#16: $0 = x\cdot(b - v) + y\cdot(u - a) + a\cdot v - b\cdot u$

A = b – v and B = u – a → v = b – A and u = B + a,
we edit A = a_ and B = b_

#17: $0 = x\cdot(b - (b - a\_)) + y\cdot((b\_ + a) - a) + a\cdot(b - a\_) - b\cdot(b\_ + a)$

#18: $0 = a\_\cdot x + b\_\cdot y - a\cdot a\_ - b\cdot b\_$

which is in our notation:

Ax + By – (A a + B b) = 0

2006 Comment: It´s much easier to work in vector form:

$$\begin{pmatrix} x-a \\ y-b \end{pmatrix}\cdot\begin{pmatrix} A \\ B \end{pmatrix} = 0 \rightarrow (x-a)A + (y-b)B = 0 \rightarrow Ax + By - (Aa + Bb) = 0$$

or: $$y - b = -\frac{A}{B}(x-a) \rightarrow B(y-b) = A(a-x) \rightarrow \ldots$$

We intersect two lines of this kind:

SOLVE$(p\cdot x + q\cdot y - (a\cdot p + b\cdot q) = 0 \wedge r\cdot x + s\cdot y - (a\cdot r + b\cdot s) = 0,\ [x,\ y])$

$x = a \wedge y = b$

The algebra shown above illustrates that the solution of the system will be ($x = a$, $y = b$),

Try solving any two of the following *linear equations*. If (a, b) is to be our solution then C = – (Aa + Bb), where A and B are arbitrary constants. For this set of problems we will let (2, 3) be the solution to the family of equations.

All *linear equations* in the form **Ax + By – (2A + 3B) = 0** will intersect at **(2, 3)**.

Systems of Equations {Set 3}

1. $x + y - 5 = 0$
2. $x + 2y - 8 = 0$
3. $-x + y - 1 = 0$
4. $x - y + 1 = 0$
5. $x - 2y + 4 = 0$
6. $3x - y - 3 = 0$
7. $x + 3y/2 - 13/2 = 0$
8. $0.1x + 0.8y - 2.6 = 0$

The following program written in *DERIVE* will generate **n** number of equations passing through the point (**a**, **b**). When solving any two of these family members the solution will be (a, b).

sys(a, b, n) ≔ VECTOR([RANDOM(10)·(–1)^RANDOM(2)·(x – a) + RANDOM(10)·(–1)^RANDOM(2)·(y – b) = 0], z, n)

Function from 1993 does not work as satisfying as recent versions:

We experience an unintended simplification (sometimes). The constant is on the right hand side – or not. Very often we meet two equal equations following – the random number generator works too slow for the CPU-velocity.

Output from

1993 (V. 2.5x)	2006 (V. 6.10)
SYS(1, -2, 10)	SYS(1, -2, 10)
9·x - 8·y - 25 = 0	x = 1
- 2·(x - 4·y - 9) = 0	x = 1
- 4·x + 5·y + 14 = 0	x - y - 3 = 0
7·x + 8·y + 9 = 0	7·x - 5·y = 17
6·(x - 1) = 0	8·x + y = 6
3·x - y - 5 = 0	x - y - 3 = 0
3·(x - 1) = 0	3·x + 8·y = -13
- 5·(x + y + 1) = 0	x - y = 3
3·(2·x + y) = 0	2·x + 3·y = -4
8·(y + 2) = 0	2·x - 3·y - 8 = 0

I tried some "tricks" and finally I found a way to avoid autosimplification applying the seldom used **quote operator '**.

See the program with an additional parameter k for rising the random range.

```
sys_6(a, b, k, n, dummy, a_, b_, aux, list, i, equ) :=
  Prog
    dummy := RANDOM(0)
    list := []
    i := 1
    Loop
      a_ := (RANDOM(k) + 1)·(-1)^RANDOM(2)
      b_ := (RANDOM(k) + 1)·(-1)^RANDOM(2)
      equ := a_·x + b_·y - (a_·a + b_·b) = 0
      equ := LHS(equ) - RHS(equ)
      list := APPEND([['(equ = 0)]], list)
      If i = n
         RETURN list
      i :+ 1
```

```
sys_6(1, -2, 20, 5)

[ 10·x - 15·y - 40 = 0 ]
[ 6·x - 15·y - 36 = 0  ]
[ 9·x + 17·y + 25 = 0  ]
[ x - 10·y - 21 = 0    ]
[ 17·x - 10·y - 37 = 0 ]
```

You can observe the arithmetic sequence for the terms A, B, C in the linear equation Ax + By + C = 0.

```
#35:  sys_6(11, -15, 50, 5)

      [ 19·x + 18·y + 61 = 0    ]
      [ 10·x + 4·y - 50 = 0     ]
#36:  [ 36·x - 49·y - 1131 = 0  ]
      [ 17·x - 2·y - 217 = 0    ]
      [ 38·x - 11·y - 583 = 0   ]

              ⎛⎡ 19·x + 18·y + 61 = 0   ⎤         ⎞
              ⎜⎢ 10·x + 4·y - 50 = 0    ⎥         ⎟
#37:  SOLVE   ⎜⎢ 36·x - 49·y - 1131 = 0 ⎥, [x, y] ⎟
              ⎜⎢ 17·x - 2·y - 217 = 0   ⎥         ⎟
              ⎝⎣ 38·x - 11·y - 583 = 0  ⎦         ⎠

#38:  [x = 11 ∧ y = -15]
```

Last summer I received a letter from Erich containing some sheets of computer paper with a DERIVE listing. It´s headline was "How to program a Maximum – Minimum Problem". The title made me curious and I followed the functions. At first I couldn´t believe the title, but it is true, indeed. DERIVE is able because of the sophisticated – but quite simple – functions to solve Maximum – Minimum problems of students´ and teachers´ daily life.

Sometimes occur problems, but they are worth to be investigated. Please notice how Erich uses ITERATE to work with the solutions of an equation.

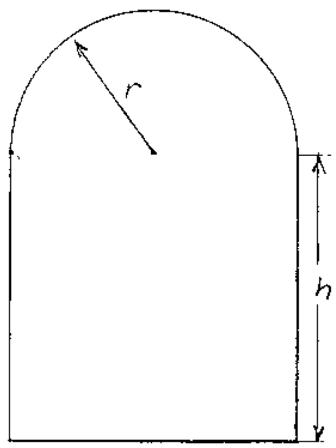
Maximum and Minimum Problems

Erich Zott, Mödling, Austria

The first example will be treated step by step demonstrating the functions which are finally used to "compose the program".

Problem 1:

Given is the circumference U of a window (rectangle + semicircle). Find the dimensions for r and h to maximize the area A of the window.



Model:

$A = \frac{r^2\pi}{2} + 2rh = \text{Maximum}$ (main condition)

$U = 2r + 2h + r\pi$ (side condition)

```
#1:  [a := r^2·π/2 + 2·r·h, s := u - 2·r - 2·h - r·π]
```

Solve the side condition for variable h:

```
#2:  ITERATE(s, h, 0, -1) = (u - r·(π + 2))/2
```

and we substitute into the goal function:

```
#3:          lim               a = r·(2·u - r·(π + 4))/2
     h→ITERATE(s, h, 0, -1)
```

We can use SUBST now!

```
#4:  SUBST(a, h, ITERATE(s, h, 0, -1)) = r·(2·u - r·(π + 4))/2
```

We obtain a function with only one variable a(r) (= F_ONE),
which then can be differentiated with respect to r:

#5: $F\_ONE(a, s, r, h) := \lim_{h \to ITERATE(s, h, 0, -1)} a$

#6: $\frac{d}{dr} F\_ONE(a, s, r, h) = u - r\cdot(\pi + 4)$

#10 = 0 and solve for r (don´t use solve!)

#7: $ITERATE\left(\frac{d}{dr} F\_ONE(a, s, r, h), r, 0, -1\right) = \frac{u}{\pi + 4}$

This is now the argument of the turning point (local Max or Min)

#8: $EXTR\_ONE(a, s, r, h) := ITERATE\left(\frac{d}{dr} F\_ONE(a, s, r, h), r, 0, -1\right)$

Substitute into the side condition to obtain value for variable h

#9: $SUBST(s, r, EXTR\_ONE(a, s, r, h)) = \frac{2\cdot u}{\pi + 4} - 2\cdot h$

#10: $ITERATE(SUBST(s, r, EXTR\_ONE(a, s, r, h)), h, 0, -1) = \frac{u}{\pi + 4}$

#11: EXTR_TWO(a, s, r, h) := ITERATE(SUBST(s, r, EXTR_ONE(a, s, r, h)), h, 0, -1)

Substitute for both variables in the main condition to obtain the extremum value:

#12: $SUBST(a, [r, h], [EXTR\_ONE(a, s, r, h), EXTR\_TWO(a, s, r, h)]) = \frac{u^2}{2\cdot(\pi + 4)}$

#13: EXTRVAL(a, s, r, h) := SUBST(a, [r, h], [EXTR_ONE(a, s, r, h), EXTR_TWO(a, s, r, h)])

Is it a local Maximum or Minimum, we need the 2nd derivative:

#14: $\left(\frac{d}{dr}\right)^2 F\_ONE(a, s, r, h) = -\pi - 4$

and we substitute the value of the first variable:

#15: $DER\_2ND(a, s, r, h) := SUBST\left(\left(\frac{d}{dr}\right)^2 F\_ONE(a, s, r, h), r, EXTR\_ONE(a, s, r, h)\right)$

#16: $DER\_2ND(a, s, r, h) = -\pi - 4$

if DER_2ND < 0 then Maximum, if > 0 then Minimum, else Point of Inflection

```
       EXTR_DEC(a, s, r, h) :=
         If DER_2ND(a, s, r, h) < 0
            "Maximum"
#17:        If DER_2ND(a, s, r, h) > 0
               "Minimum"
               "Point of Inflection"
```

#18: EXTR_DEC(a, s, r, h) = Maximum

Now we put all functions together into one function EXTREM(z,s,x,y):

$$\#19:\quad \text{EXTREM}(goal,\ side,\ x,\ y,\ extr) := \begin{bmatrix} x = \text{EXTR\_ONE}(goal,\ side,\ x,\ y) & y = \text{EXTR\_TWO}(goal,\ side,\ x,\ y) \\ extr = \text{EXTRVAL}(goal,\ side,\ x,\ y) & \text{EXTR\_DEC}(goal,\ side,\ x,\ y) \\ \text{F\_ONE}(goal,\ side,\ x,\ y) & \frac{d}{dx}\,\text{F\_ONE}(goal,\ side,\ x,\ y) \end{bmatrix}$$

Problem once more:

$$\#20:\quad \left[a := \frac{r^2\cdot\pi}{2} + 2\cdot r\cdot h,\ s := u - 2\cdot r - 2\cdot h - r\cdot\pi\right]$$

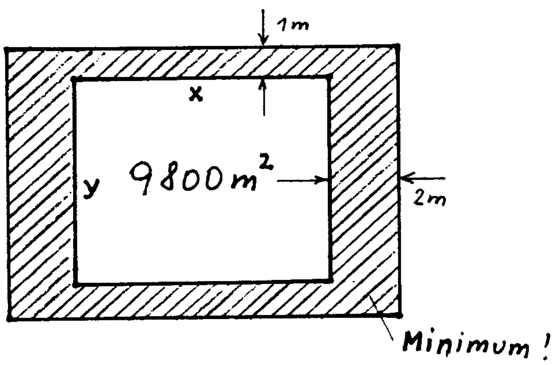
$$\#21:\quad \text{EXTREM}(a,\ s,\ r,\ h) = \begin{bmatrix} r = \dfrac{u}{\pi + 4} & h = \dfrac{u}{\pi + 4} \\ extr = \dfrac{u^2}{2\cdot(\pi + 4)} & \text{Maximum} \\ \dfrac{r\cdot(2\cdot u - r\cdot(\pi + 4))}{2} & u - r\cdot(\pi + 4) \end{bmatrix}$$

Let´s change the order! (Which is solving the side condition for h and proceeding ....)

$$\#22:\quad \text{EXTREM}(a,\ s,\ h,\ r) = \begin{bmatrix} h = \dfrac{u}{\pi + 4} & r = \dfrac{u}{\pi + 4} \\ extr = \dfrac{u^2}{2\cdot(\pi + 4)} & \text{Maximum} \\ \dfrac{(u - 2\cdot h)\cdot(2\cdot h\cdot(\pi + 4) + \pi\cdot u)}{2\cdot(\pi + 2)^2} & \dfrac{4\cdot(u - h\cdot(\pi + 4))}{(\pi + 2)^2} \end{bmatrix}$$

All the sketches are scanned from the original DNL#12 from 1993.

Problem 2:



Modell:

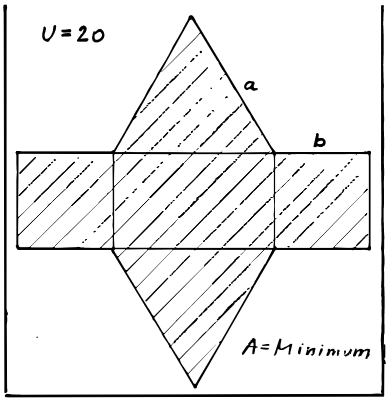
$$z = (x+4)(y+2) - x\,y = \text{Minimum}$$
$$x\,y = 9800$$

```
#1:  LOAD(D:\DfD\DNL\DNL93\MTH12\MaxMin_ut.mth)
#2:  [water := (x + 4)·(y + 2) - x·y, ar := x·y - 9800]
#3:  EXTREM(water, ar, x, y)
```

$$\#4:\quad \begin{bmatrix} x = 140 & y = 70 \\ extr = 568 & \text{Minimum} \\ \dfrac{2\cdot(x^2 + 4\cdot x + 19600)}{x} & \dfrac{2\cdot(x^2 - 19600)}{x^2} \end{bmatrix}$$

$x = 140$ and $y = 70$ make a minimum shaded area of 568 square units.

Problem 3:



The triangles are equilateral.

Modell:

$$area = ab + 2b^2 + \frac{2a^2\sqrt{3}}{4} = \text{Maximum!}$$

$$20 = 4a + 6b$$

#1: LOAD(D:\DfD\DNL\DNL93\MTH12\MaxMin_ut.mth)

#2: $area := a\cdot b + 2\cdot b^2 + \frac{2\cdot a^2\cdot\sqrt{3}}{4}$

#3: $circf := 20 - 4\cdot a - 6\cdot b$

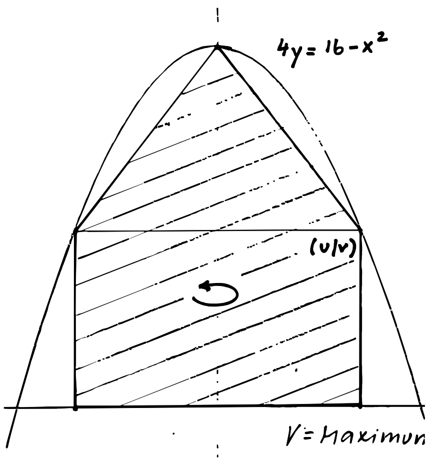
#4: EXTREM(area, circf, a, b)

#5: $\left[\begin{matrix} a = \frac{450\cdot\sqrt{3}}{227} - \frac{200}{227} & b = \frac{890}{227} - \frac{300\cdot\sqrt{3}}{227} \\ extr = \frac{5600}{227} - \frac{1250\cdot\sqrt{3}}{227} & \text{Minimum} \\ \frac{a^2\cdot(9\cdot\sqrt{3} + 4) - 100\cdot a + 400}{18} & \frac{a\cdot(9\cdot\sqrt{3} + 4) - 50}{9} \end{matrix}\right]$

#6: $\left[\begin{matrix} a = 2.552523627 & b = 1.631650915 \\ extr = 15.13187881 & \text{Minimum} \\ \frac{a^2\cdot(9\cdot\sqrt{3} + 4) - 100\cdot a + 400}{18} & \frac{a\cdot(9\cdot\sqrt{3} + 4) - 50}{9} \end{matrix}\right]$

The interesting answer is: the area of 15.13 is a **Minimum**! Check it by plotting F_ONE. (bottom left)

Problem 4:



Modell:

$$Vol = u^2\pi + \frac{u^2(4-v)\pi}{3} = \text{Maximum!}$$

$$4v = 16 - u^2$$

#1: LOAD(D:\DfD\DNL\DNL93\MTH12\MaxMin_ut.mth)

#2: $\left[vol := u^2\cdot\pi\cdot v + \frac{u^2\cdot\pi}{3}\cdot(4 - v),\ par := 4\cdot v + u^2 - 16\right]$

#3: EXTREM(vol, par, u, v)

#4: $\left[\begin{matrix} u = 0 & v = 4 \\ extr = 0 & \text{Minimum} \\ \frac{\pi\cdot u^2\cdot(24 - u^2)}{6} & \frac{2\cdot\pi\cdot u\cdot(12 - u^2)}{3} \end{matrix}\right]$

#5: EXTREM(vol, par, v, u)

#6: $\left[\begin{matrix} v = 1 & u = 2\cdot\sqrt{3} \\ extr = 24\cdot\pi & \text{Maximum} \\ \frac{8\cdot\pi\cdot(v + 2)\cdot(4 - v)}{3} & \frac{16\cdot\pi\cdot(1 - v)}{3} \end{matrix}\right]$

Problem 5 is quite interesting because it worked properly with DERIVE 2.xx and DERIVE 3.xx (DOS DERIVE) as shown below. I reactivated my DOS-DERIVE and checked it – it really works.

67: "***********"

68: "Example 5"

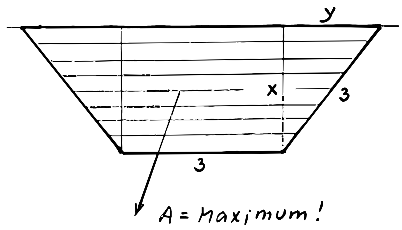
69: $\left[z := x\cdot(3 + y),\ s := x^2 + y^2 - 9\right]$

70: EXTREM(z, s, y, x)

71: $$\left[\begin{array}{cc} y = \dfrac{3}{2} & x = \dfrac{3\cdot\sqrt{3}}{2} \\ extr = \dfrac{27\cdot\sqrt{3}}{4} & \text{Maximum} \\ \sqrt{(9 - y^2)}\cdot(y + 3) & \sqrt{(9 - y^2)} - \dfrac{y\cdot(y + 3)}{\sqrt{(9 - y^2)}} \end{array}\right]$$

72: "*********** "

Problem 5:



This is a part of the DERIVE 6 output.

$$\left[\begin{array}{cc} y = -3 & x = 0 \vee x = 0 \\ extr = 0 & \text{IF(?, Maximum, IF(DER\_2ND}(x\cdot(y + 3),\ x^2 + y^2 - 9,\ y,\ x,\ 1)\ : \\ \sqrt{(9 - y^2)}\cdot(y + 3) & \sqrt{(9 - y^2)} - \dfrac{y\cdot(y + 3)}{\sqrt{(9 - y^2)}} \end{array}\right.$$

The reason for the problem lies in another internal programming of the ITERATES-function. In EXTR_ONE we solve an equation which can have more than one solution – usually in school problems might appear quadratics. As you can see I changed EXTR_ONE applying SOLUTIONS with an additional parameter k_ with defaults to 1 (1st solution) or must be entered as 2 (2nd solution). Then I adapted all other functions accordingly (see file MaxMin_ut_impr.mth).

#2: $\text{EXTR\_ONE}(a,\ s,\ r,\ h,\ k\_ := 1) := \left(\text{SOLUTIONS}\left(\dfrac{d}{dr}\ \text{F\_ONE}(a,\ s,\ r,\ h),\ r\right)\right)_{k\_}$

Then I tried once more using the improved version:

#1: LOAD(D:\DfD\DNL\DNL93\MTH12\MaxMin_ut_impr.mth)

#2: $\left[trap\_ar := \dfrac{(3 + 2\cdot y + 3)\cdot x}{2},\ pyth := 9 - x^2 - y^2\right]$

#3: $$\text{EXTREM}(trap\_ar,\ pyth,\ x,\ y) = \left[\begin{array}{cc} x = \dfrac{3\cdot\sqrt{3}}{2} & y = -\dfrac{3}{2} \\ extr = \dfrac{9\cdot\sqrt{3}}{4} & \text{Maximum} \\ x\cdot(\sqrt{(9 - x^2)} + 3) & \sqrt{(9 - x^2)} - \dfrac{x^2}{\sqrt{(9 - x^2)}} + 3 \end{array}\right]$$

#4: EXTREM(trap_ar, pyth, x, y, 2) = $\left[\begin{array}{ll} x = -\dfrac{3\cdot\sqrt{3}}{2} & y = -\dfrac{3}{2} \\ extr = -\dfrac{9\cdot\sqrt{3}}{4} & \text{Minimum} \\ x\cdot(\sqrt{(9 - x^2)} + 3) & \sqrt{(9 - x^2)} - \dfrac{x^2}{\sqrt{(9 - x^2)}} + 3 \end{array}\right]$

#5: EXTREM(trap_ar, pyth, y, x) = $\left[\begin{array}{ll} y = -3 & \\ extr = 0 & \text{IF(?, Maximum, IF(DER\_2ND(x·(y +} \\ \sqrt{(9 - y^2)}\cdot(y + 3) & \end{array}\right.$

#6: EXTREM(trap_ar, pyth, y, x, 2) = $\left[\begin{array}{ll} y = \dfrac{3}{2} & x = -\dfrac{3\cdot\sqrt{3}}{2} \\ extr = -\dfrac{27\cdot\sqrt{3}}{4} & \text{Maximum} \\ \sqrt{(9 - y^2)}\cdot(y + 3) & \sqrt{(9 - y^2)} - \dfrac{y\cdot(y + 3)}{\sqrt{(9 - y^2)}} \end{array}\right]$

The negative *x*-value is caused by the ITERATION-function – there are two solutions for the underlying equation and ITERATION returns now the negative one. Don´t know why. Problems occur in all cases with more than one solution.

We see that the first three attempts are leading to solutions which are outside of the range because of the negative y-value. The fourth try is successful.

Much later, in 2002 in wrote a book “Programmieren in DERIVE”[1]. I came back to Erich´s basic ideas and tried to program the MaxMin-procedure with DERIVE 6:

extrem($x\cdot(3 + y)$, $x^2 + y^2 = 9$, y, x)

y	x	Optimum	Art
$\dfrac{3}{2}$	$-\dfrac{3\cdot\sqrt{3}}{2}$	$-\dfrac{27\cdot\sqrt{3}}{4}$	lokales Minimum
$\dfrac{3}{2}$	$\dfrac{3\cdot\sqrt{3}}{2}$	$\dfrac{27\cdot\sqrt{3}}{4}$	lokales Maximum
−3	0	0	kein EW oder nicht def.

[1] Josef Böhm, *Programmieren in DERIVE*, bk-teachware Schriftenreihe SR-32, bk-teachware 2002 (For all of you who are owning this “masterpiece“ is an improved version of program extrem included among the DNL12-files.)

In 1993 Keith Eames gave step by step instructions how to work with Derive 2.xx. So he wrote:

Step 1: <A> r^3 <C> <S> <ENTER> <ENTER> <ENTER>

I´ll try to keep his style in best possible form but adapt it for the recent Derive versions. Josef

Use of DERIVE in Proofs by Induction

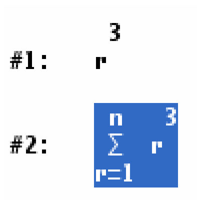
Keith Eames, Romford, UK

Assumed Knowledge: *Derive* commands, for example SUM, VECTOR, TABLE, declaring functions, Σ-notation. Introduction to method of Proof by Induction.

The following two examples will illustrate how *Derive* can help to reinforce your understanding of the Method of Proof by Induction.

Example 1

Prove by induction $1^3+2^3+3^3+\ldots+n^3=\dfrac{n^2(n+1)^2}{4}$



Step 1: <F2 (= Author Expression)> r^3 <ENTER>
<Alt + C> <Sum> <Ok> or
shortcut <Ctrl + Shift + S> <Ok>

This will give the series being looked at in summation form.

Step 2: <F2> S(n):=#2 <ENTER>
or S(n):=sum(r^3,r,1,n) <ENTER>

This will declare a function *S* which will contain the answers to the sum of r^3 for given values of n.

Step 3: <F2> T(n):=n^2 (n+1)^2/4 <ENTER>

This will declare a function *T* which will contain the answers to $1^3+2^3+3^3+\ldots+n^3$ obtainable from the quoted formula.

Step 4: <F2> TABLE([S(n),T(n)],n,1,10) <ENTER and Simplify>

This will compare the two sets of answers for each value of n in *S*(*n*) and *T*(*n*). This should lead to a comment that the result seems to be true for *n* = 1 to 10.

Now assume the statement

$$1^3+2^3+3^3+\ldots+n^3=\frac{n^2(n+1)^2}{4} \text{ is true for } n.$$

Step 5: <F2> S(n) + (n+1)^3 = <ENTER>

By adding the (n+1)th term to the original series we are required to show that

$$S(n+1) = S(n) + (n+1)^3.$$

Step 6: <F2> S(n+1) = <ENTER>

On looking at the answers to the line numbers for step 5 and step 6 (#7 & #8), the answers should be identical. User *Derive*'s FACTOR command on each answer to verify that the two statements are identical (or subtract one answer from the other one to obtain 0).

Students should be able to recognize the identity without using FACTOR, of course.

#3: $S(n) := \sum_{r=1}^{n} r^3$

#4: $T(n) := \frac{n^2 \cdot (n+1)^2}{4}$

#5: TABLE([S(n), T(n)], n, 1, 10)

#6: $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 9 & 9 \\ 3 & 36 & 36 \\ 4 & 100 & 100 \\ 5 & 225 & 225 \\ 6 & 441 & 441 \\ 7 & 784 & 784 \\ 8 & 1296 & 1296 \\ 9 & 2025 & 2025 \\ 10 & 3025 & 3025 \end{bmatrix}$

#7: $S(n) + (n+1)^3 = \frac{(n+1)^2 \cdot (n^2 + 4 \cdot n + 4)}{4}$

#8: $S(n+1) = \frac{(n+2)^2 \cdot (n^2 + 2 \cdot n + 1)}{4}$

#9: $\text{FACTOR}\left(\frac{(n+1)^2 \cdot (n^2 + 4 \cdot n + 4)}{4}\right) = \frac{(n+1)^2 \cdot (n+2)^2}{4}$

#10: $\text{FACTOR}\left(\frac{(n+2)^2 \cdot (n^2 + 2 \cdot n + 1)}{4}\right) = \frac{(n+1)^2 \cdot (n+2)^2}{4}$

Example 2
Prove by induction: $7^n + 2$ is always divisible by 3 for n ≥ 1.

Step 1: <F2 > 7^n + 2 <ENTER>

This will give the numbers being looked at for various values of *n*.

Step 2: <F2> T(n):=#1 <ENTER>
or <F2> T(n):=7^n+2 <ENTER>

This will declare a function *T* which will contain the answers to the values of $7^n + 2$ for defined values of *n*.

Step 3: <F2> VECTOR(T(n),n,1,10) <ENTER and Simplify> → giving expression #4
<F2> #4/3 <ENTER and Simplify> → resulting in integers only
or <F2> mod(#4,3) <ENTER and Simplify> → resulting in a list of zero reminders

This will show whether the conjecture seems to be true for *n* = 1 to 10. Now assume that the statement ist true for *n*.

You are required to show that *T*(*n*+1) is also divisible by 3.

Step 4: <F2> T(n+1) <ENTER and Simplify>

By looking at the expression we have to show that it is divisible by 3.

Step 5: <F2> 7T(n) <ENTER and Simplify>

Comparing this answer with *T*(*n*+1) we can show that *T*(*n*+1) = 7*T*(*n*) – 12 which is divisible by 3 under the assumption that *T*(*n*) ist divisivle by 3.

The print out of this example is given in the following, with statements added:

(We can prove this without any CAS: rewrite $T(n+1) = 7^{n+1} + 2$ as $7 \cdot 7^n + 2 = 6 \cdot 7^n + 7^n + 2$ and this is obviously divisible by 3, because $(7^n + 2)$ and 6 are both divisible by 3.)

#1: $7^n + 2$

#2: $T(n) := 7^n + 2$

#3: VECTOR(T(n), n, 1, 10)

#4: [9, 51, 345, 2403, 16809, 117651, 823545, 5764803, 40353609, 282475251]

#5: $\frac{[9, 51, 345, 2403, 16809, 117651, 823545, 5764803, 40353609, 282475251]}{3}$

#6: [3, 17, 115, 801, 5603, 39217, 274515, 1921601, 13451203, 94158417]

#7: MOD([9, 51, 345, 2403, 16809, 117651, 823545, 5764803, 40353609, 282475251], 3)

#8: [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

True for n = 1 to 10

Assume T(n) is divisible by 3

Prove that T(n+1) is also divisible by 3

#9: T(n + 1)

#10: $7^{n+1} + 2$

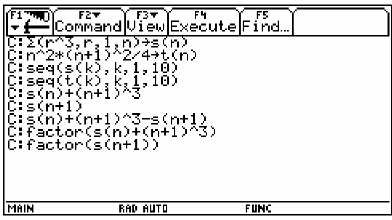
Compare with T(n) = 7^n + 2, multiplying T(n) by 7:

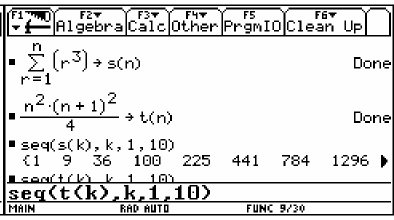
#11: $7\cdot T(n) = 7^{n+1} + 14$

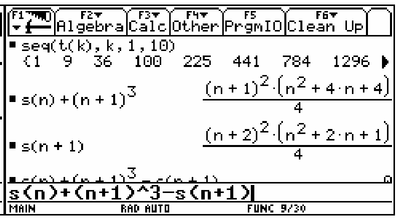
#12: $T(n + 1) = 7\cdot T(n) - 12 = (7^{n+1} + 2 = 7^{n+1} + 2)$

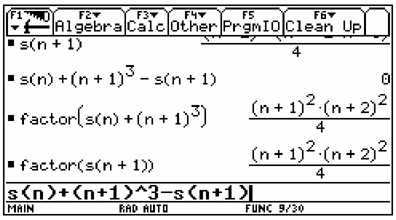
T(n) and 12 are both divisible by 3 then T(n+1) is also divisible by 3.
qued.

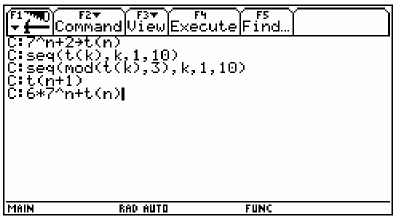
It is very easy to transfer proofs like these onto the TI-92 or the Voyage 200:











Use a CAS to help proving the following statements by the Method of Induction:

(a) $2^{3n-1} + 3$ is *divisible by* 7

(b) Show that 6 is a factor of $n^3 + 5n$

(c) $\sum_{r=1}^{n} r\cdot r! = (n+1)! - 1$

(d) $3^{2n+2} - 8n - 9$ is divisible by 64

(e) $\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n+1}$

(f) $\sum_{r=1}^{n} 4r\cdot 3^r = 3^{n+1}(2n-1) + 3$

(g) $\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

(h) 1 + 5 + 9 + … + (4n – 3) = n (2n – 1)

(i) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

(k) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

(l) Show that 19 is a factor of $5^{2n+1}\cdot 2^{n+2} + 3^{n+2}\cdot 2^{2n+1}$

(m) $1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$

(n) $10^{n+1} - 9n - 10$ is divisible by 81

(o) $1\cdot 5 + 2\cdot 5^2 + 3\cdot 5^3 + \ldots + n\cdot 5^n = \frac{5 + (4n-1)\cdot 5^{n+1}}{16}$

I am very glad to welcome the first Asian contribution in the DNL. We met Mr Chuan at the TMT Conference in Birmingham. He surprised the participants of the UK-DUG Meeting with his enthusiastic talk about using DERIVE and a spreadsheet program. He promised to submit papers for the DNL. Thanks to Taiwan.

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Playing Cards Shuffling with DERIVE

Jen-Chung Chuan, Hsinchu, Taiwan

When a deck of 52 playing cards are perfectly shuffled, there associates a permutation of the form

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & \ldots & 49 & 50 & 51 & 52 \\ 1 & 27 & 2 & 28 & \ldots & 25 & 51 & 26 & 52 \end{pmatrix}$$

describing how the position has been altered. The iterated k fold product

$$\tau^k = \underbrace{\tau \cdot \tau \ldots \tau}_{k}$$

therefore represents the position of the cards after identical shuffling has been applied k-times. The following three lines of inputs in DERIVE computes $\tau^0, \tau^1, \tau^2, \ldots$ until the identity permutation has been reached.

```
#1:   [u := VECTOR(n, n, 52), k := IF(MOD(n, 2), n/2 + 26, (n + 1)/2)]

#2:   SHUFFLE(v) := VECTOR(v_k, n, 52)

#3:   ITERATES(SHUFFLE(v), v, u)
```

Does it surprise you that the cards are back to their original position after shuffling only 8 times?

#4:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	27	2	28	3	29	4	30	5	31	6	32	7	33	8	34	9
1	14	27	40	2	15	28	41	3	16	29	42	4	17	30	43	5
1	33	14	46	27	8	40	21	2	34	15	47	28	9	41	22	3
1	17	33	49	14	30	46	11	27	43	8	24	40	5	21	37	2
1	9	17	25	33	41	49	6	14	22	30	38	46	3	11	19	27
1	5	9	13	17	21	25	29	33	37	41	45	49	2	6	10	14
1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Part of the DERIVE output