THE DERIVE - NEWSLETTER #14

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THE BULLETIN OF THE

DERIVE®

USER GROUP

Contents:

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June 1994

[1] **DERIVE in Education -Opportunities and Strategies,**
Proceedings of the Krems '93 Conference, H.Heugl & B.Kutzler, editors
Chartwell-Bratt, Bromley/UK; 1994, 302 pages, ISBN 0-86238-351-X.

[2] **Calculus Projects for DERIVE, J R Kirkwood**
Wm. C. Brown, Dubuque/USA; 1994, 190 pages, ISBN 0-697-22888-6.

[3] **Stochastik mit DERIVE, Benno Grabinger**
Dümmler, Bonn/Germany; 1994, 160 pages

[4] **Höhere Analysis mit DERIVE, Wolfram Koepf**
Vieweg, Wiesbaden/Germany; 1994,206 pages, ISBN 3-528-06594-X

[5] **Prácticas de Matemáticas con DERIVE, Alfonsa Garcia (editor)**
CLAGSA, Madrid/Spain; 1993, 418 pages, ISBN 84-604-8958-2

MNU 1995 NÜRNBERG

Zu Ostern 1995 (10. - 13. April) findet die 86. Hauptversammlung der MNU statt. Unser Mitglied StD Wolfgang Pröpper bittet um Referatsanmeldungen zum Thema DERIVE (kann natürlich auch etwas anderes sein). Unterlagen können bei ihm gerne angefordert werden. Anschrift: StD W.Pröpper, Josef-Simon-Str.59, D-90473 Nürnberg. Das wäre doch schön, wenn eine ordentliche DERIVE-Riege auftreten könnte!? Bitte bald melden!!

Sie finden eine Tastaturschablone für die Funktionstasten. Sie wurde von H.Scheuermann gestaltet und der DERIVE-Gemeinde zur Verfügung gestellt. Herzlichen Dank!!

You find enclosed a keyboard model for the function keys. It was produced by H.Scheuermann and made available for the DUG - members. Many thanks!!

Teaching Mathematics with DERIVE & DERIVE in Education

Ich habe noch einige -wenige -Exemplare von **Teaching Mathematics with DERIVE**, Tagungsband von Krems 1992 vorrätig. Sie können das Buch bei mir um öS 330.-, bzw. DM 50.- bestel1en.

Wir haben es auch wieder übernommen, für unsere deutschsprachigen Freunde den Tagungsband von Krems 93 "**DERIVE in Education**" (siehe Book Shelf) zu importieren. Das Buch ist ausgezeichnet gelungen und bietet einen hervorragenden Querschnitt über die Einsatzmöglichkeiten von DERIVE. Leider ist dieses Buch etwas teurer. Wir können es Ihnen um öS 450.-, bzw. DM 65.- anbieten. Die Preise verstehen sich inklusive Porto und Verpackung. Mit der Auslieferung des zweiten Buches, müssen Sie sich aber bis August gedulden. Jetzt kommt die Plymouth Konferenz, dann ein wenig Urlaub und dann sind wir wieder im Einsatz. Anruf, FAX oder ein kurzes Schreiben genügen.

Liebes DUG - Mitglied,

gerade noch rechtzeitig vor den großen Ferien wird der DNL# 14 fertig. Herzlichen Dank für die vielen Zuschriften und Beiträge. Ich bin sehr erfreut, dass die DUG in vermehrtem Maße kleinere Dienstleistungen für die Mitglieder verrichten konnte. Viele von Ihnen werden sich im nächsten Monat anlässlich der DERIVE-Konferenz in Plymouth zurn ersten Mal oder wieder treffen. Ich freue mich schon darauf, eine große Anzahl unserer Mitglieder kennen zu lernen, bzw. wieder zu sehen. Das Programm verspricht auch einen spannenden Tagungsverlauf.

Mit Dr. S. Biryukov aus Moskau hat die DUG nun auch ein Mitglied in Russland. Dr. Biryukov hat einen sensationellen Beitrag geschickt: einen Zeichensatz für DERIVE, mit dem sich von DERIVE aus Grafiken bezeichnen lassen. Dazu gibt es eine Utility zum Generieren eigener Zeichensätze. Zwei Screenshots können Sie auf dieser Seite bewundern. Weiteres folgt im DNL#15.

Die Auswahl der Artikel für den DNL erfolgt primär nach technischen Gesichtspunkten (Umfang, Fort-setzung, ...) und eventuell nach zusammengehörigen Themengruppen. Ich bitte die Autoren um Geduld. nichts geht verloren. Ich denke auch daran, im nächsten Jahr den DNL umfangreicher zu gestalten. Die Materialfülle lässt dies zu.

Im nächsten DNL möchte ich einige DERIVE-unterstiitzte Schularbeiten österreichischer Kollegen vorstellen: Aufgaben zu dynamischen Systemen und Schularbeiten im Teamwork!!!

Mit den besten Wünschen für einen schönen Sommer

Ihr

Dear DUG Member,

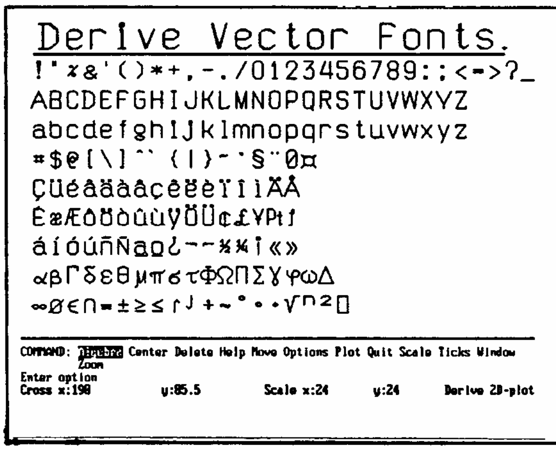
just in time before summer vacations will start DNL#14 has been finished. Many thanks for lots of letters and contributions. It is fine that we can support our members with little services more and more. Many of you will meet next month again or the first time at the DERIVE-Conference in Plymouth, UK. I am looking forward to meeting a lot of our members and seeing many of you again. The conference programme promises some thrilling days, too.

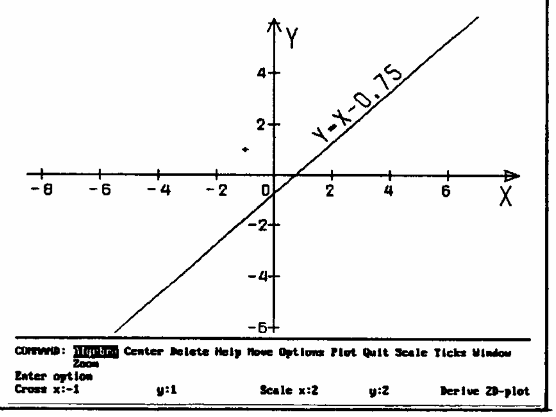
With Dr. S.Biryukov from Moscow the DUG has now its first Russian member. He has delivered a really sensational contribution: A font set for labeling and describing DERIVE plots – in the DERIVE-environment – together with a tool to create own DERIVE fonts. You can find two DERIVE-screens on this page … and wait for DNL#15, please!

Selection of contributions for the DNL is primarily a technical question (size, sequel,) and sometimes I am looking for a possible connection between the articles. I beg the authors for patience, no contribution is lost. I have the plan to give the DNL more content next year. The plenty of material will allow it.

With my best wishes for a fine summer

Sincerely yours





The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE User Group*. It is published at least four times a year with a contents of 30 pages minimum. The goals of the *D-N-L* are to enable the exchange of experiences made with *DERIVE* as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

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Contributions:

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *D-N-L*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in *D-N-L*. The more contributions you will send, the more lively and richer in contents the *DERIVE Newsletter* will be.

Preview: (Contributions for the next issues):

Stability od systems of ODEs, Kozubik, SLK
Algebraic Operations on Polynomials in DERIVE, Roanes, ESP
Prime Iterating Number Generators, Wild, UK
Graphic Integration, Probability Theory, Linear Programming, Böhm, AUT
DERIVE in Austrian Schools, some examples, Lechner & Eisler, AUT
Tilgung fremderregter Schwingungen, Klingen, GER,
Der Fermat-Punkt im Dreieck, Geyer, GER
Continued Fractions in DERIVE, Cordobá a.o., ESP
Turtle-Commands in DERIVE, Lechner, AUT
Dreieck.MTH, Wadsack, AUT
Newton Method and Ill-Conditioned Problems, Lopes, POR
2D Plots Labeling, Biryukov, RUS
Plotting t-periodic functions, Verhoosel, NED
IMP Logo and Misguided Missiles, Sawada, HAWAII
Discussion of Curves, Kuenzer & Reichel & Böhm, IT & AUT
Reichel-Splines versus Böhm-Splines, a Competition, AUT
3D-Geometry, Reichel, AUT
Parallel- and Central Projection, Böhm, AUT
Conic Sections, Fuchs, AUT

and others

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K. Herdt, Osnabrück, Germany

Approximation/Interpolation

Bei der Interpolationsaufgabe wird zu vorgegebenen Punkten (x_i,y_i), i = 0,1,…, k das Interpolationspolynom $y=a_0 + a_1x + ...+ a_kx^k$ bestimmt mit $P(x_i) = y_i$. Bekanntlich führen die Verfahren Newton-Interpolation, Lagrange-Interpolation und der direkte Ansatz über das lineare Gleichungssystem zum selben Ergebnis. Zu Zeiten der Tischrechner (und der Handrechnung) wurde das Newton-Verfahren der dividierten Differenzen vorgezogen, da unter anderem leichter die Anzahl der zu berücksichtigenden Punkte geändert werden konnte. Mit Einführung der programmierbaren Rechner wurde die Lagrange-Interpolation bedeutsamer, da ihr Algorithmus in höheren Programmiersprachen wesentlich leichter umgesetzt werden kann als der der Newton-Interpolation. Mit den Computer-Algebra-Syste-men wird nun der (direkte) Lösungsweg über das lineare Gleichungssystem zunehmend interessant, da die auftretende Vandermonde-Koeffizientenmatrix mit den Mitteln der CAS leicht aufgestellt werden kann und die Lösung linearer Gleichungssysteme ohnehin integriert ist. Hinzu kommt, dass hierbei die Interpolation deutlicher als Grenzfall der Approximationsaufgabe $\Sigma(p(x_i) – y_i)^2$ = Min. in Erscheinung tritt. Mit den Fehlergleichungen

$$V\cdot a - y = \delta$$

(V = Vandermonde-Matrix der x_i-Potenzen, a = Vektor der gesuchten Polynomkoeffizienten und y = Vektor der y_i) lässt sich die Approximationsaufgabe auch kurz mit $\delta^T\cdot\delta$ =Min. beschreiben und a ergibt sich bekanntlich als Lösung des linearen Gleichungssystems

$$V^T V\, a = V^T y \qquad (*).$$

Im Falle einer quadratischen Matrix V wird natürlich δ = 0 und die Fehlergleichungen bilden gerade das lin. Gleichungssystem der Interpolationsaufgabe, bräuchten also gar nicht über (*) gelöst zu werden. Lediglich aus Praktikabilitätsgründen wird jedoch auch in diesem Falle (*) verwendet, denn dann braucht beim Rechengang nicht zwischen Interpolation und Approximation unterschieden zu werden. Die schlechtere Kondition von (*) gegenüber V·a = y braucht in diesem Falle nicht zu stören, da sich ja die Rechengenauigkeit jederzeit erhöhen lässt. Eine „Mustersitzung“ habe ich beigefügt.

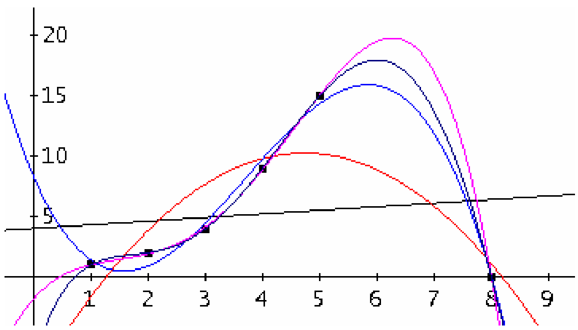
From approximation to interpolation using the Vandermonde matrix. Using this method you can see that the interpolation problem is the extreme case of the approximation problem

$$\Sigma(p(x_i) – y_i)^2 = \text{Min.}$$

With V a – y = δ we obtain $\delta^T\delta$ =Min. and a is the solution of the system of linear equations

$$V^TV\, a = V^T y.$$

(V = Vandermonde matrix of the powers of x_i, a = vector of the polynomial coefficients, y = vector of the y_i)




```
#1:   Datenmatrix der (xi,yi) - The Matrix of the data
```

$$\#2: \quad d := \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \\ 4 & 9 \\ 5 & 15 \\ 8 & 0 \end{bmatrix}$$

```
#3:   Vandermonde_Matrix and RHS (k = order of the polynomial):
```

#4: $V\_Y(d, k) := VECTOR(VECTOR(IF(j \le k, ELEMENT(d, i, 1)^j, ELEMENT(d, i, 2)), j, 0, k + 1), i, 1, DIMENSION(d))$

#5: Vandermonde - Matrix transposed:

#6: $V(d, k) := VECTOR(VECTOR(ELEMENT(d, i, 1)^j, j, 0, k), i, 1, DIMENSION(d))'$

#7: Beispiel - Example

#8: V_Y(d, 2)

#9: $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 9 & 4 \\ 1 & 4 & 16 & 9 \\ 1 & 5 & 25 & 15 \\ 1 & 8 & 64 & 0 \end{bmatrix}$$

#10: V(d, 2)

#11: $$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 8 \\ 1 & 4 & 9 & 16 & 25 & 64 \end{bmatrix}$$

#12: Lösung der Normalgleichungen - Solution of the Normal equations:

#13: $sol(d, k) := ROW\_REDUCE(V(d, k) \cdot V\_Y(d, k))$

#14: Beispiel:

#15: sol(d, 2)

#16: $$\begin{bmatrix} 1 & 0 & 0 & \frac{11}{5} \\ 0 & 1 & 0 & -\frac{29}{14} \\ 0 & 0 & 1 & \frac{13}{14} \end{bmatrix}$$

#17: Polynomkoeffizienten - Coefficients of the polynomial

#18: KOEFF(d, k) := VECTOR(ELEMENT(LÖSUNG(d, k), i, k + 2), i, 1, k + 1)

#19: Beispiel - Example

#20: KOEFF(d, 2)

#21: $$\left[\frac{11}{5}, -\frac{29}{14}, \frac{13}{14}\right]$$

#22: $POT\_X(k) := VECTOR(x^i, i, 0, k)$

#23: Endgültiges Interpol./Approx.-Polynom:

#24: $POL(d, k) := KOEFF(d, k) \cdot POT\_X(k)'$

#25: 1. Beispiel (Approximationspolynom vom Grad 2) - 1. Example (quadratic approximation)

#26: POL(d, 2)

#27: $$\left[\frac{13 \cdot x^2}{14} - \frac{29 \cdot x}{14} + \frac{11}{5}\right]$$

#28: 2. Beispiel (Interpolationspolynom) - 2. Example (interpolation polynomial)

#29: POL(d, DIMENSION(d) - 1)

#30: $$\frac{41 \cdot x^5}{2520} - \frac{23 \cdot x^4}{56} + \frac{1705 \cdot x^3}{504} - \frac{1847 \cdot x^2}{168} + \frac{20107 \cdot x}{1260} - \frac{146}{21}$$

#31: $\text{POL}(d, 1) = \frac{11\cdot x}{37} + \frac{149}{37}$

#32: $\text{POL}(d, 2) = -\frac{2485\cdot x^2}{2904} + \frac{23497\cdot x}{2904} - \frac{4297}{484}$

#33: $\text{POL}(d, 3) = -\frac{7885\cdot x^3}{19868} + \frac{65765\cdot x^2}{14901} - \frac{652211\cdot x}{59604} + \frac{82727}{9934}$

#34: $\text{POL}(d, 4) = -\frac{3959\cdot x^4}{44336} + \frac{23959\cdot x^3}{22168} - \frac{476053\cdot x^2}{133008} + \frac{44944\cdot x}{8313} - \frac{19955}{11084}$

#35: The interpolation polynomial:

#36: INT_POL(d) := POL(d, DIMENSION(d) - 1)

#37: $\text{INT\_POL}(d) = \frac{41\cdot x^5}{2520} - \frac{23\cdot x^4}{56} + \frac{1705\cdot x^3}{504} - \frac{1847\cdot x^2}{168} + \frac{20107\cdot x}{1260} - \frac{146}{21}$

#38: much more elegant: instead of functions KOEFF, POT_X, POL(d,k):

#39: $\text{POLY}(d, k) := \sum_{i=0}^{k} \text{ELEMENT}(\text{SOLUTION}(d, k), i + 1, k + 2)\cdot x^i$

#40: $\text{POLY}(d, 3) = -\frac{7885\cdot x^3}{19868} + \frac{65765\cdot x^2}{14901} - \frac{652211\cdot x}{59604} + \frac{82727}{9934}$

#41: INT_POLY(d) := POLY(d, DIMENSION(d) - 1)

#42: $\text{INT\_POLY}(d) = \frac{41\cdot x^5}{2520} - \frac{23\cdot x^4}{56} + \frac{1705\cdot x^3}{504} - \frac{1847\cdot x^2}{168} + \frac{20107\cdot x}{1260} - \frac{146}{21}$

Additional note of the Editor:
There is a contribution which is related to the topic above in DNL#63: ***Beyond Polynomial Regression***.

H. Scheuermann, Hofheim/Taunus, Germany

(I try to give a brief summary of H. Scheuermann´s letter)

1. Letter: … It would be nice if I could copy one DERIVE expression from one Algebra Window to another one (to change the expression or to try another way of solving the problem or …)

2. Letter (some days after): … By chance I´ve found a way to copy expressions using Ctrl+U.

- Open two Algebra Windows and load two files into the windows
- Highlight the expression to be copied in window #1
- AUTHOR and F3 (or F4) – copying the expression into the Author line
- Change the window pressing F1 (the expression seems to be lost but …)
- AUTHOR (in the other window) and press Ctrl+U – the expression appears in the Edit line ready for proceeding as you want.

DNL: *Thanks for the hint, and many thanks for the keyboard model.*

V. Neurath, Velbert, Germany

I would like to contact DERIVE Users dealing with problems of Applied Economics.

Message 2530: From SOFT WAREHOUSE to PUBLIC about HEAT TRANSFER FROM A FIN

File `HEATFIN.MTH` defines a function that derives the differential equation describing the temperature of a fin having variable conductivity, cross sectional area, perimeter and film coefficient. The file also includes an example. Distributed utility files `ODE2.MTH` and `ODE_APPR.MTH` may help solve the resulting equation.

`EXTENDED_SURFACE(u,x,a,p,k,h,uinf)` derives the differential equation describing the temperature u(x) of a fin having cross sectional area a(x), perimeter p(x), conductivity k(x), film coefficient h(x) and ambient temperature uinf(x)

#1: [p0 :=, p1 :=]

#2: $\text{LIN}(a0, a1, x, l) := a0 + \frac{x}{l}\cdot(a1 - a0)$

#3: $\text{EXTENDED\_SURFACE}(u, x, a, p, k, h, uinf) := \frac{d}{dx}\left(k\cdot a\cdot\frac{d}{dx}u\right) - h\cdot p\cdot(u - uinf)$

#4: T(x) :=

#5: For example, simplify the following expression:

#6: EXTENDED_SURFACE(T(x), x, LIN(a0, a1, x, l), LIN(p0, p1, x, l))

Message 2108: From JEFFCOLE to PUBLIC about ABSOLUTE VALUE BUG

There seems to be a bug in version 2.09. In a simplified form, try graphing y=abs(sqr(x)). I get a graph that includes negative values of x in the domain. Is this a known bug or just a quirk on my end?

Message 2710: From JERRY GLYNN to JEFFCOLE about #27081 ABSOLUTE VALUE BUG

I think the x has negative values which produces complex numbers and the ABS turns them back to real numbers which are plotted. ABS has two jobs ... turn negatives numbers into positive numbers and turn complex to real. I think it would be better if another function (rnaybe call it NORM) would do the second job and ABS of a complex number would be undefined.

Message 3081: From HARALD LANG to PUBLIC about SIMPLIFY AND DERIVE

Try simplifying the following expression with DERIVE:

x = (sqrt(5) + 2)^(1/3) –(sqrt(5) –2)^(1/3) (1)

You will get back the same answer. Now approximate. You will get "x = 1". This suggests that x **is** equal to 1, but recall that when approximating, DERIVE uses only a finite number of significant digits, so maybe x is only very close to 1. OK, switch to 20 significant digits and approximate; "x = 1 " again. In fact, x **is** equal to one (challenge: prove it!). It may seem a bit strange that DERIVE does not simplify (1) to "x = 1", but we shouldn't expect it to: as far as I understand, there is no general algorithm for it. If you worked out a proof, you will see how ad-hoc it is. Here are some thoughts around this example:

(1) It is a bit amazing that it is so difficult to simplify expressions only containing numbers, + , -, *, 1 and powers [it is easy to construct immensely more complicated examples than (1); in fact, (1) is the simplest I can figure out]. Since there are some very clever people at Soft Warehouse, maybe someone there [or anyone else, for that matter] can tell us a little about simplifying algorithms; what can be implemented and what can't.

(2) In teaching the nature of mathematics, examples of this kind, in combination with DERIVE, could serve as a starting point for a discussion on what a **proof** is in mathematics. Can we **prove** that x = 1 by approximating it to more and more significant digits? One could relate this to "constructive mathematics".

Has anyone on this BBS any experience or thoughts on this?

In general, we have a tendency on this BBS to discuss the user interface of DERIVE, which is fine, but I would find it interesting to discuss also other experiences of DERIVE. For instance, Roger Folsom's discussion about the chain rule on this BBS was very useful to me. Jerry Glynn asked some time ago if we were interested in discussing teaching with DERIVE. I should have answered YES, but didn't since I have no such experience, so I would have little to contribute, BUT, I do teach mathematics, and I would be very interested to "listen" to such a discussion.

Message 3082:From JERRY GL YNN to HARALD LANG about #3081 I SIMPLIFY AND DERIVE

Wonderful!! I will ignore your disclaimer and take your entire statement as a contribution to a discussion about teaching math. I will try out your provocative example and then comment.

Message 3089: From CHRISAR to PUBLIC about QUADRATIC EQUATIONS AND DERIVE

Recently some classmates and I were assigned solve some simple quadratic equations using Derive in our college's math lab. The following equation caused Derive to produce only one of two correct values for x; "x^(2/3) – 2x^(1/3) – 35 = 0." In actuality, x = {–125, 343}, but Derive will only provide 343 as a possible value. Can anyone tell me why this is, and what this means in terms of Derive's reliability? Thanks!

Message 3090: From JERRY GLYNN to CHRISAR about #3089/ QUADRATIC EQUATIONS AND DERIVE

Your question connects to complex and real numbers. If you plot the left side of your equation you'll get a graph which only appears when x > 0. Derive (and many other programs) thinks that (-125)^(2/3) is a complex number. In this case –25/2+25 sqrt(3)*i/2. The complex i can be entered by holding down the ALT key and typing I ... you get an i with a cap on it. In Derive Manage Branch will get you to a choice of Principal or Real or Any. Solve your equation with the setting on Principal (the default) and then try it with the setting at Real. Also graph the left side of your equation for each of these settings. I'll be happy to discuss this further and respond to any questions or comments. Should you question the results you get from Derive? Yes! Also question the answers from all other sources (including me). Thanks for your good question!

See how DERIVE 6 is behaving. Josef

```
              2/3       1/3
#1:   SOLVE(x    - 2·x     - 35 = 0, x, Real)

#2:   x = 343

              2/3       1/3
#3:   SOLVE(x    - 2·x     - 35 = 0, x)

#4:   x = 343
```

#5: $\text{NSOLVE}(x^{2/3} - 2\cdot x^{1/3} - 35 = 0, x, \text{Real})$

#6: $x = 342.9999923$

#7: $\text{NSOLVE}(x^{2/3} - 2\cdot x^{1/3} - 35 = 0, x, -1000, 0)$

#8: false

#9: $(-125)^{2/3} = -\frac{25}{2} + \frac{25\cdot\sqrt{3}\cdot i}{2}$

#10: $(-125)^{2/3} - 2\cdot(-125)^{1/3} - 35 = -\frac{105}{2} + \frac{15\cdot\sqrt{3}\cdot i}{2}$

#11: Branch := Real

#12: $(-125)^{2/3} - 2\cdot(-125)^{1/3} - 35 = 0$

#13: $\text{SOLVE}(x^{2/3} - 2\cdot x^{1/3} - 35 = 0, x, \text{Real})$

#14: $x = 343 \vee x = -125$

#15: $(-125)^{2/3} = 25$

Message 3091: From HADUD to HARALD LANG about SIMPLIFYING CUBE ROOTS

The equation (SQRT(5)+2)^(1/3)–(SQRT(5)-2)^(1/3) = 1 follows immediately from the fact that

SQRT(5) +\- 2 = (SQRT(5)/2 +\- 1/2)^3.

You shouldn't be "amazed" that DERIVE gives up on this expression. The problem is that **fractional** powers are involved here and, as you point out, there is no general algebraic method (other than an infinite series) to expand a fractional power of a sum. It is only in highly special cases that simplification is possible. Consider the steps DERIVE would have to go through to try and find a simplification:

1) Fractional powers are multivalued functions; which branch is to be used? In our example both arguments of the cube roots are positive. Assuming that the principal branch has been selected by the user, there is no ambiguity here.
2) Inspecting the arguments DERIVE would have to recognize that they are so-called "quadratic irrationalities", i.e. members of a field (SQRT(i) + j)/k, where i, j, k are integers and i is not a perfect square (a field is a group that is closed under addition, multiplication and division). Therefore there is a chance (no certainty !) that the arguments can be represented as the third power of a member of the same field. In that case the expression would simplify to a number which, again, belongs to the same field.
3) To check out this possibility expand

   ((SQRT(5)+u)/v)^3 = (3 u^2 + 5)/v^3 SQRT(5) + (u^3 + 15 u)/v^3

   and set up the equations

   3u^2 + 5 = v^3
   u^3 + 15u = 2v^3

   Do these nonlinear equations have a solution with integer u?
4) Eliminating v by multiplying the first equation by 2 and subtracting the second equation gives

   u^3 – 6 u^2 + 15 u – 10 = 0,

   the only real root of which is u = 1. Hence v = 2.

   It is clear from this that in the general case of a fractional exponent m/n DERIVE must find an integer solution of a n-th degree polynomial in order to simplify the expression. For n > 4 this becomes very difficult and time-consuming. Thus it is understandable (and perhaps desirable) that it doesn't bother.

Message 3098: From HARALD LANG to HADUD about #3091 / SIMPLIFYING CUBE ROOTS

Very interesting! So what to do is

(1) We consider the number field { x*sqrt(n) + y } where n is a fixed positive integer which is not a perfect square (n = 5 in my example) and x and y are arbitrary rational numbers.

(2) We check whether each of the numbers sqrt(5) +\- 2 is a perfect cube **in this field**, which amounts to solving a third-degree equation [BTW: when we 'identify coefficients', we actually use that sqrt(n) is irrational, a nice classroom problem.]

I think you have clarified to us in a very nice way what we ask DERIVE to perform when we try to simplify my expression. BTW, it seems that we have used the **failure** of DERIVE to perform a simplification to motivate the introduction of the concept 'number field' [now I am a teacher again.]

Here is another example: 8*SIN(pi/18)*COS(pi/9)*COS(2*pi/9)

This is equal to 1 (new challenge: prove it!). We can get reasonably convinced by approximating it by DERIVE, but DERIVE fails to **simplify** it to 1. A Bug!! Hello, Soft Warehouse, are you there <there...echo...echo> :-)

No, no; no bug. But what intelligent can be said about this example? Is there again some number field around to help us understand what we demand from DERIVE when we try simplifying?

[To the sysop: If you think the issue if off topic for this BBS, don't hesitate to delete this message, and give me a message to shut up.] --Harald

Message 3099: From JERRY GLYNN to HARALD LANG about #3098

But why would I tell you to shut up?? I want to hear what comes next!

This is how DERIVE 6 treats the expression now:

#1: $8\cdot \mathrm{SIN}\left(\frac{\pi}{18}\right)\cdot \mathrm{COS}\left(\frac{\pi}{9}\right)\cdot \mathrm{COS}\left(\frac{2\cdot\pi}{9}\right) = 8\cdot \mathrm{SIN}\left(\frac{\pi}{18}\right)\cdot \mathrm{COS}\left(\frac{\pi}{9}\right)\cdot \mathrm{COS}\left(\frac{2\cdot\pi}{9}\right)$

#2: Trigonometry := Expand

#3: $8\cdot \mathrm{SIN}\left(\frac{\pi}{18}\right)\cdot \mathrm{COS}\left(\frac{\pi}{9}\right)\cdot \mathrm{COS}\left(\frac{2\cdot\pi}{9}\right) = 4\cdot\sqrt{2}\cdot \mathrm{COS}\left(\frac{\pi}{9}\right)\cdot \mathrm{COS}\left(\frac{2\cdot\pi}{9}\right)\cdot\sqrt{\left(1 - \mathrm{COS}\left(\frac{\pi}{9}\right)\right)}$

#4: Trigonometry := Collect

#5: $8\cdot \mathrm{SIN}\left(\frac{\pi}{18}\right)\cdot \mathrm{COS}\left(\frac{\pi}{9}\right)\cdot \mathrm{COS}\left(\frac{2\cdot\pi}{9}\right) = 1$

Some more short notes for the DERIVE USER FORUM

Bob from Homer. NY, USA

Sorry I'm late with this (membership dues, ed.) -please don't cancel me - love your DNL issues. Thanks, Bob.

DNL: *Donl worry, we didn't cancel you. And much pleasure with this DNL.*

G. Scheu. Pfinztal, Germany

DearJosef, do you know the factorization of $a^2 + b^2 = (|a| + |b| + 2\sqrt{|a b|})\cdot(|a| + |b| - 2\sqrt{|a b|})$. DERIVE only knows expanding from right hand side to left hand side.

In one file you can find some functions for calculating the sum of the digits of an number.

DNL: *We will publish this file in one of the next DNLs.*

Dr. S.V.Biryukov. Moscow, Russia

I have a utility that supports DERIVE 2D Plots labeling and axes drawing, labeling and numbering. Dr. Kutzler told me that it will be rather interesting for DERIVE Users and suggested to describe it in a short paper for DUG Newsletter .

Ebene Algebraische und Transzendente Kurven (4)

Thomas Weth, Würzburg, Germany

Einheitliche Konstruktionen

In den ersten drei Folgen dieser Reihe wurden die (neben Kegelschnitten) bekanntesten und bedeutendsten algebraischen Kurven (zunächst dritter Ordnung) vorgestellt. Im Einzelnen handelte es sich um die Kissoide (Folge 1), die Strophoide (Folge 2) und die Trisektrix von MacLaurin (Folge 3). Für jede dieser Kurven wurde neben der Herleitung ihrer algebraischen Gleichung auch jeweils eine elementargeometrische punktweise Konstruktionsvorschrift angegeben. Obwohl sich diese Erzeugungsweisen sehr stark voneinander unterscheiden, habe ich (mehr oder weniger durch Zufall) eine Möglichkeit entdeckt, alle drei Kurventypen durch eine einzige Konstruktion herzustellen. Abgesehen von der graphischen Darstellung der Kurven war mir DERIVE vor allem bei der Herleitung der algebraischen Gleichungen (und der damit verbundenen endgültigen Identifizierung) der Kurven *die* entscheidende Hilfe. Allerdings gelang mir trotz DERIVE nicht die Klassifizierung *aller* im Folgenden auftretenden Kurven. Somit stellt diese Folge des "Kurvenlexikons" auch eine Aufforderung an die DERIVE-Spezialisten in aller Weit dar, in den genannten Fällen die Identifizierung der Kurven zu versuchen (mit der Bitte, mich bei Erfolg davon zu unterrichten).

Uniform Constructions

In the first three contributions I have introduced the best known algebraic curves (besides the conic sections). The constructions of these curves are very different but by chance I found a possibility to produce the curves by only one construction. DERIVE was very helpful not only by plotting the graphs but also by deriving the algebraic equations (connected with the identification) of the curves. Unfortunately I was not able to classify all of the curves which you will find in this series in future contributions. So this part of the "Lexicon of Curves" could be a challenge for all DERIVE-specialists to try the identification of the curves - and please contact me in case of success.

Die Konstruktionsvorschrift – The Rule of Construction

a) Die Inversions am Kreis – The Inversion wrt a Circle

Unter der Inversion am Kreis versteht man eine ebene Abbildung gemäß folgender Vorschrift:

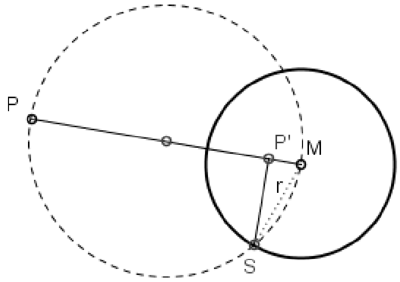
> Gegeben ist der Kreis k(M,r) und ein Punkt P ≠ M. Der Bildpunkt P' liegt auf der Halbgeraden MP und es gilt: $\overline{MP} \cdot \overline{MP'} = r^2$.

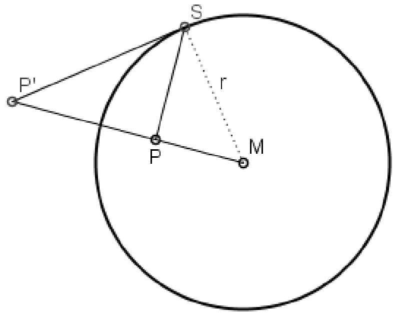
Eine elementargeometrische Konstruktion des Bildpunktes P' gelingt (unter anderen) etwa folgendermaßen:

Liegt P außerhalb der Kreislinie, so konstruiert man mit der Strecke PM als Hypotenuse das rechtwinklige Dreieck PMS (vgl. Zeichnung). Nach dem Kathetensatz im rechtwinkligen Dreieck gilt dann für den Lotfußpunkt P': $\overline{MP} \cdot \overline{MP'} = r^2$.

Liegt P innerhalb der Kreislinie, so konstruiert man das Lot durch P auf PM und erhält S. Dann vervollständigt man das rechtwinklige Dreieck P'MS und erhält nach dem Kathetenstaz wiederum $\overline{MP} \cdot \overline{MP'} = r^2$.

Inversion on a circle: P' is the image point of P. If P outside of the circle, then construct the right triangle PMS over PM and P' is the pedal point of S wrt to PM and we have $\overline{MP} \cdot \overline{MP'} = r^2$. If P inside of the circle then find the perpendicular line through P wrt to PM giving intersection point S with the circle and accomplish the right triangle MSP' with the right angle in S. Then we have again $\overline{MP} \cdot \overline{MP'} = r^2$.





a) Die achsenvariante Abbildung – The mapping with variant axis

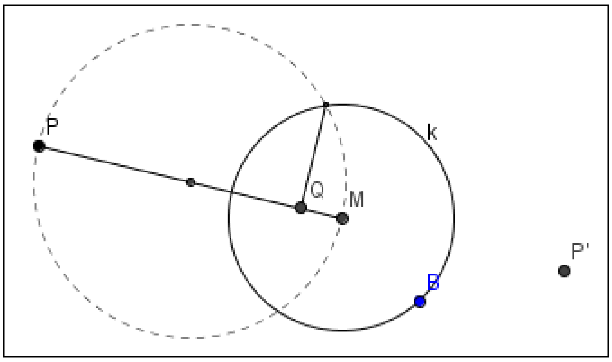
Die im Folgenden untersuchte Abbildung lässt sich nun nach folgender Vorschrift konstruieren:

Gegeben ist der Kreis k(M,r) und ein Punkt P ≠ M.

1. Schritt: Bilde P durch Inversion auf den Hilfspunkt Q ab.

2. Schritt: Bilde P durch Punktspiegelung an Q auf den Bildpunkt P' ab.

Man beachte, dass jeder Urpunkt durch diese Vorschrift sein „individuelles" Spiegelungszentrum zugeordnet bekommt.



Q is the image point of P applying the inversion wrt circle k(M,r). P' is the image point of P applying a reflection on Q. Each single point has its own reflection centre.

Herleitung der Bildpunktkoordinaten – Deriving the coordinates of the image points

Zur Herleitung einer Koordinatendarstellung für den Bildpunkt P' der Abbildung verwenden wir ein Koordinatensystem mit dem Kreismittelpunkt als Ursprung.

For finding the coordinates of P' we use a system with the centre of the inversion circle as origin. The following calculation uses vector representation:

Zunächst gelten die Vektorgleichungen:

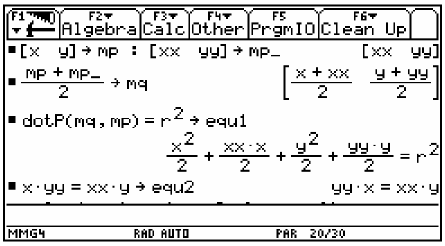
(1) $\overrightarrow{MQ} \cdot \overrightarrow{MP} = r^2$, da Q Bildpunkt von P bei der Inversion an k ist und (because of the inversion)

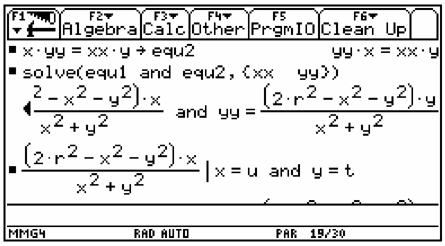
(2) $\overrightarrow{MQ} = \dfrac{\overrightarrow{MP} + \overrightarrow{MP'}}{2}$, da Q als Spiegelungszentrum die Strecke PP' halbiert.

Setzt man (2) in (1) ein, dann erhält man: (*) $MP \cdot (MP + MP') = 2r^2$. Setzt man $\overrightarrow{MP} = \begin{pmatrix} x \\ y \end{pmatrix}$ und $\overrightarrow{MP'} = \begin{pmatrix} x' \\ y' \end{pmatrix}$, so erhält man zusätzlich zu dieser vektoriellen Abbildungsgleichung (**) $\dfrac{x}{x'} = \dfrac{y}{y'}$, denn P und P' liegen auf einer Geraden, die durch M, also den Ursprung geht.

Einsetzen der Koordinaten in (*) liefert gemeinsam mit (**) ein Gleichungssystem für die Bildkoordinaten x' und y'. DERIVE liefert als Bildpunkt von P(x,y):

(Koordinatendarstellung der Bildpunkte) $$P' = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \dfrac{2r^2 x}{x^2 + y^2} - x \\ \dfrac{2r^2 y}{x^2 + y^2} - y \end{pmatrix}.$$





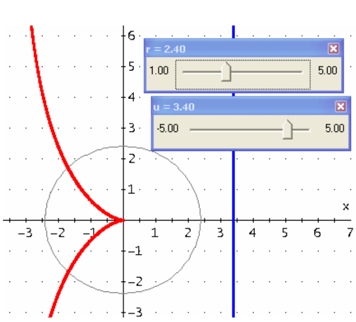
(You can find the DERIVE calculation in curve_lexicon.mth.)

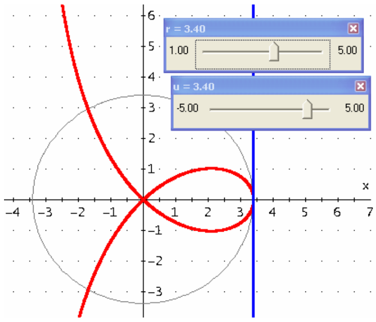
Konstruktion der Kurven – Construction of the Curves

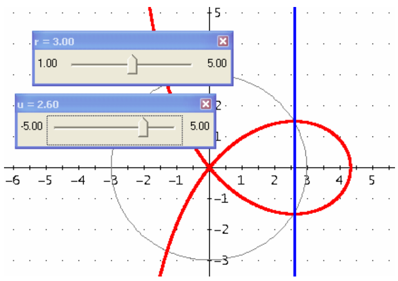
Zu den angekündigten Kurven gelangt man nun, indem man Parallele zur y-Achse in verschiedenen Abständen zum Ursprung punktweise abbildet. Bildet man eine Parallele zur y-Achse mit Abstand u zum Ursprung ab, so haben die Geradenpunkte die Koordinaten (u,t) und werden demnach abgebildet auf Kurvenpunkte mit den Koordinaten:

$$\left(\frac{2r^2u}{u^2+t^2}-u,\ \frac{2r^2t}{u^2+t^2}-t\right).$$

Mit t als Kurvenparameter liefert DERIVE als Bilder der Geraden – unter Verwendung von Schiebereglern für den Radius r des Inversionskreises und den Abstand der senkrechten Geraden u:



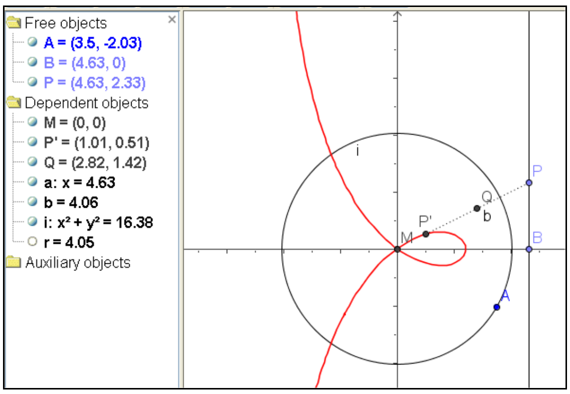


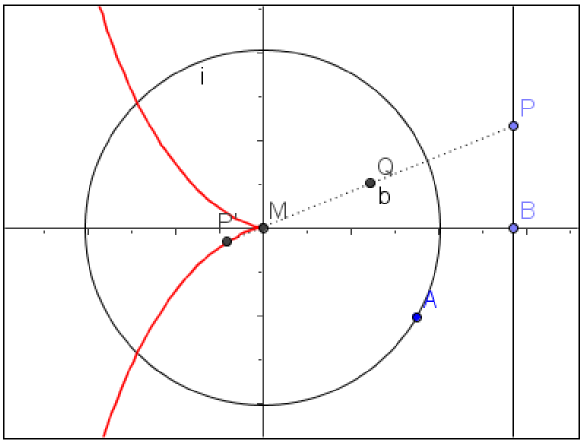


Die entstehenden Kurven ähneln nun sehr stark Kissoiden, Strophoiden und Trisektrices. Ob die abgebildeten Kurven aber wirklich die genannten sind lässt sich erst an Hand ihrer algebraischen Gleichungen entscheiden.

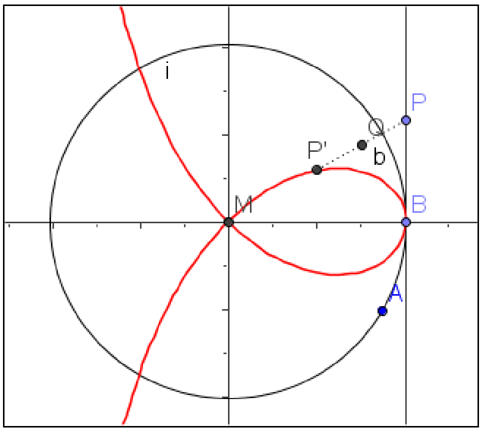
We obtain the curves mentioned above as images of vertical lines $x = u$. The resulting curves are described by the parameter representation with parameter t. We can install slider bars for r and u and the resulting curves are reminding us on Cissoids, Strophoids and Trisectrices. Whether the conjecture is true or not can be decided by inspecting their respective algebraic equations …

But first of all let me – now in 2007 – reproduce the mapping using GeoGebra. The first figure shows the general construction. The we can edit the x-coordinate of point B which is the u from above and enter $u = x = r\cdot\sqrt{2}$, $u = x = r$ and $u = x = r/\sqrt{2}$.



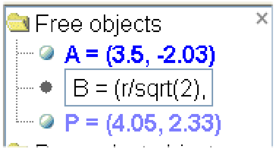


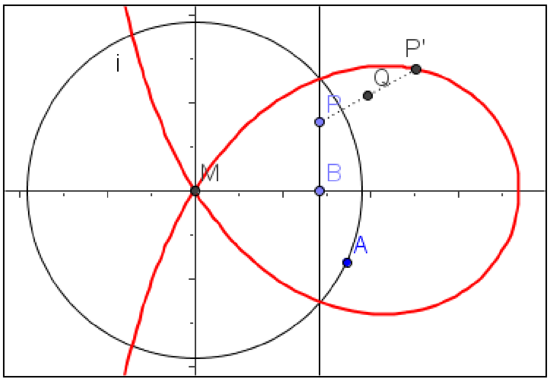
$x = u = r \cdot \sqrt{2}$



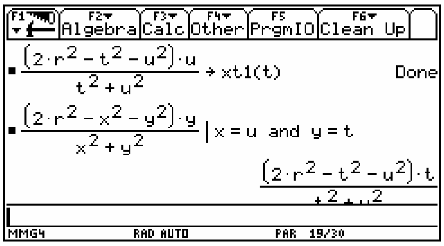
$x = u = r$

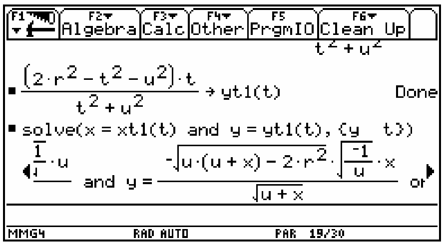
$x = u = \dfrac{r}{\sqrt{2}}$

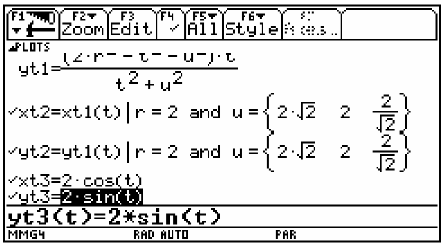


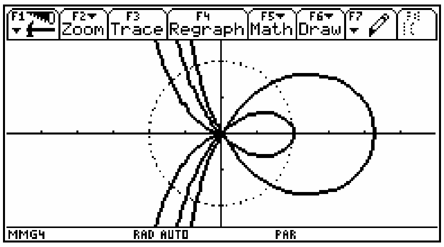


The TI-Procedure:





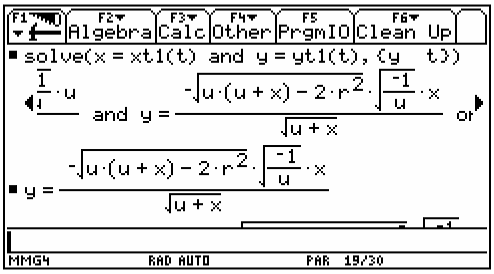


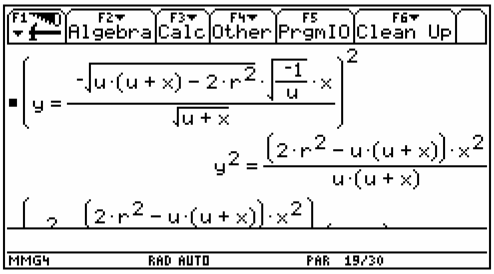


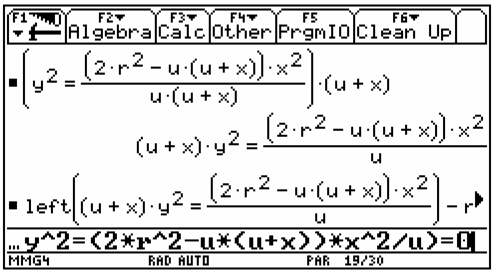
Algebraische Kurvengleichungen – Algebraic Equations of the Curves

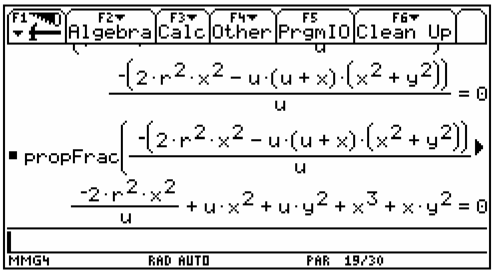
Bezeichnet man der Einfachheit wegen die Koordinaten der Kurvenpunkte mit x und y so erhält man die Parameterdarstellung $\left(x = \dfrac{2r^2 u}{u^2 + t^2} - u,\ y = \dfrac{2r^2 t}{u^2 + t^2} - t \right)$. Eliminiert man (mit DERIVE oder dem TI) den Kurvenparameter t, so erhält man als algebraische Gleichung für die Kurven:

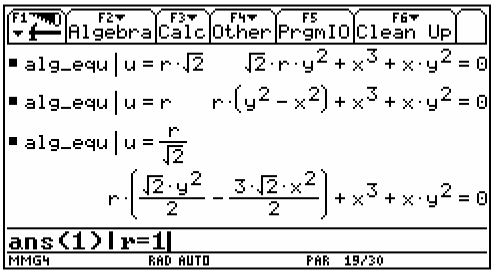
$$x^3 + x^2\left(u - \frac{2r^2}{u}\right) + y^2(x+u) = 0 \quad \text{(algebraic equation of the curves)}$$











Setzt man hierin speziell:

- $u = r \cdot \sqrt{2}$: so erhält man als Kurvengleichung $y^2(r \cdot \sqrt{2} + x) + x^3 = 0$, und nach einer Spiegelung der Kurve an der y-Achse ($x \to -x$): $y^2(r\sqrt{2} - x) - x^3 = 0$, also die algebraische Gleichung der **Kissoide. (Vgl. Lexicon of Curves (1))**
- $u = r$: so erhält man die Kurvengleichung: $y^2(r + x) + x^2(x - r) = 0$, also die algebraische Gleichung einer **Strophoide. (Vgl. Lexicon of Curves (2))**
- $u = \frac{r}{\sqrt{2}}$: so erhält man als Kurvengleichung: $y^2\left(x + \frac{r\sqrt{2}}{2}\right) - x^2\left(\frac{3r\sqrt{2}}{2} - x\right) = 0$, was mit $r = 1$ die algebraische Gleichung einer **Trisektrix von MacLaurin** liefert. **(Vgl. Lexicon of Curves (3))**

Insgesamt liefert damit eine einzige Abbildung die aufgezählten algebraischen Kurven als Bilder von speziellen Geraden.

Das Problem: Bilder von Kreisen – The Problem: Images of Circles

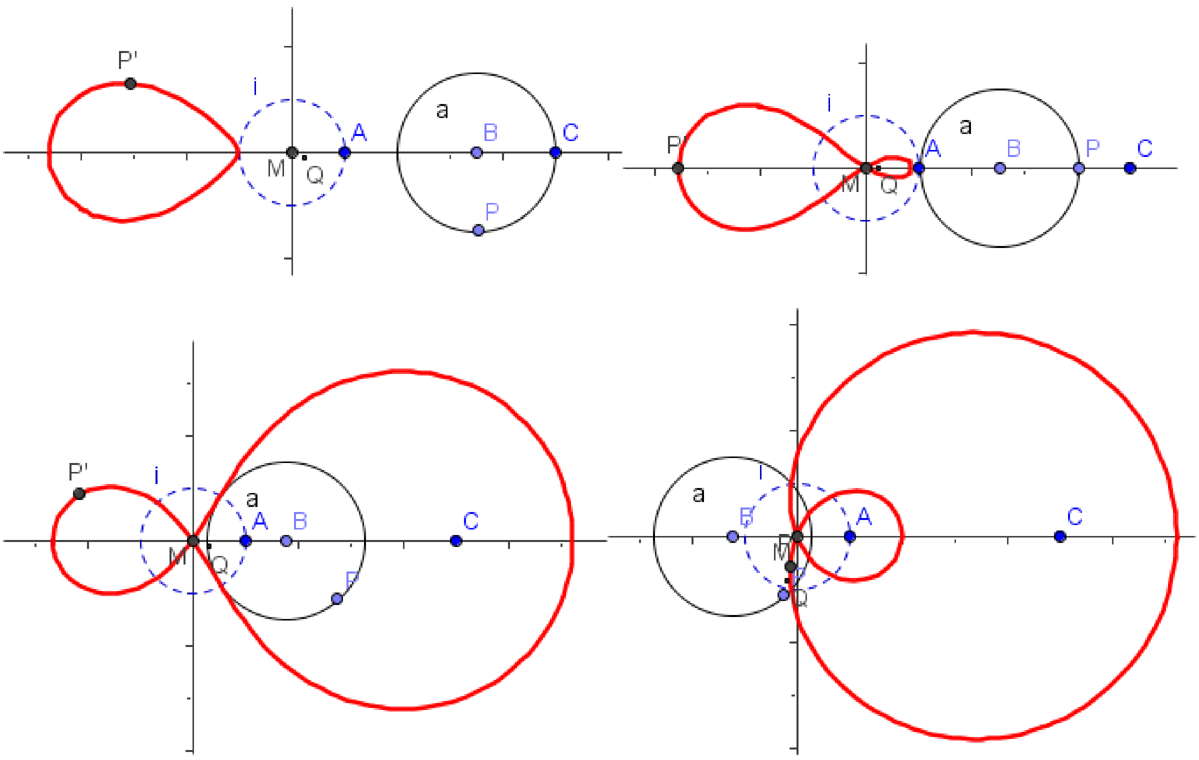
Die Untersuchung von Bildern von Kreisen unter der obigen Abbildung kann nun prinzipiell nach demselben Schema erfolgen. Allerdings ergeben sich hier algebraische Probleme, die ich trotz DERIVE nicht "in den Griff" bekam. Sicher bin ich, dass es sich bei den Bildern auch wieder um algebraische (und nicht etwa transzendente) Kurven dritter und vierter Ordnung handelt. Vermutlich finden sich unter diesen Kurven die Lemniskate von Bernoulli, die bekannten Pascalschen Schnecken sowie als Spezialfall die Kardioide. Zum Abschluss seien noch einige Kurven dargestellt, die sich als Bilder von Kreisen mit der Parameterdarstellung (t + 1.5 cos a, 1.5 sin a) ergeben. Die Mittelpunkte der Kreise liegen also auf der x-Achse, die Radien sind r = 1.5, der Radius des Inversionskreises ist 1.

Interessante Kurven ergeben sich auch beim Abbilden von Kegelschnitten. Allerdings ist eine Herleitung der algebraischen Kurvengleichungen in diesem Fall sehr (zeit-) aufwändig. Ausführlich ist die Untersuchung der oben angegebenen Abbildung und ihrer Eigenschaften dargestellt in "Didaktik der Mathematik, 2 (1991), S. 145 -164".
Allen, die sich mit dieser Konstruktion beschäftigen wollen, ware ich sehr dankbar, wenn Sie mich über ihre Entdeckungen und Ergebnisse informieren würden.

Using this mapping we can investigate the images of circles. Here we are facing algebraic problems which I couldn´t solve even using DERIVE. I am sure that the images are algebraic curves, too (of order three and four). I think that we will find the lemniscate of Bernoulli, the well known snails of Pascal with the Cardioide as a special case among them. In the figures you will find some curves which are images of circles with the parameter form [u + 1.5 cos(t), 1.5 sin(t)]. The centers of the circles are lying on the *x*-axis. The diameter of the inversion circle is 2.

Interesting curves appear as are the results of mapping conic sections. Deriving the algebraic equations of the curves is a very time consuming work. You can find the mapping introduced above in "Didaktik der Mathematik, 2 (1991), p 145 - 164".
I would appreciate each investigation and result concerning this special mapping. Thank you!

In the original version of 1994 Th. Weth added four figures. in 2007 I used the GeoGebra file from above and redefined the line *x* = u as the circle given above. In an instant I can see the new image pictures of the circles.



The next page shows the DERIVE calculation followed by the graphic representation using a slider bar for varying parameter *u*. DERIVE and GeoGebra form a wonderful pair for investigating mappings and loci.

$$x^2 + y^2 = 1$$

$$\text{circ} := \left[u + \frac{3}{2}\cdot\text{COS}(t),\ \frac{3}{2}\cdot\text{SIN}(t)\right]$$

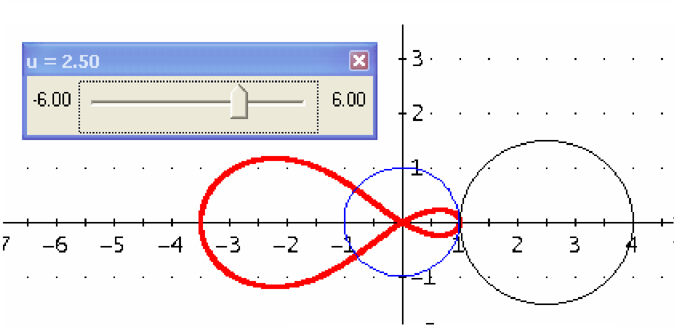
$$\text{map} := \left[\frac{2\cdot x}{x^2 + y^2} - x,\ \frac{2\cdot y}{x^2 + y^2} - y\right]$$

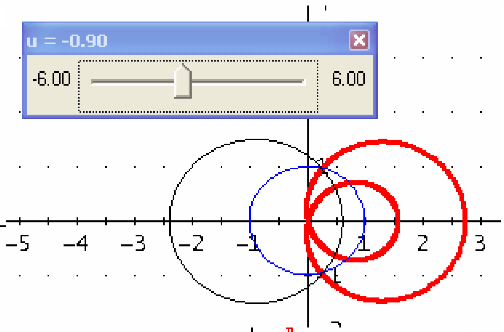
$$\text{circ\_img} := \text{SUBST}(\text{map},\ [x,\ y],\ \text{circ})$$

$$\text{circ\_img} := \left[-\frac{36\cdot u\cdot\text{COS}(t)^2 + 3\cdot(12\cdot u^2 + 1)\cdot\text{COS}(t) + 2\cdot u\cdot(4\cdot u^2 + 1)}{2\cdot(12\cdot u\cdot\text{COS}(t) + 4\cdot u^2 + 9)},\ -\frac{3\cdot\text{SIN}(t)\cdot(12\cdot u\cdot\text{COS}(t) + 4\cdot u^2 + 1)}{2\cdot(12\cdot u\cdot\text{COS}(t) + 4\cdot u^2 + 9)}\right]$$

$$\#14:\quad y^2 =$$

$$\frac{(\sqrt{(64\cdot u^2\cdot x^2 + 16\cdot u\cdot x\cdot(4\cdot u^2 - 17) + 16\cdot u^4 - 8\cdot u^2 + 1)} - 8\cdot u\cdot x - 4\cdot u^2 + 1)\cdot((8\cdot u\cdot x + 12\cdot u^2 + 1)\cdot\sqrt{(64\cdot u^2\cdot x^2 + 16\cdot u\cdot x\cdot(4\cdot u^2 - 17) + 16\cdot u^4 - 8\cdot u^2 + 1)} - 64\cdot u^2\cdot x^2 + 128\cdot u\cdot x\cdot(1 - u^2) - 80\cdot u^4 + 280\cdot u^2 - 1)}{128\cdot u^2\cdot(\sqrt{(64\cdot u^2\cdot x^2 + 16\cdot u\cdot x\cdot(4\cdot u^2 - 17) + 16\cdot u^4 - 8\cdot u^2 + 1)} - 8\cdot u\cdot x - 4\cdot u^2 + 17)}$$



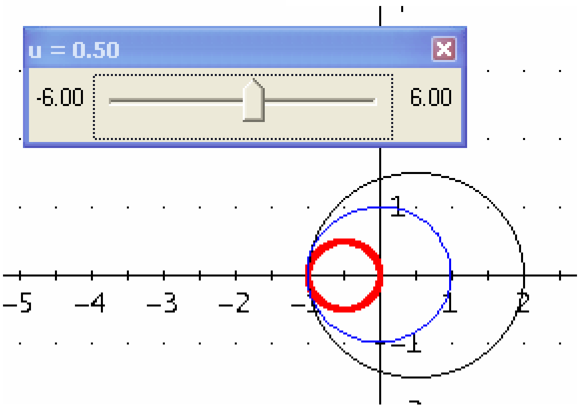


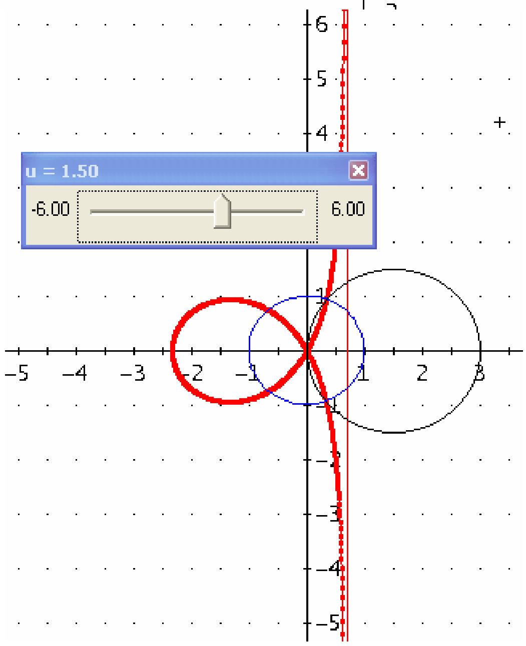
u = 3/2

$$\#15:\quad y^2 = \frac{x^2\cdot(3\cdot x + 7)}{2 - 3\cdot x}$$

u=1/2

$$\#16:\quad y^2 = -\ x\cdot(x + 1)$$



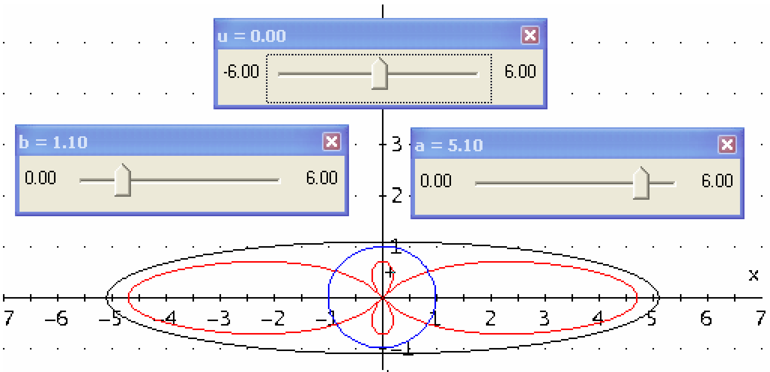


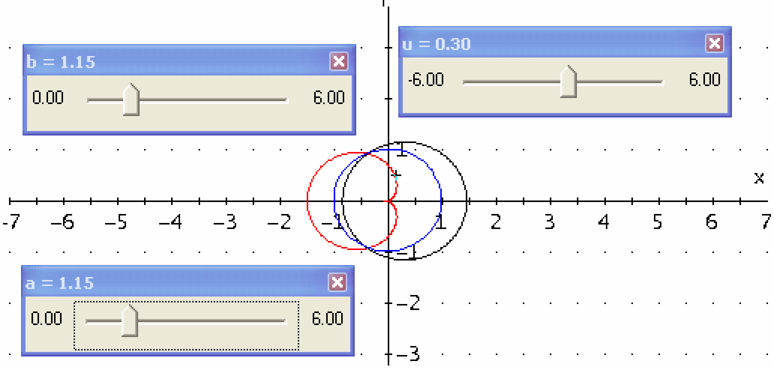
```
#20:  ell := [u + a·COS(t), b·SIN(t)]

#21:  ell_img := SUBST(map, [x, y], ell)
```

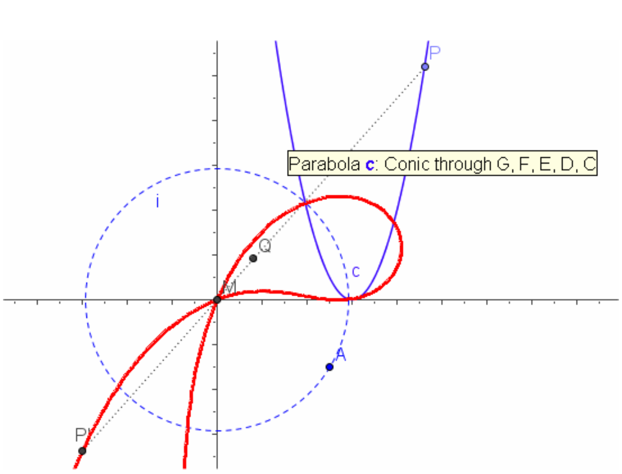
#22: ell_img :=

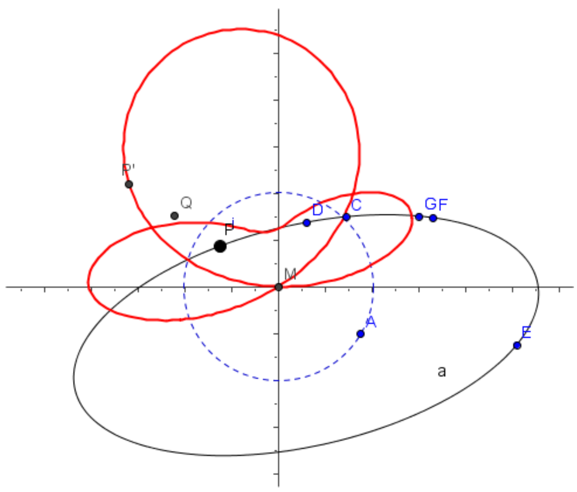
$$\left[-\frac{a^3\cdot\cos(t)^3+3\cdot a^2\cdot u\cdot\cos(t)^2+a\cdot\cos(t)\cdot(b^2\cdot\sin(t)^2+3\cdot u^2-2)+u\cdot(b^2\cdot\sin(t)^2+u^2-2)}{a^2\cdot\cos(t)^2+2\cdot a\cdot u\cdot\cos(t)+b^2\cdot\sin(t)^2+u^2},\ -\frac{b\cdot\sin(t)\cdot(a^2\cdot\cos(t)^2+2\cdot a\cdot u\cdot\cos(t)+b^2\cdot\sin(t)^2+u^2-2)}{a^2\cdot\cos(t)^2+2\cdot a\cdot u\cdot\cos(t)+b^2\cdot\sin(t)^2+u^2}\right]$$

The images of ellipses represented supported by slider bars in DERIVE and of general conics (GeoGebra):









Fluid Flow in DERIVE™

C.E. Reuther de Siqueira & J.V. Domingos, Petrópolis, Brasil

ABSTRACT

The DERIVE program shows itself as a powerful tool to the teaching of Mathematics, Physics, Thermodynamics and Strength of Materials among others. The students of Engineering courses are hopeful with the results presented in DERIVE, mainly that with graphics visualization. In this contribution we use the graphic facilities of DERIVE for the graphic visualization of the fluid flow through a cylinder.

INTRODUCTION

The Navier-Stokes equations for viscous fluid flow is very complex for the mathematical analysis. The Computational Fluid Dynamics (CDF), is using Finite Difference Methods for the solution of these equations, together with methods of grid generation. But, simplification of Navier-Stokes equations could be done without a sensible loss of reality. So, one could find analytic solutions more easily. Here we will use one of these simplifications using the potential flow in polar coordinates and in steady incompressible flow. From this the continuity equation is [1,2]:

$$\nabla \cdot V = \frac{1}{r}\left(\frac{\partial r V_r}{\partial_r} + \frac{\partial V_\theta}{\partial \theta}\right) = 0 \tag{1}$$

Figure (1) illustrates the cylinder studied and the speed of the fluid in infinity.

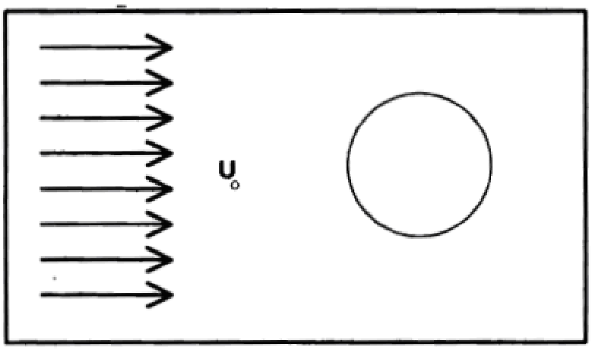


Figure 1. The physical domain

Using the potential flow, the streamfunction, which satisfies continuity equation (1), could be[3]:

$$\psi = U_0\, r \left(1 - \frac{a^2}{r^2}\right) \sin\theta \tag{2}$$

where ψ ist he streamfunction and $a^2 = -\frac{A}{U_0}$ with A being a constant and U_0 the initial speed of the fluid. The solution of equation (2), using DERIVE will be:

$$r = \frac{\frac{\psi}{U_0 \sin\theta} + \sqrt{\left(\frac{\psi}{U_0 \sin\theta}\right)^2 + 4a^2}}{2} \tag{3}$$

An idea of the streamlines could be obtained making $\psi = 2\,a\,U_0$ in equation (3) and the result will be:

$$r = a\left(\csc\theta + \sqrt{1+\csc^2\theta}\right) \qquad (4)$$

In equation (4) the function varies from infinite, in $\theta = 0$, to 1 in $\theta = \pi/2$, and then becomes ininity in $\theta = 0$ and $\theta = \pi$. The streamlines are presented in figure (2).

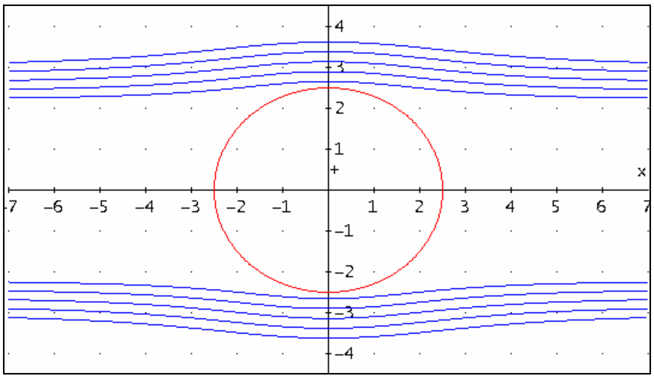


Figure 2. The streamlines around the cylinder

For figure (2) we use the following sequences in DERIVE:

```
#1:   Fluid Flow

#2:   Up lines

#3:   VECTOR(a·(CSC(θ) + √(1 + CSC(θ)^2 )), a, 1.1, 1.5, 0.1)

#4:   For plotting: π/20 <= θ <= 19π/20

#5:   Down lines

#6:   VECTOR(a·(CSC(θ) + √(1 + CSC(θ)^2 )), a, -1.1, -1.5, -0.1)

#7:   The cylinder:

#8:   2.5
```

CONCLUSION

Equation (4) is written in *polar coordinates*. If DERIVE doesn´t have the possibility for plotting these curves in these coordinates the problem would become even more complex. The development of numerical procedures and graphics for visualization of the streamlines would be necessary. In this contribution the speed and facility for visualization in the problem of Fluid Dynamics was shown. Like in this work others like Thermodynamics and Strength of Materials could be elaborated. Once more DERIVE showed itself as powerful tool.

ACKNOWLEDGEMENTS

The authors thank Universidade Católica de Petrópolis and Dr. Michael Wyles for financial support.

REFERENCES

[1] Currier, I.G. – Fundamental Mecahnics of Fluids, McGraw-Hill Book Company, 1974
[2] Fletcher, C.A.J. – Computational Techniques for Fluid Flow Dynamics Vol 1, 2 Springer 1988
[3] Hansen, A.G. – Fluid Mechanics, Wiley International Edition – John Wiley and Sons, N.Y. 1967

I didn't translate or give a short English version of Mr Ramos' contribution, because I really believe that you will understand his ideas presented in his file. As a little support I changed his DERIVE-comments, giving short translations. Spanish is a wonderful language and I hope you will enjoy his *desplazamientos.* I really did. Muchas gracias! Josef

Los desplazamientos en las funciones elementales

P. Familiar Ramos, Valencia, Spain

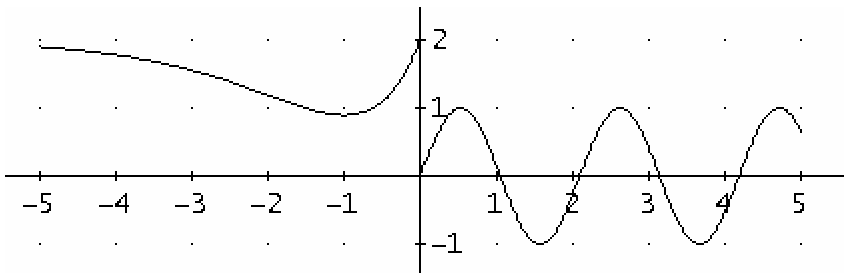
Este articulo es un estudio de las transformaciones geométricas en el plano aplicadas a una función u:=f{x), utilizando el programa *DERIVE.* Para ello se definen las respectivas funciones que corresponden a los movimientos de traslación, rotación, reflexión, o reflexión en deslizamiento (las cuatro isometrías del plano), y también se estudia la homotecia.

Al tratar de enfocar el problema, en seguida nos encontramos con una limitación: dado que incluso una pequeña rotación aplicada a la función u=f(x) origina casi con seguridad que la función deja de ser *uniforme...* lo mejor es trabajar en paramétricas al efectuar estas transformaciones.

¿Alguna otra limitación? Si. Como *DERIVE* no es capaz de representar en paramétricas funciones que tengan discontinuidades asintóticas, por ejemplo no dibuja (t,1/t) sabre un intervalo que contenga el cero (v.g.-3.1416 $\le t \le$ 3.1416), nos limitaremos a considerar funciones continuas.

Sin embargo, *DERIVE* es capaz de dibujar en paramétricas funciones con discontinuidades de salto finito. For ejemplo representa sin problemas la función

$$[x, \mathrm{IF}(x \le 0, 3x\,\hat{e}^{x} + 2, \mathrm{SIN}(3x))] \text{ en } -5 \le x \le 5,$$



de modo que también podemos aplicar las transformaciones geométricas a este tipo de funciones.

Es interesante experimentar con funciones sencillas y ver como actúan los movimientos. Presentamos ahora el listado que contiene las transformaciones geométricas, listas para ser usadas, y mostramos a continuación algunos ejemplos concretos, que sin duda pueden ser mejorados, pero que sirven para ilustrar esta teoría.

```
**************************************************************************************
        TRANSFORMACIONES GEOMETRICAS EN EL PLANO
           aplicadas a una funcion continua u:= f(x)
**************************************************************************************
```

Traslacion de vector (a,b) - Translation by the vector (a,b)

```
#1:   TRASLACION(u, a, b) := [x, u] + [a, b]
```

Traslacion horizontal del longitud |a| - Horizontal translation

```
#2:   TRASLACION_H(u, a) := TRASLACION(u, a, 0)
```

Traslacion vertical del longitud |b| - Vertical translation

#3: TRASLACION_V(u, b) := TRASLACION(u, 0, b)

****

Rotacion de centro (x0,y0) y amplitud α (en grados) -
Rotation with rotation center (x0, y0) by α degrees

#4: ROTACION(u, x0, y0, α) := $[x - x0,\ u - y0]\cdot\begin{bmatrix} \mathrm{COS}(\alpha\cdot 1°) & \mathrm{SIN}(\alpha\cdot 1°) \\ -\mathrm{SIN}(\alpha\cdot 1°) & \mathrm{COS}(\alpha\cdot 1°) \end{bmatrix} + [x0,\ y0]$

Simetria central respecto del punto (x0,y0) - Reflection with center (x0,y0)

#5: SEMIGIRO_(u, x0, y0) := ROTACION(u, x0, y0, 180)

Giro de amplitud α alrededor del origen - Rotation by α deg around origin

#6: GIRO(u, α) := ROTACION(u, 0, 0, α)

Simetria central respecto del origen - Reflection with origin as center

#7: SEMIGIRO(u) := GIRO(u, 180)

****

Reflexion respecto de la recta ax + by + c = 0 - Reflection with respect to the line ax + by + c = 0

#8: REFLEXION(u, a, b, c) := $[x,\ u]\cdot\begin{bmatrix} -\dfrac{a^2 - b^2}{a^2 + b^2} & -\dfrac{2\cdot a\cdot b}{a^2 + b^2} \\ -\dfrac{2\cdot a\cdot b}{a^2 + b^2} & \dfrac{a^2 - b^2}{a^2 + b^2} \end{bmatrix} + \left[-\dfrac{2\cdot a\cdot c}{a^2 + b^2},\ -\dfrac{2\cdot b\cdot c}{a^2 + b^2}\right]$

Simetria respecto del eje Ox - Reflection with respect to the x-axis

#9: SIMETRIA_OX(u) := REFLEXION(u, 0, 1, 0)

Simetria respecto del eje Oy - Reflection with respect to the y-axis

#10: SIMETRIA_OY(u) := REFLEXION(u, 1, 0, 0)

Simetria respecto de la diagonal principal - Reflection with respect to y = x

#11: SIMETRIA_DIAGONAL(u) := REFLEXION(u, -1, 1, 0)

Simetria respecto de la diagonal secundaria - Reflection with respect to y = -x

#12: SIMETRIA_DIAGONAL2(u) := REFLEXION(u, 1, 1, 0)

****

Homotecia de razon k con centro en el punto (x0,y0) - Enlargement by factor k with center (x0,y0)

#13: HOMOTECIA_(u, x0, y0, k) := k·[x - x0, u - y0] + [x0, y0]

Homotecia de razon k centrada en el origen - Enlargement by factor k with center (0,0)

#14: HOMOTECIA(u, k) := HOMOTECIA_(u, 0, 0, k)

****

Reflexion en deslizamiento del longitud |d| respecto de la recta a x + b y + c = 0 -
Reflection and glide by |d| with respect to the line a x + b y + c = 0

#15: de la recta ax + bx + c = 0

#16: $$\mathrm{DESLIZAMIENTO}(u, a, b, c, d) := \mathrm{REFLEXION}(u, a, b, c) + \left[\frac{b\cdot d}{\sqrt{(a^2 + b^2)}}, -\frac{a\cdot d}{\sqrt{(a^2 + b^2)}}\right]$$

****

Ejemplos - Examples

The original curves are black and the results of the transformations are red.

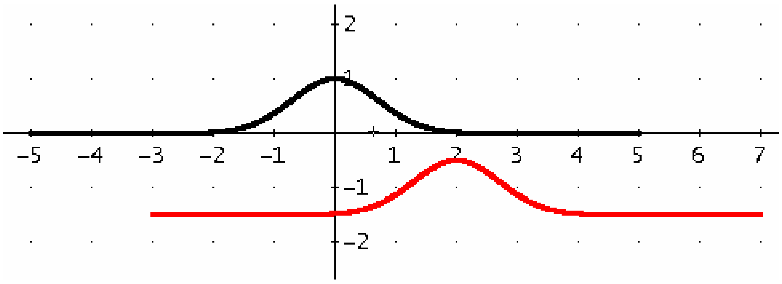
#17: $u := e^{-x^2}$

#18: TRASLACION(u, 2, -1.5)

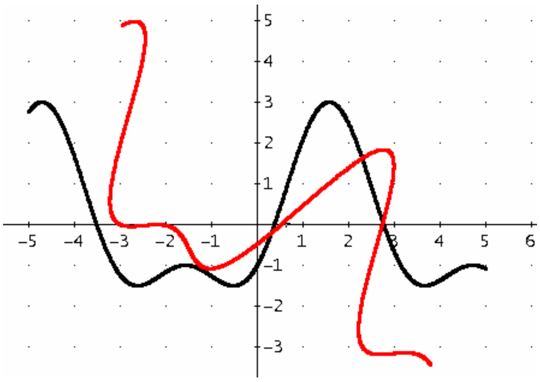
#19: $\left[x + 2, e^{-x^2} - \frac{3}{2}\right]$

#20: $\mathrm{TABLE}\left(e^{-x^2}, x, -5, 5, 0.01\right)$

#21: (TABLE(TRASLACION(u, 2, -1.5), x, -5, 5, 0.01))↓↓2



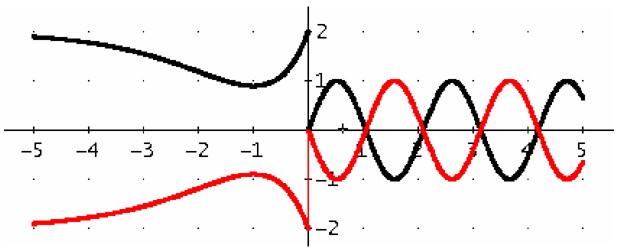
Rotation



u := 2·SIN(x) - COS(2·x)

GIRO(u, -30)

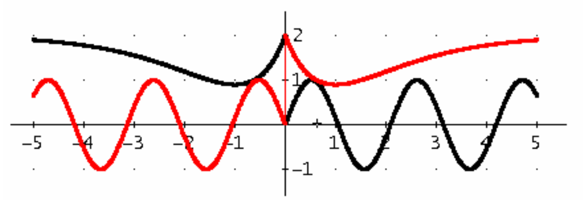
Reflection



$u := \mathrm{IF}(x \le 0, 3\cdot x\cdot e^{x} + 2, \mathrm{SIN}(3\cdot x))$

SIMETRIA_OX(u)

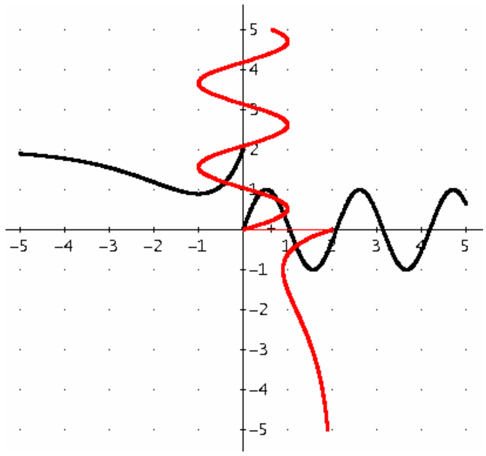
Reflection



$$u := \mathrm{IF}(x \le 0,\ 3 \cdot x \cdot e^{x} + 2,\ \mathrm{SIN}(3 \cdot x))$$

SIMETRIA_OY(u)

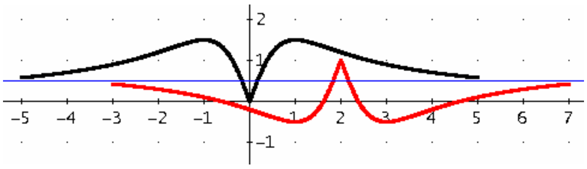
Reflection



$$u := \mathrm{IF}(x \le 0,\ 3 \cdot x \cdot e^{x} + 2,\ \mathrm{SIN}(3 \cdot x))$$

SIMETRIA_DIAGONAL(u)

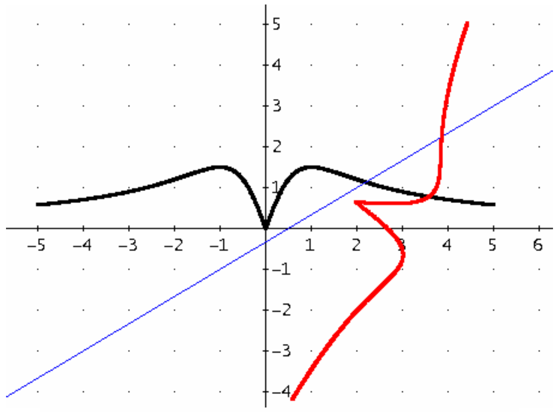
Reflection & Glide



$$u := \frac{3 \cdot |x|}{1 + x^{2}}$$

$$0 \cdot x + 1 \cdot y - \frac{1}{2} = 0$$

$$\mathrm{DESLIZAMIENTO}\left(u,\ 0,\ 1,\ -\frac{1}{2},\ 2\right)$$

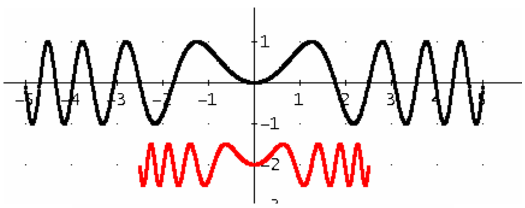


$$u := \frac{3 \cdot |x|}{1 + x^{2}}$$

$$2 \cdot x - 3 \cdot y - 1 = 0$$

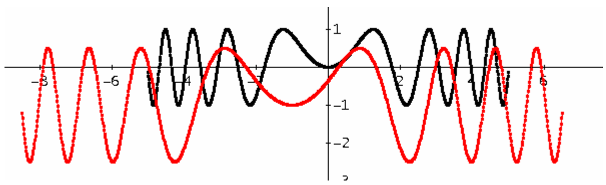
DESLIZAMIENTO(u, 2, −3, −1, −2)

Homotecia



$$u := \mathrm{SIN}(x^{2})$$

$$\mathrm{HOMOTECIA\_}\left(u,\ 0,\ -4,\ \frac{1}{2}\right)$$



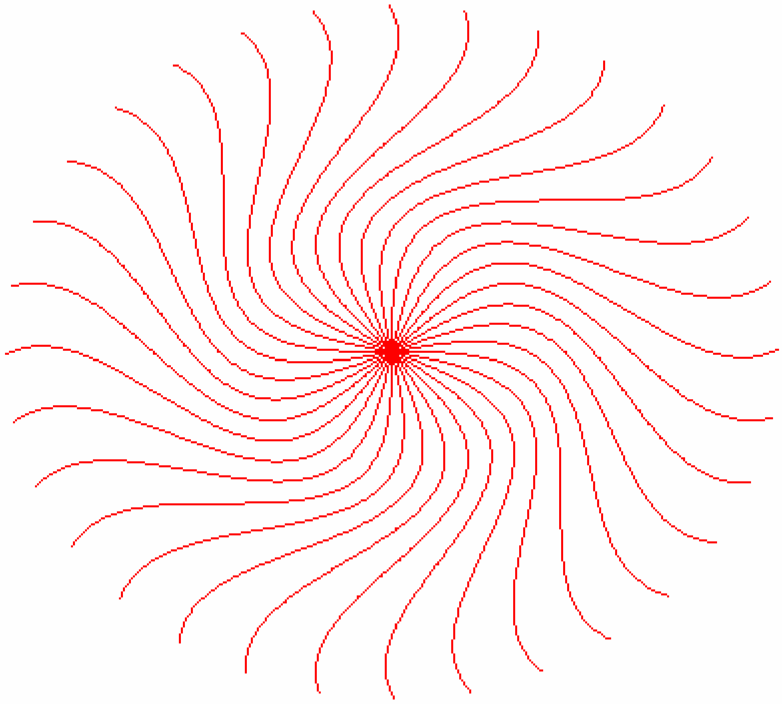
$$u := \mathrm{SIN}(x^{2})$$

$$\mathrm{HOMOTECIA\_}\left(u,\ 2,\ 2,\ \frac{3}{2}\right)$$

Ahora que conocemos major las notaciones, podemos ir más allá *jugar* un poco con expresiones del tipo

```
VECTOR(GIRO(SIN(x), k), k, 11.25, 180, 11.25)
```

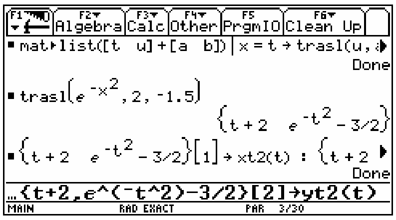
obteniendo en este caso:

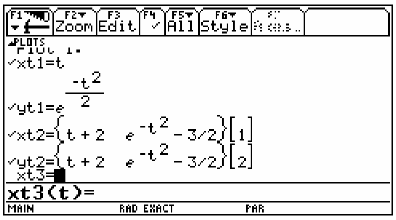


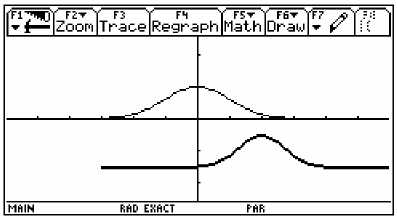
¿Qién iba a decir que este resultado es un «*conjunto de senos*»?

It is not so difficult to transfer the transformations on the CAS-TI devices. However there are some specialities which must be considered. I convert the matrix into a list – then it is easier to store the two components of the parameter representation in the Y=EDITOR. Piecewise defined functions make problems by transfering them into the Y=EDITOR (see below).

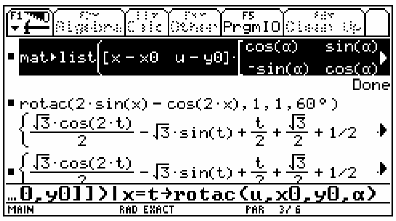
```
matúlist([t,u]+[a,b])|x=t → trasl(u,a,b)
```

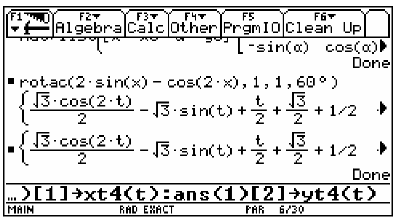


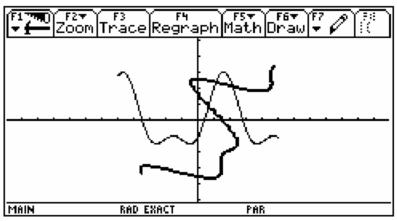




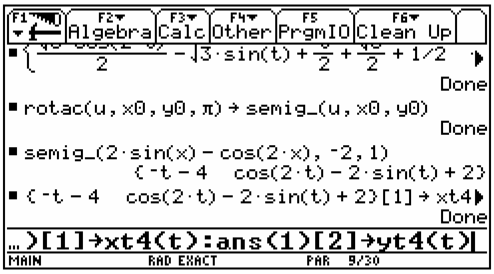

```
matúlist([x-xØ,u-yØ]*[cos(α),sin(α);-sin(α),cos(α)]+[xØ,yØ])|x=t → rotac(u,xØ,yØ,α)
```

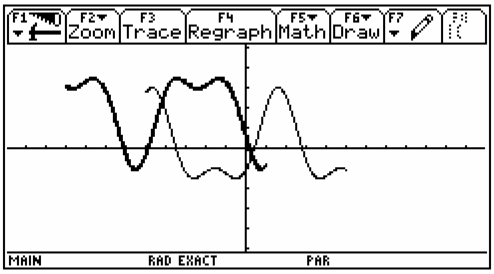




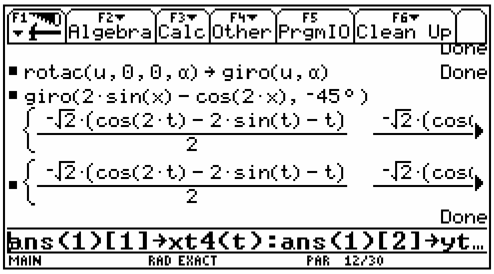


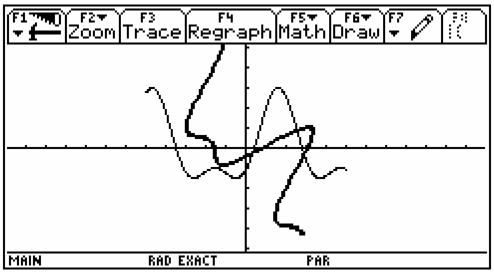
rotac(u,xØ,yØ,π) → semig_(u,xØ,yØ)



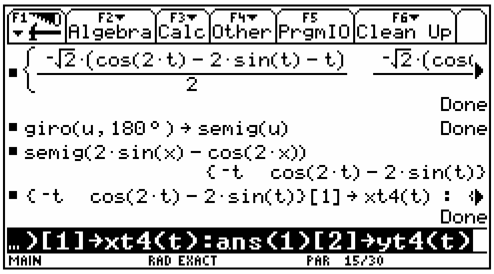


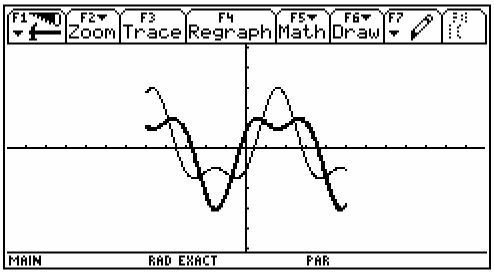
rotac(u,Ø,Ø,α) → giro(u, α)



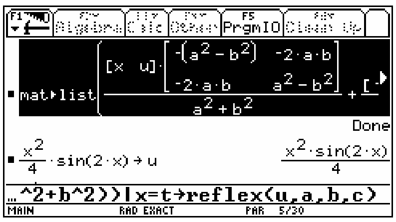


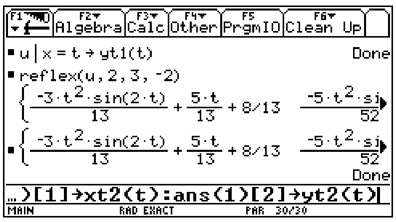
giro(u,18Ø°) → semig(u)

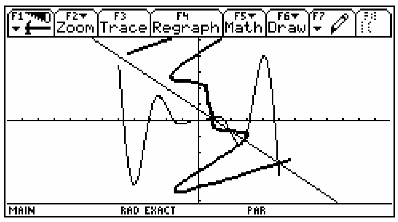


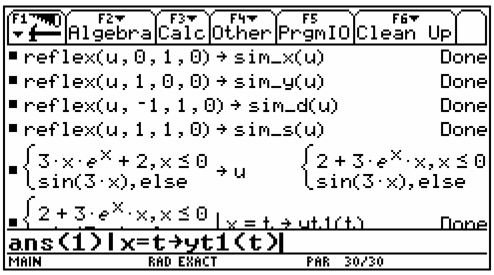


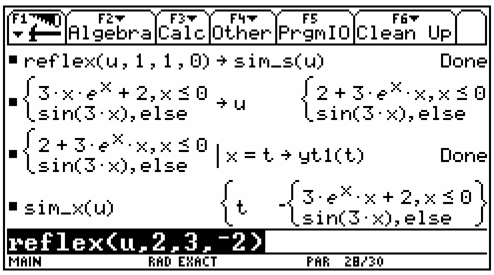
matúlist([x,u]*[-(a^2-b^2),-2a*b;-2a*b,a^2-b^2]/(a^2+b^2)+
[-2a*c,2b*c]/(a^2+b^2))|x=t → reflex(u,a,b,c)



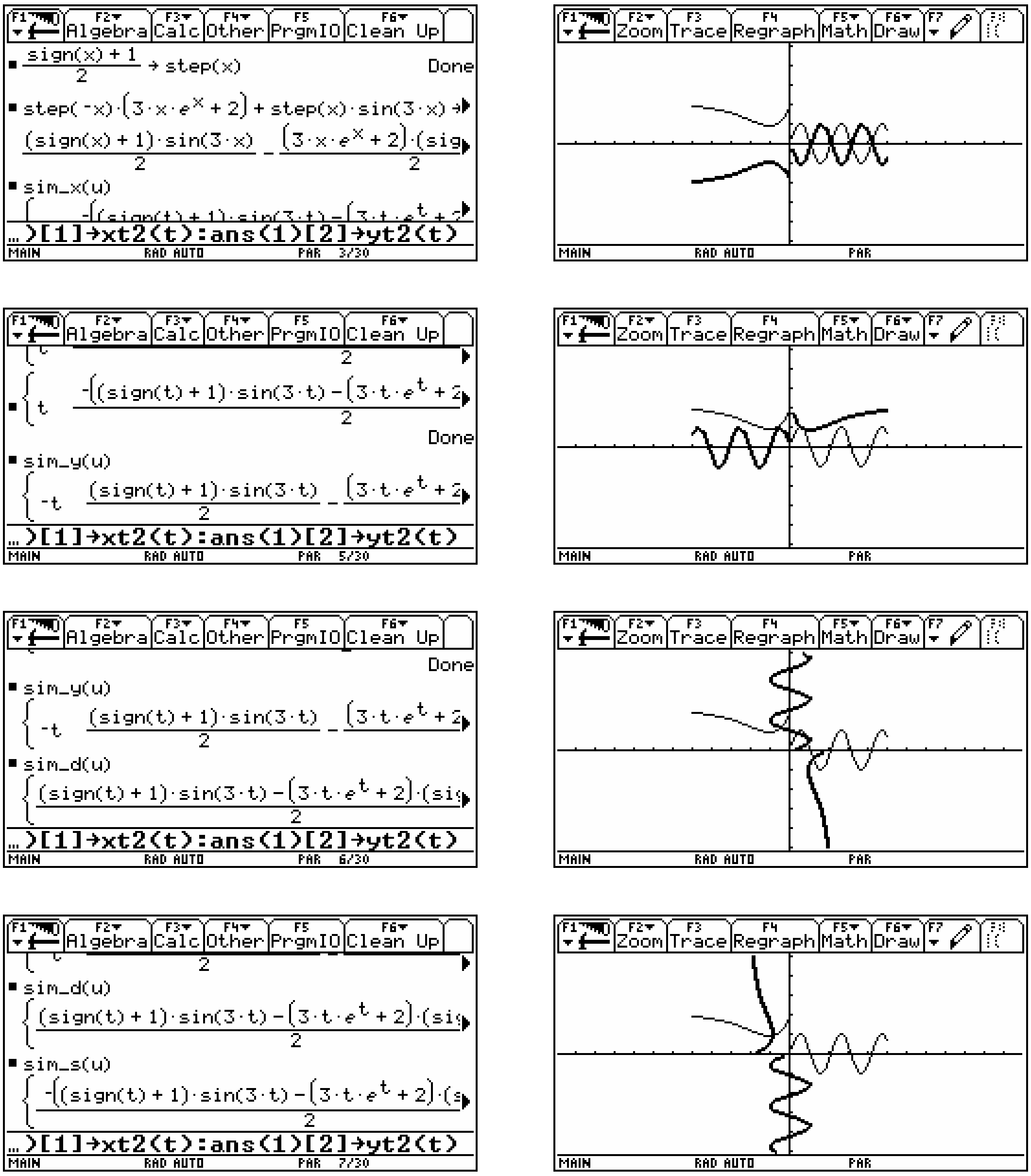




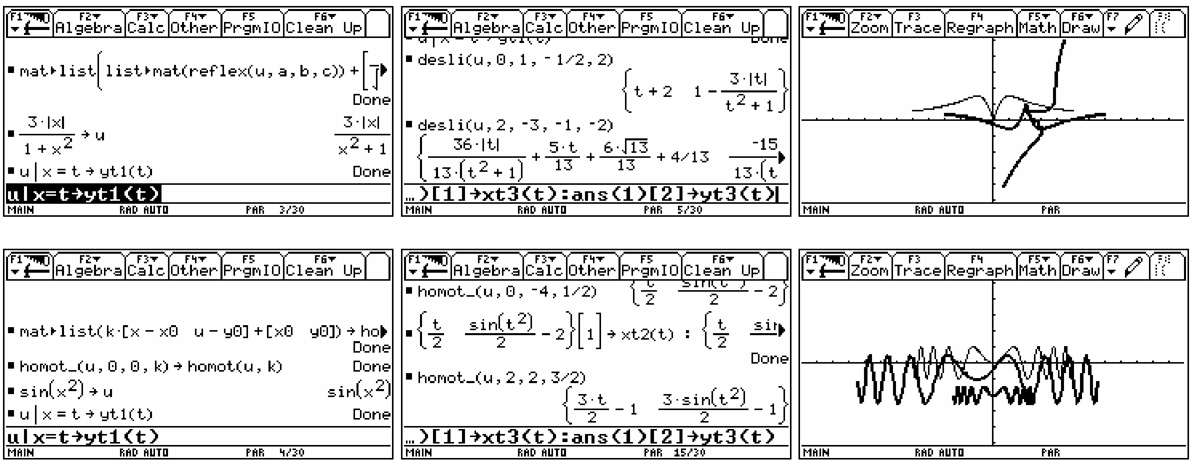




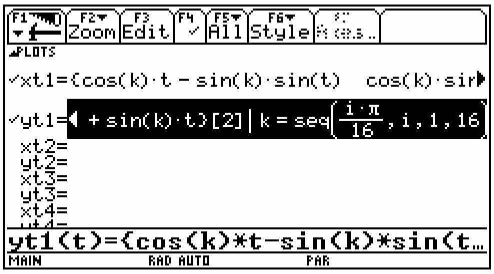
Substituting t for x does not apply for the piecewise defined function. I helped myself by defining the STEP-function,which returns 0 for x<0 and 1 for x>0. You could also define u as a function of t.

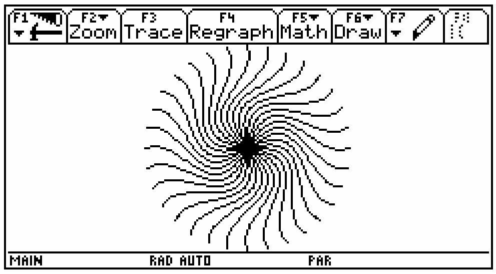


matúlist(listúmat(reflex(u,a,b,c))+[b*d,-a*d]/√(a^2+b^2))|x=t → desli(u,a,b,c,d)



See finally the family of rotated sine waves on the TI-screen. Plotting takes some time!





In my opinion it might be a nice task for students to transfer the transformations on the TI trying to overcome all problems. There are transformations left (eg shear). So they could also accomplish the collection of planar transformations. Josef

Aufgaben aus der Elektrotechnik – nicht nur für den Unterricht in beruflichen Schulformen

Problems from electrical engineering – not only for Vocational Schools

H. Scheuermann & N. Wild, Hofheim, Germany

Mit diesem Beitrag möchten wir anwendungsorientierte Beispielaufgaben aus der Elektrotechnik für den Schulunterricht unter besonderer Berücksichtigung von DERIVE™ vorstellen. Die Aufgaben stammen aus unseren Unterrichtskonzepten für die beruflichen Schulformen *Berufliches Gymnasium* und *Fachoberschule* mit den Schwerpunkten Elektrotechnik bzw. Informatik sowie (nach angemessener didaktischer Reduktion) für die Teilzeitschulform *Berufsschule* (insbesondere Kommunikationselektronik). Alle Beispiele wurden von uns im Unterricht dieser Schulformen meist mehrfach ausprobiert und evaluiert. Aufgrund der – nach unserer – Einschätzung positiven Unterrichtserfahrungen glauben wir, dass sich die folgenden Anwendungsaufgaben – zumindest partiell – auch als Unterrichtssequenzen für den Mathematik– oder Physikunterricht an den allgemeinbildenden Schulen eignen.[1]

1. Aufnahme der Kennlinien eines belasteten Spannungsteilers[2]

Characteristics of a loaded voltage divider

Aufgabenstellung:

Die Kennlinien eines belasteten Spannungsteilers (vgl Abb. 1) sollen graphisch dargestellt werden. Durch Veränderung des Lastwiderstands R_L soll dessen Einfluss auf den Verlauf der Kennlinien untersuchtwerden.

Problem: Give the graphic presentation of the characteristics of a loaded voltage divider. Investigate the influence of the load resistor R_L on the characteristics.

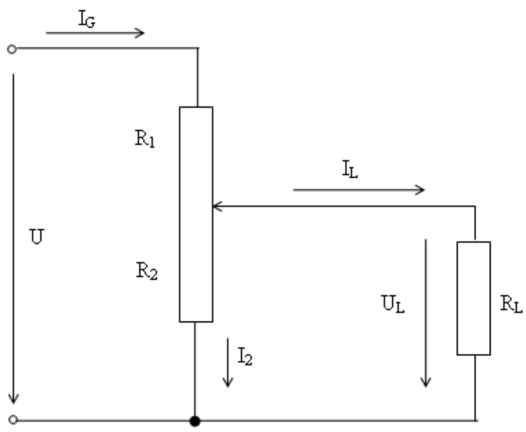


Abb. 1: Belasteter Spannungsteiler

Das DERIVE-Listing[3] könnte folgendermaßen aussehen[4] – bis zur normierten Form (Ausdruck #12)

The DERIVE-procedure could look like as follows – up to the normalized form (expr # 12)

Calculation of the total resistance Rg
with Rp = R1 + R2 as resistance of the potentiometer.

#1: CaseMode := Sensitive

#2: InputMode := Word

#3: $Rg := R1 + \frac{R2 \cdot RL}{R2 + RL}$

#4: $Rg := (Rp - R2) + \frac{R2 \cdot RL}{R2 + RL}$

#5: $Ig := \frac{U}{Rg}$

#6: $Ig := - \frac{U \cdot (R2 + RL)}{R2^2 - R2 \cdot Rp - RL \cdot Rp}$

#7: $UL := \frac{Ig \cdot R2 \cdot RL}{R2 + RL}$

#8: $UL := - \frac{R2 \cdot RL \cdot U}{R2^2 - R2 \cdot Rp - RL \cdot Rp}$

Division by U and normation by substitution:
F(x)=UL/U, R2 = x * Rp and RL = Rp/z

#9: $\frac{UL}{U} = - \frac{R2 \cdot RL}{R2^2 - R2 \cdot Rp - RL \cdot Rp}$

#10: $\frac{UL}{U} = - \frac{(x \cdot Rp) \cdot \frac{Rp}{z}}{(x \cdot Rp)^2 - (x \cdot Rp) \cdot Rp - \frac{Rp}{z} \cdot Rp}$

#11: $\frac{UL}{U} = - \frac{x}{x^2 \cdot z - x \cdot z - 1}$

Domain: [0,1]
We plot some characteristics by variation of z
and now with DERIVE 6 we add a slider bar to animate the characteristic!

```
#12:  F(x, z) :=
        If 0 ≤ x ≤ 1
           - x/(x^2·z - x·z - 1)

#13:  VECTOR(F(x, z), z, [0, 0.8, 2, 4, 8, 16, 40])

#14:  TABLE(F(x, z) :=                    , x, 0, 1, 0.001)
            If 0 ≤ x ≤ 1
               - x/(x^2·z - x·z - 1)
```

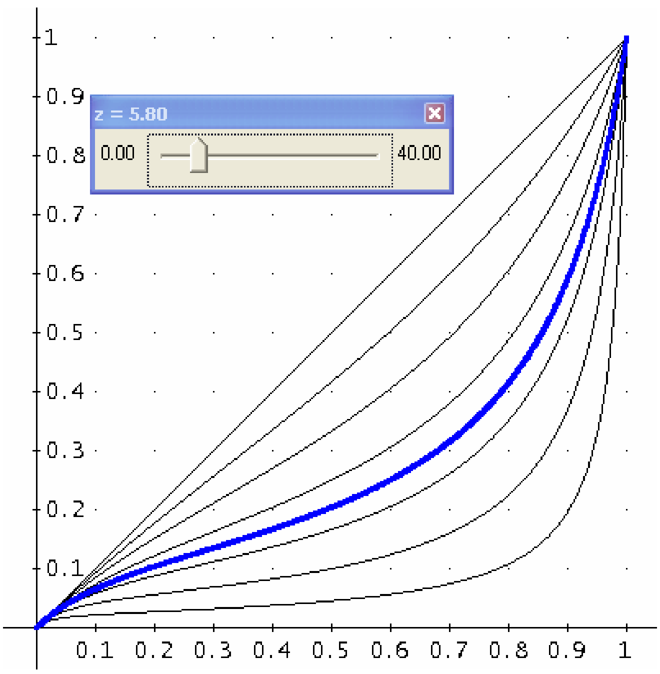


Abb. 2: Kennlinien des belasteten Spannungsteilers

Durch Variation von z (hier z ∈ {0, 0.8, 2, 4, 8, 16, 40}) erhält man die Kennlinienschar im Plot-Fenster (vgl. Abb. 2).

Variation of z - z ∈ {0, 0.8, 2, 4, 8, 16, 40} – gives the family of characteristics.

2. Die belastete Spannungsquelle – The loaded supply point

Aufgabenstellung:
Gegeben ist eine Spannungsquelle mit dem Innenwiderstand R_I und einem regelbaren Lastwiderstand R_L (vgl. Abb. 3). Die Leistung P_L soll in Abhängigkeit vom Lastwiderstand R_L untersucht werden.

Problem: Given is a supply point with an internal resistance R_I and a regulable load resistor R_L. Investigate the power P_L in dependence on R_L. (It might be easier for the students to work with special values for the source voltage U_0 and for R_I).

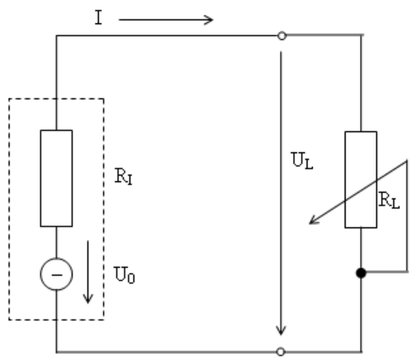


Abb. 3: Belastete Spannungsquelle

Wie bei den folgenden Aufgaben ist auch bei dieser die Wahl eines geeigneten, adressatengerechten Anspruchsniveaus unabdingbar. In diesem Sinne könnten die inhaltlichen Anforderungen reduziert werden, wenn für die Quellenspannung und den Innenwiderstand bereits in der Aufgabenstellung Zahlenwerte angegeben werden. Diese didaktische Reduktion ermöglicht die Koordinaten für die maximale Leistung ohne größeren rechnerischen Aufwand aus dem Funktionsgraphen mit Hilfe des Cursors auszumessen. Dieser Vorgehensweise entspricht u.a. der Lösungsvorschlag in Abb.4[5].

Die Auswertung der Zeilen #1 bis #9 (inkl. Funktionsgraph) legen die *Annahme* nahe, dass die Leistung am Lastwiderstand maximal wird, wenn $R_L = R_I$ gilt. Die Verallgemeinerung und damit die Richtigkeit der Annahme beweisen die folgenden Zeilen im Listing (vgl. Abb. 4[5]).

Interpretation of expressions #l to #9 including the graph gives the idea that power will be maximal at the load resistor. Generalization and with it the truth of this assumption can be shown in the following listing -see fig. 4[5].

Eine alternative Lösungsvariante weist das folgende DERIVE-Listing auf. Da hier auf das Instrumentarium der Differentialrechnung verzichtet werden kann, eignet sich dieser Lösungsweg auch für die 11te Klasse des BGs in der Oberstufe. Diesem Lösungsansatz liegen pragmatische Überlegungen zugrunde. Die Leistung P_L am Lastwiderstand kann als Produkt aus Spannung und Strom beschrieben werden, also $P_L = U_L \cdot I$. Die Variation des Lastwiderstands $R_I \in [0, \infty]$ bewirkt eine Änderung beider Faktoren. Die Grenzwerte von R_L zeichnen sich durch ihre praktische Bedeutung aus:

(I) $R_L = 0\Omega$ (Kurzschluss) → der Strom I wird maximal, die Spannung $U_L = 0V$ → $P_L = 0W$
short circuit

(II) RL → ∞ („offene“ Klemmen) → die Spannung wird maximal, der Strom I = 0A → PL = 0W
open binders

Da für $0 < P_L < \infty$ die Leistung $P_L > 0$ ist, muss mindestens ein R_L existieren, das für P_L ein Maximum besitzt (dies ist anschaulich klar; der mathematische Hintergrund, dh der Satz von Rolle, sollte aber in diesem Zusammenhang kein Thema für den Unterricht sein). Das Auflösen der Gleichung nach R_L in Zeile #3 führt zur Lösung einer quadratischen Gleichung mit positiven bzw negativen Wurzeltermen (Zeile #4). Die Existenz von zwei Lösungen wird in der graphischen Darstellungsbene plausibel, wenn das Diagramm in Abb. 4 durch eine Parallele zur ersten Achse (mit $0<P_L<P_{L\,max}$) ergänzt wird. (Die Sekante weist mit R_{L1} und R_{L2} zwei Schnittpunkte auf). Durch Verschieben dieser Parallelen in positiver Richtung der zweiten Achse geht die Sekante in eine Tangente (mit $P_L = P_{Lmax}$ und dem Argument $R_{L1} = R_{L2}$) über. Bezogen auf die Lösungen der quadratischen Gleichung bedeutet dies, dass die Diskriminante gleich Null wird (Zeilen #5 ff).

#1: [CaseMode := Sensitive, InputMode := Word]

#2: $PL = \left(\frac{U0}{RI + RL}\right)^2 \cdot RL$

#3: $\text{SOLVE}\left(PL = \left(\frac{U0}{RI + RL}\right)^2 \cdot RL,\ RL\right)$

#4: $RL = -\frac{\sqrt{(U0^2 - 4 \cdot PL \cdot RI)} \cdot |U0|}{2 \cdot PL} - \frac{2 \cdot PL \cdot RI - U0^2}{2 \cdot PL} \lor RL = \frac{\sqrt{(U0^2 - 4 \cdot PL \cdot RI)} \cdot |U0|}{2 \cdot PL} - \frac{2 \cdot PL \cdot RI - U0^2}{2 \cdot PL}$

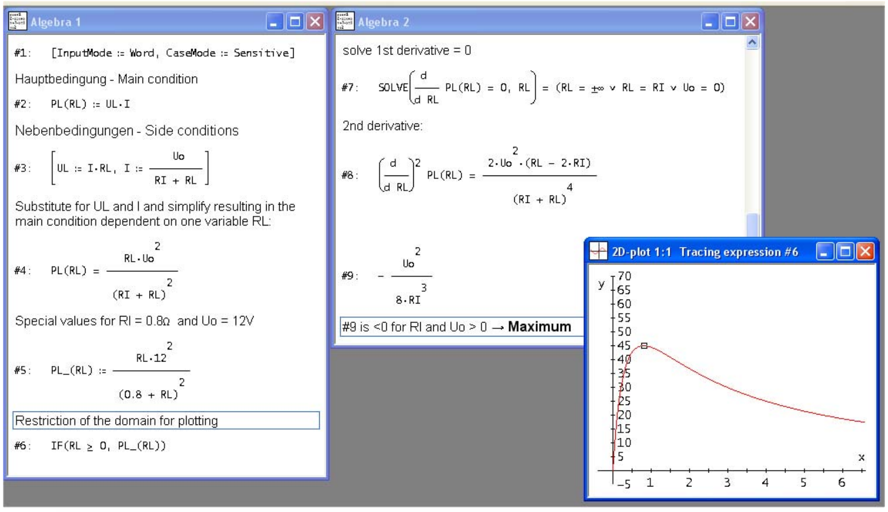
Set the discriminant = 0

#5: $\text{SOLVE}(U0^2 - 4 \cdot PLmax \cdot RI = 0,\ PLmax) = \left(PLmax = \frac{U0^2}{4 \cdot RI}\right)$

expr #3 = expr #6 and solve for RL:

#6: $\text{SOLVE}\left(\left(\frac{U0}{RI + RL}\right)^2 \cdot RL = \frac{U0^2}{4 \cdot RI},\ RL\right) = (RL = RI \lor U0 = 0)$

Compare the DERIVE screens from 2007 and 1994:



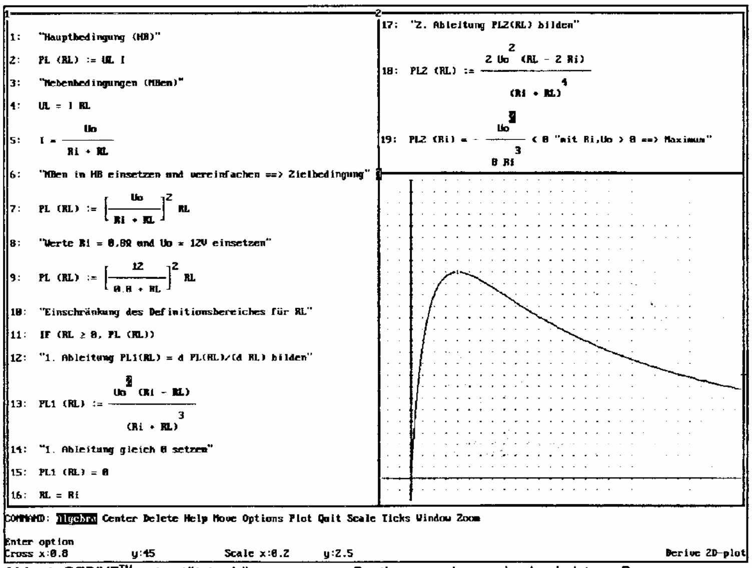


Abb. 4: DERIVE[TM]-unterstützter Lösungsweg zur Bestimmung der maximalen Leistung P_L

3. Der Effektivwert eines sinusförmigen Wechselstroms – The effective value of a sinusoidal alternating current

Aufgabenstellung:

Beweisen Sie, dass der Effektivwert[6] eines sinusförmigen Wechselstroms berechnet werden kann durch die Formel

$$I_{eff} = \frac{I_{max}}{\sqrt{2}}.$$

Problem: Proof that the effective value of a sinusoidal A.C. can be calculated by the formula given above.

#1: [InputMode := Word, CaseMode := Sensitive]

Labour of direct current at an ohmic resistor R during a period T:

$$\#2:\quad [W\_ := P\_\cdot T,\ P\_ := R\cdot Ieff^2]$$

$$\#3:\quad W\_ = Ieff^2\cdot R\cdot T$$

Function for alternating current

$$\#4:\quad I(t) := Imax\cdot SIN(w\cdot t + \phi)$$

Labour of A.C. at an ohmic resistor R during a period T:

$$\#5:\quad \left[W\omega := \int_0^T P\omega\ dt,\ P\omega := R\cdot I(t)^2\right]$$

The definition of the effective value leads to equation

$$\#6:\quad W\_ = W\omega$$

$$\#7:\quad Ieff^2\cdot R\cdot T = -\frac{Imax^2\cdot R\cdot SIN(2\cdot T\cdot w + 2\cdot\phi)}{4\cdot w} + \frac{Imax^2\cdot R\cdot SIN(2\cdot\phi)}{4\cdot w} + \frac{Imax^2\cdot R\cdot T}{2}$$

Substitute the angular frequency by $2\pi/T$

$$\#8:\quad Ieff^2\cdot R\cdot T = -\frac{Imax^2\cdot R\cdot SIN\left(2\cdot T\cdot\frac{2\cdot\pi}{T} + 2\cdot\phi\right)}{4\cdot\frac{2\cdot\pi}{T}} + \frac{Imax^2\cdot R\cdot SIN(2\cdot\phi)}{4\cdot\frac{2\cdot\pi}{T}} + \frac{Imax^2\cdot R\cdot T}{2}$$

$$\#9:\quad Ieff^2\cdot R\cdot T = \frac{Imax^2\cdot R\cdot T}{2}$$

$$\#10:\quad SOLVE\left(Ieff^2\cdot R\cdot T = \frac{Imax^2\cdot R\cdot T}{2},\ Ieff\right)$$

$$\#11:\quad Ieff = -\frac{\sqrt{2}\cdot Imax}{2} \vee Ieff = \frac{\sqrt{2}\cdot Imax}{2} \vee R = 0 \vee T = 0$$

Algebra 2

```
#1:   [CaseMode := Sensitive, InputMode := Word]

#2:   [Imax := 4, f := 50, Φ := 0, w := 2·π·f]

#3:   IΩ(t) := Imax·SIN(w·t - Φ)

                Imax
#4:   I_(t) := ────
                 √2

#5:   I_(t) := 2.828427124
```

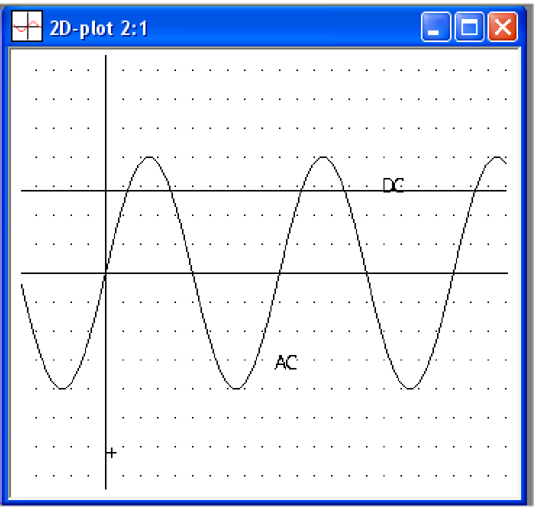


Abb. 5: Wechselstrom AC = IΩ(t), Effektivstrom I_(t)

4. Einstellung des optimalen Arbeitspunkts bei symmetrisch angesteuerten Transistorstufen – Adjustment of the optimal operating point at symmetric steered transistor steps

Aufgabenstellung:

Gegeben ist eine Transistorstufe laut Schaltbild (vgl. Abb. 6). Vom Transistor sind das Ausgangskennlinienfeld ($I_C = f(U_{CE})$, zB als Derive-Listing) und die maximale Verlustleistung P = 45W bekannt.

(I) Bestimmen Sie den Arbeitspunkt der Transistorstufe für eine maximale Leistungsverstärkung mit U_B, R_C als Parameter (Ansteuerung mit einem symm. Wechselspannungssignal).

(II) Die Betriebsspannung U_B betrage 12V. Berechnen Sie R_C unter Berücksichtigung von (I).

(III) Zeichnen Sie den Graph für die Widerstandskennlinie in das DERIVE-Ausgangs-kennlinienfeld und überprüfen Sie die Ergebnisse aus (I).

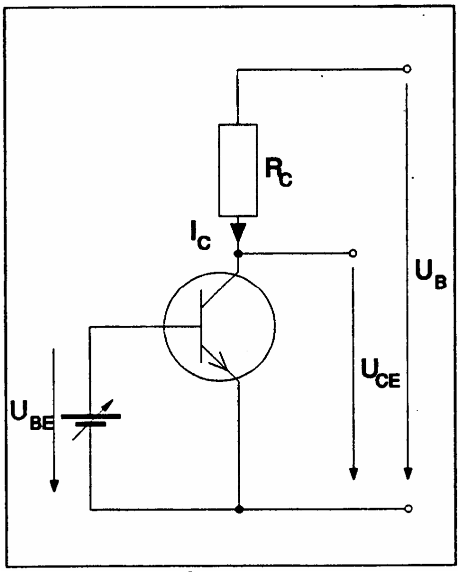


Abb. 6: Transistorschaltung

Problem: Given is a transistor step (fig. 6). We know the family of output characteristics of the transistor and its maximum power loss P = 45W.

(I) Find the operating point for a maximum power amplification.

(II) Operating voltage = 12V. Calculate R_C under consideration of (I).

(III) Plot the characteristic of the resistor to the family of output-characteristics and check the results of (I).

```
#1:   [CaseMode := Sensitive, InputMode := Word]
```

Task (I)
Equation of the hyperbola for the maximum powerloss

```
                  P
#2:   Ip(Uce) := ───
                 Uce
```

The 1st derivative of Ip(Uce) is Ip1

#3: $$Ip1(Uce) := \frac{d}{d\ Uce} Ip(Uce)$$

#4: $$Ip1(Uce) := -\frac{P}{Uce^2}$$

Equation of the resistor line

#5: $$Iw(Uce) := -\frac{1}{Rc} \cdot Uce + \frac{Ub}{Rc}$$

1st derivative of Iw is Iw1

#6: $$Iw1(Uce) := \frac{d}{d\ Uce} Iw(Uce)$$

#7: $$Iw1(Uce) := -\frac{1}{Rc}$$

The coordinate Uce in the osculation point

#8: $$[Ip(Uce) = Iw(Uce),\ Ip1(Uce) = Iw1(Uce)]$$

#9: $$\left[\frac{P}{Uce} = \frac{Ub - Uce}{Rc},\ \frac{P}{Uce^2} = \frac{1}{Rc}\right]$$

Solve this system for Uce (= volatge in the operating point) and Rc

#10: $$\mathrm{SOLVE}\left(\left[\frac{P}{Uce} = \frac{Ub - Uce}{Rc},\ \frac{P}{Uce^2} = \frac{1}{Rc}\right],\ [Uce,\ Rc]\right)$$

#11: $$\left[Uce = \frac{Ub}{2} \wedge Rc = \frac{Ub^2}{4 \cdot P} \wedge Rc \cdot Uce^2 \neq 0 \wedge Rc \cdot Uce \neq 0\right]$$

Substitute for Uce in #2 = (collector) current Ic in the operation point

#12: $$Ip\left(\frac{Ub}{2}\right) = \frac{2 \cdot P}{Ub}$$

Task (II)
Replace Uce in #9 (1st component) by Ub/2, solve for Rc and substitute the given values

#13: $$\frac{P}{Uce} = \frac{Ub - Uce}{Rc}$$

#14: $$\frac{P}{\frac{Ub}{2}} = \frac{Ub - \frac{Ub}{2}}{Rc}$$

#15: $$\frac{2 \cdot P}{Ub} = \frac{Ub}{2 \cdot Rc}$$

#16: $\text{SOLVE}\left(\dfrac{2\cdot P}{Ub} = \dfrac{Ub}{2\cdot Rc},\ Rc\right)$

#17: $Rc = \dfrac{Ub^2}{4\cdot P}$

#18: $Rc = \dfrac{12^2}{4\cdot 45}$

#19: $Rc = 0.8$

Task (III)
Resistorline: replace Rc in #5 by Rc of #17

#20: $Iw(Uce) = \dfrac{Ub - Uce}{Rc}$

#21: $Iw(Uce) = \dfrac{Ub - Uce}{\dfrac{Ub^2}{4\cdot P}}$

#22: $\dfrac{Ub - Uce}{Rc} = \dfrac{4\cdot P\cdot(Ub - Uce)}{Ub^2}$

#23: $\dfrac{4\cdot 45\cdot(12 - Uce)}{12^2}$

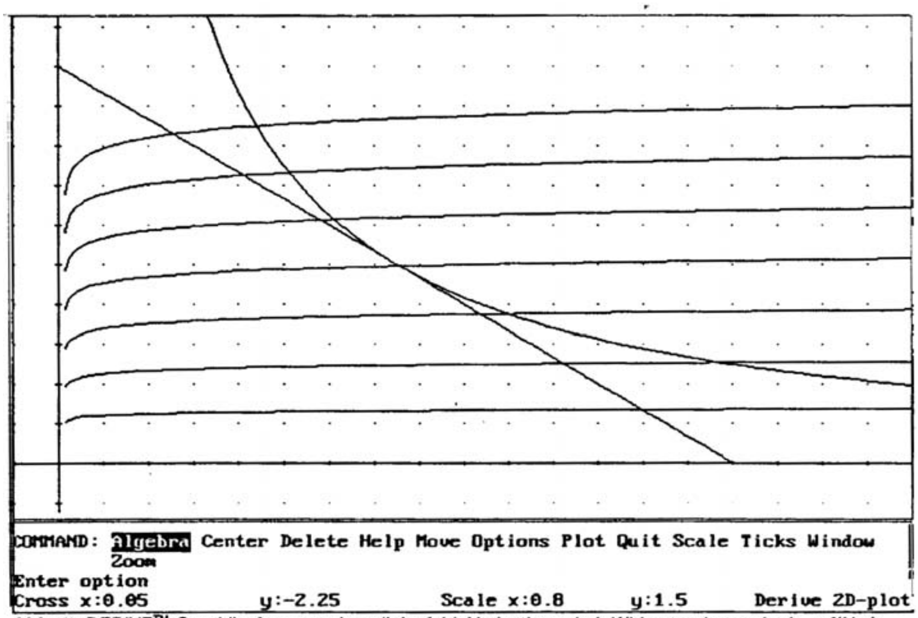


Abb. 7: DERIVE™-Graphik: Ausgangskennlinienfeld, Verlusthyperbel, Widerstandsgerade, $I_c = f(U_{CE})$

Original screen shot from DNL#14

[1] Den fachlichen Hintergrund beschreiben zB [1] und [2].

[2] Eine ausführliche kommentierte Unterrichtssequenz über den belasteten Spannungsteiler in [6].

[3] Die Bezeichnung DERIVE-Listing beinhaltet die Gesamtheit aller Schreibzeilen zur Lösung einer Aufgabenstellung.

[4] Selbstverständlich können bzw. müssen bereits bei der Aufgabenstellunge der Lerngruppe entsprechende angemessene Vereinfachungen wie zB konkrete Zahlenwerte für U, R_P, R_L vorgenommen werden.

[5] Das Cursor-Kreuz im Plot-Fenster weist das Maximum des Graphen der Funktionsgleichung $P_L=f(R_L)$ aus; die Koordinaten des Kreuzes können links unten abgelesen werden, also $x = R_{LM} \approx 0{,}8$ und $y = P_L(R_{LM}) \approx 45$W.

[6] Effektivwert: Der Effektivwert I_{eff} eines Wechselstroms ist der sich aus den Augenblickswerten I(t) ergebende Dauerwert, der in einem ohmschen Widerstand die gleiche Wärmearbeit erzeugt, wie ein Gleichstrom der gleichen Höhe (aus [4], Kapitel 2, S 33).

will be continued in DNL#15

Josef Lechner realized J.H.Conway´s famous Game of Life for Derive. I copy the first lines of the 1994 DERIVE code together with one result. In the recent Derive version we can use the Derive Unicode font, which shows the “living cells” in a better way. Josef

Conway´s CAME of LIFE

Josef Lechner, Viehdorf, Austria

(In 1994 the SUB-Operator was not available. We had to enter `ELEMENT(a,i,j)` instead of `a↓i↓j.)`

#1: LIFE – A Game from J.H.Conway

#2: An attempt with DERIVE (Josef Lechner)

#3: We have three creatures in our miniworld

#4: $\text{world} := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

```
      CREA(a, i, j) :=
        If a↓i↓j = 1
#5:        "Yes"
           "No"
```

#6: CREA(world, 2, 3) = Yes

#7: CREA(world, 1, 1) = No

#31: $t := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

#32: EVOL(t, 8)

#33: $$\left[\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}\right]$$

#34: took 114.8 sec in 1994, takes now 0.55 sec!

You can find numerous great websites dealing with the Game of Life:

```
http://www.radicaleye.com/lifepage/
http://www.ibiblio.org/lifepatterns/
http://de.wikipedia.org/wiki/Conways Spiel des Lebens
```

1 and 0 (living cell and free cell) are replaced by two Unicode symbols ■ and ·. You can get these two symbols by typing 25A0 Alt-C and 2027 Alt-C and formatting the characters as DERIVE Unicode in a Word document. In Derive you can produce them by calling

```
CODES_TO_NAME([127, 50, 53, 97, 48, 127, 50, 48, 50, 55]) = ■·
```

Now you can copy and paste the two symbols. For easier editing the various miniworlds I am using l and f for living and free cells.

```
#1:   CODES_TO_NAME([127, 50, 53, 97, 48, 127, 50, 48, 50, 55]) = ■·
```

Life - A Game from J.H.Conway
An Attempt with DERIVE (Josef Lechner)

We have three creatures - living cells - in our miniworld

```
#2:   [l := ■, f := ·]

               ┌ f  f  f  f  f ┐
               │ f  f  l  f  f │
#3:   world := │ f  f  l  f  f │
               │ f  f  l  f  f │
               └ f  f  f  f  f ┘

               ┌ ·  ·  ·  ·  · ┐
               │ ·  ·  ■  ·  · │
#4:   world := │ ·  ·  ■  ·  · │
               │ ·  ·  ■  ·  · │
               └ ·  ·  ·  ·  · ┘
```

Some functions in the miniworld:

```
      CREA(a, i, j) :=
        If a↓i↓j = ■
#5:        "Yes"
           "No"
           "No"

#6:   CREA(world, 2, 3) = Yes

#7:   CREA(world, 1, 1) = No

      num(a, i, j) :=
        If a↓i↓j = ■
#8:        1
           0
           0
```

NB(a,i,j) looks for existing neighbours

```
      NB(a, i, j) :=
        If i > 0 ∧ (j > 0 ∧ (i ≤ DIMENSION(a↓1) ∧ j ≤ DIMENSION(a)))
#9:        num(a, i, j)
           0
```

N(a,i,j) counts the number of neighbours

```
#10:  N(a, i, j) := NB(a, i - 1, j - 1) + NB(a, i - 1, j) + NB(a, i - 1, j + 1) + NB(a,
        i, j - 1) + NB(a, i, j + 1) + NB(a, i + 1, j - 1) + NB(a, i + 1, j) + NB(a, i +
        1, j + 1)
```

There are three functions for the game:

```
       LIFE(a, i, j) :=
         If num(a, i, j) = 1 ∧ (N(a, i, j) = 2 ∨ N(a, i, j) = 3)
#11:        1
            f

       DEATH(a, i, j) :=
         If num(a, i, j) = 1 ∧ (N(a, i, j) > 3 ∨ N(a, i, j) < 2)
#12:        1
            f

       BIRTH(a, i, j) :=
         If num(a, i, j) = 0 ∧ N(a, i, j) = 3
#13:        1
            f
```

NG(a,i,j) determins whether the cell is living in the next generation or not.

```
       NG(a, i, j) :=
         If num(a, i, j) = 1 ∧ (N(a, i, j) = 2 ∨ N(a, i, j) = 3)
            1
#14:        If num(a, i, j) = 0 ∧ N(a, i, j) = 3
               1
               f
```

NG1(a) finds the full next generation.

```
#15:   NG1(a) := VECTOR(VECTOR(NG(a, j, i), i, 1, 3), j, 1, 3)
```

#16: Example 1:

#17: $s := \begin{bmatrix} f & 1 & f \\ f & 1 & f \\ f & 1 & f \end{bmatrix}$

#18: $s := \begin{bmatrix} \cdot & \blacksquare & \cdot \\ \cdot & \blacksquare & \cdot \\ \cdot & \blacksquare & \cdot \end{bmatrix}$

#19: $NG1(s) = \begin{bmatrix} \cdot & \cdot & \cdot \\ \blacksquare & \blacksquare & \blacksquare \\ \cdot & \cdot & \cdot \end{bmatrix}$

We want to observe the evolution of the population of miniworld s

#20: $NG2(a) := VECTOR(VECTOR(NG(a, j, i), i, 1, DIMENSION(a_1)), j, 1, DIMENSION(a))$

```
#21:   EVOL(world, steps) := ITERATES(NG2(m), m, world, steps)
```

#22: $EVOL(s, 3) = \left[\begin{bmatrix} \cdot & \blacksquare & \cdot \\ \cdot & \blacksquare & \cdot \\ \cdot & \blacksquare & \cdot \end{bmatrix}, \begin{bmatrix} \cdot & \cdot & \cdot \\ \blacksquare & \blacksquare & \blacksquare \\ \cdot & \cdot & \cdot \end{bmatrix}, \begin{bmatrix} \cdot & \blacksquare & \cdot \\ \cdot & \blacksquare & \cdot \\ \cdot & \blacksquare & \cdot \end{bmatrix}, \begin{bmatrix} \cdot & \cdot & \cdot \\ \blacksquare & \blacksquare & \blacksquare \\ \cdot & \cdot & \cdot \end{bmatrix} \right]$

Example 2:

#23: $t := \begin{bmatrix} f & f & f & f & f \\ f & f & 1 & f & f \\ f & 1 & 1 & 1 & f \\ f & f & f & f & f \\ f & f & f & f & f \end{bmatrix}$

#24: EVOL(t, 8)

#25:

$$\left[\begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot \\ \blacksquare&\cdot&\cdot&\cdot&\blacksquare \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \blacksquare&\blacksquare&\cdot&\blacksquare&\blacksquare \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \blacksquare&\cdot&\cdot&\cdot&\blacksquare \\ \blacksquare&\cdot&\cdot&\cdot&\blacksquare \\ \blacksquare&\cdot&\cdot&\cdot&\blacksquare \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \blacksquare&\cdot&\blacksquare&\cdot&\blacksquare \\ \blacksquare&\blacksquare&\cdot&\blacksquare&\blacksquare \\ \blacksquare&\cdot&\blacksquare&\cdot&\blacksquare \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \end{bmatrix}\right]$$

This took 115 seconds in 1994, now it needs only one second for evaluation. A famous pattern is the following – see first one gliding step and then starting the GLIDER from left above gliding gently down:

This is the famous GLIDER

#26: $g1 := \begin{bmatrix} f & f & f & f & f & f \\ f & f & 1 & f & f & f \\ f & f & f & 1 & f & f \\ f & 1 & 1 & 1 & f & f \\ f & f & f & f & f & f \\ f & f & f & f & f & f \end{bmatrix}$

#27: EVOL(g1, 4) =

$$\left[\begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\blacksquare&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\blacksquare&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\blacksquare&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\blacksquare&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}, \begin{bmatrix} \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \\ \cdot&\cdot&\cdot&\blacksquare&\cdot&\cdot \\ \cdot&\cdot&\cdot&\cdot&\blacksquare&\cdot \\ \cdot&\cdot&\blacksquare&\blacksquare&\blacksquare&\cdot \\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot \end{bmatrix}\right]$$

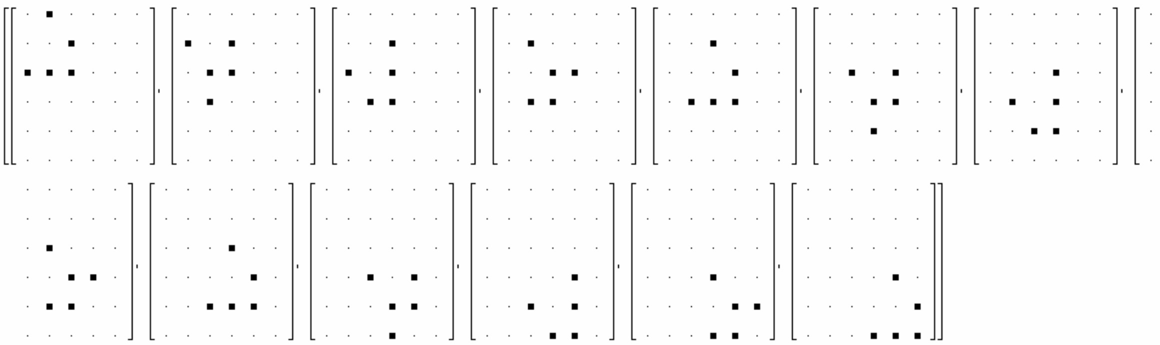
I allocate the glider in the left upper corner of our 6 by 6 miniworld and then start the gliding process:

You can experiment with various patterns starting in a larger world – see the “globe”.

#29: $g12 := \begin{bmatrix} f & 1 & f & f & f & f \\ f & f & 1 & f & f & f \\ 1 & 1 & 1 & f & f & f \\ f & f & f & f & f & f \\ f & f & f & f & f & f \\ f & f & f & f & f & f \end{bmatrix}$

#30: EVOL(g12, 12)

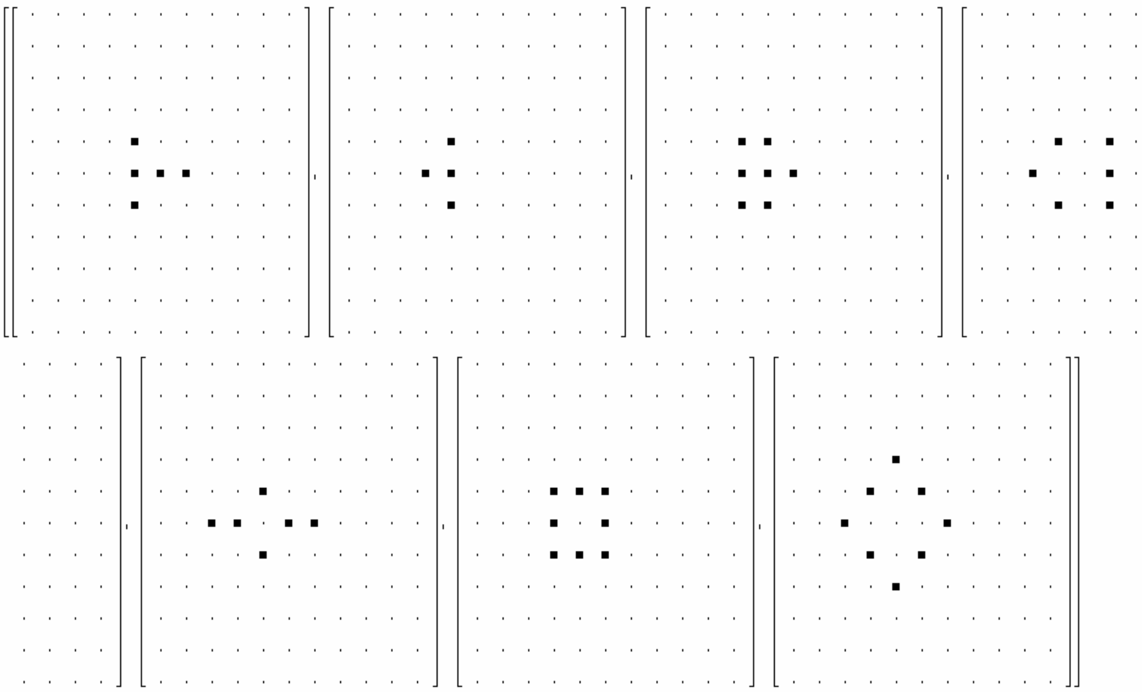
#31:



$$\#32: \quad \text{globe} := \begin{bmatrix} f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & 1 & f & f & f & f & f & f \\ f & f & f & f & 1 & 1 & 1 & f & f & f & f \\ f & f & f & f & 1 & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \\ f & f & f & f & f & f & f & f & f & f & f \end{bmatrix}$$

#33: EVOL(globe, 6)

#34:



As Josef Lechner wrote in a private communication he used Conway´s Game of Life in Information Technology as an example for programming with two-dimensioned arrays.

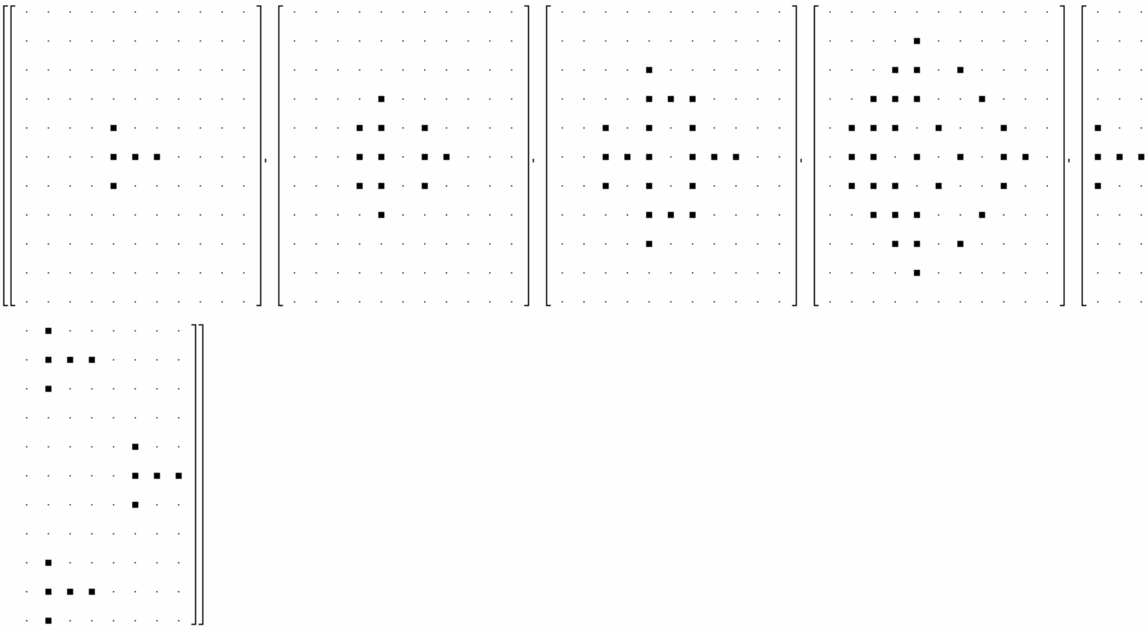
I remembered that I should have a paper on the Game of Life among my many collected articles from various newspapers and journals and finally I found an article from Scientific American 1984 where another cellular automat was presented:

The Fredkin-Automat:

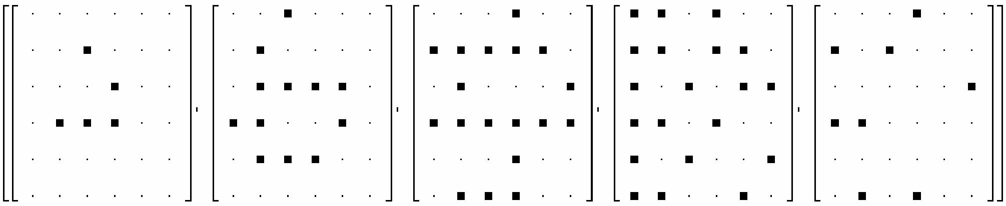
Only the neighbour cells in horizontal and vertical direction are responsible for the next generation.
If the number of "living" neighbour cells is even then the actual cell will die,
if the number of is odd, then the cell remains living or comes into life.

```
#35:  NF(a, i, j) := NB(a, i - 1, j) + NB(a, i, j - 1) + NB(a, i, j + 1) + NB(a, i + 1, j)

      NFG(a, i, j) :=
        If MOD(NF(a, i, j), 2) = 1
#36:       L
           F

#37:  NFG2(a, i, j) := VECTOR(VECTOR(NFG(a, j, i), i, 1, DIMENSION(ELEMENT(a))), j, 1, DIMENSION(a))

#38:  EVOLF(world, steps) := ITERATES(NFG2(m), m, world, steps)

#39:  EVOLF(globe, 4)
```

#40:



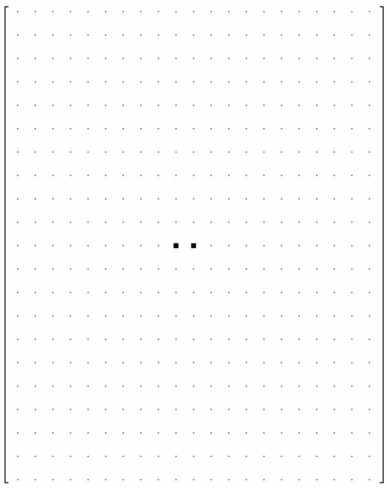
#41: EVOLF(gl, 4) =



Have a 21 by 21 globe with two living cells and follow how this world will develop.

The two Josefs wish much fun creating your own patterns of life.

#57: globe_12 :=



http://www.univ-lemans.fr/enseignements/physique/02/divers/fredkin.html

Titbits from Algebra and Number Theory (2)

by Johann Wiesenbauer, Vienna, Austria

Hi, out there! Josef Böhm did a fabulous job when acting as my ghost writer last time, but here I am myself -eius ipsissimus, as it were. Before going to new stuff, I'd like to add a few comments to the DERIVE listing in DNL #13 (revised version) on page 31. Its main purpose was to show that Euclid's algorithm for the computation of the greatest common divisor of two natural numbers a and b (in symbols gcd(a,b)) can be easily implemented in DERIVE - even in its extended form which is often used to solve linear congruences, as shown below.

For didactic reasons I started with the function `euclid(a,b)` which yields the divisions of Euclid's algorithm for a and b in a neatly arranged form. In (5) you can see its output for the values a = = 1234567891 and b = 234567891. In particular, the last nonvanishing remainder, which is 1 in this case, is the gcd(a,b). (Here and hereafter, if necessary , consult any textbook on Number Theory for the mathematical background, for example [1].)

What about the number of steps needed for Euclid's algorithm? It is easy to see that it is strongly connected with the magnitude of the quotients which are all 1 in the worst case as e.g. in (19). Therefore you get an upper bound for the number of steps needed to calculate the gcd(a,b) for $a,b \le s$ by counting the number of steps of the algorithm of Euklid for the largest adjacent pair of the Fibonacci sequence

0,1,1,2,3,5,8,13,21,34,...

below s.

```
euclid(fib1(14), fib1(13))
```

$$\begin{bmatrix} 377 & = & 1 & \cdot & 233 & + & 144 \\ 233 & = & 1 & \cdot & 144 & + & 89 \\ 144 & = & 1 & \cdot & 89 & + & 55 \\ 89 & = & 1 & \cdot & 55 & + & 34 \\ 55 & = & 1 & \cdot & 34 & + & 21 \\ 34 & = & 1 & \cdot & 21 & + & 13 \\ 21 & = & 1 & \cdot & 13 & + & 8 \\ 13 & = & 1 & \cdot & 8 & + & 5 \\ 8 & = & 1 & \cdot & 5 & + & 3 \\ 5 & = & 1 & \cdot & 3 & + & 2 \\ 3 & = & 1 & \cdot & 2 & + & 1 \\ 2 & = & 2 & \cdot & 1 & & \end{bmatrix}$$

Exactly this is done in (9) – (26) for $s = 10^{100}$. (Note that the disappointing calculation time in (26) is by no means representative for actual applications of Euclid's algorithm since it is caused almost totally by the unnecessary storing of the whole tableau! In 1994 it needed 65 sec, now we need only 0.05 sec.) In the following the result is derived in another way.

For this purpose the explicit formula for Fibonacci numbers is needed, namely

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$

Since

$$\left(\frac{1-\sqrt{5}}{2}\right) \approx 0.618 < 1 \text{ and } \left(\frac{1+\sqrt{5}}{2}\right)^2 \approx \sqrt{5}$$

this yields

$$F_n \approx \left(\frac{1+\sqrt{5}}{2}\right)^{n-2}.$$

On the other hand, $n-2$ is exactly the number of steps for the calculation of the gcd(F_n ,F_{n-1}) (see above). Therefore its value is about ${}_\lambda\log s$, where $\lambda = \frac{1+\sqrt{5}}{2}$ is the number known as the "golden ratio" (cp. (24), (26)). In particular, it depends only linearly on the number of digits of s which is the best we could expect!

On the other hand, the number of steps you get on average are not much better than those in the worst case (see below). It can be shown that in some sense the most likely value is about

$$\frac{12 \ln 2}{\pi^2} \ln s \quad \text{(again this } \pi!) \text{ which is about 40\% of the maximum.}$$

```
DIM(euclid(RANDOM(10^100), RANDOM(10^100))) = 180

VECTOR(DIM(euclid(RANDOM(10^100), RANDOM(10^100))), k, 5) = [198, 169, 178, 205, 191]
```

$$\frac{12 \cdot LN(2)}{\pi^2} \cdot LN(10^{100}) = \left(\frac{12}{\pi^2} \cdot 100\right) \cdot LN(2) \cdot 100 \cdot LN(5)$$

$$\frac{12 \cdot LN(2)}{\pi^2} \cdot LN(10^{100}) = 194.0540228$$

As for the EEA applied to a and b, each line

$$[r_{i-1} \quad r_i \quad x_{i-1} \quad x_i \quad y_{i-1} \quad y_i]$$

of the output (cp. 31)) has the following meaning

$$r_i = x_{i-1}a + y_{i-1}b, \quad r_i = x_i a + y_i b.$$

In the last row, apart from r_n = gcd(a,b) (2. entry) also the coefficients x_n and y_n (4. and 6. entry, respectively) can be seen. Again, the whole tableau is presented here only for didactical reasons, usually we are only interested in a subset of these three numbers! For example, we could use the EEA to compute the inverse of a mod m, in case gcd(a,m)=1:

INV(a,m) ≔ IF(GCD(a,m) = 1,
MOD(ELEMENT(ITERATE(IF(MOD(ELEMENT(x,1), ELEMENT(x,2)) = 0, x,
[ELEMENT(x,2), MOD(ELEMENT(x,1), ELEMENT(x,2)), ELEMENT(x,4),
ELEMENT(x,3) – FLOOR(ELEMENT(x,1)/ELEMENT(x,2))·ELEMENT(x,4)]), x,
[a,m,1,0]),4),m))

For those of you who are still here I am going to unpack now a little present as a reward: A truly powerful powermod-function! One day in the not too distant future the programmers of DERIVE will be merciful to those of us who are in desperate need of a fast and efficient built-in routine to compute large powers a^n mod m, but until then take this one:

PM(a,n,m) ≔ IF(n < 0, PM(INV(a,m), –n,m),
ELEMENT(ITERATE([IF(MOD(ELEMENT(x,3),2) = 1,
MOD(ELEMENT(x,1)·ELEMENT(x,2), m), ELEMENT(x,1)), IF(ELEMENT(x,3) > 0,
MOD(ELEMENT(x,2)^2,m), ELEMENT(x,2)), FLOOR(ELEMENT(x,3),2)],x,
[1,a,n]),1))

There are a lot of useful applications for this utility function. For instance, it can be used to emulate RSA public-key cryprosystem with realistic numbers, as it was shown in [2]. Furthermore many primality tests require the computation of large powers a^n mod m. Let me conclude with some examples of this kind:

Some examples of primality testing using the subsequent powermod-function PM

```
      INV(a, m) :=
#1:     If GCD(a, m) = 1
           MOD(ELEMENT(ITERATE(IF(MOD(ELEMENT(x, 1), ELEMENT(x, 2)) = 0


        2)), ELEMENT(x, 4), ELEMENT(x, 3) - FLOOR(ELEMENT(x, 1)/ELEMENT

      PM(a, n, m) :=
        If n < 0
#2:        PM(INV(a, m), -n, m)
           ELEMENT(ITERATE([IF(MOD(ELEMENT(x, 3), 2) = 1, MOD(ELEMENT(x


        IF(ELEMENT(x, 3) > 0, MOD(ELEMENT(x, 2)^2, m), ELEMENT(x, 2)), I
```

10^100+9 has no prime divisor < 1000, as it is shown by

```
      PRIM(n) :=
        If NEXT_PRIME(n - 1) = n
#3:        n
           1
```

$$\#4: \quad \mathrm{GCD}\left(10^{100} + 9, \prod_{n=2}^{1000} \mathrm{PRIM}(n)\right) = 1$$

Even so, it is not a prime by Fermat´s theorem:

$$\#5: \quad \mathrm{IF}(\mathrm{PM}(2, 10^{100} + 8, 10^{100} + 9) = 1, 1, 0) = 0$$

needs 0.031 sec - in 1994 2.0 sec.

And with the built-in NEXT_PRIME function ...

$$\#6: \quad \mathrm{IF}(\mathrm{NEXT\_PRIME}(10^{100} + 8) = 10^{100} + 9, 1, 0) = 0$$

0.041 sec against 3.7 sec.

On the other hand ...

$$\#7: \quad \mathrm{NEXT\_PRIME}(10^{100}) - 10^{100} = 267$$

$$\#8: \quad \mathrm{IF}(\mathrm{PM}(2, 10^{100} + 266, 10^{100} + 267) = 1, 1, 0) = 1$$

It passes also the Miller-Rabin test for base 2:

$$\#9: \quad \mathrm{PM}\left(2, \frac{10^{100} + 266}{2}, 10^{100} + 267\right) - 10^{100} - 267 = -1$$

Johann Wiesenbauer provided a DERIVE 6 file for his Titbits 2. You can download it together with all other files connected with contributions of DNL#13.

You can find here an addition to the file of the last DNL.

Some new functions and results. (ed.)

TEILER(n,s) returns the smallest non trivial divisor of $n < s$ if there is one, otherwise it returns 1.

PRIM(n) returns n, if n is a prime number, otherwise 1.

PRODUCT(PRIME(k),k,2,1000) is the product of all prime numbers ≤ 1000.

FERMAT(v,a,b) returns a list of all elements from v, which satisfy the so called "Little Fermat" for base a after adding the base of number b.

INV and PM must be preloaded as a Utility file.

```
#1:  INV(1234567, 234567) = 9640

#2:  INV(1234567, 254) = ?

#3:  GCD(1234567, 254) = 127

#4:  TEILER(n, s) := ITERATE(IF(t > s, 1, IF(MOD(n, t) = 0, t, NEXT_PRIME(t))), t, 2)
```

#5: $\text{list1} := \left(\text{ITERATE}\left(\left[\text{IF}\left(\text{TEILER}\left(10^{100} + x_2,\ 1000\right) = 1,\ \text{APPEND}\left(x_1,\ \left[x_2\right]\right),\ x_1\right),\ x_2 + 2\right],\ x,\ [[],\ 1],\ 499\right)\right)_1$

```
#6:  list1 := [9, 37, 39, 61, 63, 99, 103, 111, 123, 133, 151, 159, 177, 207, 211, 223, 229, 253, 261, 267, 271, 273, 277,
       289, 291, 321, 331, 363, 369, 391, 411, 433, 453, 463, 499, 511, 529, 531, 547, 559, 579, 583, 589, 597, 609, 613,
       627, 639, 643, 649, 663, 667, 669, 679, 691, 697, 711, 721, 729, 733, 741, 753, 757, 763, 771, 781, 783, 793, 811,
       817, 831, 849, 853, 867, 889, 919, 921, 937, 939, 949, 957, 973, 981, 991, 993]

#7:  DIM(list1) = 85

     prim(n) :=
       If NEXT_PRIME(n - 1) = n
#8:       n
          1

#9:  prim(100) = 1

#10: prim(101) = 101
```

#11: $\prod_{k=2}^{1000} \text{prim}(k) = 1$

#12: $\prod_{k=2}^{102} \text{prim}(k) = 2328623643584973609000633168805073630 70$

#13: $\prod_{k=2}^{103} \text{prim}(k) = 1$

Derive has problems with upper bounds > 102!! This does not work!! Let´s try another way:

```
#14: VECTOR(prim(k), k, 50) = [1, 2, 3, 1, 5, 1, 7, 1, 1, 1, 11, 1, 13, 1, 1, 1, 17, 1, 19, 1, 1, 1, 23, 1, 1, 1, 1, 1,
       29, 1, 31, 1, 1, 1, 1, 1, 37, 1, 1, 1, 41, 1, 43, 1, 1, 1, 47, 1, 1, 1]

#15: p1000 := Π(k, k, VECTOR(prim(k), k, 1000))

#16: p1000 :=
```

Advice from Johann Wiesenbauer: take 1 as last argument!

```
#18: Π(prim(k), k, 2, 1000, 1) =

       1959034064499908343126250819820638104612397239058936822388260532896866631637987066185195164878948232159622955911 54~
       3601914918952972521526672829228299085264902336273139240401793914201095826139363495947148375719672167224341006711 85~
       1622766113313519248884898991489215718830867989687513743951933890396809490554975038640710603383658666068353920101 16~
       359179000399044950652032997495429859931346698148053184740805812078911259 10
```

#19: $\text{list2} := \left(\text{ITERATE}\left(\left[\text{IF}\left(\text{GCD}\left(10^{100} + x_2,\ \text{p1000}\right) = 1,\ \text{APPEND}\left(x_1,\ \left[x_2\right]\right),\ x_1\right),\ x_2 + 2\right],\ x,\ [[],\ 1],\ 499\right)\right)_1$

#20: list2 := [9, 37, 39, 61, 63, 99, 103, 111, 123, 133, 151, 159, 177, 207, 211, 223, 229, 253, 261, 267, 271, 273, 277, 289, 291, 321, 331, 363, 369, 391, 411, 433, 453, 463, 499, 511, 529, 531, 547, 559, 579, 583, 589, 597, 609, 613, 627, 639, 643, 649, 663, 667, 669, 679, 691, 697, 711, 721, 729, 733, 741, 753, 757, 763, 771, 781, 783, 793, 811, 817, 831, 849, 853, 867, 889, 919, 921, 937, 939, 949, 957, 973, 981, 991, 993]

#21: DIM(list2) = 85

#22: p10000 := ∏(k, k, VECTOR(prim(k), k, 10000))

#23: p10000 :=

```
594906795799986358685640222425034456596803224406793279383097039161404056383834347448332205554521305581407728940461697007471864099413~
175506658216194791400987432883351062728565339787197696777947372874194618999224566142568267097271335158855818786148462982514280965124~
955905459895362397392852905098708607324351097940454916405610117757446172184260117932833304978784911379996737607407179049797301069941~
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```

0.5 sec!!

#24: $\text{p10000} := 5.949067957 \cdot 10^{4297}$

#25: $\text{fermat}(v,\ a,\ b) := \left(\text{ITERATE}\left(\left[\text{IF}\left(\text{PM}\left(a,\ b + v_{x\downarrow 2} - 1,\ b + v_{x\downarrow 2}\right) = 1,\ \text{APPEND}\left(x_1,\ \left[v_{x\downarrow 2}\right]\right),\ x_1\right),\ x_2 + 1\right],\ x,\ [[],\ 1],\ \text{DIM}(v)\right)\right)_1$

#26: $\text{fermat}(\text{list2},\ 2,\ 10^{100}) = [267,\ 949]$

Dr. Wiesenbauer´s email address, if anyone wants to contact him is

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[1] Nöbauer W. – Wiesenbauer J., Zahlentheorie, Prugg Verlag, Eisenstadt 1981

[2] Heugl H. – Kutzler B. (ed.), DERIVE in Education – Opportunities and Strategies, Chartwell-Bratt Ltd,
1994, p 51 – 61

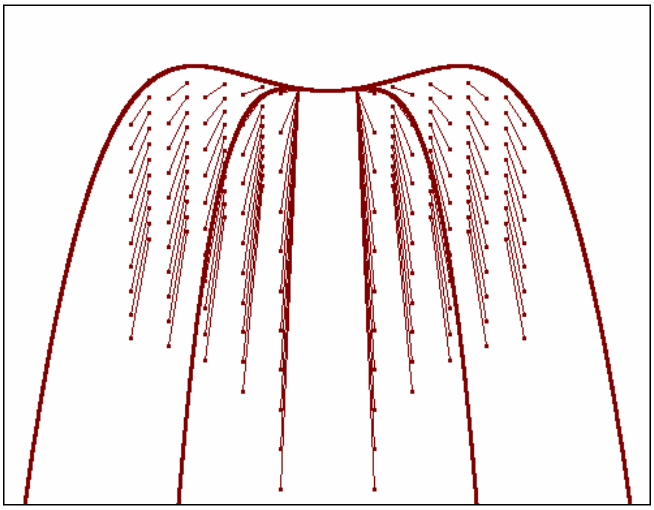
The Girl with Long Hair

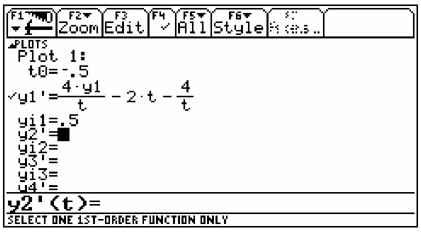
J.M. Cardia Lopes, Porto, Portugal

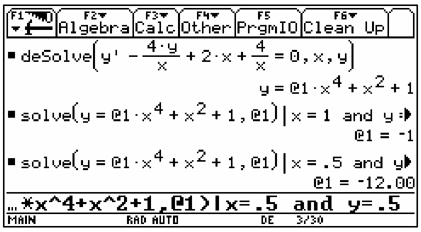
Sometimes DERIVE gives unexpected but funny pictures with PLOT. Please see what DERIVE returns when we try to display the direction field for the equation together with two particular solutions:

$$y' - \frac{4y}{x} + 2x + \frac{4}{x} = 0$$

$$\mathrm{DIRECTION\_FIELD}\left(\frac{4\cdot y}{x} - 2\cdot x - \frac{4}{x},\ x,\ -1,\ 1,\ 10,\ y,\ -1,\ 1,\ 10\right)$$







#2: $\mathrm{DSOLVE1}\left(-\frac{4\cdot y}{x} + 2\cdot x + \frac{4}{x},\ 1,\ x,\ y,\ 1,\ 1\right) = \left(\frac{x^4 - x^2 + y - 1}{x^4} = 0\right)$

#3: $\mathrm{SOLVE}\left(\frac{x^4 - x^2 + y - 1}{x^4} = 0,\ y\right) = (y = -x^4 + x^2 + 1)$

#4: $\mathrm{TABLE}(-x^4 + x^2 + 1,\ x,\ -3,\ 3,\ 0.001)$

#5: $\mathrm{DSOLVE1}\left(-\frac{4\cdot y}{x} + 2\cdot x + \frac{4}{x},\ 1,\ x,\ y,\ 0.5,\ 0.5\right) = \left(\frac{12\cdot x^4 - x^2 + y - 1}{x^4} = 0\right)$

#6: $\mathrm{SOLVE}\left(\frac{12\cdot x^4 - x^2 + y - 1}{x^4} = 0,\ y\right) = (y = -12\cdot x^4 + x^2 + 1)$

#7: $\mathrm{TABLE}(-12\cdot x^4 + x^2 + 1,\ x,\ -3,\ 3,\ 0.001)$