THE DERIVE - NEWSLETTER #21

ISSN 1990-7079

THE BULLETIN OF THE

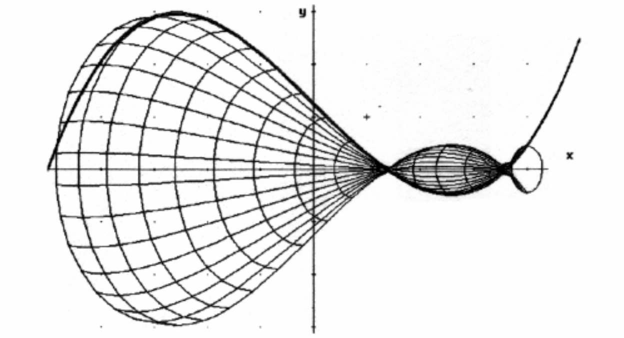
DERIVE®

USER GROUP

Contents:

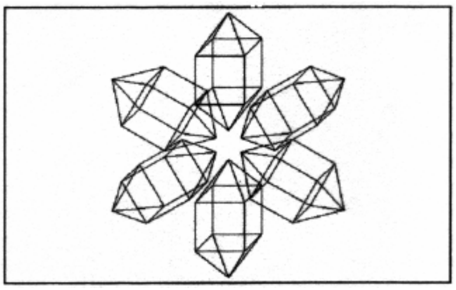
revised Version 2009 January 1996

[1] **Analyse Bekijken en Begrijpen met *DERIVE*,** Jan Vermeylen & Marcel Dams
Uitgeverij Rhombus, Kapellen (Belgium), 1996, 82 pages + diskette.
(*Although this is a Dutch book, it an easily be understood by non Dutch readers. It contains a lot of ideas to forward the concepts of calculus. The figure shows one example. Josef*)



[2] **Matematica con il PC, Introduzione a DERIVE,** Bernhard Kutzler,
Media Direct, Bassano del Grappa, (VI), 1995, 159 pages.

[3] **Neue Medien im Mathematikunterricht, *DERIVE* mehr als ein Assistent**, H. J. Kayser,
Verlag für Schule und Weiterbildung, Druck Verlag Kettler, 1995, ISBN 3 8165 1782 X, 187 pages.



[4] **Mastering the TI-92: Explorations from Algebra through Calculus**, N. Rich, J. Rose, L. Gilligan,
GILMAR, Cincinnati, OH, 1996, 208 pages.

[5] **Symbolrechner TI-92,** Bernhard Kutzler, Addison Wesley, ISBN 3 89319 952 7, 192 pages.

[6] **Atlas mathematischer Bilder,** Leo Klingen, Addison Wesley, ISBN 3 89319 947 0, 224 pages + diskette.

Exchange for DERIVE Teaching materials in the DNL

The wheel has not to be invented twice.

Börse für DERIVE Unterrichts-materialien im DNL

Das Rad muss nicht zweimal erfunden werden.

I can offer:

Binomial Theorem, GCD & LCM, System of Coordinates (in English and in German as well), Modeling Word Problems with *DERIVE*, SET.EXE, MENGE.EXE

International DERIVE and TI-92 Conference
Computeralgebra and Matheducation
Schloß Birlinghoven, Bonn, 2. - 6. Juli 1996

7 Keynote Lectures (M.Artigue, J.Berry, B.Buchberger, W.Herget, W.Koepf, J.Palmiter, B.Waits)
57 Lectures (from Carmen Arriero to Nurit Zehavi)
14 DERIVE Workshops(from Josef Böhm to Helen Surovyatkina)
12 TI-92 Workshops (from Roger Brown to Thomas Weth)
Banquet, Choice of 9 coach tours on Thursday, Family programme for Wednesday and Friday.
Information: Bärbel Barzel, Heinrich-Könn-Str.225, D-40625 Düsseldorf

Liebe DUG Mitglieder,

Ungebrochen sind der Ideenreichtum und die Schaffenskraft vieler DERIVE-Freunde aus aller Welt auch im 6. Jahr des Bestehens der DUG, zu dem ich Sie alle recht herzlich begrüße. Der Umfang der diesmal angebotenen (Sport-)Artikel machte es nötig, die versprochene Konstruktion des 17-Ecks zu den letzten Titbits auf den DNL#22 zu verschieben. Außerdem ließ mir J. Wiesenbauer noch eine zweite Konstruktionsvorschrift zukommen, die ich gerne vergleichsweise darstellen mochte. Leider muss auch die nächste Folge des Kurvenlexikons auf DNL#22 warten, dafür auch mit dem TI-92 Cabri!

In diesem Newsletter finden Sie zwei Neuerungen. "Carl und Marvin's Laboratory" ist eine Sammlung von nicht alltäglichen DERIVE Aktivitäten und es freut mich, dass Carl sofort seine Zustimmung zum Abdruck gegeben hat. "What an honor to have a section entitled 'Carl and Marvin's Laboratory '. I don't know if I will be able to keep my head in my hat, ", Ich denke, Du solltest Deinen Kopf dort behalten, wo er ist, Pat braucht sicher keinen kopflosen Carl. Und wo würdest Du Deine Tauchermaske und den Schnorchel fest machen. (Wir freuen uns auf ein Wiedersehen in Bonn!)

Zweitens werden Sie eine neuerliche Erweiterung des DNL bemerken können. Vier zusätzliche Seiten sind vor allem dem CAS im TI-92 gewidmet. Natürlich sollen auch die weiteren TI-92 Möglichkeiten in Zukunft behandelt werden. Dank gilt Bert Waits und Bernhard Kutzler, die mitgeholfen haben, die erste TI-92 Sektion zu gestalten. Ich erwarte gerne Beiträge und Anfragen zum TI-92.

Eine meiner persönlichen Zielsetzungen für die DUG im Jahr 1996 ist es, das TI-92 DERIVE in den DNL zu integrieren. Damit habe ich ein zweites Ziel verknüpft: die Anzahl unserer Mitglieder sollte von mehr als 500 auf annähernd 600 angehoben werden können. Wenn wir alle ein wenig mithelfen, könnte das gelingen.

Bitte beachten Siebitte die Materialienbörse auf der Infoseite.

Ich kann Ihnen für 1996 einige sehr interessante Beiträge im DNL versprechen und freue mich mit Ihnen auf ein spannendes DERIVE (for WINDOWS & TI-92) Jahr 1996. Außerdem hoffe ich, viele von Ihnen im Juli in Bonn zu treffen.

Mit den besten Grüßen, Ihr

Dear DUG Members,

Unbroken is the ingenuity and the creative power of so many DERIVE friends from all over the world even in the 6th year of the DUG's existence. I would like to welcome you all, old and new members from all continents. The volume of the (sportive) contributions published in this issue makes it necessary to show the construction of the 17-edge in the next DNL. In addition J. Wiesenbauer has sent me another instruction for drawing this figure. I will compare both constructions next time. Unfortunately the next part of Th. Weth's Lexicon of Curves has to wait for DNL#22, but then with the TI-92 Cabri!

You will find two innovations in this Newsletter. "Carl and Marvin's Laboratory" is a collection of some out-of-the-ordinary DERIVE activities and I am glad that Cart agreed the labs to be published in the DNL. "What an honor to have a section entitled 'Cart and Marvin's Laboratory'. I don't know if I will be able to keep my head in my hat!" I think you should keep your head just where it is. Pat will have no use with a headless Carl. And where would you fix your diving mask and snorkel? (We are looking forward to meeting you both in Bonn!)

You will also note another expansion of the DNL. Four additional pages are dedicated to the TI-92. Of course, we will deal with the other features of this magic machine in the future, too. My special thank goes to Bert Waits and Bernhard Kutzler who helped to produce the first TI-92 section. I will encourage you to submit papers and requests concerning the TI-92.

One of my personal goals for the DUG in 1996 is to integrate TI-DERIVE into the DNL. This is closely associated with the second goal to increase the number of our members from more than 500 up to approximately 600. I think, we may master this ambitious task.

Please take notice of our Materials Exchange on the Information-page.

For 1996 I can promise a couple of very interesting contributions and together with you I am looking forward to having a thrilling DERIVE (for WINDOWS & TI-92) year 1996. I hope to meet many of you in July in Bonn.

Sincerely yours

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE User Group*. It is published at least four times a year with a contents of 30 pages minimum. The goals of the *D-N-L* are to enable the exchange of experiences made with *DERIVE* as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.
We include a section dealing with the use of the TI-92.

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Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *D-N-L*. It must be said, though, that non-English articles are very welcome nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *D-N-L*. The more contributions you will send, the more lively and richer in contents the *DERIVE Newsletter* will be.

Preview: (Contributions for the next issues):

Graphic Integration, Probability Theory, Linear Programming, Böhm, A
LOGO in DERIVE, Lechner, A
DREIECK.MTH, Wadsack, A
IMP Logo and Misguided Missiles, Sawada, HAWAII
3D Geometry, Reichel, A
Parallel- and Central Projection, Böhm, A
Vector and Vector Indices Sorting, Biryukov, RUS
Algebra at A-Level, Goldstein, UK
Tilgung fremderregter Schwingungen, Klingen, GER
Utility for Complex Dynamic Systems, Lechner, A
Some notes on DERIVE 2.6 functions and limits, Speck, NZL
Linear Mappings and Computer Graphics, Kümmel, GER
Julia Sets, Kümmel, GER
Solving Word Problems with DERIVE, Böhm, A
DERIVE and ACROSPIN, Schorn & Böhm, A/GER
The TI-92 Section, Bert Waits, Bernhard Kutzler, Frank Demana
and
Setif, FRA; Vermeylen, BEL; Leinbach, USA, Aue, GER; Halprin, AUS;
Weth, GER; Wiesenbauer, A; Keunecke, GER; Weller, GER; Zehavi, ISR; …

Impressum:
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G P Speck, Wanganui, New Zealand

I have received DERIVE NEWSLETTER#20 which draws attention to several matters of interest to me.
Many thanks for the diskette which you forwarded with DNL#20. It provides some very useful additions to the DERIVE programs.
Last year I was in some doubt as to whether or not joining the DERIVE User Group would prove worthwhile. This year I can say without hesitation that the DERIVE Newsletters alone provide excellent value for subscription money! Keep up your good work!
I have a number of mathematics articles at various stages of completion and several of these employ DERIVE in useful ways which may be of interest to DERIVE users. I will forward some of these uses of DERIVE from time to time for your consideration for publication in the DNL.

Lorenz Kopp, Neumarkt, Germany

Ich unterrichte an einer bayerischen Fachoberschu1e Mathematik, Physik und Informatik. Seit einigen Monaten bereite ich an Hand von Büchern den Einsatz von DERIVE für den Unterricht vor. Als Fachbetreuer muss ich auch meine Kollegen informieren. Ihre Newsletter sind mir dabei eine wertvolle Hilfe.

Wilfried Wieland, Wüstenrot, Germany

Vielleicht können Sie in der nächsten Ausgabe etwas über die versprochene Windows-Version yon DERIVE sagen?

DNL: *Fortunately I received yesterday an e-mail from AI Rich, which might be of interest for you - and not only for you, I'm sure:*

A Windows version of DERIVE (DfW) will soon be available!

DfW will carry on the tradition by combining the "point and click" ease of Windows with the reliability users have come to expect from DERIVE. SWHH has targeted Summer 1996 as the release date for DfW. Its suggested retail price will be $250. Registered users of DERIVE and DERIVE XM will soon be receiving their discount offer. The following are a few of the many innovations incorporated into Derive for Windows:

Command toolbar: Issuing frequently used DERIVE commands is as easy and effortless as a mouse click. Just highlight an expression and click on the desired operation on the command toolbar. The algebra window toolbar includes buttons for authoring, simplifying, solving, differentiating, and integrating expressions. Helpful tools indicate the purpose of each button.

Substitution dialogs: Simply click on the variables from the list that DfW presents to you and then enter the desired value for that variable. If you want to change a value for a variable already entered, simply click again on that variable and edit its value.

File menu commands: Navigate through directory trees using standard Windows dialog boxes to load, save, and print files.

Context sensitive help: Press the F1 key to instantly get detailed help on any command dialog box. Extensive on-line help is available on DERIVE's built-in mathematical functions and operators. The utility files distributed with DERIVE are also fully documented on-line.

Expression Entry and Editing Greek toolbar: Entering Greek variable names and mathematical symbols on the author line is as easy as clicking on the desired name or symbol in the Greek toolbar. Or, if you prefer, enter them using Ctrl key combinations.

2D matrix input: After entering the dimensions of the matrix, DfW displays a two dimensional expression entry array ready for your input. You can move around the array using the mouse or direction keys.

Multiple expression highlighting: Click and drag with the mouse to highlight multiple expressions. Then you can easily remove them, or drag and drop them at a new location.

Subexpression highlighting: Click on an expression to highlight the entire expression. Then click on a subexpression to highlight it. Just keep clicking to highlight more deeply nested subexpressions.

Printing Print page setup: Customize the header and footer lines of DERIVE expressions and plot printouts. Using &-macros it is easy to include the file name, page number, date and time in headers and footers. You can also specify the page size, margins and orientation of DERIVE printouts.

Print preview: Preview the print image of DERIVE expressions and plots before actually sending them to the printer. Zoom the preview in and out and view one or two pages at a time.

Printer support: If your printer is set up for Windows, it is set up for DERIVE. Support is provided for all types of black&white and color printers.

Plotting fast and accurate plotting: Plotting is as easy as highlighting an expression and clicking on the plot command. Almost instantly your 2D- or 3D- plot appears.

Cross positioning: With just a click of the mouse, move the cross to any point in a 2D-plot window and view the point´s coordinates displayed on the status bar.

Plot range: Use a drag box to specify the boundaries of a new 2D-plot range or enter boundary coordinates in a dialog box.

Command toolbar: The 2D-plot window toolbar makes it easy to annotate and print plots, position the plot region, and scale the plot.

Plot annotation: Annotations can be placed at any location in a 2D-or 3D-plot window. They can include Greek letters or mathematical symbols as well as normal text. Each annotation can be displayed in any color, for example, to match the color of a particular plot line.

Albert Rich, Soft Warehouse Hawaii

Dear Josef, Got DUG Newsletter#20. Looks great! I wonder if anyone will respond to my mathematical challenges that you have published.

The Soft *Warehouse/DERIVE* Web page has been getting a lot of activity since it came on-line. A number of web page visitors have suggested that we include more information on the DUG.

If we have enough room, perhaps we could also put MTH files from the recent Newsletters on-line so they could easily be downloaded. The MTH files would be useful to someone only if he or she was a member of the DUG and had a copy of the corresponding Newsletter.

DNL: *Your challenge was faced by J. Wiesenbauer. Look at Johann's Titbits in this issue. Your idea concerning the MTH files is great. So e-mail users have not to wait until the dikette of the year. I'll forward you a diskette with all MTH files immediately after having mailed the DNLs.* So *you can put them on your Web page.*

Torbjörn Alm, Ekerö, Sweden

A happy New Year to DUG. The ODE file was the Gem of the Year. A real Herculean effort by Professor Douros.

Karl-Heinz Keunecke, Kiel, Germany

Ha1lo Josef, ich habe kürzlich die Arbeit von Douros in DNL#20 gelesen. Die darin besprochene Funktion ODE(...) aus dem Utility file ODE.MTH hat mich begeistert und ich wollte sie gerne haben. Der Hinweis auf den Listserver von mailbase war wenig ergiebig. Da ich annahm, dass es sich um eine Utility von DERIVE 3 handelte, habe ich bei Bernhard (Kutzler) angefragt. Er verwies mich auch auf mailbase, weil Douros die Utility dort abgelegt hat. Weißt Du Doch eine andere Möglichkeit an die Utility heranzukommen?

Lieber Josef, danke für Deinen Anruf. Ich habe nicht erwartet, dass die von mir gesuchten Files bereits auf der Diskette 95 sind. Ich habe sie bereits gefunden und werde damit loslegen. Aus meiner vielleicht einseitigen Sicht ist das eine der besten Arbeiten, die im DNL veröffentlicht wurden.

Vielleicht können die Autoren in Zukunft ihre e-mail Adresse angeben. Man kommt dann leichter mit Ihnen ins Gespräch. Douros hat z.B. eine, ich habe sie erfahren und ihn schon angeschrieben. Es grüßt aus eingeschneitem Haus Karl-Heinz.

Hallo Josef, inzwischen habe ich die Arbeit von Douros mit seinem Utility file nachvollzogen, besser es nur mit mäßigem Erfolg versucht. Häufig erhalte ich nicht seine Lösung, sondern die Fehlermeldung "No ordinary differential equation ". Einiges in der Nomenklatur erscheint mir unklar und wenigstens in einem Falle erhalte ich mit den Douros-Funktionen eine falsche Lösung einer DGL. Wie weit hast Du Dich damit befasst? Hast Du Reaktionen von anderen Lesern dieser Arbeit erhalten?

DNL: *Karl-Heinz had some problems to find Prof Douros' Utility file containing ODE() from DNL#20. He was surprised that it was already on the diskette of the year 95 (in subdirectory <DOUROS>). Karl-Heinz estimates this contribution as one of the best articles published in the DNL so far. In the meanwhile he has worked with this file but the results are not as good as he had expected. Are there any experiencies of other DUG members with Prof. Douros' files? I called Karl-Heinz and he told, that he had contacted Prof Douros because of some problems and he will submit the outcome of this ODE-discussion for the next DNL. By the way Karl-Heinz suggested that the authors of DNL articles should add their e-mail address in the future if they have one.*

Hellmut Scheuermann, Hofheim/Taunus, Germany

Hellmut Scheuermann wrote a long letter which could be of interest for many of you. So I try to translate and summarize his ideas. Josef:

I sent my last letter in March 95. So it is obvious that some *DERIVE* questions have arised since then:

- Is it intentional, that the SIGN-function is not defined at location x = 0?
- Is there a possibility to transform the quadratic using *DERIVE* step by step:

  f(x) = a x^2 + b x + c ⇒ f(x) = a (x– n)^2 + h?

- Is it possible to perform a^x · b^x ⇔ (a · b)^x similar to a^(x+y) ⇔ a^x · a^y?
- I have learned in the *DERIVE* manual that it is able to create more special characters using the keys Ctrl + P followed by Alt + ASCII-Code (e.g. Alt + 127 for A). I liked this idea, because I often use Δx, Δy etc. But unfortunately even in *DERIVE's* Word Mode Δx looks like Δ · x. But even more interesting is the following: If you enter in a *DERIVE* comment the character → (Ctrl + P, Alt 026), e.g.

  ```
  #26 "function f leads for x → ∞ towards 1",
  ```

  save the file and reload it, then the file appears until the x. All the following part of the file seems to be abandoned, but if you will view it in a text editor you will find the complete file. Maybe ASCII 026 is interpreted as a loading stop character.

- The last point: During my last end examinations I found myself in big troubles and I sweated a lot. I had installed DERIVE.INI in the net with Precision Mixed. In this case *DERIVE* was unable to solve the system of simultaneous linear equations. Why that?

#1: $F(x) := a\cdot x^4 + b\cdot x^3 + c\cdot x^2 + d\cdot x + e$

#2: $F\_1(x) := 4\cdot a\cdot x^3 + 3\cdot b\cdot x^2 + 2\cdot c\cdot x + d$

#3: $\left[F(0) = 0,\ F(2) = 2,\ F\_1(0) = 5,\ F\_1(2) = -7,\ \int_0^2 F(x)\,dx = 10\right]$

#4: Precision := Mixed

#5: Notation := Mixed

#6: $\text{APPROX}\left(\text{SOLVE}\left(\left[F(0) = 0,\ F(2) = 2,\ F\_1(0) = 5,\ F\_1(2) = -7,\ \int_0^2 F(x)\,dx = 10\right], [a, b, c, d, e]\right)\right)$

#7: [ ]

#8: Precision := Exact

#9: Notation := Rational

#10: $\text{SOLVE}\left(\left[F(0) = 0,\ F(2) = 2,\ F\_1(0) = 5,\ F\_1(2) = -7,\ \int_0^2 F(x)\,dx = 10\right], [a, b, c, d, e]\right)$

#11: $\left[a = \frac{15}{4} \wedge b = -16 \wedge c = 15 \wedge d = 5 \wedge e = 0\right]$

This is Hellmut´s problem executed with DERIVE 6. In Precision Mixed the SOLVE-Button automatically switches to approximative solving which results in presenting no solution! Josef

- There is an extended discussion on sign(0) in DNL#55 which can be downloaded from our website.
- Transforming the quadratic is a nice challenge. In original DNL#21 I didn´t provide any answer. Now, many years later I tried and can offer a solution using DERIVE´s capability simplifying and factoring subexpressions and finally programming the procedure:

#1: $2\cdot y^2 + 11\cdot y - 7$

#2: $2\cdot\left(y^2 + \frac{11}{2}\cdot y\right) - 7$

#3: $2\cdot\left(y^2 + \frac{11}{2}\cdot y + \left(\frac{11}{4}\right)^2\right) - 7 - 2\cdot\left(\frac{11}{4}\right)^2$

#4: $2\cdot\frac{(4\cdot y + 11)^2}{16} - 7 - 2\cdot\left(\frac{11}{4}\right)^2$

#5: $2\cdot\frac{(4\cdot y + 11)^2}{16} - 7 - \frac{121}{8}$

#6: $2\cdot\frac{(4\cdot y + 11)^2}{16} - \left(7 + \frac{121}{8}\right)$

#7: $2\cdot\frac{(4\cdot y + 11)^2}{16} - \frac{177}{8}$

#8: $2\cdot\frac{(4\cdot y + 11)^2}{16} - \frac{177}{8} - (2\cdot y^2 + 11\cdot y - 7) = 0$

The last expressions shows the identity. Now let me collect all steps into one program:
(The tool consists of two programs and one assignment.)

```
transform(q, v, c2, c1, c0, c4, c5) :=
  Prog
    v := (VARIABLES(q))↓1
    c2 := ∂(q, v, 2)/2
    c1 := ∂(q, v)
    c1 := SUBST(c1, v, 0)
    c0 := SUBST(q, v, 0)
    c4 := c1/(2·c2)
    c5 := c4^2
    steps := '['(c2·(v^2 + c1·v/c2) + c0); '(c2·(v^2 + c1/c2·v + c4^2) + c0 - c2·(c1/(2·c2))^2);
              '(c2·(v + c4)^2) + '(c0 - c2·c5); '(c2·(v + c4)^2) + c0 - c2·(c1/(2·c2))^2;
             "fertig - ready!"]
    stn := 0
    DISPLAY("Vereinfache step auf step - Simplify step by step")
    DISPLAY("")

step_(dummy) :=
  Prog
    stn := stn + 1
    steps↓stn↓1

step := step_()
```

How to run this tool?

$$\#13:\quad \text{transform}(2\cdot y^2 + 11\cdot y - 7)$$

Vereinfache step auf step – Simplify step by step

I enter **transform(2y^2+11y-7)** and simplify this expression. I receive the instruction to simplify "step by step". I follow this instruction and enter **step=** followed by pressing ENTER-button. I repeat and repeat … until I reach the final result:

$$\#15:\quad 2\cdot\left(y^2 + \frac{11\cdot y}{2}\right) - 7$$

$$\#16:\quad 2\cdot\left(y^2 + \frac{11\cdot y}{2} + \left(\frac{11}{4}\right)^2\right) - 7 - 2\cdot\left(\frac{11}{2\cdot 2}\right)^2$$

$$\#17:\quad 2\cdot\left(y + \frac{11}{4}\right)^2 - 7 - \frac{2\cdot 121}{16}$$

$$\#18:\quad 2\cdot\left(y + \frac{11}{4}\right)^2 - \frac{177}{8}$$

#19: fertig – ready!

Repeating once more delivers the list of all intermediate results as a whole which is stored in the global variable **steps**. It looks a little bit better than the step by step procedure from above (page 6) because I can influence the factorization of the square.

$$\text{steps} = \begin{bmatrix} 2\cdot\left(y^2 + \frac{11\cdot y}{2}\right) - 7 \\ 2\cdot\left(y^2 + \frac{11\cdot y}{2} + \left(\frac{11}{4}\right)^2\right) - 7 - 2\cdot\left(\frac{11}{2\cdot 2}\right)^2 \\ 2\cdot\left(y + \frac{11}{4}\right)^2 - 7 - \frac{2\cdot 121}{16} \\ 2\cdot\left(y + \frac{11}{4}\right)^2 - \frac{177}{8} \\ \text{fertig – ready!} \end{bmatrix}$$

Please not the use of the quote-Operator ' – which is used to suppress evaluating and simplifying expressions. You can use any variables. This program could be extended to transform randomly generated quadratics.

Random_poly(variable,2,integer) has the disadvantage that the leading coefficient is always 1. See an example:

#33: qu := RANDOM_POLY(u, 2, 20)

#34: $qu := u^2 + 19\cdot u + 18$

#35: transform(qu)

Vereinfache step auf step – Simplify step by step

#36:

#37: $u^2 + \frac{19\cdot u}{1} + 18$

#38: $u^2 + 19\cdot u + \left(\frac{19}{2}\right)^2 + 18 - \left(\frac{19}{2}\right)^2$

#39: $\left(u + \frac{19}{2}\right)^2 + 18 - \frac{361}{4}$

#40: $\left(u + \frac{19}{2}\right)^2 - \frac{289}{4}$

#41: fertig – ready!

However, I will include this tool into my "Training-Skills-Library". Josef

You can influence the exponential expressions using Manage Logarithm Collect & Expand, but you have to declare the bases as Positive. See the following:

#1: $a^x\cdot b^x$

#2: a :∈ Real (0, ∞)

#3: b :∈ Real (0, ∞)

#4: Logarithm := Collect

Now simplify #1

#5: $(a\cdot b)^x$

#6: a :∈ Real

#7: b :∈ Real

Simplify #1 again

#8: $a^x\cdot b^x$

Albert Rich. Soft Warehouse Hawaii

Hello Josef, thank you for sending your ACD.EXE program for transforming MTH into ACD files. Dave looks forward to seeing your demonstration of it at the Bonn conference in July. You also mentioned that R. Schorn had produced another tool for creating analglyphs of space curves. Will that also be demonstrated?

(*Y es, of course, Josef*)

I enjoyed using your set theory teaching program SET.EXE. I am happy to report that the next version of DERIVE will include finite sets as a data type and have operators for all the usual set operations. *DERIVE* will use a syntax for enter sets similar to that used by the popular ISETL programming language. For example,

`{2,3,5,7,11,13} INTER {3,...,20}`

will simplify to `{3,5,7,11,13}`.

In response to H. Scheuermann: Unfortunately, the character (ASCII 26) that displays as a right arrow is also the character used to indicate the logical end of a file. Therefore, *DERIVE* quits reading the MTH-file when it encounters this logical EOF-character.

Using several different versions of *DERIVE,* I had no trouble solving systems of simultaneous linear equations in Exact, Mixed, and Approximate Mode. If you are still encountering problems, please let me know exactly how to reproduce the problem.

Dave, Theresa and I are overloaded rigth now working on *DERIVE* for Windows. Dave is also continuing to work with TI on improving TI-92. As soon as we release DfW, we should have more time to write some interesting articles for inclusion in the DUG Newsletter. Thanks for all your help and enthusiasm.

(These are good news concerning set theory, indeed. Using SET.MTH from the 92-diskette <MTH07> *and extending this file I helped myself to work with DERIVE in set theory. I hope to include this in one of the next DNLs. I offer SET.EXE - MENGE.EXE is the German version - mentioned by Albert among other Teaching Materials. See the information page. Josef.*

I produced an extended set theory teaching and training program based on DERIVE 6 which was presented in the frame of my ACA09 presentation. It will be published in DNL#75 or DNL#76. The DOS programs SET.EXE / MENGE.EXE are still working – and are still available. Josef

Recent versions of DERIVE provide set operations UNION (∪), INTERSECTION (∩) and DIFFERENCE (\). I miss the complement of a finite set with respect to a given universal set. (See the help file: `since the complement of a finite set is not representable.`) We have the function MEMBER?(u,v) but we don´t have a function testing if a set u is a subset of set v.

So let´s add these two functions.

In the next file I define a universal set and some subsets. **`co(subset,universal set)`** gives the complementary set and **`subset?(u,v)`** gives “true” if u is subset of v and “false” otherwise.

```
#1:   u := {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

#2:   s1 := {1, 2, 3, 5, 6, 7, 8, 9, 10}

#3:   s2 := {2, 3, 4, 6, 10}

#4:   s3 := {0, 1, 3, 7, 8, 9}

#5:   s4 := {2, 5, 10}

#6:   s5 := {-2, 3, 4, 9, 10, 11, 15}

#7:   co(s, g) := g \ s

#8:   co(s1, u) = {0, 4, 11, 12}

      subset?(u, v) :=
        If u ∩ v = u
#9:        "true"
           "false"
           "false"

#10:  subset?(s4, u) = true

#11:  subset?(s5, u) = false

#12:  subset?({}, u) = true

#13:  subset?(s2, s2) = true
```

Some examples:

```
#14:  A verification of De Morgan´s Rule:

#15:  co(s1 ∩ s2, u) = {0, 1, 4, 5, 7, 8, 9, 11, 12}

#16:  co(s1, u) ∪ co(s2, u) = {0, 1, 4, 5, 7, 8, 9, 11, 12}

#17:  co(s1 ∪ s2, u) = {0, 11, 12}

#18:  co(s1, u) ∩ co(s2, u) = {0, 11, 12}

#19:  s2 ∪ s3 ∪ s5 = {-2, 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 15}

#20:  s2 ∩ s3 ∩ s5 = {3}

#21:  De Morgan´s Rule generalized:

#22:  (a ∩ b)' = a' ∪ b' = true

#23:  (a ∪ b)' = a' ∩ b' = true
```

Jan Vermeylen. Kapellen, Belgium

…… Of course you can reprint any material from ANALYSE for the DNL. As my favorites:

I am a bit proud of the 3D projection I used in "Volume Omweltelingslichaam" - (Volume of a solid of revolution) page 80.(*See information page*) I modified a function from GRAPHICS.MTH. And also the trick with *i* in "Opwinfunctie"(page 14) and in "Integraal van Leibniz" (page 73) to produce a "cartoon-like" simultaneous plotting of two figures which gives a special effect. Maybe this trick was already published before, I don't know.

DERIVE: Automatic or Semi-automatic Mode?

Dominique Lymer, Vieux-Condé, France

I teach mathematics in France, to "Lycee" students (16-22 years old).
If I use Derive to solve an equation, to factorize an algebraic expression, build a tangent to a curve, calculate an integral, or what else, I get the correct result (almost) at once.

This is already great !

But if I wish to explain and to make the pupils understand how one goes about it (and this is an important part of a teachers job), the raw and even brutal use of the program does not bring me anything, since Derive gives the result too fast, and chiefly since the result flashing on the sceen explains nothing about the method used.

Starting from this consideration, I have tried to conceive for pupils pedagogical sequences targetting a precise mathematical objective, where they are asked to use Derive in "semi-automatic" mode.

I speculate thus: if the student manages to make the hardware doing the job, then he has somehow understood the mathematical notions and he has made headway.

What a computeralgebra system can bring in a classroom remains to be studied more accurate. This is one of the aims of the study currently conducted in France with the project of utilisation of Derive led by the Ministry of National education.

Here are two examples to illustrate uses of Derive in automatic and in semi-automatic mode.

Example 1: The notion of tangent to a curve

The DIF_APPS.MTH utility file includes a tangent function which automates the calculation of equations of tangents, but does not teach how to get them.

In class of "Premiere" (one year before the "Baccalaureat"), where pupils discover differentiation, I think that, having explained the formula $y-y_0 = f1(x_0) \cdot (x-x_0)$, one can suggest the following activity with Derive:

Define the function f by $f(x) := 2x^2-3x+1$ (for example)

Calculate the derivative function f1 (Calculus, Derive, Simliplify, then f1(x) :=...) State a real variable x_0, then assign y_0 (y0 := f(x0)). After solving for y, the formula gives the general equation of a tangent to the curve.

Then questions in French (or in English!) can be asked, which the students must first understand, then rephrase to make Derive finding the answer :

- What is the tangent to the abscissa point 2 (x_0 := 2) ? ordinate 4 ? (First solve the equation f(x) = 4 for x, then assign the result to x0).
- Find the tangents at the points of intersection with the axes (solve the equation y = 0, then assign the solutions to x0).

- Is there a tangent parallel to the straight line of equation y = -2x+ 7?
  (solve the equation f1(x) = -2).
- Is there a tangent which passes point (1/2,-2)? (Use Substitution to replace x and y by 1/2 and -2, solve for x0, and assign the solution you have found to x0).

Of course, one draws on the screen the straight lines obtained (fig 1) and this graphic capability of Derive is here most interesting, because the pupil can thus illustrate and check his result by himself, and do it again in case of mistake, with his teacher's support.

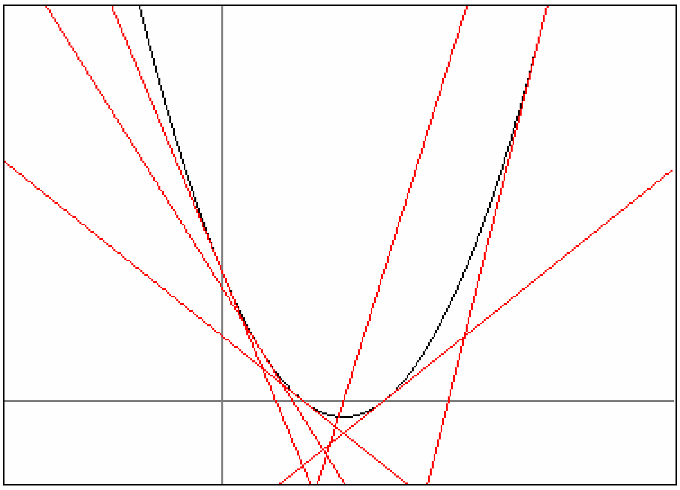

Figure 1: Some tangents to a curve

Here Derive is used in *semi-automatic mode*: the student unburdens himself onto the computer of the details of calculation (developing, simplifying, solving, which are not the aim of the lesson), but he keeps the leading role of the mathematical operations by commanding the program and by making it finding the answers.

On the other hand, the tangent function will be very useful if my pedagogical objective is different: for example in the class of "Terminale", calculating the approximated value of an integral with the method of tangents, and visualizing the trapezia calculating an area (fig 2), solving an equation with the Newton Method and illustrating graphically the procedure(fig 3), and making a curve appearing as the envelope of its tangents (fig 4).

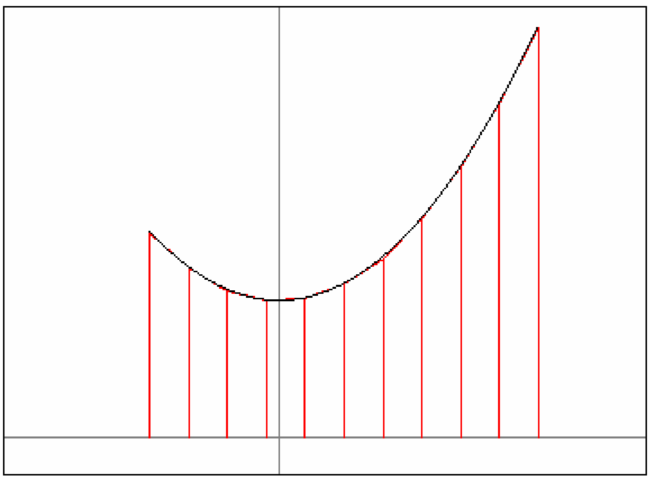

Figure 2: Approximation of an integral using tangents
(f(x) = $x^2/2 + 1$, a = –1, b=2, 10 tangents, area ~ 4.48875

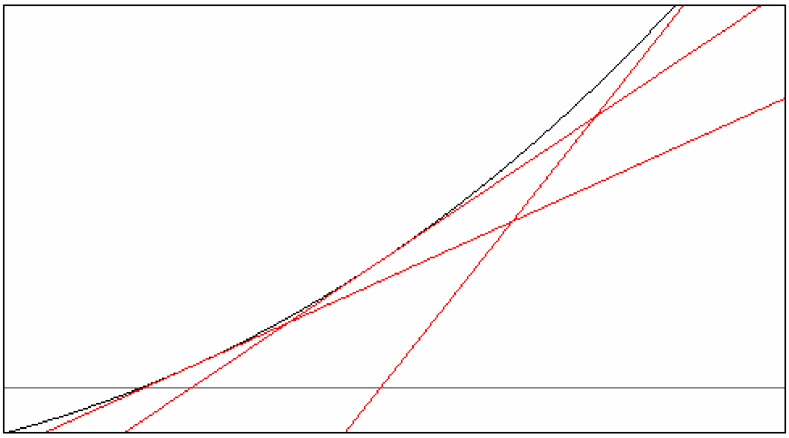

Figure 3: Visualizing Newton´s method for solving $x^3 - 3x - 1 = 0$

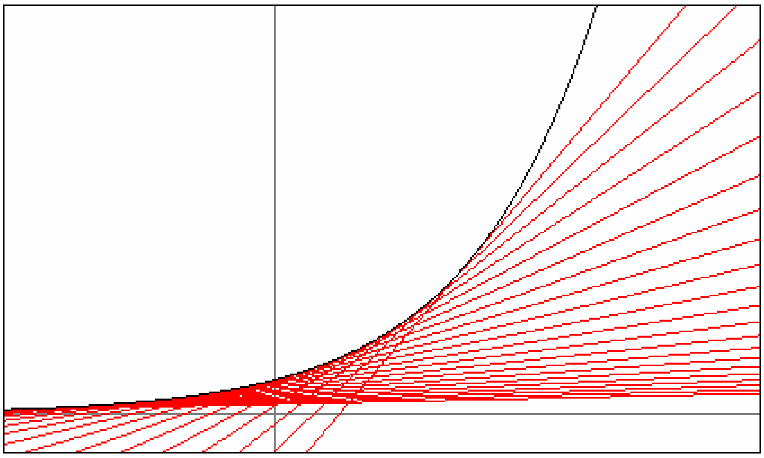

Figure 4: The curve of f(x) = exp(x) as the envelope of its tangents

Example 2 : Solving a differential equation

The study of the hit of a hammer onto an anvil leads to the particular solution of the differential equation

15 z" + 24 z' + 3660 z = 0 with z(0) = -50.7*10^(-3) and z'(0) = 0.

The calculations end up becoming simpler, but are difficult for the students and many do not know how to solve it in manual mode (no use of a CAS).

If my goal is to visualize the oscillations caused by the hit on the screen, I author with DERIVE (*automatic mode*) :

```
DSOLVE2_IV(24/15, 3660/15, 0, t, 0, -50.7*10^(-3), 0)
```

and represent the solution graphically.

If my goal is to teach the pupils to solve a differential equation, I can use this example, but the automatic mode does not suit me, because it pulls the solution out of too dark a box as if by miracle.

The following activity (*semi-automatic mode*) can help me to convey the technique for solving :

Write the characteristic equation and solve it.

Using the lesson´s formulas, write the general solution of the differential equation. Calculate the first derivative z(t) , then define z1(t). Then set the systeme of equations which fits the initial conditions, and solve this system.

Replace the constants by the values found. Check whether the function found satisfies the differential equation as well as the initial conditions.

$$\mathrm{SOLVE}(15\cdot x^2 + 24\cdot x + 3660 = 0,\ x)$$

$$x = -\frac{4}{5} - \frac{78\cdot i}{5} \vee x = -\frac{4}{5} + \frac{78\cdot i}{5}$$

$$Z(t) := \mathrm{EXP}\left(-\frac{4}{5}\cdot t\right)\cdot\left(a\cdot\mathrm{COS}\left(\frac{78}{5}\cdot t\right) + b\cdot\mathrm{SIN}\left(\frac{78}{5}\cdot t\right)\right)$$

$$\frac{d}{dt} Z(t) = -e^{-4\cdot t/5}\cdot\left(\left(\frac{4\cdot a}{5} - \frac{78\cdot b}{5}\right)\cdot\mathrm{COS}\left(\frac{78\cdot t}{5}\right) + \left(\frac{78\cdot a}{5} + \frac{4\cdot b}{5}\right)\cdot\mathrm{SIN}\left(\frac{78\cdot t}{5}\right)\right)$$

$$Z1(t) := -e^{-4\cdot t/5}\cdot\left(\left(\frac{4\cdot a}{5} - \frac{78\cdot b}{5}\right)\cdot\mathrm{COS}\left(\frac{78\cdot t}{5}\right) + \left(\frac{78\cdot a}{5} + \frac{4\cdot b}{5}\right)\cdot\mathrm{SIN}\left(\frac{78\cdot t}{5}\right)\right)$$

$$\mathrm{SOLVE}\left(\left[Z(0) = -50.7\cdot 10^{-3},\ Z1(0) = 0\right],\ [a,\ b]\right)$$

$$[a = -0.0507 \wedge b = -0.0026]$$

$$Z(t) := \mathrm{EXP}\left(-\frac{4}{5}\cdot t\right)\cdot\left((-0.0507)\cdot\mathrm{COS}\left(\frac{78}{5}\cdot t\right) + (-0.0026)\cdot\mathrm{SIN}\left(\frac{78}{5}\cdot t\right)\right)$$

$$Z1(t) := \frac{793\cdot e^{-4\cdot t/5}\cdot\mathrm{SIN}\left(\frac{78\cdot t}{5}\right)}{1000}$$

$$[Z(0),\ Z1(0)] = \left[-\frac{507}{10000},\ 0\right]$$

$$Z2(t) := \left(\frac{d}{dt}\right)^2 Z(t)$$

$$15\cdot Z2(t) + 24\cdot Z1(t) + 3660\cdot Z(t) = 0$$

Draw the Curve representing the solution (fig 5).

Thus the student unburdens himself onto the computer of the laborious and boring part of the calculations that his knowledge or abilites in maths do not allow him to deal with, but keeping the lead in the calculations and making the machine work, he can concentrate more on the mathematical method of solving as well as the interpretation of the results attained.

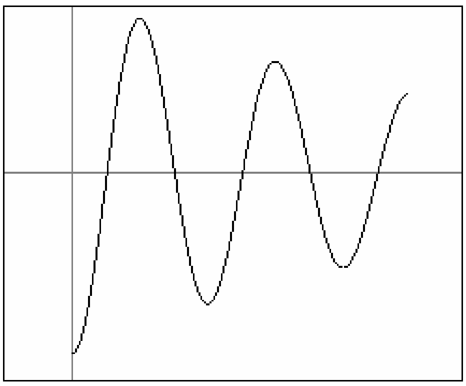

Figure 5: The hit of a hammer

About the Tennis Net (with DERIVE)*

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* Translation of an article published in the Boletin de la Soc. "Puig Adam", with permission.

** *As* a qualified tennis coach, this co-author is familiar with the Tennis Rules and regulations, and hence the tennis analogy.

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ABSTRACT

Computer Algebra Systems are sometimes unable to completely solve some problems. In some of these cases, it is desirable to merge Symbolic and Numerical computing. A simple example is given here.

To calculate the area covered by a tennis net, in the position which it is usually to be found in tennis clubs, and to compare this with the theoretical position, as recommended by the Tennis Federation. Surprisingly, the difference is not small, as will be shown using elementary mathematics and DERIVE 3.

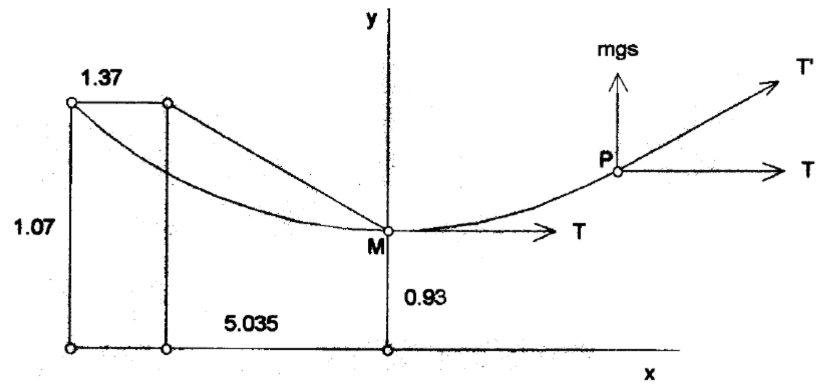
INTRODUCTION

On a tennis court, the posts of the net should be positioned differently for singles (2 players) and doubles (4 players). Official tennis rules state that, to play singles the posts should be in the singles position (i.e. closer to each other). The wire from which the net hangs should be very taut and it's height controlled at the center by a vertical tape (the other end of the tape is fixed to the surface of the court). This is how the net is positioned for championships. Therefore, we shall call it *championship position.* In this position, the shape of the upper border of the net is that of two equal and consecutive segments, forming a wide open V.

But the more common net position for singles is with the net posts in the doubles position (which is further apart), the net slack rather than taut, and without a vertical tape at the center. The net height at the center can be controlled by the players, by adjusting the tension of the net. In this position, the shape of the upper border of the net is a catenary curve. This position is what is found in most tennis clubs, so we shall refer to it as the *club position.*

1. DIMENSIONS

Height of the posts (singles and doubles): 1.07*m*
Height of the net at it´s center (singles and doubles): 0.93*m*
Distance from the center of the net to the posts in singles: 4.115*m* + 0.93*m* = 5.035*m*
Distance from the center of the net to the posts in doubles: 4.115*m* + 1.37*m* + 0.93*m* = 6.405*m*



1. EQUATIONS

Let us introduce a coordinate system in the vertical plane, that contains the net. The *x*–axis is the ground line, and the *y*–axis is the symmetry axis of the net.

2.1. CLUB POSITION

For the sake of simplicity, the weight of the nylon net will be incorporated with that of the iron wire which supports it. Let us denote by *T* the horizontal tension at the midpoint, *M*, of the wire. If *P* is an arbitrary point of the wire; *s* is the length of the arc of wire whose extreme points are *M* and *P*; and *m* is the mass per unit of length of wire; then the weight of the arc of wire whose extreme points are *M* and *P* will be *–mgs*. Therefore, the tension, *T*, of the wire in *P* will be obtained by adding *T* (horizontal) with the reaction *mgs* (vertical) against the weight of the arc of wire. When balanced, the direction of the wire in *P* will be the direction of *T′*. Therefore, denoting by *y* the function that gives the position of the wire when balanced and denoting by φ the angle of *T′* with *T* , results in

$$y' = \frac{dy}{dx} = \tan\varphi = \frac{mgs}{T}.$$

Differentiating with respect to *x*

$$y'' = \frac{mg}{T}\cdot\frac{ds}{dx} = \frac{mg}{T}\sqrt{1+y'^2}.$$

So, denoting by *k* the quotient $\frac{mg}{T}$, the differential equation of the wire of the tennis net is obtained as $y'' = k\sqrt{1+y'^2}$.

With the change of variable $y' = z$, it is transformed into the equation of separable variables

$$z' = k\sqrt{1+z^2}$$

that will be integrated with the help of DERIVE. The previous equation can be integrated by issuing the expression

```
                    2 1/2
SEPARABLE(k, (1 + z )    , x, z, 0, 0)
```

as $z = y' = 0$ when $x = 0$ (horizontal incline).

```
             2 1/2
#1:   SEPARABLE(k, (1 + z )     , x, z, 0, 0)

           2
#2:   LN(√(z  + 1) + z) = k·x

                  2
#3:   SOLVE(LN(√(z  + 1) + z) = k·x, z)
```

$$\#4:\quad z = \frac{e^{k\cdot x}}{2} - \frac{e^{-k\cdot x}}{2}$$

$$\#5:\quad \text{SEPARABLE}\left(\text{RHS}\left(z = \frac{e^{k\cdot x}}{2} - \frac{e^{-k\cdot x}}{2}\right), 1, x, y, 0, 0.93\right)$$

$$\#6:\quad y = \frac{e^{k\cdot x}}{2\cdot k} + \frac{e^{-k\cdot x}}{2\cdot k} - \frac{1}{k} + \frac{93}{100}$$

Simplifying #1 gives equation #2 which is solved wrt variable *z*. By undoing the change of variable $y' = z$, another equation of separable variables is obtained. As $y = 0.93$ when $x = 0$ (height of the net at its center), then this can be integrated by simplifying expression #5 (which can be entered as

`SEPARABLE(RHS(#4),1,x,y,0,0.93)`

where RHS is the "right hand side" of an equation (which is new in DERIVE 3). Simplifying #5 results in the following

$$y - 0.93 = \frac{e^{k\cdot x}}{2\cdot k} - \frac{e^{-k\cdot x}}{2\cdot k} - \frac{1}{k}$$

i.e. the catenary of equation

$$y = 0.93 + \frac{1}{k}\cdot\cosh(k\,x) - \frac{1}{k}.$$

Now the constant *k* has to be obtained from the condition $y = 1.07$ when $x = 6.405$ (which is the height of the net at the posts):

$$1.07 - 0.93 = \frac{e^{k\cdot 6.405}}{2\cdot k} - \frac{e^{-k\cdot 6.405}}{2\cdot k} - \frac{1}{k}.$$

Let us observe, that unlike other CAS, DERIVE transforms all numerical inputs (even those written in floating point) into fractions.

Unfortunately, DERIVE cannot solve the previous transcendental equation with respect to variable k. So, a numerical approximation will be computed.

To obtain this numerical approximation, the bisection method will be used. This method is based on Bolzano´s Theorem, and due to its recursive nature, can easily be implemented in DERIVE.

```
        BOLZANO(f, x, p1, p2, e) :=
          If ABS(p1 - p2) < e ∨ LIM(f, x, (p1 + p2)/2) = 0
             (p1 + p2)/2
#8:          If LIM(f, x, p1)·LIM(f, x, (p1 + p2)/2) > 0
                BOLZANO(f, x, (p1 + p2)/2, p2, e)
                BOLZANO(f, x, p1, (p1 + p2)/2, e)
```

The inputs to the BOLZANO-function are: *f* (function), *x* (variable), *p*1 (lower extreme), *p*2 (upper extreme) and *e* (upper boundary for the error). It is supposed that *F*(*p*1) and *F*(*p*2) are of different sign. In this case *f* will be the function given in #9, which has different signs in 0.001 and 0.02 (as can be checked with DERIVE´s command SIGN).

```
#9:   w(k) := 1/k·COSH(6.405·k) + (0.93 - 1/k) - 1.07

#10:  BOLZANO(w(k), k, 0.001, 0.02, 0.00001)

#11:  0.006821533203
```

Therefore, the recursice process can start from *p*1 = 0.001 and *p*2 = 0.02. If *e* = 0.00001 is chosen as upper boundary for the error, when DERIVE is asked to calculate #10 the system returns 0.00682153… . So we shall consider as an approximative value for *k*

$$k_0 = 0.00682.$$

Comment of the editor:
It is no problem to solve the equation from above numerically:

```
#12:  NSOLVE(1.07 = e^(k·6.405)/(2·k) + e^(- k·6.405)/(2·k) - 1/k + 93/100, k, 0, 1)

#13:  k = 0.006824182346
```

We consider the function of the net in club position as

$$F(x) := \frac{1}{k_0} \cdot \cosh(k_0 \cdot x) + 0.93 - \frac{1}{k_0}.$$

DERIVE returns approximations for *F*(0) and *F*(6.405):

```
#14:  k0 := 0.00682

#15:  F(x) := 1/k0·COSH(k0·x) + 0.93 - 1/k0

#16:  [F(0), F(6.405)] = [0.93, 1.069914171]
```

2.2. CHAMPIONSHIP POSITION

According to the introduction, the upper border of the right half of the net is a segment whose extreme points are (0,0.93) and (5.035,1.07). Therefore, the equation of the straight line that contains this segment is

$$G1(x) = 0.93 + m\,x$$

where according to paragraph 1, the inclination of the straight line (*m*) is

$$m = \frac{1.07 - 0.93}{5.035}.$$

As a consequence, the equation of the upper border of the whole net (in the championship position) is

$$G(x) = 0.93 + m \cdot \text{ABS}(x).$$

2.2. DIFFERENCE OF AREAS

The difference between both areas can be calculated by integrating the difference of functions F and $G1$. The lower and upper limits of integration are 0 and the distance from the center of the net to one of the single posts respectively. As the net is symmetric, the area to be calculated will be twice the value of this definite integral. Calculating it with the help of DERIVE, i.e. integrating between the theoretical positions of the single posts, we obtain

$$2 \cdot \int_0^{+5.035} (F(x) - G(x))\, dx$$

```
-0.4147068548
```

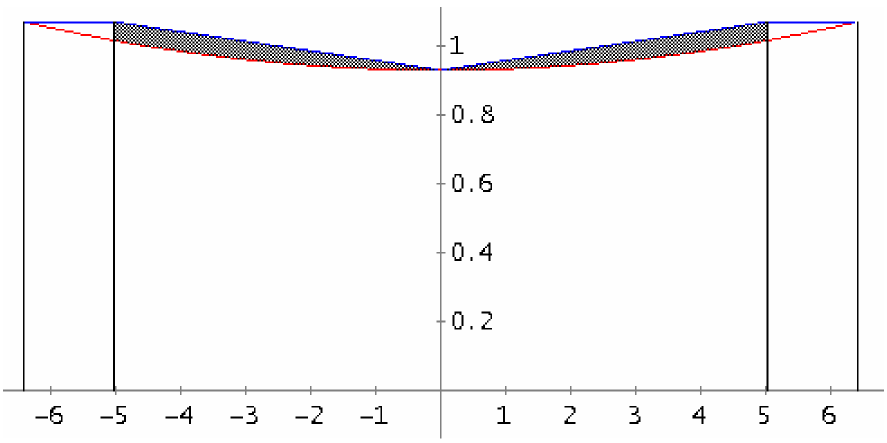
which is as expected, a negative value.

3. GRAPHIC REPRESENTATION OF THE NET

The shape of the net can be obtained in DERIVE with the following functions, defined using function F and G from above:

```
FF(x) :=
  If ¬ ABS(x) > 6.405
     F(x)
     ?

GG(x) :=
  If ABS(x) < 5.035
     G(x)
     If ¬ (ABS(x) < 5.035 ∨ ABS(x) > 6.405)
        1.07
        ?
```




```
#21:  FF(x) ≤ y ≤ GG(x) ∧ |x| ≤ 5.035

#22:  [[-6.405  0   ], [6.405  0   ], [5.035  0   ], [-5.035  0   ]]
      [[-6.405  1.07], [6.405  1.07], [5.035  1.07], [-5.035  1.07]]
```

#21 gives the shading of the difference area and #22 gives the plot of the posts.

4. CONCLUSION

The correct combination of Computer Algebra and numerical techniques enabled us to solve automatically this simple problem with the help of DERIVE. The conclusion is that the area covered by the tennis net (for singles) in its recommended position is bigger than that usually found in tennis clubs by

more tan 0.4 square meters!

Therefore, those who practise this sport at a certain level must check more than just the mere height at the center of the net.

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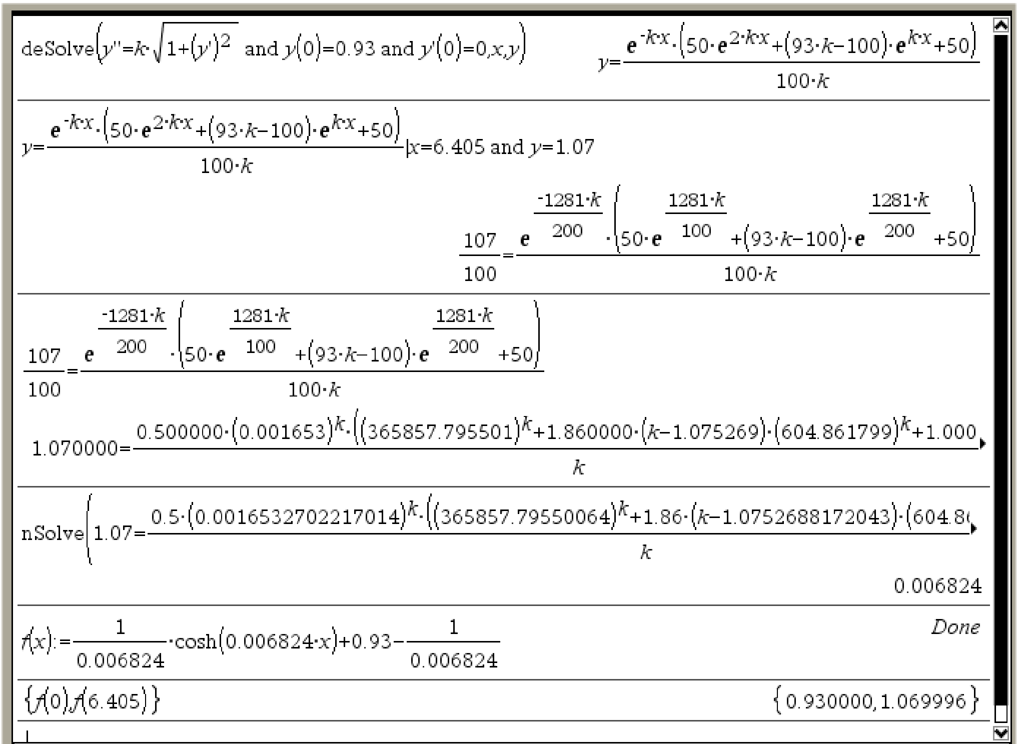
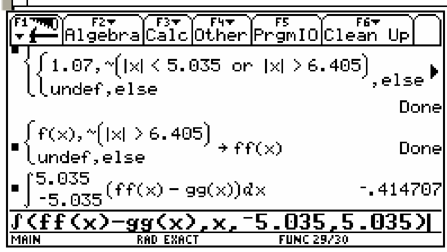
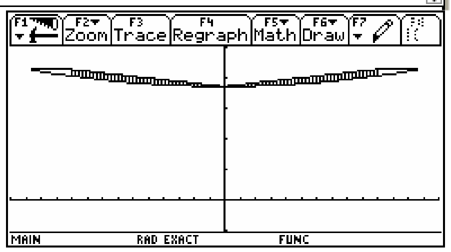
Appendix from 2009

TI-CAS implemented in Voyage 200 and TI-NspireCAS as well is able to solve the differential equation in one single step. Using the shade-function we can visualize the difference in areas on the V200. (See the respective TI-Nspire and V200 screen shots.)







Skispringen im Blickpunkt der Mathematik

Ski Jumping in the Focus of Mathematics

Hellmut Scheuermann, Hofheim/Taunus, Germany

Wer kennt sie nicht: Spektakuläre Fernsehbilder von Skispringern, die sich ganz von oben aus der Anlaufluke in die Tiefe stürzen. Dennoch und trotz zunehmender Variationen der Bildeinstellungen durch immer aufsehenerregendere Kamerapositionen können reale Eindrücke nur eingeschränkt über- und vermittelt werden. Der Fernsehzuschauer kann sich kaum vorstellen, wie steil es auf der Anlaufstrecke heruntergeht oder wie weit der Skispringer nach unten fliegt.

Die Fernsehübertragung des Weltcup-Skispringens in Willingen im Januar 1995 waren für mich der Anstoß, u.a. diesen Fragen nachzugehen und speziell diese Schanzenanlage auf ihre spezifischen Kenndaten (Länge, Höhe und insbesondere die Steigungen an verschiedenen markanten Stellen) zu analysieren. Darüber hinaus wollte ich auch erkunden, "wie viel Mathematik" zur Planung und Konstruktion – abgesehen von den üblichen Berechnungen zur Baustatik – einer Skisprungschanze benötigt wird.

Das *Organisationskomitee Weltcup Willingen*[1] (übersandte mir auf Anfrage detaillierte Unterlagen zur Sprungschanze, die die angeführten Fernsehbilder durch beeindruckende Daten der Schanze ergänzen und somit zu einer realistischeren Einschätzung der Wahrnehmung des Skispringens beitragen können. Die zusätzlich beigelegte *Internationale Skiwettkampfordnung für Skisprung und Skifliegen* führt in einem Kapitel die Normen zum Bau von Sprungschanzen aus. Diese Richtlinien ordnen (u.a.) jedem Teilstück der Schanze einen speziellen Funktionstyp zu und beschreiben Vorgaben und Kriterien zur Bestimmung der Funktionsparameter.

Natürlich interessierte bei Bearbeitung dieser Fragestellung auch, inwieweit sich der Problemkontext zur Formulierung einer anwendungsorientierten Aufgabe für den Mathematikunterricht eignet. Als Adressatenkreis stellte ich mir Schüler aus den Klassen 11/II oder 12/I des Gymnasiums oder vergleichbarer Schulformen (z.B. Fachoberschule) vor. Die mathematischen Anforderungen werden durch die traditionellen Oberstufeninhalte abgedeckt. Aufgrund der Komplexität der Aufgabe erscheint mir der Einsatz des Computers mit entsprechender Software (z.B. DERIVE™) sinnvoll, ja prädestiniert. Damit kann sich der Schüler im Wesentlichen auf den Modellbildungsprozess konzentrieren.

Zur Nachbildung der Willinger Skisprungschanze an einem Computer werden vom Schüler Modellbildungsqualitäten gefordert, wie sie z.B. bei Kaiser/Blum/Schober 1982 beschrieben werden. Das Homologierungszertifikat (vgl. Abb. 1 der Aufgabenblätter) stellt bereits eine Idealisierung des realen Modells dar. Die Lösungen zu verschiedenen Funktionsfindungsaufgaben (Mathematisierung) eröffnen den Übergang zum mathematischen Modell. Der Einsatz des Computers zum einen als Graphikwerkzeug (er zeichnet die mathematische Nachbildung der Schanzenprofillinie) und zum anderen als Rechenhilfe (zur Bearbeitung der weiteren Fragen aus dem Aufgabenblatt; mathematische Überlegungen) führen zu mathematischen Resultaten. Die Rückinterpretation der erzielten Ergebnisse auf das reale Modell, aber vor allem auf die reale Situation, sollte u.a. zur Verbesserung der eingangs erwähnten Vorstellungen vom Skispringen beitragen.

Who does not know them: spectacular TV-pictures of ski jumpers, who plunge into the depth. Although we can see more and more thrilling views the real facts yet cannot be transferred unrestricted. The person sitting in front of a TV-set can neither imagine the steepness of the approach nor how far the jumpers are gliding down through the air.

The telecast of the World Cup ski jumping from Willingen 1995 gave the idea to investigate these questions and to analyse the parameters of this special ski jump. I wanted to explore how much mathematics is necessary for planning and constructing such a facility.

The organizing committee sent detailed materials of the jump and enclosed the *International Competition Rules for Ski Jumping and Ski Flying.* These rules define each single part of the ski jump by a special function and describe the criteria to determine the parameters of the functions.

I was interested to find out if the problem would be able to give an application oriented problem for the maths class. The mathematical requirements are covered by the traditional SII-level. The complexity of the task seemed to make the use of the PC not only useful but predestined. So the students can concentrate on the modelling process.

They are forced to show modelling abilities (Kaiser/Blum/Schober, 1982). The Certificate of Jumping Hill is already an *idealisation* of the *real model.* The construction of several functions (*mathematization process)* opens the change to the *mathematical model.* The use of the PC to plot the profile and to work on the other problems given on the work sheet leads to *mathematical results. Back interpretation* of these results for the *real model,* but especially for the *real situation* should contribute to a much more better imagination of ski jumping.

[1] Meinen Dank mochte ich insbesondere dem Sekretär des Organisationskomitees Weltcup Willingen, Herrn Manfred Stede, aussprechen, der mir spontan alle Unterlagen zuschickte und der im weiteren durch zahlreiche fachliche Erläuterungen zum Entstehen dieser Anwendungsaufgabe beitrug.

Die einzelnen Phasen des Modellbildungsprozesses lassen sich wie folgt präzisieren:
The phases of the modelling process can be defined as follows:

Mathematisierung / Mathematization

- Mathematische Beschreibung des realen Modells durch ganzrationale Funktionen (bis 3. Grades) und Kreisfunktionen / Mathematical description of the real model by polynomial functions (up to cubis) and trig functions
- Lösung von Funktionsfindungsaufgaben zur Bestimmung der Parameter / Solution of problems finding the functions in order to determine the parameters

Mathematische Überlegungen und Rechnungen mit DERIVE™ / Mathematical considerations and calculations supported by DERIVE™

- Graphische Darstellung der abschnittsweise definierten Funktionen (Längenbestimmung von Geraden- und Kreisstücken) / Graphic representation of the piecewise defined functions (including finding the lengths of line segments and arcs of circles)
- Verifikation der (Funktions-) Gleichungen zur Beschreibung des Schanzenvorbaus laut Wettkampordnung / Verification of the (functions´) equations desribing the jump´s front part (according to FIS rules)

- Überlegungen zur Messung von Sprungweiten (vgl. Definition der Weite W → Modellvorstellung: Ein Maßband wird mit dem Anfang an der Schanzentischkante befestigt und zur Messung der Weite so stramm den Berg herunter gezogen, dass es nicht durchhängen kann / Considerations for measuring of the jump widthd (see W on the certificate)
- Bestimmung der Bogenlänge für einen Teil des Schanzenvorbaus / Calculation of the arc length for a part of the front part of the ski jump

Rückinterpretation / Back Interpretation

- Vergleich der zum einen theoretisch ermittelten und zum anderen der real gemessenen Länge W / Comparison of the theoretical and the real measured width W
- Interpretation der Koordinaten des Schanzenrekords verdeutlichen die Leistungen des Springers aus einem anderen Blickwinkel: der Rekordsprung impliziert eine ca. 67m tiefe, vertikale Fallstrecke (ganz abgesehen von der wirklichen Flugkurve, die ja am Anfang des nach oben gerichteten Absprungs sogar noch ansteigt). Die Markierung des Rekords weist eine Aufsprungstelle mit deutlich geringerem Gefälle als vor dem kritischen Punkt K auf. Dies ist ein Indiz für die zunehmende Belastung des Springers bei der Landung / Interpretation of the coordinates of the record jump shows the athlete´s performance, which is a fall of 67m. The landing position has an incline which is significantly less than close to the critical point K, which indicates growing physical stress for the jumper

The next two pages show the work sheets which were given to the students.

This is a picture of the ski jump which was renovated in 2000.

The technical details of the recent jump (right)

Picture and table from:

http://de.wikipedia.org/wiki/Mühlenkopfschanze

Große Mühlenkopfschanze[1]	
Anlauf	
Anlauflänge	107 m
Neigung des Anlaufs (γ)	35°
Anlaufgeschwindigkeit	26 m/s (93,6 km/h)
Schanzentisch	
Tischhöhe	3,25 m
Tischlänge	6,7 m
Neigung des Schanzentisches (α)	11°
Aufsprung	
Hillsize	145 m
Konstruktionspunkt	130 m
Höhendifferenz Tischkante bis K-Punkt (h)	65,73 m
Längendifferenz Tischkante bis K-Punkt (n)	111,41 m
Verhältnis Höhen- zu Längendifferenz (h/n)	0,590
K-Punkt Neigungswinkel (β)	35°
Auslauf	
Länge des Auslaufs	118 m
Größe	

Brühlwiesenschule Hofheim

Fach:	Mathematik	Aufgabenblatt:	Sprungschanze/2
Thema:	Analysis	Klasse:	12/I

FÉDÉRATION INTERNATIONALE DE SKI
INTERNATIONAL SKI FEDERATION
INTERNATIONALER SKI-VERBAND

No. 27 /GER 10

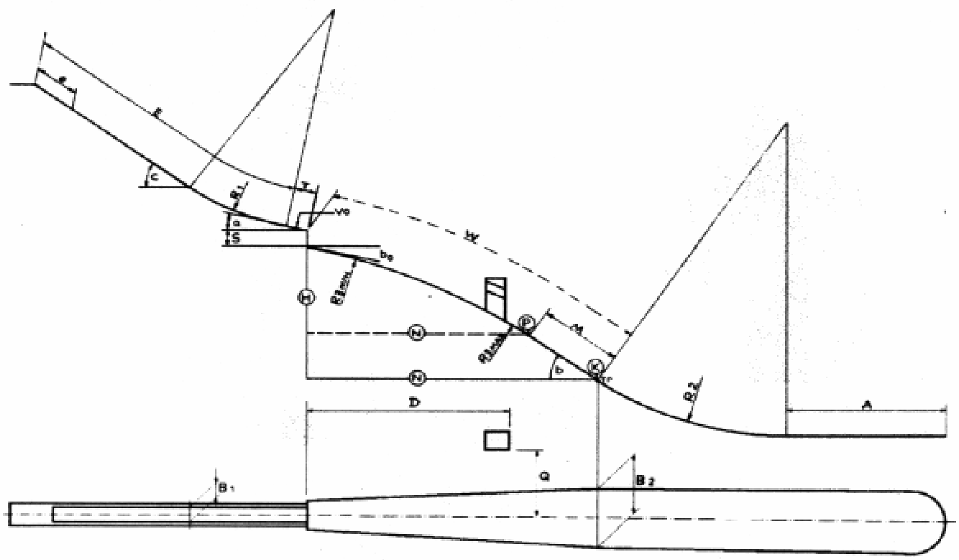
CERTIFICATE OF JUMPING HILL
CERTIFICAT DE CONFORMITÉ
SCHANZENPROFILBESTÄTIGUNG

Date of issue / Établi le / Ausgestellt am: 1.10.94

Valid till / Valable jusq' au / Gültig bis: 31.12.99

Place: WILLINGEN

Name: GROSSE MÜHLENKOPF-SCHANZE



K/W/TP	120	a	10,75°	T	6,45
P		b	37°	S	3,15
H (K) 60,74 H (P)		c	35°	E	84,85
N (K) 102,96 N (P)		R 1	82	e	10
H : N (K)	0,59	R 2	125	Vo	25 m/s
H : N (P)		R 3		D	59,5
M	30	B 1	220/250	Q	30
		B 2	22,80	A	180

Abb. 1: Homologationszertifikat der *Grossen Mühlkopf-Schanze* in Willingen

Brühlwiesenschule Hofheim

Fach: Mathematik **Aufgabenblatt:** Sprungschanze/1

Thema: Analysis **Klasse:** 12/I

Die Große Mühlenkopf-Schanze

Die Konstruktion und der Bau von Skisprungschanzen sind genauen Vorgaben des Internationalen Skiverbandes (FIS) unterworfen. Eine Freigabe zur Durchführung nationaler und internationaler Wettbewerbe erhalten Skisprungschanzen nur dann, wenn die Schanze von Sachverständigen der FIS abgenommen wurde und eine Schanzenprofilbestätigung erteilt werden konnte. Dieses sog. Homologationszertifikat führt alle spezifischen Daten der jeweiligen Schanze auf (vgl. Abb. 1, Erläuterungen in Abb. 3).

Aufgaben

1. Nähern Sie unter Berücksichtigung der Daten aus Abb. 1 die Anlauf- und Aufsprungbahn durch abschnittweise definierte Funktionen (Gleichungen aus Abb. 2 anwenden!). Nehmen Sie das Diagramm der kompletten Sprunganlage auf (Auslauf kürzen!).
2. Ermitteln Sie mit Hilfe der Differentialrechnung eine ganzrationale Funktionsgleichung 3. Grades (**allgemein**) zur Näherung des Schanzenvorbaus. Vergleichen Sie Ihr Ergebnis mit den Vorgaben der FIS (siehe Abb. 2).
3. Überprüfen Sie, inwieweit die Länge W (Definition!) aus der Schanzenprofilbestätigung mit der Länge Ihrer "Schanzennachbildung" übereinstimmt (prozentualer Fehler).
4. Der Schanzenrekord wurde während des Weltcup-Springens 1995 aufgestellt. Der Japaner Funaki sprang 132m weit. Markieren Sie den Rekord im Diagramm aus 1.

Der Schanzenvorbau beginnt am Schanzentischfuss [...] mit einer Tangentenneigung von b_0 (Richtwert $b_0 = b/6$) und endet bei P mit der Tangentenneigung b. Das Profil des Vorbaues soll bei kurzen Sprungweiten gute Landebedingungen und gleichzeitig bei langen Flügen möglichst geringe Flughöhen gewährleisten. Das Profil kann mit einer kubischen Parabel definiert werden. Mit den Koordinaten gemäss Abb. 1 ist

$$y = px^3 + qx^2 + tgb_0 x + S \quad \text{wobei}$$

$$p = \frac{-2u + vN_p}{N_p^3}, \qquad q = \frac{3u - vN_p}{N_p^2}, \quad \text{und}$$

$$u = H_p - S - N_p \cdot tgb_0, \quad v = tgb - tgb_0 \quad \text{ist.}$$

Abb. 2: Auszug aus FIS 1992, S. 37

Geometrische Elemente einer Sprungschanze

P	Beginn der Landestrecke
K	Kritischer Punkt am Ende der Landestrecke
M	Länge der Landestrecke
W	Entfernung von Schanzentischkante bis zum kritischen Punkt K
H	Höhendifferenz zwischen Schanzentischkante und K
N	Horizontaldifferenz zwischen Schanzentischkante und K
H_p	Höhendifferenz zwischen Schanzentischkante und P
N_p	Horizontaldifferenz zwischen Schanzentischkante und P
a	Neigung des Schanzentisches
b	Neigung der Landebahn
b_0	Neigung des Vorbauprofils am Schanzentischfuß
c	Neigung des geradlinigen Teils der Anlaufbahn
R_1	Radius des Bogens von der Anlaufbahn zum Schanzentisch
R_2	Radius des Bogens vom K zum Auslauf
T	Länge des Schanzentisches
S	Höhe des Schanzentisches
E	Länge der Anlaufbahn vom obersten Startplatz bis zum Beginn des Schanzentisches
e	Bereich der Startplätze
A	Länge des Auslaufes
v_0	Projektierte Geschwindigkeit auf dem Schanzentisch

Abb. 3: Erläuterungen (FIS 1992, S. 34) zur Abb. 1 (Schanzenprofilbestätigung)

Brühlwiesenschule Hofheim

Subject:	**Mathematics**	**Work sheet:**	**Ski Jump/1**
Object:	**Calculus**	**Class:**	**12/1**

The Big Mühlenkopf-Ski Jump

Construction and building of ski jumps has to follow precise rules of the International Ski Federation (FIS). The permission for execution of national and international competitions will be given only if FIS specialists have inspected the jump and given a jump profile certificate. This so called "Homologation Certificate" contains all specific data of the respective jumping hill (see fig. 1 and fig. 3)

Tasks

1. Considering the data from fig. 1 approximate the inrun and the landing run by piecewise defined functions (use the equations from fig. 2). Plot the profile of the complete ski jump facility (cut the run-out).
2. Use Calculus to find a cubic (general) for approximating the front structure of the jump. Compare your results with the FIS rules (see fig. 3).
3. Check if length W (definition!) from the Certificate of Jumping Hill matches with your "Jump Hill Model" (percentage error).

The front structure of the jump starts at the base of the jumping hill platform […] with a slope b_0 (approximate value b_0 = b/6) and ends at point P with a slope *b*. The profile of the front building should ensure good landing conditions for short jumps and minimum flight heights for long jumps as well. The profile can be defined by a cubic. Using the coordinates from fig. 1 we define

$$y = p\,x^3 - q\,x^2 - \tan b_0 x + S \qquad \text{with}$$

$$p = \frac{-2u - v\,N_P}{N_P{}^3}, \quad q = \frac{3u - v\,N_P}{N_P{}^2}, \quad \text{and}$$

$$u = H_P - S - N_p \cdot \tan b_0, \quad v = \tan b - \tan b_0.$$

Figure 2: Excerpt from FIS 1992, page 37

4. The jump hill record of 132m was reached at the world cup competition 1995 by the Japanese athlete Kozuyashi Funaki. Mark this record jump on your plot from task 1.

Geometric elements of a ski jump

P	begin of the landig section	*b*	slope of the landing run
K	critical point at the end of the landing section	b_0	slope of the profile at the platform base
M	length of the landing section	*c*	slope of the linear part of the inrun
W	distance between platform edge and critical point K	R_1	radius of arc between inrun and platform
H	height difference between platform edge and K	R_2	radius of arc between K and landing run
N	horizontal difference between platform edge and K	T	length of the jumping platform
		S	heigth of the jumping platform
H_P	height difference between platform edge and P	E	length of the inrun from the topmost starting position to the begin of the starting position
N_P	horizontal difference between platform edge and P	*e*	zone of starting positions
		A	length of the landing run
a	slope of the landing hill	v_0	assumed velocity on the jumping platform

Figure 3: Explanations (FIS 1992, page 34) to Fig. 1 (Certificate of Jumping Hill)

```
#1:   [Precision := Mixed, Notation := Decimal]

#2:   [InputMode := Word, CaseMode := Sensitive]

#3:   NotationDigits := 6
```

Solution of problem 1
Part 1: Equation of the ramp
(I) Equation for segment T (length T = 6.45m, <a = -10.75°)

```
#4:   [a := - 10.75·1°, T := 6.45]
```

TF(x) = equation for T, guT = lower bound for T;
then plot in the interval [guT,0]

```
#5:   [m_T := TAN(a), TF(x) := m_T·x, guT := - T·COS(a)]

#6:   [m_T := -0.189855, TF(x) := - 0.189855·x, guT := -6.3368]

#7:   [TF(x) := - 0.189855·x, m_T := -0.189855, guT := -6.3368]

      TF_(x) :=
#8:     If -6.3368 ≤ x ≤ 0
           TF(x)
```

(II) Equation for the arc KR1 (radius R1 = 82m)

```
#9:   R1 := 82
```

Starting point of KR1 is endpoint of segment T

```
#10:  TR1 := [guT, TF(guT)]

#11:  TR1 := [-6.3368, 1.20307]
```

The lower half of the circle and its first derivative

$$\#12:\quad R1F(x) := -\sqrt{R1^2 - (x - x0)^2} + y0$$

$$\#13:\quad R1F\_1(x) := \frac{d}{dx} R1F(x)$$

$$\#14:\quad R1F\_1(x) := \frac{x - x0}{\sqrt{-x^2 + 2\cdot x0\cdot x - x0^2 + 6724}}$$

We know the osculation point of KR1 with tangent T
and we know that TF is tangent of the circle - this leads to two equations

```
#15:  [R1F(guT) = TF(guT), R1F_1(guT) = m_T]

#16:  APPROX(SOLVE([R1F(guT) = TF(guT), R1F_1(guT) = m_T], [x0, y0]))

#17:  [x0 = 8.95809 ∧ y0 = 81.7640, x0 = -21.6316 ∧ y0 = 81.7640]
```

The center of the circle

```
#18:  [x0 := 8.95809, y0 := 81.764]
```

This is another way to find the center M(xM,yM):

```
#19:  [x_TM := R1·SIN(a), y_TM := R1·COS(a)]

#20:  [xM = -6.3368 - x_TM, yM = 1.20307 + y_TM]

#21:  [xM = 8.95817, yM = 81.7640]
```

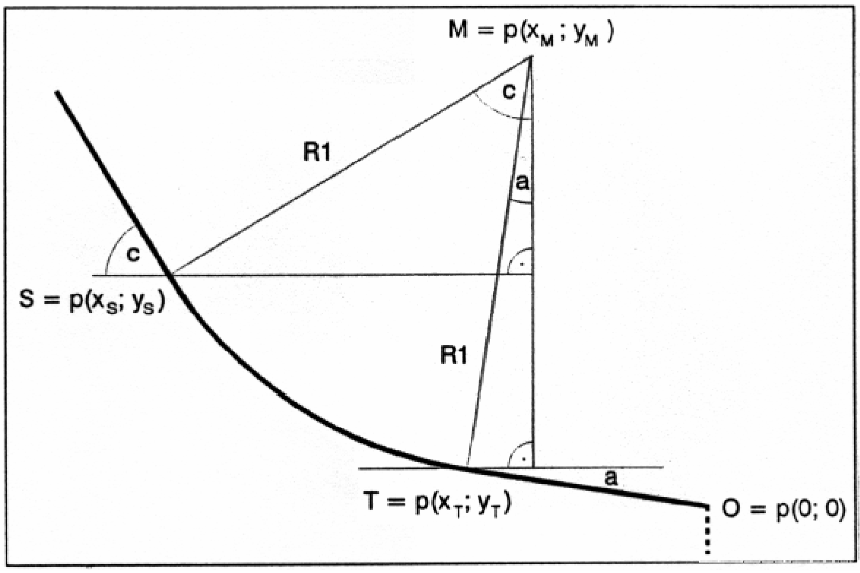


Figure 4: Ideas and illustration for calculating the center of the circle

Finding point S(xS,yS)

```
#22:  [c := 35°, x_SM := R1·SIN(c), y_SM := R1·COS(c)]

#23:  [xS = xM - x_SM, yS = yM - y_SM]

#24:  [xS = 8.95817 - x_SM, yS = 81.764 - y_SM]

#25:  [xS = -38.075, yS = 14.5935]

#26:  [-38.075, 14.5935]
```

End of this alternative

(III) Equation for the line EG (length E, <c1 = -35°)

We have to find the parameters m_E and b_E:

```
#27:  EGF(x) := m_E·x + b_E

#28:  [c1 := -c, m_E := TAN(c1)]
```

EF ist tangent of the circle, thus:

```
#29:  SOLVE(R1F_1(x) = m_E, x)

#30:  x = 55.9913 ∨ x = -38.0751
```

Begin of the arc

```
#31:  guR1 := -38.0751

#32:  R1F(guR1) = 14.5934
```

Point R1E (-38.0751, 14.5934) is osculation point of EGF with KR1

```
#33:  R1E := [-38.0751, 14.5934]
```

2nd equation to find b_e

```
#34:  SOLVE(EGF(-38.0751) = R1F(-38.0751), b_e) = (b_E = -12.0669)

#35:  b_E := -12.0669

#36:  EGF(x) := - 0.700207·x - 12.0669
```

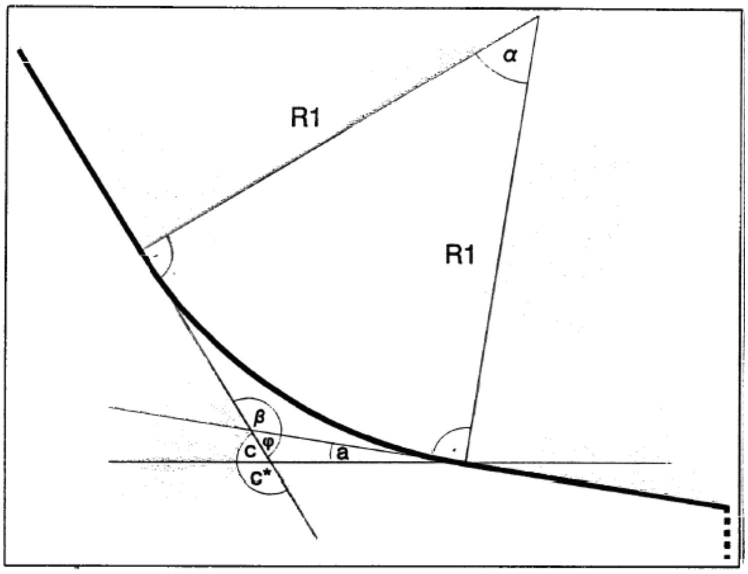


Figure 5: Illustration for calculation of the arc length

Calculation of the arc length

#37: $LR1 = \int_{-38.0751}^{-6.3368} \sqrt{(1 + R1F\_1(x)^2)}\ dx$

#38: LR1 = 34.7058

Alternative way to find the arc length:

#39: $\Phi := \frac{180° - (180° - c + -a)}{1°}$

#40: Φ := 24.25

#41: α := 24.25°

Arc length

#42: lg = 82·α

#43: lg = 34.7058

Length EG = 85.85m - 34.7058m = 50.1442m, <c = -35°

#44: EG := 50.1442

#45: guE := − EG·COS(c) + guR1

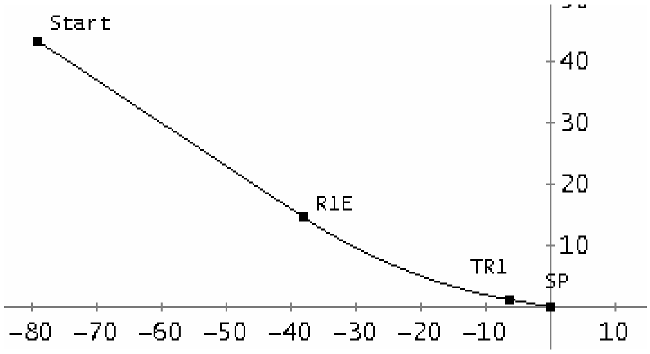
#46: guE := −79.1508

```
#47:  EGF_(x) :=
        If −79.1508 ≤ x < −38.075
           EGF(x)

#48:  R1F_(x) :=
        If −38.075 ≤ x ≤ −6.3368
           R1F(x)
```

#49: [Start := [guE, EGF(guE)], SP := [0, 0]]

#50: [Start, R1E, TR1, SP, TF_(x), R1F_(x), EGF_(x)]

Plot the complete hill:



Part 2: Equation of the landing

(I) Equation for the line segment M (length M = 30m, <b = -37°, point K (102.96m, -60.74m))

```
#51:  [K ≔ [102.96, -60.74], M ≔ 30, b ≔ - 37°]

#52:  m_M ≔ TAN(b)
```

Equation for MF:

```
#53:  MF(x) ≔ m_M·x + b_M

#54:  MF(102.96) = b_M - 77.5859

#55:  SOLVE(MF(102.96) = -60.74, b_M)

#56:  b_M = 16.8459

#57:  b_M ≔ 16.8459

#58:  goM ≔ 102.96

#59:  guM ≔ goM - M·COS(b)

#60:  guM ≔ 79.0009

      MF_(x) ≔
#61:    If 79.0009 ≤ x ≤ 102.96
           MF(x)
```

(II) Equation for the arc KR2 (radius R2 = 125m, Point K)

```
#62:  R2 ≔ 125
```

The lower half of the circle and its 1st derivative:

$$\#63: \quad R2F(x) := -\sqrt{R2^2 - (x - x02)^2} + y02$$

$$\#64: \quad R2F\_1(x) := \frac{d}{dx} R2F(x)$$

$$\#65: \quad R2F\_1(x) := \frac{x - x02}{\sqrt{- x^2 + 2 \cdot x02 \cdot x - x02^2 + 15625}}$$

Now similar to Part 1 #15 - #18

K is osculation point qtih tangent M

```
#66:  [R2F(goM) = MF(goM), R2F_1(goM) = m_M]

#67:  APPROX(SOLVE(R2F_1(goM) = m_M, x02))

#68:  x02 = 27.7331 ∨ x02 = 178.186
```

```
#69:  x02 ≔ 178.186

#70:  APPROX(SOLVE(R2F(goM) = MF(goM), y02))

#71:  y02 = 39.0900

#72:  y02 ≔ 39.09

#73:  R2F(178.186) = -85.91
```

Center of circle R2F = (178.186, 39.0900)

Finding the endpoint R2A of the arc:
We find the x-coordinate by shifting the circle x02 = 178.186m and the y-coordinate -85.91

```
#74:  R2A ≔ [178.186, -85.91]

      R2F_(x) ≔
#75:    If 102.96 ≤ x ≤ 178.186
           R2F(x)
```

(III) Equation for line segment A (length = 180m, point R2A)
A is a horizontal tangent ot arc KR2:

```
      AF_(x) ≔
#76:    If 178.186 ≤ x ≤ 178.186 + 180
           R2F(x02)
```

(IV) Equation for the part beween the ramp and point P
First finding point P:

```
#77:  P ≔ [guM, MF(guM)]

#78:  P ≔ [79.0009, -42.6855]
```

Approximation by a cubic (according to the FIS rules)

#79: $\left[Np ≔ guM,\ Hp ≔ MF(guM),\ S ≔ -3.15,\ b0 ≔ \frac{b}{6}\right]$

#80: $[u ≔ Hp - S - Np\cdot TAN(b0),\ v ≔ TAN(b) - TAN(b0)]$

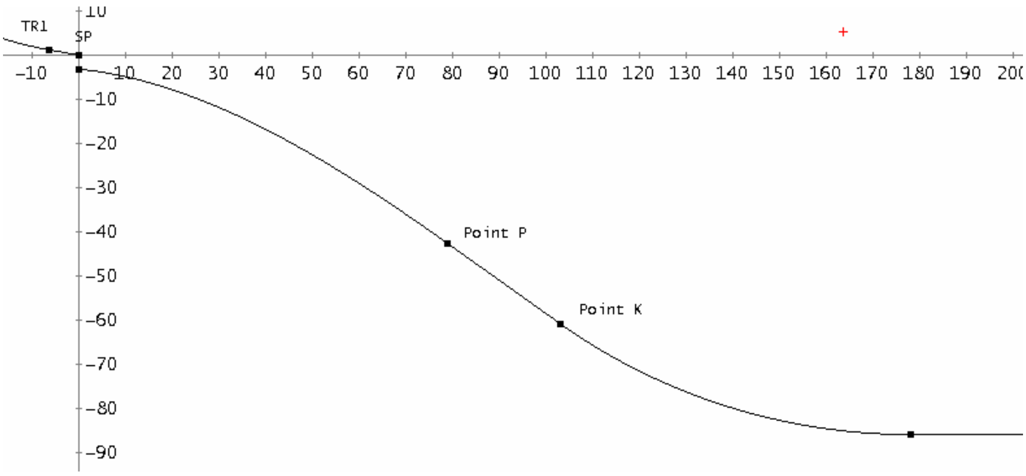
#81: $\left[p ≔ \frac{-2\cdot u + v\cdot Np}{Np^3},\ q ≔ \frac{3\cdot u - v\cdot Np}{Np^2}\right]$

#82: $SPF(x) ≔ p\cdot x^3 + q\cdot x^2 + TAN(b0)\cdot x + S$

#83: $SPF(x) ≔ 0.0000223177\cdot x^3 - 0.00673013\cdot x^2 - 0.108046\cdot x - 3.15$

```
      SPF_(x) ≔
#84:    If 0 ≤ x ≤ 79.0009
           SPF(x)

#85:  AS ≔ [0, S]

#86:  [K, R2A, P, AS, SPF_(x), MF_(x), R2F_(x), AF_(x)]
```

Plot the landing zone



Solution of problem 2

Given are: S, P(Np,Hp), <b, <bo = <b/6

#87: $$F(x) := a3\cdot x^{3} + a2\cdot x^{2} + a1\cdot x + a0$$

#88: $$F\_1(x) := 3\cdot a3\cdot x^{2} + 2\cdot a2\cdot x + a1$$

m_Np = TAN(b) and m_0 = TAN(b0)

#89: [S :=, Hp :=, Np :=]

#90: SOLVE([F(0) = S, F(Np) = Hp, F_1(Np) = m_Np, F_1(0) = m_0], [a0, a1, a2, a3])

#91: $$\left[a0 = S \wedge a1 = m\_0 \wedge a2 = \frac{3\cdot Hp - Np\cdot(2\cdot m\_0 + m\_Np) - 3\cdot S}{Np^{2}} \wedge a3 = -\frac{2\cdot Hp - Np\cdot(m\_0 + m\_Np) - 2\cdot S}{Np^{3}}\right]$$

Equation according to the FIS rules

#92: $$F\_FIS(x) := p\cdot x^{3} + q\cdot x^{2} + m\_0\cdot x + S$$

#93: [u := Hp − S − Np·m_0, v := m_Np − m_0]

#94: $$\left[p := \frac{-2\cdot u + v\cdot Np}{Np^{3}}, q := \frac{3\cdot u - v\cdot Np}{Np^{2}}\right]$$

#95: $$\left[p := -\frac{2\cdot Hp - Np\cdot(m\_0 + m\_Np) - 2\cdot S}{Np^{3}}, q := \frac{3\cdot Hp - Np\cdot(2\cdot m\_0 + m\_Np) - 3\cdot S}{Np^{2}}\right]$$

Comparing the coefficients we see the identity

Solution of Problem 3

See the definition for measuring the width of a jump.
We need a tangent from the edge of the ramp to the cubic of the landing section (see Figure 6)

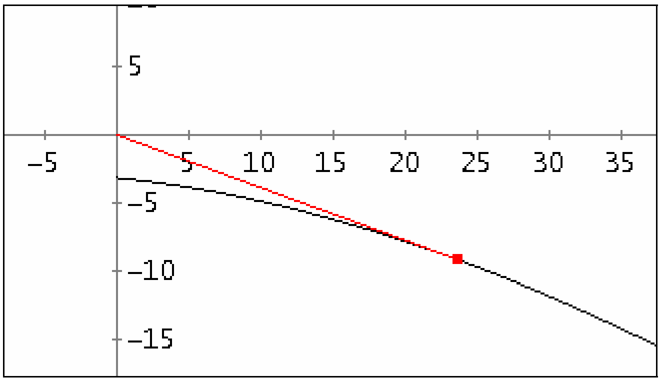


Figure 6: Illustration for calculation of width W

Osculation point BP(xT,SPF(xT))

#96: $\left[\text{Np} := \text{guM},\ \text{Hp} := \text{MF(guM)},\ \text{S} := -3.15,\ \text{b0} := \frac{\text{b}}{6}\right]$

#97: [m_Np := TAN(b), m_0 := TAN(b0)]

Equation of the "measurement tangent"

#98: MTan(x) := m_MT·x

#99: $\text{SPF\_1(x)} := \frac{d}{dx}\ \text{SPF(x)}$

#100: $\text{SPF\_1(x)} := 0.0000669531 \cdot x^2 - 0.0134602 \cdot x - 0.108046$

#101: MTan(x) := SPF_1(xT)·x

#102: SOLVE(SPF(xT) = MTan(xT), xT)

#103: xT = 147.538 ∨ xT = -20.3096 ∨ xT = 23.5517

#104: BP := [23.5517, SPF(23.5517)]

#105: m_MT := SPF_1(23.5517)

#106: m_MT := -0.38792

#107: MTanF(x) := m_MT·x

#108: MTanF_(x) :=
  If 0 ≤ x ≤ 23.5517
    MTanF(x)

Length of segment O BP

#109: $\text{L\_MTan} := \sqrt{(23.5517^2 + \text{SPF}(23.5517)^2)}$

#110: L_MTan := 25.2616

Arc length of the front part from PT to Normalpoint P

#111: $LR1 := \int_{23.5517}^{79.0009} \sqrt{(1 + SPF\_1(x)^2)}\, dx$

#112: LR1 := 65.01

Calculation of width W_Rech from the ramp to the K-point

#113: W_Rech := L_MTan + LR1 + 30

#114: W_Rech := 120.271

Error in percent is:

#115: $\frac{W\_Rech - 120}{120} \cdot 100 = 0.226441$

The error is 0.23%.

Solution of problem 4

#116: [Rek := 132, LSK := 120.271]

Distance to point K on the arc

#117: LKRek := Rek − LSK

#118: LKRek := 11.729

Calculation of the upper boundary (needs some calculation time)

Solution of problem 4

#116: [Rek := 132, LSK := 120.271]

Distance to point K on the arc

#117: LKRek := Rek − LSK

#118: LKRek := 11.729

Calculation of the upper boundary (needs some calculation time)

#119: $SOLVE\left(\int_{goM}^{x} \sqrt{(1 + R2F\_1(z)^2)}\, dz = LKRek,\ x\right) = (x = 112.644)$

Landing point for the world record

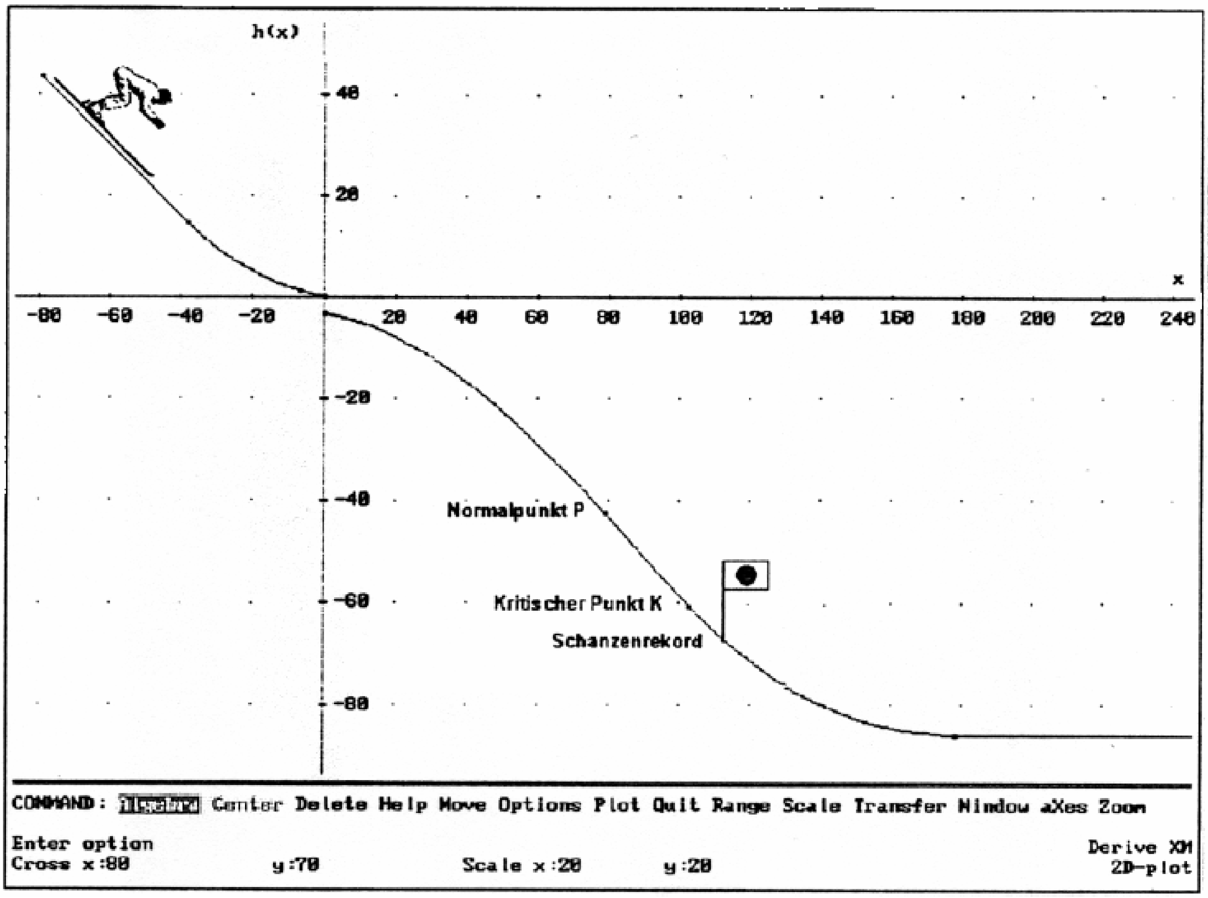
#120: [112.644, R2F(112.644)] = [112.644, −67.3489]

#119: $SOLVE\left(\int_{goM}^{x} \sqrt{(1 + R2F\_1(z)^2)}\, dz = LKRek,\ x\right) = (x = 112.644)$

Landing point for the world record

#120: [112.644, R2F(112.644)] = [112.644, −67.3489]

This is the original plot from 1996:



In the meanwhile the record was improved by Janne Ahonen. He jumped 152 m in January 2005 (on a renovated jump hill).

It was last Sunday when I watched a ski jump competition which was held in Hakuba, Japan, in the frame of the Summer Grand Prix 2009 and I saw Kazuyoshi Funaki jumping. He is now 13 years later among the seniors of the jumpers. And he performed very well.
Josef

Zur C_{60} – Modifikation des Kohlenstoffs

Richard Schorn, Kaufbeuren, Germany

Historisches / Historical

Im Jahre 1985 entdeckten Harold W. Kroto (Universität von Sussex) und Richard E. Smalley (Rice Universität) eine neue Modifikation des Kohlenstoffs. Fünf Jahre später konnte Wolfgang Krätschmer (Max-Planck-Institut für Kernphysik, Heidelberg) den neuen „Stoff" in größeren Mengen herstellen. Die Modifikation besteht aus 60 Kohlenstoffatomen, die im molekularen Bereich einen Fußball aufspannen.

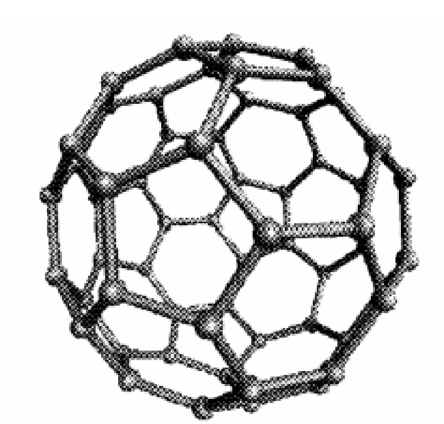
Harold W. Kroto (University of Sussex) and Richard E. Smalley (Rice University) discovered a new modification of carbon in 1985. Five years later W. Krätschmer (Max-Planck-Institute for Nuclear Physics, Heidelberg) was able to produce larger amounts of this "new stuff".

Mathematisches

Als Modell verwendet man einen üblichen Fußball (in Spielzeuggeschäften gibt es auch kleine Plastikbälle, die beim Tischfußball verwendet werden). Die Oberfläche besteht aus 12 (meist schwarz gefärbten) Fünfecken und 20 (weißen) Sechsecken. An den Ecken sitzen dann die insgesamt 60 Kohlenstoffatome.

Mathematical

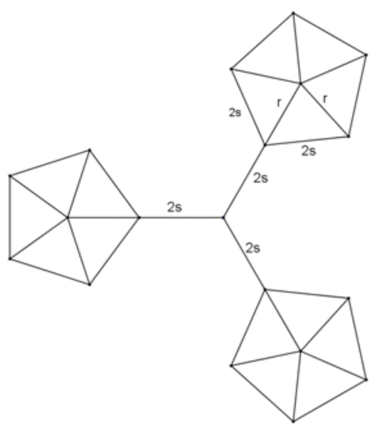
Use a regular soccer ball as a model. Its surface consists of 12 (mostly black coloured) pentagons and 20 (white) hexagons. In the vertices are all the 60 carbon atoms located.



Im Folgenden sei der Radius der Kugel 1. Der „Radius" des (sphärischen!) Fünfecks ist ein Großkreisbogen der Länge r und die Seitenlängen der regelmäßigen Vielecke seien $2s$ (natürlich auch, wie alle weiteren Angaben, als Großkreisbögen gedacht).
Die Gleichungen zur Berechnung der beiden Größen s und r erhält man aus sphärischen Dreiecken.

Let the radius of the sphere 1. The "radius" of the (spherical) pentagon is an arc of a great circle with length r and the edges of the regular polygons have length $2s$ (also arcs of great circles).
Spheric triangles lead to the equations to find r and s.

1. Verbindet man den Mittelpunkt eines Fünfecks mit zwei benachbarten Ecken des Fünfecks, ergibt sich ein gleichschenkliges Dreieck, in dem (1) gilt.
   (1) $\sin s = \sin 36° \cdot \sin r$
2. Verbindet man die Mittelpunkte **dreier** benachbarter Fünfecke, erhält man ein gleichseitiges Dreieck mit der Seite $2r+2s$ und den Winkeln 72°. Nach dem Winkel-Kosinussatz gilt (2).
   (2) $\cos(2r+2s) = \dfrac{\cos 72° \cdot (1+\cos 72°)}{\sin^2 72°}$



1. Connecting the center of a pentagon with two adjacent vertices gives an isosceles triangle which leads to equation (1)

(1) $$\sin s = \sin 36° \cdot \sin r$$

2. Connecting the centers of three adjacent pentagons results in n equilateral triangle with side length 2*r* + 2*s* and angles 72° which leads to equation (2) (angle-cosine-rule).

(2) $$\cos(2r+2s) = \frac{\cos 72° \cdot (1+\cos 72°)}{\sin^2 72°}$$

As there is a closed form for 36° and 72° both equations (1) and (2) can be simplified to (1') and (2').

The next steps are obvious: one has to apply the the trigonometric sum rules in (2) in order to substitute for sin *s* from (1). For avoiding uncomfortable roots we will try to have only one variable $x = \sin^2 r$. The boring "simplifications" can easily be done using DERIVE and they finally lead to a quadratic (3). Only one of its two solution is valid for our problem (4). This solution gives the result which is presented in the box (5).

Da die trigonometrischen Funktionen für den Winkel 36° und damit natürlich auch für 72° in geschlossener Form angegeben werden können, lassen sich beide Beziehungen vereinfachen zu (1') und (2'):

(1') $$\sin s = \frac{1}{4} \sin r \sqrt{10 - 2\sqrt{5}}$$

(2') $$\cos(2r+2s) = \frac{1}{\sqrt{5}}$$

Der weitere Gang ist klar: man wendet auf (2) die Additionstheoreme der Trigonometrie so weit an, dass man schließlich für die Größe sin *s* aus (1') substituieren kann. Damit keine unerfreulichen Wurzelterme auftreten, wird man die Gleichung so umformen, dass neben den Konstanten nur noch $x = \sin^2 r$ vorkommt. Diese Umformungen sind „zu Fuß" äußerst mühsam, können aber mit DERIVE in kürzester Zeit bewerkstelligt werden. Sie führen schließlich zu einer quadratischen Gleichung (3):

(3) $$\left(65\sqrt{5} - 275\right)x^2 + \left(280 - 72\sqrt{5}\right)x + 16\sqrt{5} - 48 = 0.$$

Die Gleichung besitzt zwei Lösungen

(4) $$x_1 = \frac{4}{5},\ x_2 = \frac{100 - 16\sqrt{6}}{545},$$

von denen die erste ausscheidet. Sie wird durch das Quadrieren (Vermeidung von Wurzeln) produziert. Die zweite Lösung führt zu folgendem Ergebnis:

(5) $$\boxed{\begin{array}{l} r = \arcsin\sqrt{\dfrac{100 - 16\sqrt{5}}{545}} = \arctan\dfrac{4\sqrt{5} - 2}{19} \approx 0{,}3504054 \\ s = \arcsin\sqrt{\dfrac{29 - 9\sqrt{5}}{218}} = \arctan\dfrac{\sqrt{5} - 1}{6} \approx 0{,}2031689 \end{array}}.$$

Derivisches

Die oben angeführten Ergebnisse können mit den folgenden DERIVE-Anweisungen ermittelt werden.

Im Anhang gibt es eine kürzere Version mit DERIVE 6.

Derivical

The results given above can be obtained as follows.

The appendix will show a shorter version performed with DERIVE 6.

#1: $$\left[\mathrm{SIN}(36°),\ \frac{\mathrm{COS}(72°)\cdot(1+\mathrm{COS}(72°))}{\mathrm{SIN}(72°)^2}\right] = \left[\frac{\sqrt{(10-2\cdot\sqrt{5})}}{4},\ \frac{\sqrt{5}}{5}\right]$$

#2: $$\mathrm{SIN}(s) = \frac{\sqrt{(10-2\cdot\sqrt{5})}}{4}\cdot\mathrm{SIN}(r)$$

#3: $$\mathrm{COS}(2\cdot r + 2\cdot s) = \frac{\sqrt{5}}{5}$$

Forcing to express the equation in sines:

#4: Trigonometry := Expand

#5: Trigpower := Sines

Simplification of expression #3 followed by rewriting in order to prepare for squaring:

#6: $$-4\cdot\mathrm{SIN}(r)\cdot\mathrm{COS}(r)\cdot\mathrm{SIN}(s)\cdot\mathrm{COS}(s) + \mathrm{SIN}(r)^2\cdot(4\cdot\mathrm{SIN}(s)^2 - 2) - 2\cdot\mathrm{SIN}(s)^2 + 1 = \frac{\sqrt{5}}{5}$$

#7: $$-4\cdot\mathrm{SIN}(r)\cdot\mathrm{COS}(r)\cdot\mathrm{SIN}(s)\cdot\mathrm{COS}(s) = -\mathrm{SIN}(r)^2\cdot(4\cdot\mathrm{SIN}(s)^2 - 2) + 2\cdot\mathrm{SIN}(s)^2 - 1 + \frac{\sqrt{5}}{5}$$

#8: $$-4\cdot\mathrm{SIN}(r)\cdot\mathrm{COS}(r)\cdot\mathrm{SIN}(s)\cdot\mathrm{COS}(s) = \mathrm{SIN}(r)^2\cdot(2 - 4\cdot\mathrm{SIN}(s)^2) + 2\cdot\mathrm{SIN}(s)^2 + \frac{\sqrt{5}}{5} - 1$$

#9 : $$\left(-4\cdot\mathrm{SIN}(r)\cdot\mathrm{COS}(r)\cdot\mathrm{SIN}(s)\cdot\mathrm{COS}(s) = \mathrm{SIN}(r)^2\cdot(2 - 4\cdot\mathrm{SIN}(s)^2) + 2\cdot\mathrm{SIN}(s)^2 + \frac{\sqrt{5}}{5} - 1\right)^2$$

#10: $$\mathrm{SIN}(r)^4\cdot(16\cdot\mathrm{SIN}(s)^4 - 16\cdot\mathrm{SIN}(s)^2) + \mathrm{SIN}(r)^2\cdot(16\cdot\mathrm{SIN}(s)^2 - 16\cdot\mathrm{SIN}(s)^4) =$$
$$\mathrm{SIN}(r)^4\cdot(16\cdot\mathrm{SIN}(s)^4 - 16\cdot\mathrm{SIN}(s)^2 + 4) - \mathrm{SIN}(r)^2\cdot\left(16\cdot\mathrm{SIN}(s)^4 + \left(\frac{8\cdot\sqrt{5}}{5} - 16\right)\cdot\mathrm{SIN}(s)^2 - \frac{4\cdot\sqrt{5}}{5} + 4\right) + 4\cdot\mathrm{SIN}(s)^4 + \left(\frac{4\cdot\sqrt{5}}{5} - 4\right)\cdot\mathrm{SIN}(s)^2 - \frac{2\cdot\sqrt{5}}{5} + \frac{6}{5}$$

Expression #8 squared, all terms collected on the left side and then simplified once more:

$$\#12:\quad -4\cdot\mathrm{SIN}(r)^4+\mathrm{SIN}(r)^2\cdot\left(\frac{8\cdot\sqrt{5}\cdot\mathrm{SIN}(s)^2}{5}-\frac{4\cdot\sqrt{5}}{5}+4\right)-4\cdot\mathrm{SIN}(s)^4+\left(4-\frac{4\cdot\sqrt{5}}{5}\right)\cdot\mathrm{SIN}(s)^2+\frac{2\cdot\sqrt{5}}{5}-\frac{6}{5}$$

Substitution for sin(s) according #2 and then simplified:

$$\#13:\quad \left(\frac{13\cdot\sqrt{5}}{8}-\frac{55}{8}\right)\cdot\mathrm{SIN}(r)^4+\left(7-\frac{9\cdot\sqrt{5}}{5}\right)\cdot\mathrm{SIN}(r)^2+\frac{2\cdot\sqrt{5}}{5}-\frac{6}{5}$$

Substitution $\sin^2 r = x$:

$$\#14:\quad \left(\frac{13\cdot\sqrt{5}}{8}-\frac{55}{8}\right)\cdot x^2+\left(7-\frac{9\cdot\sqrt{5}}{5}\right)\cdot x+\frac{2\cdot\sqrt{5}}{5}-\frac{6}{5}$$

$$\#15:\quad \mathrm{SOLVE}\left(\left(\frac{13\cdot\sqrt{5}}{8}-\frac{55}{8}\right)\cdot x^2+\left(7-\frac{9\cdot\sqrt{5}}{5}\right)\cdot x+\frac{2\cdot\sqrt{5}}{5}-\frac{6}{5},\ x\right)$$

$$\#16:\quad x=\frac{20}{109}-\frac{16\cdot\sqrt{5}}{545}\ \vee\ x=\frac{4}{5}$$

Only the first solution is valid for our problem.
Then back substitution and calculation of *r* and *s*:

$$\#17:\quad \frac{20}{109}-\frac{16\cdot\sqrt{5}}{545}=\mathrm{SIN}(r)^2$$

$$\#18:\quad r=\mathrm{ASIN}\left(\sqrt{\left(\frac{20}{109}-\frac{16\cdot\sqrt{5}}{545}\right)}\right)$$

$$\#19:\quad r=\mathrm{ATAN}\left(\frac{4\cdot\sqrt{5}}{19}-\frac{2}{19}\right)$$

$$\#20:\quad r=0.3504054128$$

$$\#21:\quad s=\mathrm{ASIN}\left(\frac{\sqrt{(10-2\cdot\sqrt{5})}}{4}\cdot\mathrm{SIN}(r)\right)$$

$$\#22:\quad s=\mathrm{ATAN}\left(\frac{\sqrt{5}}{6}-\frac{1}{6}\right)$$

$$\#23:\quad s=0.2031689460$$

Physikalisch-Chemisches

Bei diesen Berechnungen des „Fußballs“ wird nur der mathematische Aspekt angesprochen. Die chemisch-physikalischen Untersuchungen und Anwendungen (etwa Erzeugung von Supraleitern bei „hohen“ Temperaturen oder Verwendung bei neuen Arzneimitteln) bleiben hier außer Betracht. Eine gute Übersicht über diese neue Stoffklasse der „Fullerene“ ist in einem Artikel von R. F. Curl und R. E. Smalley im Scientific American (10/91) enthalten. Die Bezeichnung „Fulleren“ (oder auch der saloppe Name „Bucky-ball“) ist dem Architekten Buckminster Fuller zu Ehren gewählt worden, weil dieser schon Kuppeln („geodesic domes“) nach dem Muster der C_{60}-Modifikation entwickelt hatte. Aktuelle Informationen – und interessante Bilder – zu diesem Themenbereich kann man an vielen Stellen im Internet finden. Hier ist eine kleine Auswahl:

Physical-Chemical

At these computations of the “soccer ball“ we only refer to the mathematical aspect. Chemical-physical investigations and applications (e.g. production of superconductors at “high“ temperatures or its use in connection with new drugs) cannot be considered at this place. You can find a survey about the “Fullerenes” in an article written by R. F. Curl and R. E. Smalley in Scientific American (10/91). The name “Fulleren“ or “Buckyball” was chosen to honour the architect Buckminster Fuller who developed “geodesic domes” modelled after the C_{60}-modification. There are a lot of Internet-resources for information (together with interesting pictures) about this topic.

In the following you can find a selection:

http://www.chemie-im-alltag.de/articles/0079/

http://www.godunov.com/bucky/fullerene.html

http://images.google.at/images?client=firefox-a&rls=org.mozilla:de:official&channel=s&hl=de&source=hp&q=Buckyball&um=1&ie=UTF-8&ei=0TOiSrj1G9aMsAbn-N3SBA&sa=X&oi=image_result_group&ct=title&resnum=4

http://en.wikipedia.org/wiki/Fullerene

http://www.physik.uni-oldenburg.de/bucky/htmls/buckintro.html

Geographisches

Mit den Ergebnissen kann man nun die Koordinaten der Eckpunkte als „geographische Koordinaten“ (Länge λ, Breite φ) im Gradmaß oder wie in C60LAPHI.MTH im Bogenmaß angeben. Für Plotterausgaben, Raytracing-Verfahren oder auch ACROSPIN-Dateien empfehlen sich Berechnungen von kartesischen Koordinaten.

(Mit dem Radius 1 erhält man die Koordinaten $x = \cos\varphi\cos\lambda$, $y = \cos\varphi\sin\lambda$, $z = \sin\varphi$.)

Die Berechnungen sind nicht nur mit DERIVE, sondern auch mit Hilfe von Turbo-Pascal-Programmen durchgeführt worden. Das „Netz“ soll eine Übersicht zu den 60 Koordinatenpaaren und –tripeln geben. Außer diesen Angaben benötigt man natürlich auch Informationen über die Verbindungslinien der einzelnen Punkte.

Geographical

All these results obtained so far allow to present the coordinates of the vertices in “geographic coordinates“ (longitude λ, latitude φ).

The computations were performed not only using DERIVE but also using Turbo-Pascal-programs. The “net” gives a survey of the 60 pairs and triples of coordinates. In addition to the points you will need information how to connect the points by segments. I produced two ACROSPIN-files C60MONO.ACD and C60RAUMB.ACD which contain the respective data.

Using a similar program I obtained the Cartesian coordinates for the “world radius” 1000 given in C60R1000.TXT.

Die ACROSPIN-Dateien C60MONO.ACD and C60RAUMB.ACD enthalten entsprechende Angaben, die durch Pascal-Programme erzeugt worden sind. C60ACD.PAS berechnet die Koordinaten für das Mono-Bild. Mit einem ähnlichen Programm wurden die kart. Koordinaten für den „Erdradius“ 1000 ermittelt (C601000.TXT). Für die „Rot-Grün-Darstellung“ ist es notwendig, dass man zwei Ansichten (linkes Auge - rechtes Auge) erstellt. Am einfachsten geht das, indem man aus vorhandenen Koordinaten durch geringfügige Änderung der Werte einen zweiten Datensatz erzeugt. Auch hier hat ein Pascal-Programm geholfen.

For the “Red-Green-Representation” it is necessary to produce two views (left eye – right eye), which can be achieved by slight change of the given coordinates.

This was also done applying a Pascal-program.

You will find the ACD-files and the PASCAL-program among the files collected in the compressed file MTH21.ZIP. ACROSPIN is still running in the DOS-Window. You might try to execute `acrospin c60mono` *and* `acrospin c60raumb`*. It should work (if you have still ACROSPIN). In one of the next DNLs I will publish a contribution how to produce ACD-files for polyhedrons, solids given in parameter form and others. ACROSPIN was distributed together with the DERIVE-DOS versions in order to enable 3D-plots. As we do have 3D-plotting in the recent versions of DERIVE we can do without ACROSPIN but it has its own charm to have a look back to the nineties.*
The 3D-plot of the bucky ball is presented at the end of this contribution. Josef.

Here are the first and the last lines of C60LAPHI.MTH [longitude λ, latitude φ] of the 60 vertices of the bucky ball.

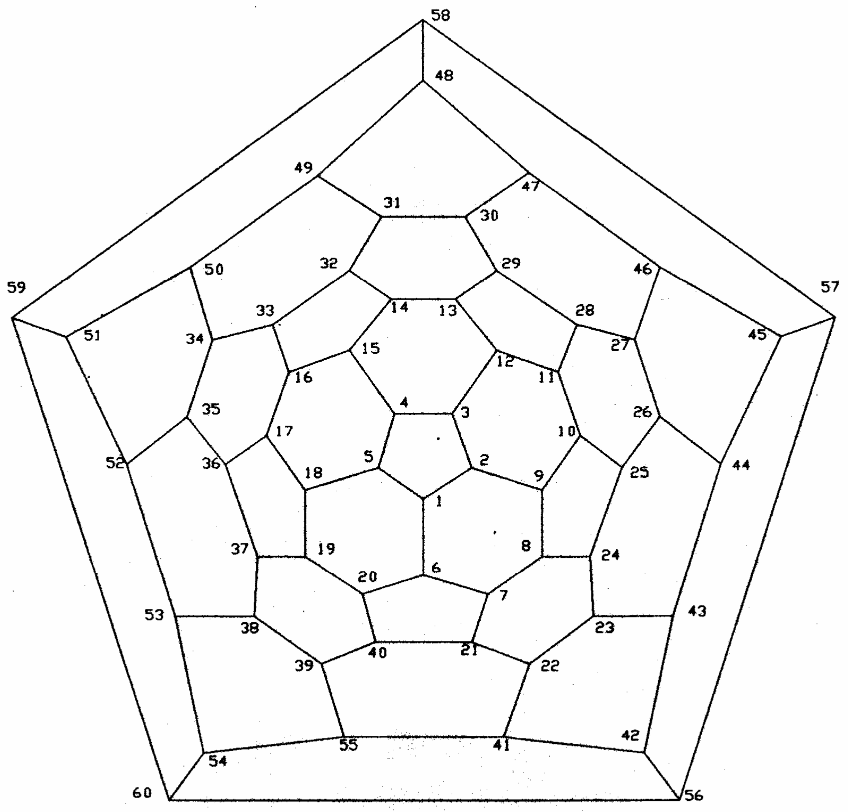
```
#1:  [0,ATAN((2*SQRT(5)+1)/2)]

#2:  [2*pi/5,ATAN((2*SQRT(5)+1)/2)]

#3:  [4*pi/5,ATAN((2*SQRT(5)+1)/2)]

…

#58:  [pi,ATAN((2*SQRT(5)+1)/2)-pi]

#59:  [7*pi/5,ATAN((2*SQRT(5)+1)/2)-pi]

#60:  [9*pi/5,ATAN((2*SQRT(5)+1)/2)-pi]
```

The 60 points given in world-coordinates in a text file C60R1000.TXT:

```
P 1 ( 343,   0, 939)    P21 ( 964, 202, 172)    P41 ( 833, 202,-515)
P 2 ( 106, 326, 939)    P22 ( 899, 404,-172)    P42 ( 555, 404,-727)
…
…
P19 ( 555,-653, 515)    P39 ( 899,-404,-172)    P59 (-106,-326,-939)
P20 ( 793,-326, 515)    P40 ( 964,-202, 172)    P60 ( 278,-202,-939)
```

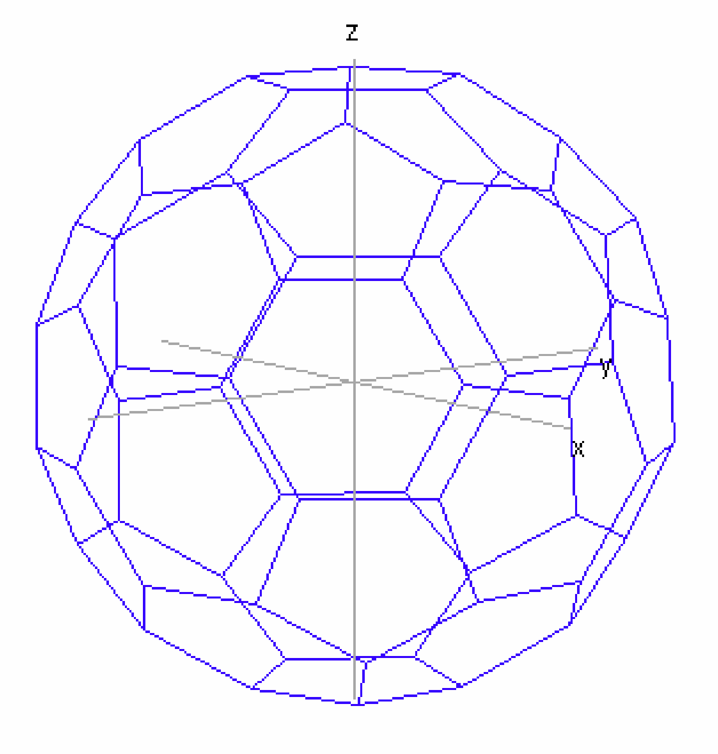
My short utility function converts the 60 points from C60LAPHI.MTH into the coordinates given in C60R1000.TXT. So simply add:

```
#61:  p:=[#1,#2,#3,...,#60]

#62:  H(x):=FLOOR(x+0.5)

#63:  pts:=VECTOR([H(1000*COS(p SUB i SUB 2)*COS(p SUB i SUB 1)),H(1000*
COS(p SUB i SUB 2)*SIN(p SUB i SUB 1)),H(1000*SIN(p SUB i SUB 2))],
i,1,DIMENSION(p))

#64:  pts:=[[343,0,939],[106,326,939],[-278,202,939],[-278,-202,939],[106,-
326,939],[687,0,727],[793,326,515],[555,653,515],[212,653,727],[-
66,855,515],[-449,730,515],[-555,404,727],[-833,202,515],[-833,-202,515],[-
555,-404,727],...
```

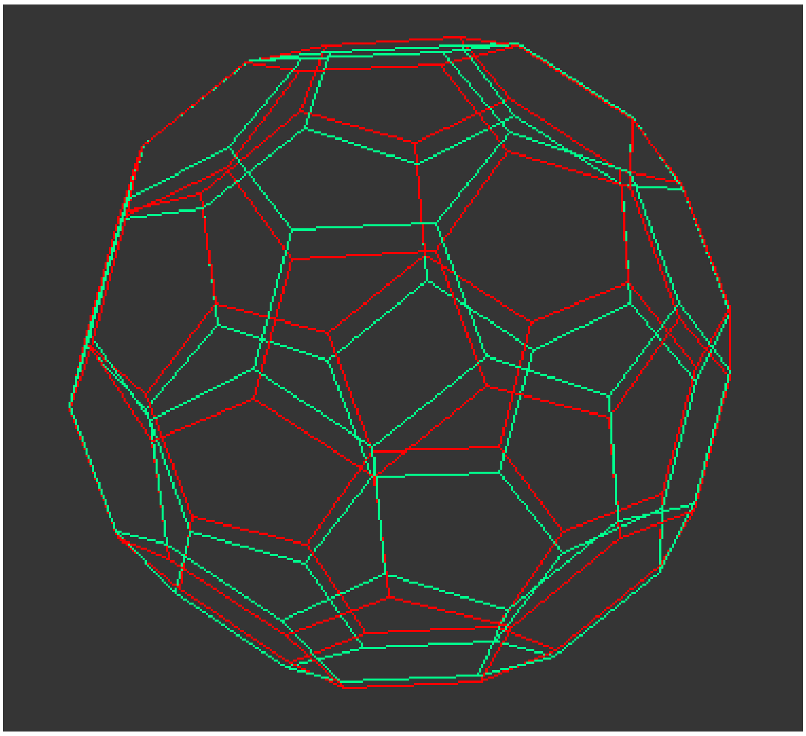


The next two expressions are from 2009 in order to produce the 3D-plot in DERIVE 6:

```
#65:  l:=[[1,2],[1,5],[1,6],[2,3],[2,9],[3,4],[3,12],[4,5],[4,15],[5,18],
[6,7],[6,20],[7,8],[7,21],[8,9],[8,24],[9,10],[10,11],[10,25],[11,12],[11,2
8],[12,13],[13,14],[13,29],[14,15],[14,32],[15,16],[16,17],[16,33],[17,18],
[17,36],[18,19],[19,20],[19,37],[20,40],[21,22],[21,40],[22,23],[22,41],[23
,24],[23,43],[24,25],[25,26],[26,27],[26,44],[27,28],[27,46],[28,29],[29,30
],[30,31],[30,47],[31,32],[31,49],[32,33],[33,34],[34,35],[34,50],[35,36],[
35,52],[36,37],[37,38],[38,39],[38,53],[39,40],[39,55],[41,42],[41,55],[42,
43],[42,56],[43,44],[44,45],[45,46],[45,57],[46,47],[47,48],[48,49],[48,58]
,[49,50],[50,51],[51,52],[51,59],[52,53],[53,54],[54,55],[54,60],[56,57],[5
6,60],[57,58],[58,59],[59,60]]

#66:  VECTOR([[pts SUB (l SUB i SUB 1),pts SUB (l SUB i SUB 2)]],i,DIM(l))
```



I used the respective ACD-file (which is plain ASCII-text) and imported the data to DERIVE 6 to reproduce the red-green-plot of the bucky ball (something similar to an analglyph).



On the next page you will find the short DERIVE 6 version of the calculation followed by a screen shot for our TI-Nspire users.

Information about ACROSPIN can be found at:

http://www.nordwest.net/doering/math/lk1/acrospin/index.htm

#1: $$\left[\mathrm{SIN}(36°),\ \frac{\mathrm{COS}(72°)\cdot(1+\mathrm{COS}(72°))}{\mathrm{SIN}(72°)^2}\right]=\left[\frac{\sqrt{10-2\cdot\sqrt{5}}}{4},\ \frac{\sqrt{5}}{5}\right]$$

#2: $$\mathrm{SIN}(s)=\mathrm{SIN}(36°)\cdot\mathrm{SIN}(r)$$

#3: $$\mathrm{SIN}(s)=\frac{\sqrt{10-2\cdot\sqrt{5}}}{4}\cdot\mathrm{SIN}(r)$$

#4: $$\mathrm{COS}(2\cdot r+2\cdot s)=\frac{\sqrt{5}}{5}$$

#5: [Trigonometry := Expand, Trigpower := Sines]

#6: $$-4\cdot\mathrm{SIN}(r)\cdot\mathrm{COS}(r)\cdot\mathrm{SIN}(s)\cdot\mathrm{COS}(s)+\mathrm{SIN}(r)^2\cdot(4\cdot\mathrm{SIN}(s)^2-2)-2\cdot\mathrm{SIN}(s)^2+1=\frac{\sqrt{5}}{5}$$

#7: $$-4\cdot\mathrm{SIN}(r)\cdot\mathrm{SIN}(s)\cdot\sqrt{1-\mathrm{SIN}(s)^2}\cdot\sqrt{1-\mathrm{SIN}(r)^2}+\mathrm{SIN}(r)^2\cdot(4\cdot\mathrm{SIN}(s)^2-2)-2\cdot\mathrm{SIN}(s)^2+1=\frac{\sqrt{5}}{5}$$

#8: $$-\frac{\sqrt{5-\sqrt{5}}\cdot\mathrm{SIN}(r)^2\cdot\sqrt{1-\mathrm{SIN}(r)^2}\cdot\sqrt{(\sqrt{5}-5)\cdot\mathrm{SIN}(r)^2+8}}{2}+\left(\frac{5}{2}-\frac{\sqrt{5}}{2}\right)\cdot\mathrm{SIN}(r)^4+\left(\frac{\sqrt{5}}{4}-\frac{13}{4}\right)\cdot\mathrm{SIN}(r)^2+1=\frac{\sqrt{5}}{5}$$

#9: $$-\frac{x^2\cdot\sqrt{5-\sqrt{5}}\cdot\sqrt{1-x^2}\cdot\sqrt{x^2\cdot(\sqrt{5}-5)+8}}{2}+x^4\cdot\left(\frac{5}{2}-\frac{\sqrt{5}}{2}\right)+x^2\cdot\left(\frac{\sqrt{5}}{4}-\frac{13}{4}\right)+1=\frac{\sqrt{5}}{5}$$

#10: $$\mathrm{SOLVE}\left(-\frac{x^2\cdot\sqrt{5-\sqrt{5}}\cdot\sqrt{1-x^2}\cdot\sqrt{x^2\cdot(\sqrt{5}-5)+8}}{2}+x^4\cdot\left(\frac{5}{2}-\frac{\sqrt{5}}{2}\right)+x^2\cdot\left(\frac{\sqrt{5}}{4}-\frac{13}{4}\right)+1=\frac{\sqrt{5}}{5},\ x\right)$$

#11: $$x=-\frac{2\cdot\sqrt{13625-2180\cdot\sqrt{5}}}{545}\ \vee\ x=\frac{2\cdot\sqrt{13625-2180\cdot\sqrt{5}}}{545}$$

#12: $$\left[\mathrm{ASIN}\left(-\frac{2\cdot\sqrt{13625-2180\cdot\sqrt{5}}}{545}\right),\ \mathrm{ASIN}\left(\frac{2\cdot\sqrt{13625-2180\cdot\sqrt{5}}}{545}\right)\right]$$

#13: $$\left[-\mathrm{ATAN}\left(\frac{4\cdot\sqrt{5}}{19}-\frac{2}{19}\right),\ \mathrm{ATAN}\left(\frac{4\cdot\sqrt{5}}{19}-\frac{2}{19}\right)\right]$$

#14: [−0.3504054128, 0.3504054128]

#15: $$\mathrm{ASIN}\left(\frac{\mathrm{SIN}(36°)\cdot 2\cdot\sqrt{13625-2180\cdot\sqrt{5}}}{545}\right)$$

#16: $$\mathrm{ATAN}\left(\frac{\sqrt{5}}{6}-\frac{1}{6}\right)$$

#17: 0.203168946

$$\left\{\sin(s)=\sin(36°)\cdot\sin(r),\cos(2\cdot r+2\cdot s)=\frac{\cos(72°)\cdot(1+\cos(72°))}{(\sin(72°))^2}\right\}\qquad\left\{\sin(s)=\frac{\sqrt{-2\cdot(\sqrt{5}-5)}\cdot\sin(r)}{4},\cos(2\cdot r+2\cdot s)=\frac{\sqrt{5}}{5}\right\}$$

$$\mathrm{tExpand}\left(\cos(2\cdot r+2\cdot s)=\frac{\sqrt{5}}{5}\right)\qquad 4\cdot(\cos(r))^2\cdot(\cos(s))^2-2\cdot(\cos(r))^2-4\cdot\sin(r)\cdot\cos(r)\cdot\sin(s)\cdot\cos(s)-2\cdot(\cos(s))^2+1=\frac{\sqrt{5}}{5}$$

$$4\cdot(\cos(r))^2\cdot(\cos(s))^2-2\cdot(\cos(r))^2-4\cdot\sin(r)\cdot\cos(r)\cdot\sin(s)\cdot\cos(s)-2\cdot(\cos(s))^2+1=\frac{\sqrt{5}}{5}\,|\,r=\sin^{-1}(x)\text{ and }s=\sin^{-1}(y)$$

$$-4\cdot x\cdot\sqrt{1-x^2}\cdot y\cdot\sqrt{1-y^2}+2\cdot x^2\cdot(2\cdot y^2-1)-2\cdot y^2+1=\frac{\sqrt{5}}{5}$$

$$-4\cdot x\cdot\sqrt{1-x^2}\cdot y\cdot\sqrt{1-y^2}+2\cdot x^2\cdot(2\cdot y^2-1)-2\cdot y^2+1=\frac{\sqrt{5}}{5}\,|\,y=\sin(36°)\cdot x$$

$$\frac{-x^2\cdot\sqrt{1-x^2}\cdot\sqrt{-(\sqrt{5}-5)\cdot((\sqrt{5}-5)\cdot x^2+8)}}{2}-\frac{(\sqrt{5}-5)\cdot x^4}{2}+\frac{(\sqrt{5}-13)\cdot x^2}{4}+1=\frac{\sqrt{5}}{5}$$

$$\mathrm{solve}\left(\frac{-x^2\cdot\sqrt{1-x^2}\cdot\sqrt{-(\sqrt{5}-5)\cdot((\sqrt{5}-5)\cdot x^2+8)}}{2}-\frac{(\sqrt{5}-5)\cdot x^4}{2}+\frac{(\sqrt{5}-13)\cdot x^2}{4}+1=\frac{\sqrt{5}}{5},x\right)$$

$x=-0.343278613032$ or $x=0.343278613032$

$\sin^{-1}(0.343278613032)$ 0.350405412847

$\sin(36°)\cdot 0.35040541284736$ 0.205963133995

7/99

An Optimization Problem
A Non Calculus Example

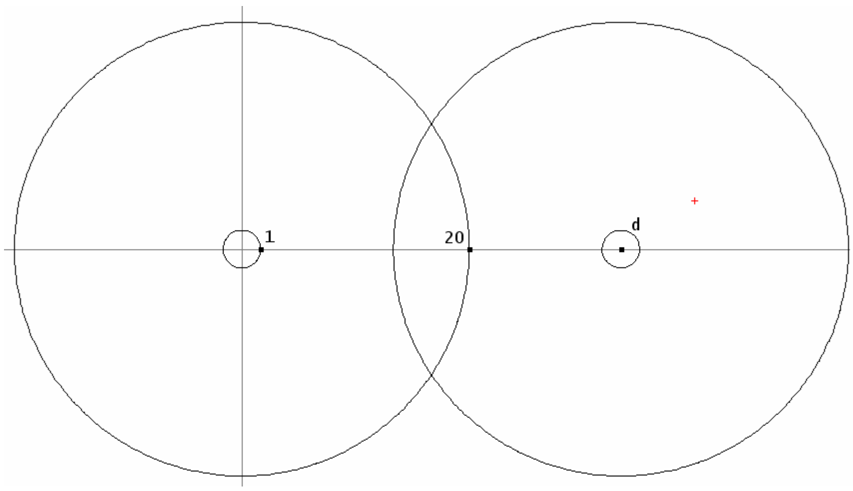
Carl Leinbach & Marvin Brubaker, USA

Every optimization problem requires a careful understanding of underlying method and definition of the objective function. Whenever possible, it is wise to plot this objective function in the region defined by the constraints. Sometimes this is the only mathematical tool that we have at our disposal. The following is an illustration of such a problem. First we begin with a description of another of *DERIVE*'s plotting features.

DERIVE will do a 2-dimensional plot of any expression that requires a single numerical input and returns a unique numerical value. This, it can plot a function defined as an IF statement. This is an obvious way to do a piecewise defined function. Another way is to use *DERIVE*'s built in "CHI" (characteristic) function. This function is authored in the following way:

```
CHI(<left end>,<variable>,<right end>).
```

We will consider a situation based on a farm irrigation device that consists of several nozzles mounted on a pipe that rolls along a field. Each nozzle is a foot long and sprays water in a circle of radius 20 feet with uniform wetting of the area covered. The sprayers are to be positioned on the pipe to give the most uniform coverage possible as the pipe is rolled along the field. The criterion we use for uniform is the time under the spray multiplied by the number of nozzles reaching a point. The figure below is a picture of a stationary position of part of the sprayer. We are assuming that at most two sprayers overlap.



Our problem gives rise to the following function to describe (within a constant multiple) the amount of water sprinkled on a point.

$$F(x,d)=\begin{cases}\sqrt{400-x^2}-\sqrt{1-x^2} & 0\le x\le 1\\ \sqrt{400-x^2} & 1\le x\le d-20\\ \sqrt{400-x^2}+\sqrt{400-(x-d)^2} & d-20\le x\le \dfrac{d}{2}\end{cases}$$

This can be entered in *DERIVE* in the following way

```
#1:   F(x,d):=CHI(0,x,1) (√(400-x^2)-√(1-x^2))+CHI(1,x,d-20)
      √(400-x^2)+CHI(d-20,x,d/2) (√(400-x^2)+√(400-(x-d)^2))
```

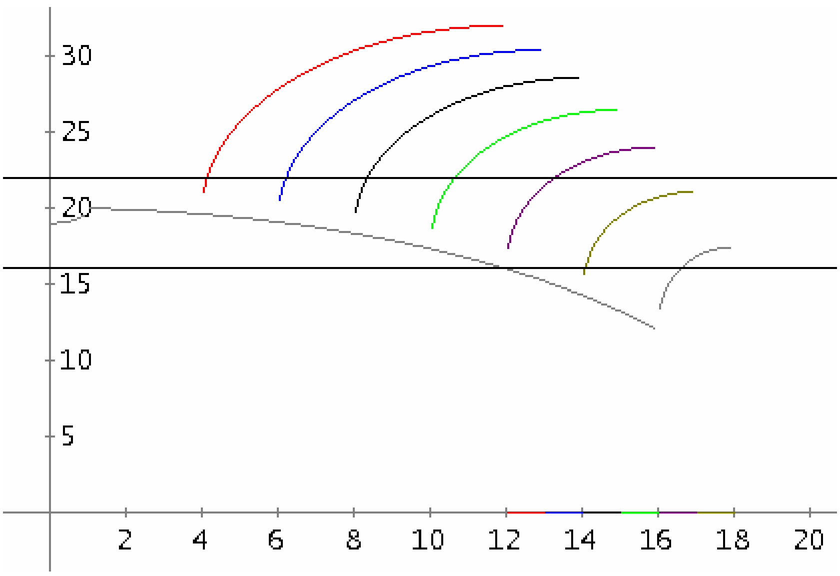
The question is now what value of d, i.e. for what spacing of the sprayers, will we get the "best" coverage of the area?

This is a very "fuzzy" question. We will graph the function for several values of the variable, d, to try and get a handle on it. Beneath is the graph of $F(x,d)$ for several values of d (24, 26, 28, 30, 32, 34 and 36) together with the lines $y = 19 \pm 3$. We chose 19 since that is the value for each function at $x = 0$ (the position under the nozzle).

To save time we use the VECTOR function. Author and Plot

```
#2:   VECTOR(F(x,y),d,24,36,2)
```

DERIVE will plot this array of functions, one function at a time. Unless we change the defaults, it plots each function in a different color.



It appears that when $d = 34$, the spraying best fits our (arbitrary) criterion.

(I like this laboratory because of some aspects:

(1) it is an unusual – but daily life problem,

(2) it emphasizes the modelling process,

(3) it is really a "fuzzy" problem, we call it an "open ended question", and

(4) the use of the computer supports in a high degree to find a solution.

Josef

Titbits from Algebra and Number Theory (7)

by Johann Wiesenbauer, Vienna

This time I am going to put special emphasis on examples which are very illustrative of what one might call "the art of programming in DERIVE". Yes, it is true, programming is certainly an art, both in general and particularly in DERIVE in view of the few tools which are available for this purpose. On the other hand, it´s always a great feeling if a self-made DERIVE-program works perfectly at last and usually very fast into the bargain!

In the following I have put together some nice programming examples and I strongly recommend that you have a try at some of them on your own before looking at the solutions given afterwards. Whether you are an inexperienced programmer or a dab hand at programming, at any rate you will profit from it! In the first case it will help you memorize some common "tricks of the trade" whereas in the second case you might be given a booster by coming up with a solution that is equivalent or even better (Wow!) than mine. Well, let´s go down to brass tacks! Here are the tasks you are expected to do if you feel like it:

1. Let *u* be a list of nonnegative inegers. Write a DERIVE-function DIFF(*u*) that returns the Boolean value true, if all elements of *u* are different, and false otherwise, as well as a DERIVE-function SORT(*u*) that sorts *u* in ascending order in the first cas, e.g.

   DIFF(VECTOR(MOD(51·k_, 101), k_, 100)) = true

   SORT(VECTOR(MOD(51·k_, 101), k_, 100)) = [1, 2, 3, 4, 5, 6, 7, …, 98, 99, 100]

2. Define a DERIVE-function PF(*n*) that returns a list of all prime factors of the natural number together with their multiplicities, eg:

   PF(360) = [[2,3],[3,2],[5,1]]. (360 = $2^3 \cdot 3^2 \cdot 5$)

   PF(*n*) can be used as an auxiliary function in a lot of other important functions of number theory. Give as an example the definitions of the DERIVE-routines for $\tau(n)$, i.e. the number of positive divisors of *n*, and $\Omega(n)$, which is the number of prime divisors of *n* each one counted with its multiplicity, e.g. $\tau(360) = 24$ and $\Omega(360) = 6$.

3. And here a really tough one for those readers who fear neither death nor devil when it comes to programming in DERIVE: Define a DERIVE-routine DIVS(*n*) that returns a list of all positive divisors of *n* in ascending order. Furthermore write a DERIVE-routine for $\sigma(n)$, i.e. the sum of all positive divisors of *n*. Try to avoid all auxiliary functions if possible! (To exclude trivial solutions I should also mention that all your routines in 2. and 3. are supposed to take only a few seconds for numbers with up to 20 digits.)

4. Write an efficient DERIVE-routine LUCAS(n) for calculating the n-th Lucas number L_n based on the recursion

$$L_{2n} = L_n^2 - 2(-1)^n,\ L_{2n+1} = L_n L_{n+1} - (-1)^n;\ L_0 = 2, L_1 = 1.$$

Furthermore use the formula

$$F_n = \frac{2L_{n+1} - L_n}{5}$$

to derive from it a DERIVE-routine FIB(n) for the n-th Fibonacci number and compare the calculation times with the corresponding function in the utility file NUMBER.MTH

5. You say you have had enough? Let us conclude then with two easy routines, namely POL_COEFF(u,x,n), which yields the coefficient of the term x^n of the polynomial expression u in x, and POL_DEG(u,x), which computes the degree of the polynomial expression u in x. (By a "polynomial expression u in x" we mean here that expand(u) is a polynomial in x in the strict sense of the word.) You say those functions are already contained in the utility file MISC.MTH? That´s handy! Thus they could serve us well as a benchmark for our own routines.

Now it´s high time we turned to my solutions. Here you are!

1. Comment in 2009: The SORT-routine is now a built-in command DERIVE.

```
DIFF(v) :=
  If DIM(v) = DIM(TERMS(v·VECTOR(x_^k_, k_, v)))
     true
     false
```

2. Comment in 2009: Now we have the FACTORS-command, which does the job:

$$\text{FACTORS(360)} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}$$

Comment in 2009: We do now have DIVISOR_TAU(n), which can easily be defined by using FACTORS(n). This can be found in *NumberTheoryFunctions.mth* which is one of the files in the Math-directory and is replacing the old NUMBER.MTH. Ω(n) is not contained but it is no problem to define it (= DIVISOR_OMEGA(n)):

```
DIVISOR_TAU(n) := ∏(v_  + 1, v_, FACTORS(n))
                     2

DIVISOR_TAU(360) = 24

DIVISOR_OMEGA(n) := ∑((FACTORS(n))↓↓2)

DIVISOR_OMEGA(360) = 6
```

Johann´s original solutions for tasks 2, 3, and partially 5 are using FACTORS from 1996. Recent versions of DERIVE have another FACTORS-function so the solutions from 1996 don´t work in DERIVE 5 or DERIVE 6.

3. Comment in 2009: DIVISORS(*n*) and DIVISOR_SIGMA(*n*) are implemented now.

$$\text{DIVISORS}(n) := \text{SORT}\left(\text{VECTOR}\left(\prod(u\_), u\_, \{[1]\}\cdot\prod\left(\text{VECTOR}\left(\text{MAP\_LIST}\left(\left[v\_{}_1^{\,k\_}\right], k\_, \{0, ..., v\_{}_2\}\right), v\_, \text{FACTORS}(n)\right)\right)\right)\right)$$

DIVISORS(360) = [1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360]

$$\text{DIVISOR\_SIGMA}(k, n) := \prod\left(\sum_{j\_=0}^{v\_\downarrow 2} v\_{}_1^{\,k\cdot j\_}, v\_, \text{FACTORS}(n)\right)$$

DIVISOR_SIGMA(1, 360) = 1170

4. Comment in 2009: LUCAS_(*n*) and FIB(*n*) are Johann´s routines from 1996 while LUCAS(*n*) and FIBONACCI(*n*) are the respective functions of DERIVE 6.

LUCAS_(n) := (ITERATE(IF(c_ = n, [a_, b_, c_, d_, e_], IF(e_ > d_, [a_^2 - 2·(-1)^c_, a_·b_ – (-1)^c_, 2·c_, d_, e_/2], [a_·b_ – (-1)^c_, b_^2 - 2·(-1)^(c_ + 1), 2·c_ + 1, d_ – e_, e_/2])), [a_, b_, c_, d_, e_], [2, 1, 0, n, ITERATE(IF(2·x_ > n, x_, 2·x_), x_, 1)]))↓1

LUCAS_(10000) – LUCAS(10000) = 0

VECTOR(LUCAS_(n), n, 0, 1000)

needs 1.56 sec

VECTOR(LUCAS(n), n, 0, 1000)

needs 0.53 sec

FIB(n) := (ITERATE(IF(c_ = n, [a_, b_, c_, d_, (2·b_ - a_)/5], IF(e_ > d_, [a_^2 - 2·(-1)^c_, a_·b_ – (-1)^c_, 2·c_, d_, e_/2], [a_·b_ – (-1)^c_, b_^2 - 2·(-1)^(c_ + 1), 2·c_ + 1, d_ – e_, e_/2])), [a_, b_, c_, d_, e_], [2, 1, 0, n, ITERATE(IF(2·x_ > n, x_, 2·x_), x_, 1)]))↓5

FIB(10000) and FIBONACCI(10000) give in an instant a 2090 digit number.

5. Comment in 2009: MISC.MTH from the earlier DERIVE versions has been replaced by MiscellaneousFunctions.mth. Johann´s POL_DEG from original DNL#21 does not work because of FACTORS, but we can compare his POL_COEFF with the implemented POLY_COEFF-function:

$$\text{POL\_COEFF}(u, x, k) := \text{QUOTIENT}(\text{REMAINDER}(u, x^{k+1}), x^k)$$

$$\text{POL\_COEFF}\left(\prod_{i=1}^{20}(1 + x^i), x, 20\right) = 64$$

needs 0.031 sec

$$\text{POLY\_COEFF}\left(\prod_{i=1}^{20}(1 + x^i), x, 20\right) = 64$$

needs 0.188 sec

$$\text{POLY\_DEGREE}\left(\prod_{i=1}^{20}(1 + x^i), x\right) = 210$$

The last examples remind me of Albert Rich´s appeal to the DERIVE-community in DNL#20 to improve his implementation

```
PARTS_AUX(n, m) :=
  If n < 2·m
     1
     1 + Σ(PARTS_AUX(n - k_, k_), k_, m, FLOOR(n, 2))

PARTS_(n) :=
  If n < 1
     0
     PARTS_AUX(n, 1)
```

for the number $p(n)$ of partitions of a natural number n. Well, there is an obvious improvement, namely to remove the IF-part in PARTS_(n), since the correct value of $p(0)$ is definitely 1!. (Ah, those pesky initial values! I remember with a smile that at one time mod(n,0) was simplified to 0 by DERIVE!)

DERIVE 6 contains a really improved function PARTS(n) in its utility file CombinatoricsFunctions.mth which works very fast.

```
PARTS_(50) = 204226
```

needs 8.39 sec, try PARTS_(100) !!

```
PARTS(50) = 204226

PARTS(100) = 190569292
```

in an instant

A more serious approach could be based on Euler´s formula for the partition function $p(n)$, namely

$$p(n) = \sum_{m=1}^{\infty} (-1)^{m+1} \left[p\left(n - \frac{3m^2 - m}{2} \right) + p\left(n - \frac{3m^2 + m}{2} \right) \right] \quad (n \geq 1,\ p(n) = 0 \ \text{ for } \ n < 0).$$

First a less memory-consuming implementation of it:

P0(n) := (ITERATE(APPEND([(ITERATE(IF(k_ ≤ DIMENSION(t_),
[s_ + (-1)^(d_ + 1)·(t_↓k_ + t_↓(k_ – (d_ –2)/3)), k_ + d_, d_ + 3],
IF(k_ –(d_ –2)/3 ≤ DIMENSION(t_), [s_ + (-1)^(d_ + 1)·t_↓(k_ –(d_ –2)/3), k_ + d_, d_ + 3],
[s_, k_, d_])), [s_, k_, d_], [0, 2, 5]))↓1], t_), t_, [1], n))↓1

And now a second try applying brute force:

P(n) ≔ (ITERATE(IF(n < DIMENSION(p_), [p_, v_, m_], [APPEND([p_·v_], p_),
APPEND(v_, [IF((3·m_^2 –m_)/2 = DIMENSION(p_) + 1 OR3·m_^2 + m_)/2 =
DIMENSION(p_) + 1, (-1)^(m_ + 1), 0)]), IF((3·m_^2 + m_)/2 = DIMENSION(p_) + 1,
m_ + 1, m_)]), [p_, v_, m_], [[1], [1], 1], n))↓1↓1

```
P0(100) = 190569292
```

0.078 sec

```
P(100) = 190569292
```

0.047 sec

I don´t know whether the performance of these implementations lives up to Albert´s expectations but one thing is for sure: They are both considerably faster than the one above (PARTS_(*n*). Check it. (Any comments or improvements? This is my email address: j.wiesenbauer@tuwien.ac.at)

Applications of Johann´s RED-function from DNL#20

Please remember Johann´s RED-function from the last DNL. It is indeed a remarkable invention. I want to illustrate this using four examples. Johann and I talked about the next DNL and by the way he mentioned that his RED-function would be able to solve nonlinear systems of equations and that he had found a fine example with the *van der Waals* equation. He promised to fax the example. In the meanwhile I checked his words with a problem from one of my textbooks: finding the intersection points of two conics, which is example 1 in the following. Example 2 will be Johann´s contribution.

By chance I found one problem in a *Maple*-Newsletter. I refer to *Maple Surgery Q&A*:

> *How can I get Maple to substitute a mathematical expression? For example I have the following:*
>
> ```
> > f:=x^3-y^4+3*x^2*y+3*x*y^2+y^3-y^2
> ```
>
> *I want to substitute all occurrences of* **x + y** *with* **z**. *If I use* **subs**, *nothing changes.*
>
> *The answer: … use the* **siderels** *option of the* **simplify** *command. For example*
>
> ```
> > simplify (f,{x+y}=z});
> -y^4-y^2+z^3
> ```

In example 3 I will show how to tackle this problem with Johann´s RED-function. And finally I found a suitable fourth example. In the last edition of “Spektrum der Wissenschaft” (German edition of Scientific American) contains some contributions focussing on various computeralgebra systems. Among other I read one sentence about *Groebner bases*:

> *Damit kann man sogar* $a^4 + b^4 + c^4$ *berechnen, wenn a, b und c nur implizit, nämlich über die Bedingungen* $a + b + c = 3$, $a^2 + b^2 + c^2 = 9$ *und* $a^3 + b^3 + c^3 = 24$ *bekannt sind.*
>
> *Das Ergebnis ist 69.*

In 1996 Groebner Bases were not implemented in DERIVE. In the meanwhile they are as you will see later.

In the following you can find the above described applications of the RED-function.

#1: RED(u, v) ≔ ITERATE(RHS(v)·QUOTIENT(u_, LHS(v)) + REMAINDER(u_, LHS(v)), u_, u)

Example 1

#2: $[u ≔ x^2 - x·y - y - 113, v ≔ y^2 - x·y + x = -41]$

#3: $RED(u, v) = \frac{126·(y + 13)}{(y - 1)^2} - 70$

#4: $y = -\frac{16}{5} ∨ y = 7$

#5: $SOLVE(RED(u, v), y) = \left(y = \frac{x^2 - 113}{x + 1}\right)$

It would be fine if RED and SOLVE could be combined.
This could be a challenge for Albert Rich to improve DERIVE.

Now we can solve the system because of the implemented Groeber Bases:

#6: $SOLVE([x^2 - x·y - y - 113, y^2 - x·y + x = -41], [x, y])$

#7: $\left[x = 15 ∧ y = 7, x = -\frac{61}{5} ∧ y = -\frac{16}{5}\right]$

Example 2:
See Johann´s Example together with his comments.
We start with the Van de Waals equation.

#8: [u ≔, v ≔]

#9: $\frac{p(+a)}{v^2}·(v - b) = r·t$

We solve for p and then introduce p as a function of v alone:

#10: SOLVE((p+a/v^2)*(v-b)=r*t,p)

#11: p=(a*(b-v)+r*t*v^2)/(v^2*(v-b))

#12: $p(v) ≔ \frac{a·(b - v) + r·t·v^2}{v^2·(v - b)}$

The critical values of v, t are obtained by solving the following system of nonlinear equations:

#13: $\left[\frac{d}{dv} p(v) = 0, \left(\frac{d}{dv}\right)^2 p(v) = 0\right]$

#14: $\left[\frac{2·a·(b^2 - 2·b·v + v^2) - r·t·v^3}{v^3·(b - v)^2} = 0,\right.$

$$\left.\frac{3\cdot a\cdot(b^3 - 3\cdot b^2\cdot v + 3\cdot b\cdot v^2 - v^3) + r\cdot t\cdot v^4}{v^4\cdot(b - v)^3} = 0\right]$$

The resulting system (#14) looks rather frightening, doesn´t it?
This is where our RED-function from DNL#20 comes into play:

#15: $\text{RED}\left(\frac{d}{dv}\, p(v),\ \left(\frac{d}{dv}\right)^2 p(v)\right) = \frac{r\cdot t\cdot(3\cdot b - v)}{3\cdot(v - b)^3}$

#16: $\text{SOLVE}\left(\frac{r\cdot t\cdot(3\cdot b - v)}{3\cdot(v - b)^3},\ v\right) = (v = \pm\infty \lor v = 3\cdot b \lor r\cdot t = 0)$

Surprisingly enough, the last two steps cannot be combined:

#17: $\text{SOLVE}\left(\text{RED}\left(\frac{d}{dv}\, p(v),\ \left(\frac{d}{dv}\right)^2 p(v)\right),\ v\right)$

#18:

$v = \pm\infty \lor v = -$

$$\frac{2\cdot\sqrt{3}\cdot\sqrt{(a\cdot(a - 3\cdot b\cdot r\cdot t))}\cdot\text{COS}\left(\frac{\text{ASIN}\left(\frac{(8\cdot a^2 - 36\cdot a\cdot b\cdot r\cdot t + 27\cdot b^2\cdot r^2\cdot t^2)\cdot\sim}{8\cdot a\cdot(a - 3\cdot b\cdot r\cdot t)\sim}\right.}{3}\right.}{3\cdot|r\cdot t|}\sim$$

. . . .

I omit the rest of this expression because it is too large for printing here.

Example 3:

#19: $\text{RED}(x^3 - y^4 + 3\cdot x^2\cdot y + 3\cdot x\cdot y^2 + y^3 - y^2,\ x + y = z)$

#20: $-\ y^4 - y^2 + z^3$

needs 0.015 sec

Example 4:

#21: $[x := a^4 + b^4 + c^4,\ u := a^2 + b^2 + c^2 - 9]$

#22: $[v := a^3 + b^3 + c^3 - 24,\ w := a + b + c = 3]$

#23: RED(x, w)

#24: $2\cdot b^4 + b^3\cdot(4\cdot c - 12) + b^2\cdot(6\cdot c^2 - 36\cdot c + 54) + b\cdot(4\cdot c^3 - 36\cdot c^2 + 108\cdot c - 108) + 2\cdot c^4 - 12\cdot c^3 + 54\cdot c^2 - 108\cdot c + 81$

#25: $RED(u, w) = 2\cdot b^2 + b\cdot(2\cdot c - 6) + 2\cdot c^2 - 6\cdot c$

#26: $RED(v, w) = 3\cdot b^2\cdot(3 - c) - 3\cdot b\cdot(c^2 - 6\cdot c + 9) + 3\cdot(3\cdot c^2 - 9\cdot c + 1)$

#27: $x\_new := 2\cdot b^4 + b^3\cdot(4\cdot c - 12) + b^2\cdot(6\cdot c^2 - 36\cdot c + 54) + b\cdot(4\cdot c^3 - 36\cdot c^2 + 108\cdot c - 108) + 2\cdot c^4 - 12\cdot c^3 + 54\cdot c^2 - 108\cdot c + 81$

#28: $u\_new := 2\cdot b^2 + b\cdot(2\cdot c - 6) + 2\cdot c^2 - 6\cdot c$

#29: $v\_new := 3\cdot b^2\cdot(3 - c) - 3\cdot b\cdot(c^2 - 6\cdot c + 9) + 3\cdot(3\cdot c^2 - 9\cdot c + 1)$

Next step:

#30: $RED(x\_new, u\_new = 0) = 12\cdot c^3 - 36\cdot c^2 + 81$

#31: $RED(v\_new, u\_new = 0) = 3\cdot c^3 - 9\cdot c^2 + 3$

#32: $\left[x\_next := 12\cdot c^3 - 36\cdot c^2 + 81,\ v\_next := 3\cdot c^3 - 9\cdot c^2 + 3\right]$

#33: RED(x_next, v_next) = 69

No try applying the implemented GROEBNER_BASIS-function:

#34: $GROEBNER\_BASIS(\left[a^2 + b^2 + c^2 - 9,\ a^3 + b^3 + c^3 - 24,\ a + b + c - 3\right], [a, b, c]) = \left[c^3 - 3\cdot c^2 + 1,\ b^2 + b\cdot(c - 3) + c^2 - 3\cdot c,\ a + b + c - 3\right]$

Solving vor c and considering the symmetry of all solutions I try a combination of the solutions:

#35: $SOLVE(c^3 - 3\cdot c^2 + 1, c) = \left(c = 1 - 2\cdot COS\left(\frac{2\cdot\pi}{9}\right) \lor c = 2\cdot COS\left(\frac{\pi}{9}\right) + 1 \lor c = 1 - 2\cdot SIN\left(\frac{\pi}{18}\right)\right)$

#36: $\left(1 - 2\cdot COS\left(\frac{2\cdot\pi}{9}\right)\right)^4 + \left(2\cdot COS\left(\frac{\pi}{9}\right) + 1\right)^4 + \left(1 - 2\cdot SIN\left(\frac{\pi}{18}\right)\right)^4$

#37: $- 2\cdot COS\left(\frac{2\cdot\pi}{9}\right) + COS\left(\frac{\pi}{9}\right)\cdot\left(16\cdot SIN\left(\frac{\pi}{18}\right) - 20\right) + 22\cdot\sqrt{3}\cdot SIN\left(\frac{2\cdot\pi}{9}\right) - 16\cdot SIN\left(\frac{\pi}{18}\right) + 65$

#38: 69

We try solving the system with DERIVE 6 in one single step:

$$\#39: \quad \text{SOLVE}(a^2 + b^2 + c^2 - 9 = 0 \wedge a^3 + b^3 + c^3 - 24 = 0 \wedge a + b + c - 3 = 0, [a, b, c])$$

$$\#40: \quad \left[a = -\frac{\sqrt{2}\cdot\sqrt{4\cdot\text{COS}\left(\frac{2\cdot\pi}{9}\right) - \text{COS}\left(\frac{\pi}{9}\right) + 4\cdot\text{SIN}\left(\frac{\pi}{18}\right) + 3}}{2} + \text{SIN}\left(\frac{\pi}{18}\right) + 1 \right.$$

.....

This is again a huge expression, which gives approximated:

```
#41:  (((a = 2.879385241 ∧ b = -0.5320888862) ∨ (a = -0.5320888862 ∧ b =
        2.879385241)) ∧ c = 0.6527036446) ∨ (a = 0.6527036446 ∧ ((b =
        -0.5320888862 ∧ c = 2.879385241) ∨ (b = 2.879385241 ∧ c =
        -0.5320888862))) ∨ (((a = -0.5320888862 ∧ c = 2.879385241) ∨ (a =
        2.879385241 ∧ c = -0.5320888862)) ∧ b = 0.6527036446)
```

One solution combination for a, b, and c leads to the final result 69.

THE TI-92 CORNER
(edited by Bert Waits, Bernhard Kutzler and Frank Demana

Introduction

We have waited many years for a powerful, easy to use, inexpensive, hand-held *DERIVE* like computer symbolic algebra (CSA) system like the TI-92. The algorithms of *DERIVE* are the foundation of the TI-92 CSA system. We are very pleased that Josef has allowed us to offer this new "TI-92 CORNER" as an addition to his wonderful International *DERIVE* Newsletter. We recognize that the CSA is not a complete *DERIVE* system but the TI-92 is sufficiently powerful to be of great practical and pedagogical value to both students and teachers in many levels of mathematics.

The CORNER is for YOU – we need your contributions! We hope to have enough reader contributions to make the TI-92 CORNER a regular feature of the International *DERIVE* Newsletter. We solicit TI-92 contributions in the following areas:

1. Mathematical examples
2. Pedagogical examples
3. Interactive scripts using the TI-92 text editor
4. Programs
5. Any combination of any above!
6. Letters to the editor (including questions about the TI-92 CSA)

Please send the contributions to the TI-92 Corner Editors c/o Bert Waits, The Ohio State University, Department of Mathematics, 231 W. 18th Ave., Columbus Ojio 43210, USA. Contributions may also be sent electronically to waitsb@math.ohio-state.edu

Bert Waits, Berhnard Kutzler, and Frank Demana, March 1996

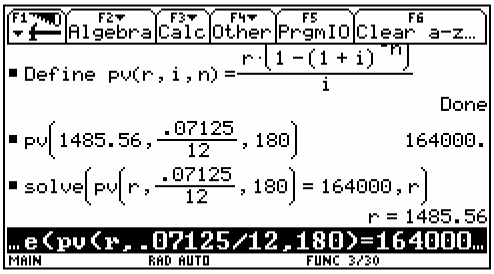
You can teach the TI-92 mathematics it does not know!

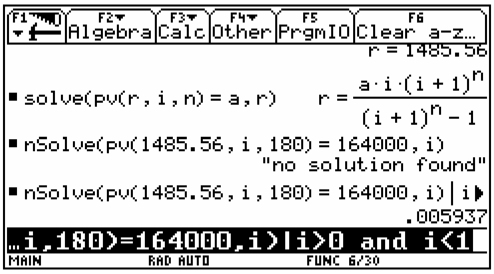
by Bert Waits and Frank Demana

The ability to easily great user defined functions on the TI-92 is a very useful feature. For example, the “present value” (value today) of a series of future payments is not a built-in function on the TI-92. However, such a mathematics of finance function can become a “permanent” feature of the TI-92!

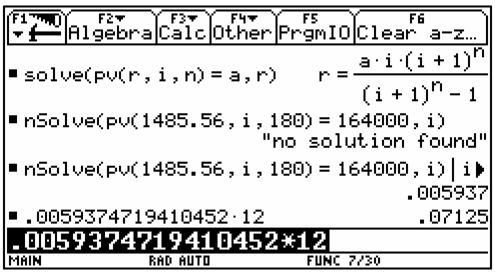
Consider a series of n monthly payments of R – at the end of each month – each at an annual percentage rate (APR) of $12*i$. The basic present value annuity formula is given by $PV = \frac{R(1+(1+i)^{-n}}{i}$.

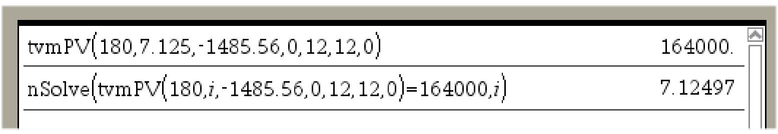
The following TI-92 screens show the results of using the TI-92 “Define” command along with illustrating other TI-92 CSA commands.





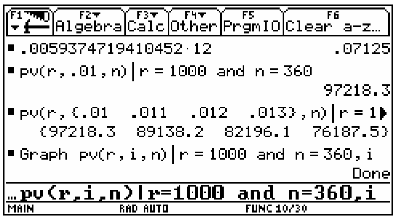
Note in the last line of the second panel the use of the vertical bar | (= with”) ([2nd] [K] to apply bounds ($i > 0$ and $i < 1$) for the nSolve algorithm to find a numerical solution for i (there is no explicit closed form solution for i). We are using the default “display digits” setting of 6. However, when selected and pasted in the entry line all digits available are shown and used in the computation of $12*i$. It is very useful to note that any expression or answer in the “history” screen can be selected and pasted in the entry line by repeated pressing of the “up” cursor key (on the large wheel) and then pressing “ENTER” in the desired line.

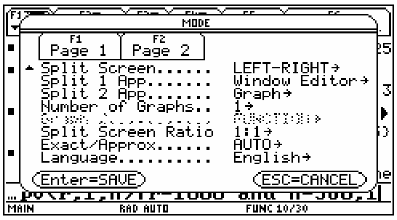


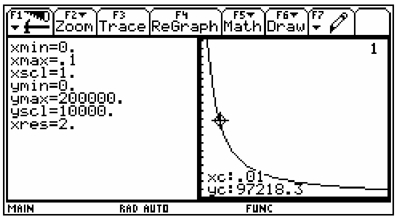


As you can see in the screen shot above TI-NspireCAS (and the Voyage 200 / TI-92 PLUS as well) have the finance mathematics functions implemented (Time-Value-Money-Solver).

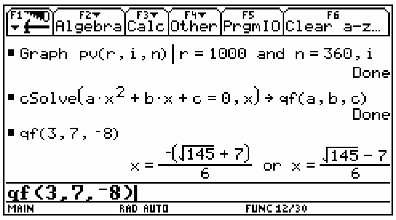
Lists and the Graph command can be used to illustrate interesting relationships between the variables (in this case between the present value, *pv*, and the interest rate, *i*). We also are illustrating a use of the split screen mode as well (Page 2 of the MODE menu).

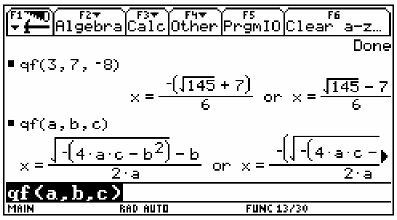






Just for fun we "teach" the TI-92 the quadratic formula!



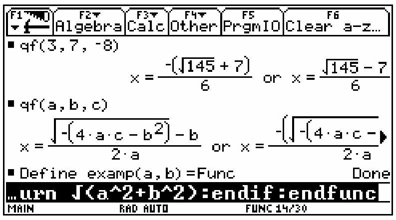


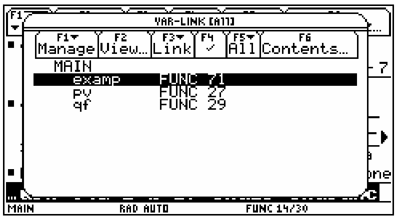
Note the use of the STO▶ key rather than the "Define" command to define functions. Either method may be used on the TI-92.

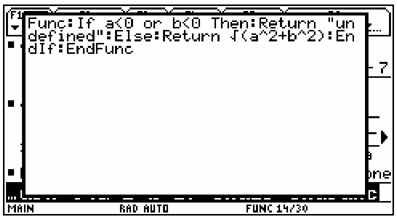
Both the "pv" and "qf" user defined functions can be placed in a special folder and on a custom menu as well. Also user defined functions can involve multi-line statements. The following is an example of such a user defined function. It is typed directly in the entry line.

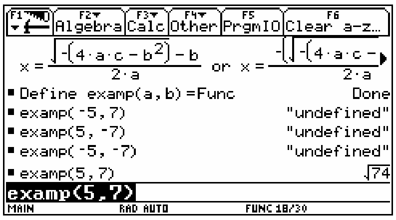
Define examp(a,b)=Func:If a<0 or b<0 Then:Return "undefined":Else:Return √(a^2+b^2):EndIf:EndFunc

There is a lovely "contents" function [F6] found in the VAR-LINK menu that shows exactly what you defined.









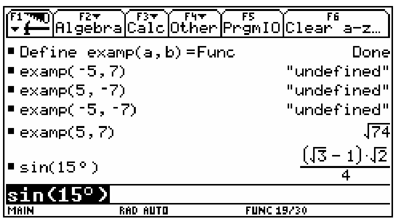
Here is an exercise:

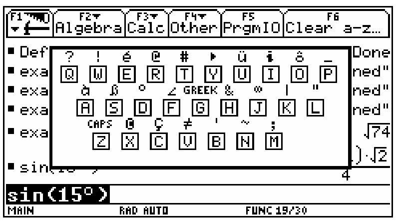
Eigenvalue is not a built-in command on the TI-92.

Create a definition for the TI-92 that solves this problem.

(See page 59.)

TI-92 Tipp of the day: Leave your calculator in the radian mode. When you want "degrees" paste in the degree symbol by pressing [2nd] [D] after the number. Hint: A "keyboard map" of the [2nd] function keys can be found by pressing [◆] [K].



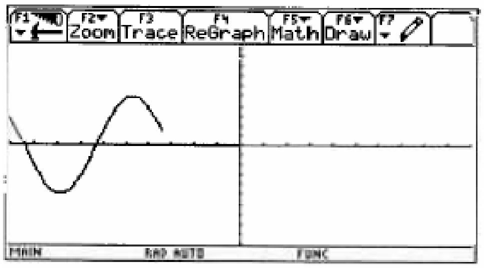


TI-92 – TIPPS AND TRICKS
(BY BERNHARD KUTZLER)

In this article we will provide you with useful hints on the use of the TI-92. Some of the examples are taken from our work at the *bk teachware* product hotline. The other examples are taken from a FAQ (= frequently asked questions) document produced by David Stoutemyer.

☹ How can I interrupt the plotting of a graph?

☺ Simply press [ON].

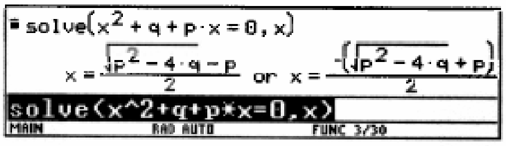


☹ How can one plot the family of curves [sin*x*, sin(2*x*), sin(3*x*)]?

☺ Use one of the following commands:
- Graph sin({1,2,3}x)
- Graph seq(sin(k*x),k,1,3)

☹ Can the TI-92 solve a generic quadratic equation? I get the following result:

☺ Obviouslx you have entered ‘px’ instead of ‘p*x’. ‘px’ designates a 2-character variable different. (There is no character mode for entering variables like in DERIVE, so you have to use the multiplication symbol or a space.)



☹ How can one see all the second functions of the QWERTY keyboard?

☺ Simply press [◆] [K] to obtain the respective keyboard map (see page 57).

☹ What do ‘cos(inf)’, ‘cos(-inf)’ or ’cos(undef)’ mean in an answer?

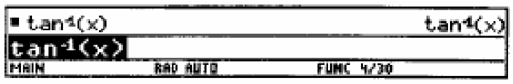
☺ These designate the interval -1 through 1.

☹ Why can´t one use curly braces to delimit function arguments?

☺ Unlike elementary arithmetic and algebra, curly braces are used only for lists. Use only parentheses to delimit function arguments.

☹ Why doesn’t ‘atan(…)’ or ‘arctan(…)’ simplify to …?

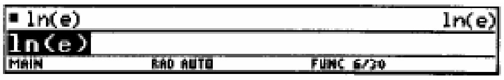
☺ These do not designate the inverse of the tangent function. Use [2nd] [TAN] for the arc-tangent function.



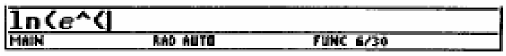
☹ Why doesn’t ‘lim(…)’ simplify to …?

☺ Although it is abbreviated to ‘lim’ on the home screen, the name of the function is ‘limit’ (see line editor).

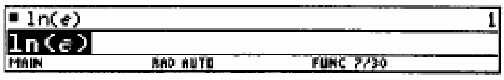
☹ Why doesn’t ‘ln(e)’ simplify to 1?



☺ ‘e’ is an ordinary variable, just like ‘x’. To denote the exact base of the natural logarithm, italic *e*=2.7182…, press [2nd] [LN],



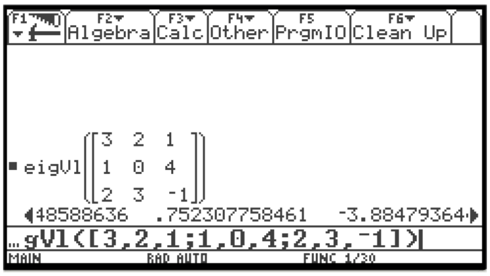
then press the [←] key twice to delete the ‘^(’ and enter a closing parenthesis.



☹ Why does ‘solve(a*x=0,x)’ return the following result?

☺ $a = 0$ means that when $a = 0$, any value of x satisfies the equation.

Eigenvectors and Eigenvalues are now built-in functions in the TI-handheld devices (and in TI-NspireCAS as well).



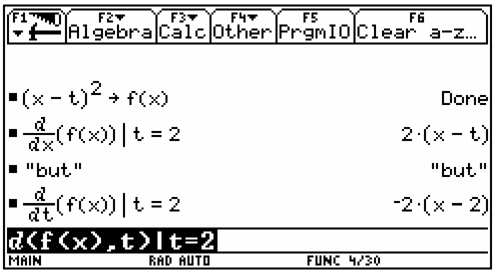
Wolfgang Pröpper, Nürnberg, Germany

The TI-92 seems not to like families of curves. I define a family of parabolas: Define f(x)=(x-t)^2. If I want to substitute *t* by 2 using the wonderful with-operator then I am failing: f(x)|t=2 does not return $(x-2)^2$, but $(t-x)^2$. And solve(f(x)=0,x)|t=2 returns *t* instead of 2. And if I want to plot graph f(x)|t=2 then I receive an error message.

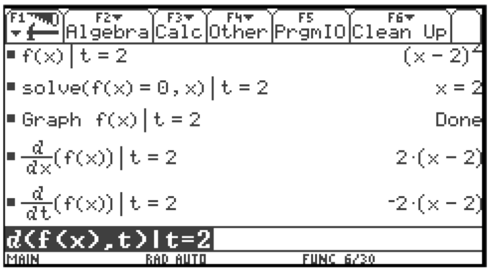
Yes, I know, there are existing other ways:

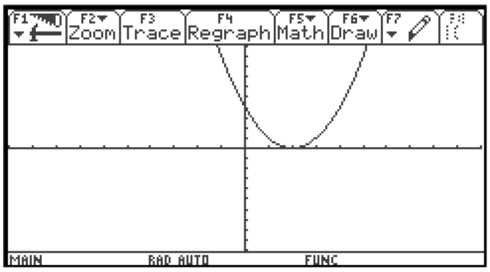
- solve((x–t)^2=0,x)|t=2 or graph (x–t)^2|t=2. But why did I define the function earlier?
- It is nice to have a list for the values {-1,0,2}→t and then be able to produce all solutions and all the graphs in one step, but having done the job I have to delete the variable with DelVar t.
- Or I define f=(x–t)^2 and go on. But doing so I will miss the functional point of view.
- Or I define f(x,t)=(x–t)^2 and use the with-operator. I regret that the function variable *x* lies in the same logical level as the parameter *t*.

I would like the idea that D. Stoutemyer could teach the chip to make all the variables of the function term local to an actual folder in order that all the variables could be substituted applying the with-operator. This is possible. Look at the following:



These problems (bugs?) have been fixed in later Operations Systems. See the respective screens of the TI-92+ (or Voyage 200). There are no problems with TI-NspireCAS.





Another thing: One problem of the last Bavarian end examination was to investigate the function $f(x) = \frac{4x-4}{x^2-2x+2}$. In one part of this problem the students had to show that $f(x)$ is reversible in $]-\infty,0]$, and after verifying $f(-\sqrt{2}) = -\sqrt{2}$ give reasons why $S(-\sqrt{2}, -\sqrt{2})$ is the intersection point of f and its inverse f^{-1}. The students were not asked to find the inverse function. But … the TI-92 freak tries to let the machine do this work. Let´s go on:

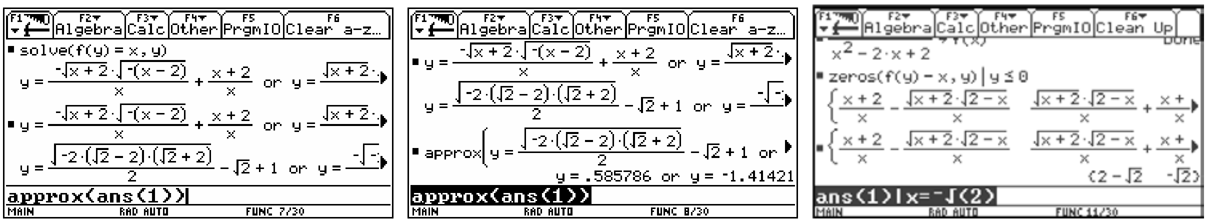
solve(f(y)=x,y)|y≤0 should help??? We receive the following result:

$$y = \frac{-\sqrt{x+2}\cdot\sqrt{-(x-2)}}{x} + \frac{x+2}{x} \text{ or } y = \frac{\sqrt{x+2}\cdot\sqrt{-(x-2)}}{x} + \frac{x+2}{x}.$$

Each other restriction behind the with-operator will lead to the same results. The mathematician in the teacher will see – immediately? – that only the second solution contains the required solution. Using *Copy* and *Paste* we could define g(x) = 2nd solution. Now our little machine should evaluate the function value for $x = -\sqrt{2}$, so ans(1)|x=-√(2). What has happened now? Has the TI-92 really lost its mind? It presents two remarkable expressions:

$$y = \frac{\sqrt{-2(\sqrt{2}-2)(\sqrt{2}+2)}}{2} - \sqrt{2} + 1 \text{ or } y = \frac{-\sqrt{-2(\sqrt{2}-2)(\sqrt{2}+2)}}{2} - \sqrt{2} + 1!!!$$

After some attempts it maybe that you try solve(f(y)=x,y)|x≤0. You will find again the two solutions but now ans(1)|x=-√(2) gives y = -√(2) + 2 or y= -√(2). Could anybody give an explanation why the TI-92 is calculating the same problem in different ways?



You can see two screens from the TI-92 (left) and one screen from the Voyage 200 (right). Later Operating Systems behaved better. The values given by the first TI-92 generation were correct – see the approximated values. Evaluating nested roots form a serious problem for computeralgebra systems[1]. The DERIVE expressions below confirm the identity of the expressions. Josef

$$\left[\frac{\sqrt{(-2\cdot(\sqrt{2}-2)\cdot(\sqrt{2}+2))}}{2} - \sqrt{2} + 1,\ -\frac{\sqrt{(-2\cdot(\sqrt{2}-2)\cdot(\sqrt{2}+2))}}{2} - \sqrt{2} + 1\right]$$

$$[2 - \sqrt{2},\ -\sqrt{2}]$$

SWHH (Albert Rich)

Concerning the TI-92 tech support question from W. Pröpper: TI wants TI-92 tech support questions to go through their designated tech support so that they can keep records of requests for future products. They in turn occasionally refer new questions to Dave (Stoutemyer) that only he can answer, and then he replies to TI.

[1] D. Jeffrey and Albert Rich: Simplifying Square Roots of Square Roots by Denesting in Computer Algebra Systems, ed. by Michael J. Wester, Wiley, 1999