THE DERIVE - NEWSLETTER #36

THE BULLETIN OF THE

DERIVE®

USER GROUP

+ TI 92

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Revised 2016 **December 1999**

[1] **Experiments in Motion Using the CBR and TI-89**, A Manual of Activities and their Analysis, H-D Hinkelmann, bk teachware 1999, SL-10, 52 pages and diskette.

[2] **Chaos erforschen mit dem TI-92**, R. Albers, bk teachware 1999, SR-12, 62 pages

[3] **Computational Methods for Representations of Groups and Algebras**, P Dräxler a.o., Ed, Birkhäuser 1999, ISBN 3-7643-6063-1
This is a collection of refereed papers from the Euroconference in Essen, 1997. There are a number of invited and survey contributions which provide an overview of this active area of research at the intersection of mathematics and computer science. In 3 extended introductory and 20 keynote articles 35 experts among others deal with Noncommutative Gröbner Bases, Construction of Finite Matrix Groups, ...

Cornell University Library Math Book Collection
http://moa.cit.cornell.edu/dienst-data/cdl-math-browse.html (not valid)

The Cornell University Library has scanned over 570 original mathbooks from their collection and placed them online. The volumes can be accessed here in their entirety. The collection may prove especially useful to mathematicians without access to a first-rate math library. A great many of the books are in languages other than English, notably French and German. Among these historically significant books are a number of works by Bernoulli, Descartes, Hardy, and Poincare. This is an interesting resource which offers a glimpse into the possible future of libraries, and books, for that matter.

Announcement of two "Must"-Conferences for the DUG-family – to exchange experiences, to meet old friends and to make new friends. Let us again enjoy the atmosphere and the spirit of DERIVE/TI-92-89 Conferences.

4th International DERIVE – TI92/89 Conference
Computer Algebra in Math Education
12 – 15 July 2000
Liverpool John Moores University, UK
inform and register at: http://www.livjm.ac.uk/derive2k/

Exam Questions and Basic Skills in a Technology-Supported Mathematics
2 – 5 July 2000
Portoroz, Slovenia
inform and register at: http://www.kutzler.com/acdca-00/

A Collection of interesting web sites :(many of them recommended by Tony Watkins, thanks.)

www.caacentre.ac.uk/math.shtml	Computer-assisted Assessment Centre
www.teleregion.co.uk/mateacher	Demoversion of an educational Software (Leonardo)
archives.math.utk.edu/ICTCM/	Proceedings of the ICTCMs (valid 2016)
webwork.math.rochester.edu/	Login as "practise 1" (or 2 or 3)
www.collegeboard.org	Samples from the US SATs and PSATs
gretton.net	More about Assessment (valid 2016)
math.bu.edu/DYSYS	Tools to teach Dynamical Systems
www.math.ohio-state.edu/~elaughbaum/	Ed Laughbaum´s Academic Page (valid)
www.exeter.edu/~rparris	A lot of useful tools for the Math teacher
aurora.tuwien.ac.at/~sleska	200 maths programs for DOS through WIN 9x
www.lindalv.skola.kungsbacka.se/ElofLindalv/Program/Nv/datamatte/datamatte.html	Computer Math page of D. Sjöstrand´s school

Happy Memories on the ACDCA Summer Academy 1999, Gösing

David and Karen Stoutemyer

Bea and Eugenio, both talking about DERIVE 5???

Hubert, the Leonardo Man , together with HR Geyer

Theresa and Nelson from SWWH

While two third of the organization commitee were working hard .....
(Bernhard Kutzler & Helmut Heugl)

.... co-chair Josef had a fine time on the tennis

What are they laughing about: Irish Coffee or Russian Roulette? Enoch and Sergey

Noor Böhm, the strong woman behind the DUG and the spouses program.

Hartmut Kümmel

Many thanks to Richard Schorn who sent so many photographs and made it possible to produce this page.

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE & TI-92 User Group*. It is published at least four times a year with a contents of 44 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE* and the *TI-92/89* as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

As many of the *DERIVE* Users are also using the *TI-92/89* the *DNL* tries to combine the applications of these modern technologies.

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Contributions:

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE & TI-92 Newsletter* will be.

Next issue: March 2000
Deadline 15 February 2000

Preview: Contributions for the next issues

Inverse Functions, Simultaneous Equations, Nested IF() versus ...., Speck, NZL
Reflections, Voigt, AUT
Symbolic Computation of exp(At) Matrix in DERIVE, Scheiber, ROM
Akima-Interpolation, Geruschkat, GER
3D-Geometry, Reichel, AUT
Graphic Integration, Linear Programming a.o., Böhm, AUT
A Utility file for complex dynamic systems, Lechner, AUT
Examples for Statistics, Roeloffs, NL
Quaternion Algebra, Sirota, RUS
Various Training Programs for the TI
A critical comment on the "Delayed Assignation" :==, Kümmel, GER
Sand Dunes, River Meander and Elastica, ..., Halprin, AUS
Type checking, Finite continued fractions, Winding number, Welke, GER
Share Holder´s Considerations using a CAS, Böhm, AUT
Kaprekar´s "Self numbers", GER
ODEs with Constant Coefficients, Fernandez, ARG
and
Setif, FRA; Vermeylen, BEL; Leinbach, USA; Wiesenbauer, AUT;
Aue, GER; Koller, AUT, Ibrahim & Córdoba, ESP, ...

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Allan S. Hugo allanh@usls.edu

Greetings DERIVERs!!

I searched the *DERIVE* archives for "chemical engineering" and rarely found few matches and surprising they are all posts coming from me. :)

Anyway, I have found a website

http://www.polymath-software.com/ASEE/

It compares the use of different mathematical packages namely EXCEL, MAPLE, MATHCAD, MATHEMATICA, MATLAB and POLYMATH. All these are compared on how they are able to solve ten representative problems in chemical engineering.

The problems are described and solutions are discussed. Along with them are found the solutions using the six packages. (See one - easy - TI- and *DERIVE* treated example on page 36, Josef)

May I invite anyone interested to visit the site and download the problems and their solutions. (ACROBAT READER needed). I would like to see how DERIVE with its versatility solves the problems? I haven´t mastered DERIVE yet and am still waiting for version 5 to come out. But, for me, DERIVE is still the best!

For chemical engineers in the list, may I also invite you to join computer applications in chemical engineering electronic group.

Perform a web search "Computer Applications in Chemical Engineering"

Hubert Voigt, Perg, Austria voigthubert@hotmail.com

Lieber Josef, Dear Josef,

Just now I came across a bad bug (???) on the TI-92. We explored the discontinuities of

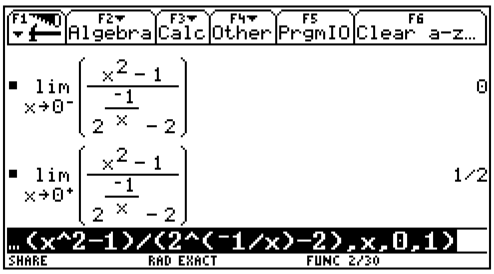
$$f(x) = \frac{x^2 - 1}{2^{-\frac{1}{x}} - 2}$$

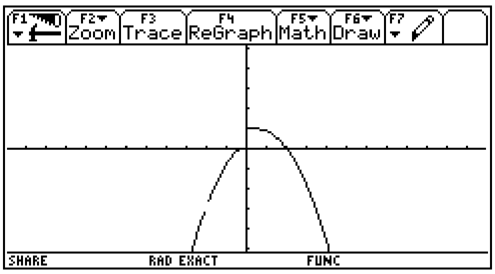
at x = -1 and x = 0. As we have the case of an indeterminate form 0/0 for x = 0 we calculate the left-hand- and right-hand limit. In Mode APPROXIMATE we obtain solutions 0!!!, in Mode EXACT the correct solutions. The same problem occurs with

$f(x) = e^{\frac{1}{2x-4}}$ for $x = 2$. In Mode APPROXIMATE the answer is undefined.

Best regards, Hubert

DNL: In the manual you can find the hint:.....When possible avoid the APPROX setting and approximate numbers when computing limits ....





DERIVE and TI-NspireCAS return the correct results in both simplification modes.

Volker Loose, Halter, Germany vwloose@cityweb.de

Hello all,
is it possible to convert with *DERIVE* numbers to the corresponding ASCII-characters and vice versa like the functions `chr` and `ord` in Pascal?
Thanks, Volker

DNL: A bad and a good answer, first the bad one: it is not with *DERIVE* version 4. The good one: it will be able in *DERIVE 5* and in *DERIVE 6*, of course.

See the following example:

```
NAME_TO_CODES(Happy New Year 2000)

[72,97,112,112,121,32,78,101,119,32,89,101,97,114,32,50,48,48,48]

CODES_TO_NAME([72,97,112,112,121,32,78,101,119,32,89,101,97,114,32,50,48,48
,48])

Happy New Year 2000
```

These functions are useful for string processing and cryptographic applications.

David Stegenga, Hawaii dave@math.hawaii.edu

Ralph Freese and I have written a new lab manual for calculus that contains a lot of the things you mentioned in your mail.

A free copy (227 pages) can be downloaded in PDF format from the web at:

http://www.math.hawaii.edu/RDPublishing/CalcLabBook/main-rev.pdf

On Primality test

John Feth John.Feth@cas.honeywell.com

I'm a happy user of Derive XM v3.14 and am curious about the NEXT_PRIME(m,n) function. How does the "probabilistic Monte Carlo primality test" work, and after testing n times, what is the confidence level that the identified number is prime?

I read somewhere (MMA documentation if memory serves) that the Rabin-Miller algorithm is shown to be valid only for primes less than some (large) finite number, and although I don't have any idea why this assertion is true, it prompts the question: Is there any size limit to primes discovered with the probabilistic Monte Carlo primality test method, beyond which the method may not reliably produce primes? I've used Derive to find primes thousands of digits in length and they seem to act like primes when I use them, but I'm effectively taking their primality on faith. Of course, Derive has never let me down.

Thanks for your time and attention,

John Feth

Ignacio Larrosa Cañestro, A Coruña, Spain ilarrosa@lander.es

I also am very interesting in know more of DERIVE primality test, as John Feth. I am using version 4.11 and so I can use de Prime(m,n) function. What is the 'Monte Carlo primality test and how work?

Two consecutive tests Prime(m) are identicals, or are there randomize in some sense?

What is the confidence level with a some thousands decimal digit number?

There are knew examples of 'Derive-probable-primes' that are really compose?
Thanks in advance,

Johann Wiesenbauer, Vienna, Austria J.Wiesenbauer@tuwien.ac.at

According to what was published by Soft Warhouse itself ("Inside the DERIVE Computer Algebra System", Int. DERIVE Journal, Vol.1, Nr.1, 1994) NEXT_PRIME(m,n) (as well as PRIME(m,n) for thast matter) uses trial division by all primes <2^10 followed by n Rabin-Miller tests (if n is not specified the default value 6 is used in DFW 4.11)

The Rabin-Miller Test for an ODD (!) m>1 with respect to the base a works as follows. If

m=s2^t+1 for positive integers s,t, where s is odd

then m passes the Rabin-Miller test w.r.t. a, if and only if the sequence

a^s, a^(2s),a^(4s),a^(8s),...,a^((m-1)/2) mod m

starts with 1 or contains -1. A DERIVE-implementation could look like this (default a = 6):

```
rabin_miller(m,a:=6):=IF((ITERATE(IF(k_=2∨a_=-1,[a_,k_],
      [MODS(a_^2,m),k_/2]),[a_,k_],ITERATE([-ABS(MODS(a^o_,m)),
      (m-1)/o_],o_,ITERATE(IF(MOD(m_,2)=1,m_,m_/2),m_,m-1),1)))↓1
       =-1,true,false)
```

```
#2:   NEXT_PRIME(100000000000) = 100000000003

#3:   rabin_miller(100000000003) = true

#4:   rabin_miller(100000000007) = false
```

As it is well-known for ANY given set S of bases there are INFINITELY many composite numbers which will pass the Rabin-Miller test for all bases in S. As a consequence, there are also infinitely many composite numbers which pass the built-in primality test of DERIVE, e.g. 22564845703. It is not known in which way the bases are chosen by DERIVE, but at any rate this choice is the same in every run.

According to a well-known theorem by Rabin-Monier a number m that passes n Rabin-Miller tests is prime with a probability of error less than 4^(-n). Hence if you are a "safety addict" you should adjust n accordingly (for most purposes the default value n=6 is perfectly ok though as the bound given above is extremely bad in favour of the user!) Thus if wisely used DERIVE won't let you down!

Hope this answered some of the questions,

Cheers, Johann

Alfonso Población, Valladolid, Spain alfonso@gauss.mat.eup.uva.es

Dear Josef,

How were your summer holidays? I hope perfect.

I send you this mail to give you the references of a new Spanish DERIVE book for the Book-Shelf of the DNL:

> Prácticas de Matemáticas de Bachillerato con DERIVE para WINDOWS
> M. Ibañes, M.F. Pérez, A.J. Población, A. Suárez.
> 288 pages, Editorial Ra-Ma, ISBN 978-84-7897-367-2

This book shows 40 activities through the full mathematics syllabus for the Spanish Secondary level. It contains an introductory activity in the use of the Spanish version of DfW, 35 thought to be made in an hour class, and 4 miscellaneous ones concerning visualization in 3D and recreational mathematics aspects. The book includes the worksheets to answer the questions involved. Students may work through the problems individually or under the teacher's supervision. The authors have a wide experience in teaching using DERIVE at different levels.

The Editorial Web: http://www.ra-ma.es (valid 2016)

Well,that's all. Cheers from here.

Jan Vermeylen, Kapellen, Belgium math@rhombus.be

Hi all Derivians,

Where can I find the best documentation about solving partial differential equations with Derive ?

Best regards, Jan Vermeylen

Sergey Biryukow, Moscow, Russia ciprel@cityline.ru

Partial Differential Equations (PDE) in DERIVE

Dear Jan Vermeylen,

I do not know any books about solving Partial Differential Equations (PDE) in DERIVE but I think you can use nearly any University Textbook for numerical solutions (Daniel D. McCracken & William S. Dorn. Numerical methods and Fortran programming. John Willey & Sons, 1965, for instance) and other CAS oriented textbooks for symbolic solutions. Algorithms translation from one CAS to another is not too difficult as it seems to be.

The attached file GALERKIN.MTH is the solution of the regular PDE problem - Viscous Liquid Flow Velocity Distribution in a Square Tube. It is from Russian Textbook by

Govorukhin V.N. & Tsybulin V.G.

"**Introduction to Maple - Mathematical package for everybody**", Moscow, "Mir", 1997, 206 p.

Galerkins' method is used to find 4- and 25-terms solutions for trigonometric and polynomial basis functions. Basis functions are also used as checking ones. The file shows that in spite of the fact that Galerkins' method is developed for orthogonal functions, it works also with others sometimes. Most of the lines in the MTH-file are annotated with the description of definitions and transformations. Try it, please! The algorithm can be generalized to the arbitrary convex boundary. It will be a nice puzzle for top ten students to find an algorithm for generating basis functions for such a boundary.

Some useful information can be found at

http://www.physique.fundp.ac.be/physdpt/administration/convode.html

where you can get numerical & symbolic solutions of Differential Equations including Partial ones. This server is not far from you.

Sincerely, Sergey V. Biryukov

DNL: File GALERKIN.MTH mentioned by Sergey is on diskette. Josef

Munroe Clayton m_clayton@dreamscape.com

Differential Equations with Derive by David C. Arney ISBN 0-9623629-3-X chapter 7 "Partial Differential Equations" pages 253-274 is the only thing that I'm aware of and it's pretty brief.

Best regards, M.C."Moe" Clayton

Reinhard Schaeck schaeck@ibm.net

Following is an extract of a Derive-session (DfW 4.11):

```
K0(Kn, i, n) := Kn/(1 + i·n)
K0(40000, 4%, 6)

                4
3.225806451·10
```

I would prefer the result as:

```
32258.06451
```

Is there any way to define the output format this way?

Al Rich, SWHH, Hawaii Albert@derive.com

Hello Reinhard Schaeck,

Yes! First, you use the Options>Mode settings pull down menu to set the Precision to Approximate, and the Digits of Precision to 10 on the Simplification register card and the Notation to Decimal on the Output register card (for DERIVE 6). Finally, you simplify your expression to get the result displayed in decimal notation. The following is a transcript of the session.

```
Precision:=Approximate
PrecisionDigits:=10
Notation:=Decimal
K0(kn,i,n):=kn/(1+i*n)
K0(40000,4%,6)
32258.06451
```

Hope this helps. Aloha, Albert D. Rich (Co-author of DERIVE)

On the FIT-function

Allan Hugo allanh@usls.edu

I am having a little trouble using the FIT function.

From what I understand I will be using this function if I want a least squares fit.

And I would need to create a label vector and data sets.

What is a label vector? How do I create it? Or generally, can you give me an example on how to use the FIT function?

Say for instance, I have these sample (x,y) data:

(1,1.5), (2,3.6), (3,5.2), (4,7.8) (5,10.1).

And I want to find a function y=a*exp(x^b) where a and b are real constants. What I want is, after using the FIT function I should be able to get the appropriate value for a and b.

Thank you very much for your help.

Johann Wiesenbauer, Vienna, Austria J.Wiesenbauer@tuwien.ac.at

Hi Allan,
It looks as if you have overlooked the following passage in the DERIVE manual concerning `FIT()`:

The expression should depend on the data variables and one or more parametric variables. The dependence on the parametric variables should be LINEAR (...) The expression's dependence on the data variables needs not be linear.

This doesn't necessarily mean that the original expression must fulfill these conditions. If you take e.g. the similar looking though totally different function y=a exp(b*x) then by setting eta = ln y, alpha = ln a you get the new function

eta = alpha +b*x

which is clearly linear in the parameters alpha and b. Hence `FIT` can be applied to eta(alpha,b) to find alpha and b, and subsequently a, in the following way:

```
#1:   [Precision := Approximate, Notation := Decimal, NotationDigits := 6]

                ⎡ 1   1.5 ⎤
                ⎢ 2   3.6 ⎥
#2:   data :=   ⎢ 3   5.2 ⎥
                ⎢ 4   7.8 ⎥
                ⎣ 5  10.1 ⎦

#3:   APPROX(FIT([x, α + b·x], VECTOR([data   , LN(data   )], i, 5)))
                                        i,1           i,2

#4:   0.458733·x + 0.164144

#5:   APPROX(EXP(0.164144)) = 1.17838

                           0.458733·x
#6:   y1(x) := 1.17838·e
```

Well, as shown above everything is extremely simple when taking the common exponential function y = a exp(b*x) instead though it is also originally not of the required form! However, in the case of your function (which looks a little bit far-fetched to me, to be honest) I haven't the faintest idea how to tackle this problem!

Cheers, Johann

Some days later:

Contrary to what I said in a previous email there might be a chance to solve this problem using FIT() alone.
Taking the logarithm of y = a exp(b*x) twice leads to ln(ln(y/a)) = b*ln(x). so we define a function b_opt(a) that computes the optimal b for any fixed a, namely

$$\#7: \quad \text{b\_opt}(a) := x \cdot \frac{d}{dx}\, \text{APPROX}\left(\text{FIT}\left([x,\ b \cdot \text{LN}(x)],\ \text{VECTOR}\left(\left[\text{data}_{i,1},\ \text{LN}\left(\text{LN}\left(\frac{\text{data}_{i,2}}{a}\right)\right)\right],\ i,\ 5\right)\right)\right)$$

This is what you get after simplifying the last assignment:

$$\#8: \quad \text{b\_opt}(a) := 0.259607 \cdot \text{LN}\left(5.48535 \cdot 10^{6} \cdot \text{LN}\left(\frac{1}{a}\right) + 1.2685 \cdot 10^{7}\right) + 0.223613 \cdot \text{LN}\left(2.42111 \cdot 10^{6} \cdot \text{LN}\left(\frac{1}{a}\right) + 4.97327 \cdot 10^{6}\right) + 0.111806 \cdot \text{LN}\left(1.28171 \cdot 10^{6} \cdot \text{LN}\left(\frac{1}{a}\right) + 1.64178 \cdot 10^{6}\right) + 0.177209 \cdot \text{LN}\left(7.07036 \cdot 10^{5} \cdot \text{LN}\left(\frac{1}{a}\right) + 1.16566 \cdot 10^{6}\right) - 11.2747$$

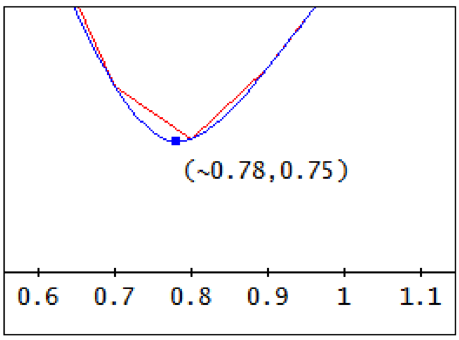
Now we define a function that measures the "deviation" of a EXP(x^b_opt(a)) from the set of given points:

$$\#9: \quad \text{dev}(a) := \left|\text{VECTOR}\left(a \cdot \text{EXP}\left(x^{\text{b\_opt}(a)}\right),\ x,\ 1,\ 5\right) - [1.5,\ 3.6,\ 5.2,\ 7.8,\ 10.1]\right|$$

Of course, we want dev(a) to become a minimum. After simplifying and plotting the expressions …

```
#10:  VECTOR([a, dev(a)], a, 0, 10, 0.1)

#11:  VECTOR([a, dev(a)], a, 0, 1, 0.01)
```



… you see that the minimum should be near a = 0.78. As a matter of fact

$$\#12: \quad y2(x) := 0.78 \cdot e^{x^{\text{b\_opt}(0.78)}}$$

$$\#13: \quad y2(x) := 0.78 \cdot e^{x^{0.591612}}$$

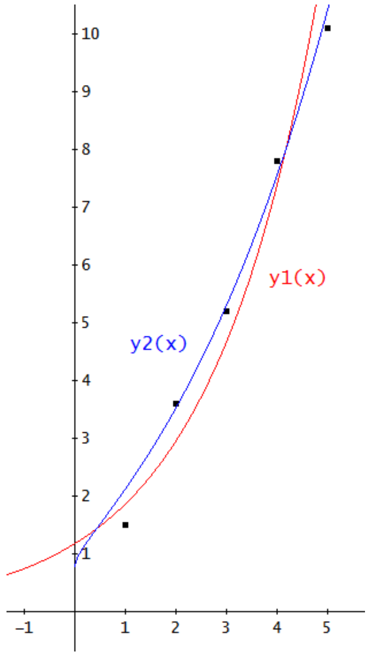
…

… is a nice approximation for the given set of points, which seems to indicate that the solution above makes sense!

Now compare both approximations:

Cheers, Johann

(fitfunc1.dfw)



Ivo de Pauw, Belgium depauwi.ph@hogeschool-wvl.be

Can you give me an answer concerning those number systems?
Does *DERIVE* e.g. facilitate (10)HEX = (16)DEC? I need it for my lessons here.
Thanks !

Yves De Racker, Antwerpen, Belgium yves_de_racker@hotmail.com

When we are working in school with numbers in a radix different from decimal (transformations from binary in octal, and then in hexadecimal for example), it seems there is a problem after reloading the *DERIVE* worksheet with <File-Load-Math> or <File-Open>. See details in the attachment!

```
#1:  Precision := Approximate
#2:  PrecisionDigits := 50
#3:  Notation := Decimal
#4:  InputBase := Hexadecimal
#5:  OutputBase := Hexadecimal
#6:  0A3FB.C98D
#7:  OutputBase := Octal
#8:  121773.623064
#9:  OutputBase := Binary
#10: 1010001111111011.1100100110001101
#11: "--------------------"
#12: Precision := Approximate
#13: PrecisionDigits := 10000000
#14: Notation := Decimal
#15: InputBase := Hexadecimal
#16: OutputBase := Hexadecimal
#17: 41979.7873077392578125
#18: OutputBase := Octal
#19: 1014571.360714073471112740224
#20: OutputBase := Binary
#21: 1000001100101111001.01111000011100110000011101110011100
#22: "--------------------"
```

```
#1:  Precision := Approximate
#2:  PrecisionDigits := 30
#3:  Notation := Decimal
#4:  InputBase := Binary
#5:  OutputBase := Binary
#6:  1110111110110100 1.10001110101
#7:  OutputBase := Octal
#8:  357551.4352
#9:  OutputBase := Hexadecimal
#10: 1DF69.8EA
#11: "------------------------------"
#12: Precision := Approximate
#13: Notation := Decimal
#14: InputBase := Binary
#15: OutputBase := Binary
#16: OutputBase := Octal
#17: OutputBase := Hexadecimal
#18: "------------------------------"
```

In Line ***#1*** to ***#10*** you can see the input of a hexadecimal number, followed by a transformation to an octal respectively a binary number . You can clearly remark the change of the statusvariable "***Radix***"***. Well, everything went normal during the input! No problem at that time!***
But then, I saved it on a disc; after reloading with <**File-Load-Math**> (or <**File-Open**>) I discovered a problemin the lines ***#12*** until ***#21***. You will see that the lines ***#17, #19*** and ***#21*** are completely different from ***#6, #8*** and ***#10***. It even can happen that you loose all the data lines; that happens when you start with a binay number followed with a octal and hexadecimal transformation (***2**[e]* ***image right hand side***).

Every time the computer tries to read such "***a lost line***" you see a small warning window with a message, ***"Error parsing expression # … in file ……."***

DNL: Dear Yves,
you posted a very interesting problem dealing with changing the base - by the way, I received a question concerning how to convert numbers from one base to the other from Belgium - and DERIVE´s strange behaviour reloading the file.

I think that this is easy to explain. Look at the *DERIVE* session below. Reloading this file I got the "error in parsing"-message together with expression #16, but why??

```
#1:  InputBase := Decimal                                          User
#2:  OutputBase := Decimal                                         User
#3:  Precision := Approximate                                      User
#4:  Notation := Decimal                                           User
#5:  NotationDigits := 30                                          User
#6: [100, 100000, 845.567]                                         User
#7: OutputBase := Hexadecimal                                      User
#8: [64, 186A0, 34D.91111111111111111111111111]           0.0s Simp(#6)
#9: OutputBase := Octal                                            User
#10: [144, 303240, 1515.44210421042104210421042104]       0.0s Simp(#6)
#11: OutputBase := Binary                                          User
0.0s Simp(#6)
#12: [1100100, 11000011010100000,
              1101001101.10010001000100010001]
#13: "To enter binary numbers I change the Input Base:"
#14: InputBase := Binary                                           User
#15: OutputBase := Binary                                          User
#16: [10000, 10111110, 10.1]                                       User
#17: OutputBase := Decimal                                         User
#18: [16, 190, 2.5]                                      0.0s Simp(#16)
#19: OutputBase := Octal                                           User
#20: [20, 276, 2.4]                                      0.0s Simp(#16)
#21: OutputBase := Hexadecimal                                     User
#22: [10, 0BE, 2.8]                                      0.0s Simp(#16)
```

During the session everything was ok. I printed the file and then saved it as `bases2.mth`. Now let´s look what *DERIVE* is doing and investigate how this file is saved. I opened `bases2.mth` with a text-editor:

```
#14: InputBase:=Binary
#15: OutputBase:=Binary
#16: [16,381,5.6875]
#17: OutputBase:=Decimal
#18: [16,381,5.6875]                                        Simp(#16)
#19: OutputBase:=Octal
#20: [16,381,5.6875]                                        Simp(#16)
#21: OutputBase:=Hexadecimal
#22: [16,381,5.6875]                                        Simp(#16)
```

Do you see what has happened? *DERIVE* saved the numbers as decimal numbers. In expression #14 we forced to read the input as binary numbers and in this "mood" nobody can understand the digit in "16", and so on. I think that this is the secret behind the story.

So unfortunately you cannot save your changes in bases for the "next generation".
Regards, Josef

Al Rich, SWHH, Hawaii Albert@derive.com

Hello all you radix base changers,

Yes, Josef has got it right. DERIVE needs a standard radix base when saving numbers in MTH files, so it uses decimal (i.e. base 10D), which is what the input radix base will normally be when loading MTH files.
However, the user can change the input base before loading the file or an assignment to the "Input-Base" control variable in the file itself can change the radix base used when reading numbers in the file.

This ability to change in radix base used when loading an MTH file makes it possible for DERIVE to load files created by other programs that store the numbers in bases other decimal.

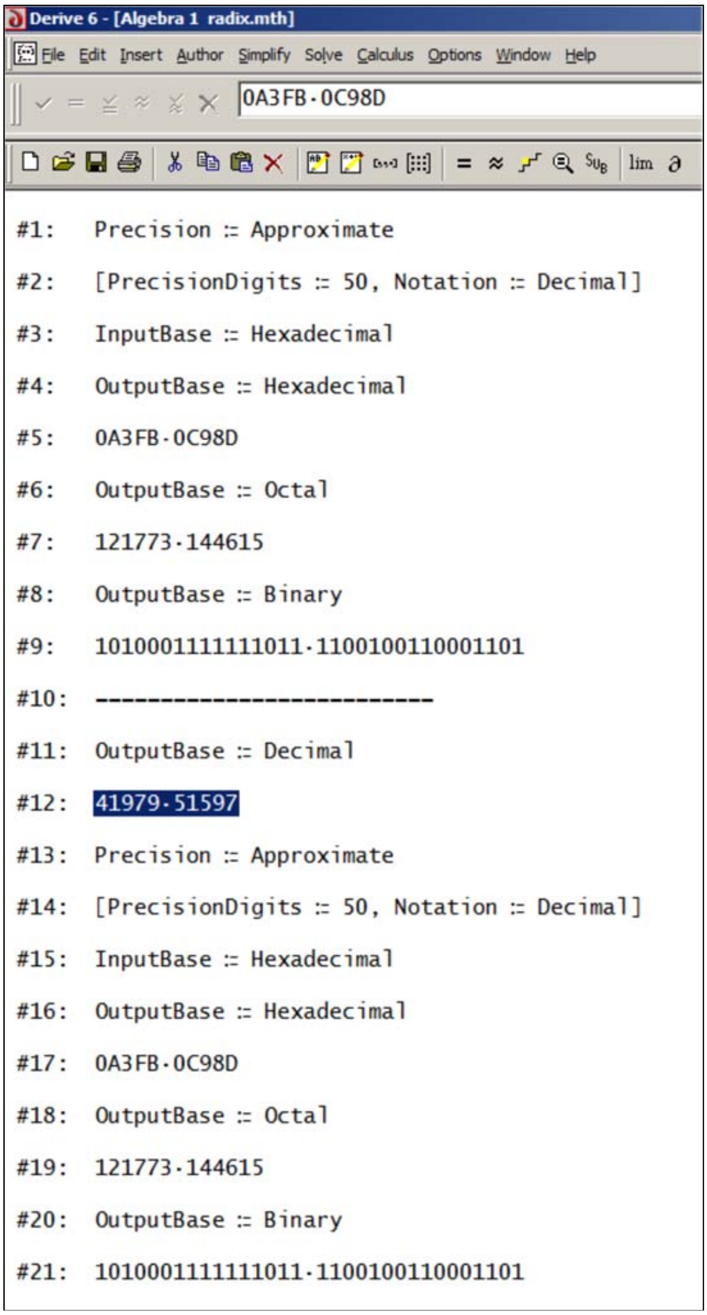
Aloha, Albert

This were the mails in 1999. I reproduced Yves' session now using DERIVE 6. There were no problems reloading the saved file.

I saved expressions #1 through #9 as radix.mth and then loaded the the so stored file. It appears identically (left screen shot). Same happened with the other procedure: No error message at all.

Please notice the entry line: 0A3FB·0C98D, which is the result of copying expression #12 via F3 from the Algebra Window into the entry line. Input base didn't change from Hexadecima!

Then I opened radix.mth with an editor and faced again all numbers in decimal base.




```
Precision:=Approximate
[PrecisionDigits:=50,Notation:=Decimal]
InputBase:=Hexadecimal
OutputBase:=Hexadecimal
41979*51597
OutputBase:=Octal
41979*51597
OutputBase:=Binary
41979*51597
OutputBase:=Decimal
41979*51597
```

Spheres tangents to four given planes

Staffan Björkenstam,
Elof Lindälvs gymnasium,
Kungsbacka, Sweden.

You can always find spheres tangent to four given nonparallel planes. The formula for the shortest distance from a point to a plane gives us the following system of equations.

$$r = \frac{|a_i \cdot x_0 + b_i \cdot y_0 + c_i \cdot z_0 + d_i|}{\sqrt{(a_i{}^2 + b_i{}^2 + c_i{}^2)}}, \; i = 1,2,3,4$$

Center of the sphere is the point (x_0, y_0, z_0) and the shortest distance from this point to the planes is the radius *r*.

$a_i x + b_i y + c_i z + d_i = 0$, $i = 1,2,3,4$ are the equations of the given planes.

Solving this system of equations algebraically would take to much time and the expression of the solution would be too big. Therefore, I decided to keep the coefficients algebraic for just one of the planes and then give numerical values to the rest of the three planes.
To solve the system of equations I used Derive. Derive could not solve the equations with the absolute value signs but by removing I received a solution. The system of equations was solved with respect to x_0, y_0, z_0, r. The expressions for x_0, y_0, z_0, r was put into the equation for a sphere:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

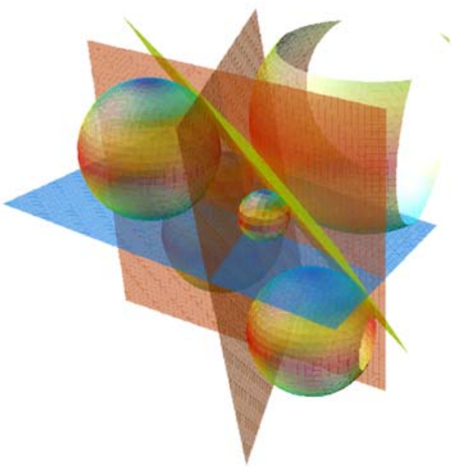
To plot the planes and the sphere I used DPGraph 2000 which is a superb tool for plotting 3-dimensional surfaces.
By defining a_1, b_1, c_1, d_1 (the coefficients I kept algebraic) in DPGraph 2000 you can change these values with a scrollbar and watch how one of the plane moves and how the sphere changes.

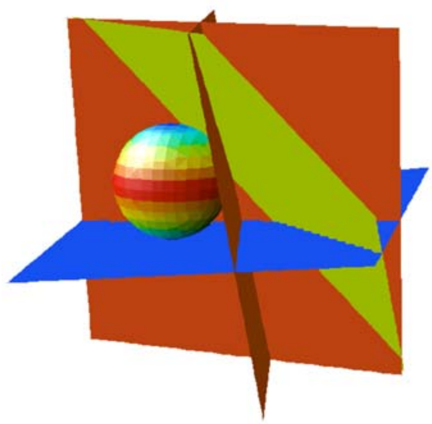
I also made a numerical solution. For that the Solver in Excel was a great tool. By giving different starting values you get the solution closest to these values. I managed to find eight solutions.

Pictures of my graphs and my DPGraph 2000 files are published at

http://www.davidparker.com/library.html (highly recommended, Josef)

If you have comments or if you would like to have a look at my files please send an email to stbjm@hotmail.com.





It is a special pleasure for me to present a contribution from Colombia in South America. It must be a good feeling for many DNL-authors and contributors to know, that the DUG is now also represented in Colombia.

LU-Factorization of a Matrix

Julio Cesar Morales C. Prof. Universidad Nacional de Colombia - Medellin branch.
Email: jcmorale@perseus.unalmed.edu.co

Help for the matrix functions package: LU_FACTR.MTH.

If you want to see examples about the use of the functions described in this document, you should load the demostration file LU_FACTR.DMO using the transfer/Demo command (in DfW it is File>Load-Demo).

-

Let A be an $m \times n$ real matrix. Gauss elimination to solve a system $A \cdot x = b$ is related to a **factorization** $\boldsymbol{A = L \cdot U}$ or to a factorization $P \cdot A = L \cdot U$.

Here U is an $m \times n$ echelon upper triangular matrix obtained by using Gauss elimination on the rows of the matrix A. When a zero pivot arises it searches for the first nonzero element below it, it exchanges the respective rows and proceeds.
L is a $m \times m$ lower triangular matrix with 1's on the main diagonal and P is a $m \times n$ permutation matrix (a matrix that has exactly one element equal to 1 in each row and column and zeros elsewhere). If row interchanges are made and P_i denotes the permutation matrix corresponding to the interchange required at the ith stage, then conceptually we are generating the triangular factorization of the matrix $P_{n-1} P_{n-2} \ldots P_2 P_1 = P \cdot A = P \cdot U$ rather than A itself. Thus the factorization is
$A = (P^{-1} \cdot L) \cdot U$.

The package LU_FACTR.MTH includes the functions **U_P(a)** and **L_U(a)**. They simlipify to the matrices [[U], [P]] and [[L], [U]] respectively, where $P \cdot A = L \cdot U$. If no row interchanges are made, P is an identity matrix.

Let me present a LU-decomposition using the functions:

$$a3 := \begin{bmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot a3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{true}$$

Here is another example:

```
#1:   LOAD(I:\DOKUS\DNLs\DNL99\MTH36\LU_FACTR_new.mth)
```

#2: example :=
$$\begin{bmatrix} 6 & 5 & 3 & -15 \\ 3 & 7 & -3 & 5 \\ 12 & 4 & 14 & 4 \\ 0 & 12 & 3 & -8 \end{bmatrix}$$

#3: U_P(example) =
$$\left[\begin{bmatrix} 6 & 0 & 0 & 0 \\ 5 & \frac{9}{2} & 0 & 0 \\ 3 & -\frac{9}{2} & 2 & 0 \\ -15 & \frac{25}{2} & \frac{152}{3} & -\frac{1264}{3} \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right]$$

#4: L_U(example) =
$$\left[\begin{bmatrix} 1 & \frac{1}{2} & 2 & 0 \\ 0 & 1 & -\frac{4}{3} & \frac{8}{3} \\ 0 & 0 & 1 & \frac{15}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 6 & 0 & 0 & 0 \\ 5 & \frac{9}{2} & 0 & 0 \\ 3 & -\frac{9}{2} & 2 & 0 \\ -15 & \frac{25}{2} & \frac{152}{3} & -\frac{1264}{3} \end{bmatrix}\right]$$

#5:
$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ 5 & \frac{9}{2} & 0 & 0 \\ 3 & -\frac{9}{2} & 2 & 0 \\ -15 & \frac{25}{2} & \frac{152}{3} & -\frac{1264}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{2} & 2 & 0 \\ 0 & 1 & -\frac{4}{3} & \frac{8}{3} \\ 0 & 0 & 1 & \frac{15}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 12 & 0 \\ 5 & 7 & 4 & 12 \\ 3 & -3 & 14 & 3 \\ -15 & 5 & 4 & -8 \end{bmatrix}$$

Challenge?

It would be great if a TI-Nspire programmer could develop a tool for LU-factorization with TI-NspireCAS?

Calculation meets Representation

Various Projections for 3D-Objects

Josef Böhm, Würmla, Austria

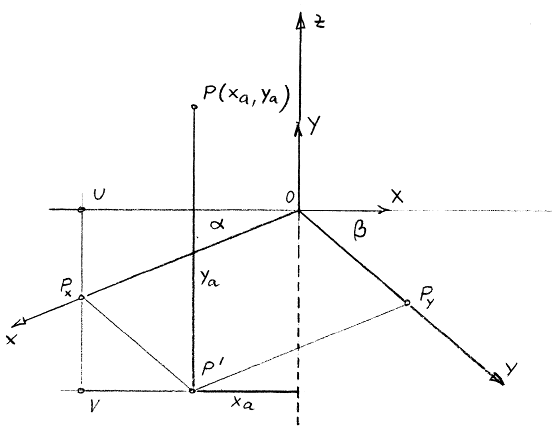
In many earlier DNLs you met contributions dealing with representations of 3D-objects, resulting from a mapping of the object into the *xy*-plane. There might be two reasons to use such mappings, the first one could be that we have not been able – up to now – to represent many types of 3D-objects in DERIVE´s 3D-Plot-Window and the second one is, that the mappings are an interesting and important application of working with matrices. In DERIVE´s utility file GRAPHICS.MTH you will find only one mapping to obtain a sort of an isometric picture. In DNL#27 Hubert Weller introduced two kinds of "Military Projections", Isometric and Dimetric Projection and a Cavalier Projection. They all are special variations of the general principle of parallel projection, called "Axonometric Projection". I will add the "Oblique View" and let us not forget the "basic" projections: Top View, Front View and Side View.

Additionally, it would be nice to have a tool to produce "Central Perspective Views" – i.e. projections of an object from one eye-point onto a plane.

The file PROJECT.MTH provides all the tools to create the mappings from above. All what you have to do is to produce – supported by DERIVE´s calculation power – lists of points in the right order.

One example follows: DERIVE helps to calculate the polygon created by intersecting a pyramid with a plane. Shading of the plane and the intersection figure support 3D-imagination and confirm the results.

In this article I´ll show how to find the axonometric and the perspective projection matrix, give several examples and show you by some figures how DERIVE for Windows 5 will handle 3D-objects – represent and animate them.



Let P(x_a, y_a) in *xy*-plane the projection of a point P(*x*,*y*,*z*) ∈ R^3. The sketch shows the projections of the axes. The projection is defined by angles α and ß between the mappings of *x*- and *y*-axis and *z*-axis and by factors v_x, v_y and v_z which are the truncation-factors for the *x*,*y*,*z*-coordinates of the points in space.

It is easy - even for students - to obtain x_a and y_a from the sketch – a nice trig application!

($OP_x = x \cdot v_x$ and $OP_y = y \cdot v_y$)

$x_a = -OU + VP' = -x \cdot v_x \cos\alpha + y \cdot v_y \cos ß$

$y_a = -UP_x - P_xV + P'P =$

$= -x \cdot v_x \sin\alpha - y \cdot v_y \sin ß + z \cdot v_z$

The projection can be written in matrix form: $(x_a, y_a) = (x, y, z) \cdot \begin{pmatrix} -v_x \cos\alpha & -v_x \sin\alpha \\ v_y \cos ß & -v_y \sin ß \\ 0 & v_z \end{pmatrix}$.

So we have AXOPIC(obj,vx,xy,vz,α,β) function to produce an arbitrary axonometric picture of object obj, which should be a list of points to be connected.

Derivation of the formula for central perspective mapping is given at the end of the contribution!

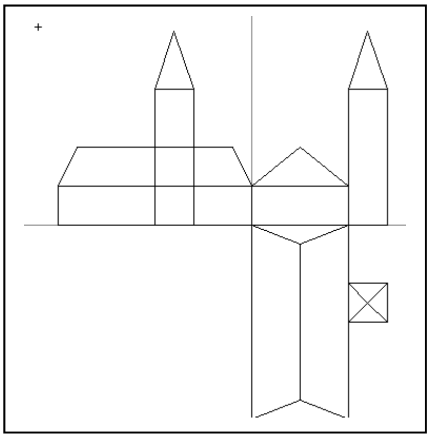
Special choices for the parameters give special projections:

Projection	v_x	v_y	v_z	α	ß
Oblique View	$0.5 \le v_x \le 0.75$	$0.5 \le v_y \le 0.75$	1	any	any
Isometric Projection	1	1	1	30°	30°
Dimetric Projection	0.5	1	1	41.4°	7.18°
Cavalier Projection	$\sqrt{5}/4$	1	1	atan(1/2)	0
Military Projection I	1	1	0.2	30°	60°
Military Projection II	1	1	0.5	30°	30°
Top View	1	1	0	90°	0
Front View	0	1	1	0	0
Side View	1	0	1	0	0

I don´t want to show the whole file listing. You can find it on the diskette. I recommend to load the file PROJECT.MTH as MTH-file or as utility file in the background. Then load PROJDEMO.DMO as a Demo-file and work through this demonstration, leave it whenever you want and plot the figures in DERIVE´s 2D-Plot Window.

The file shows two different ways to create a polyhedron). You can easily follow the procedure running the DMO-file. See also DNLs #24, #26, #27, #28, #29, #35 - Weller, Kümmel, Geyer & Böhm.

Project_new.dfw is the file which can be opened with DERIVE 6 and enables all plots.

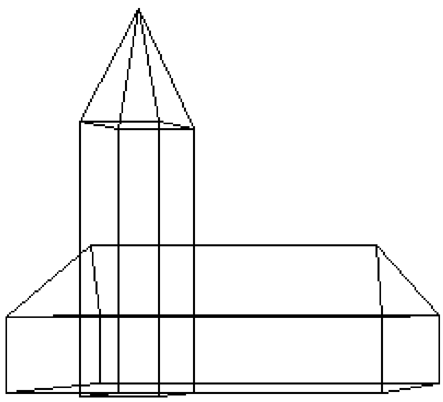


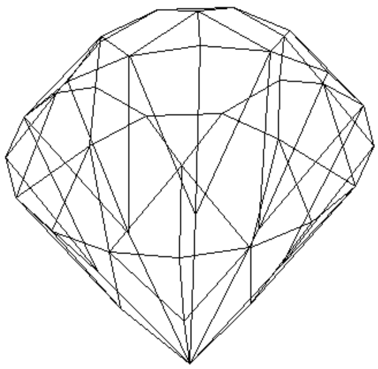
He conours of a house and a tower are given by their points and lists how to connect them; in a combination they form a church.

[TOPV(church),FRONTV(church),SIDEV(church)]

returns a nice drawing of the building. To have a more realistic picture call a perspective view (see below):

CENTR1(church, [50, 100, 5])

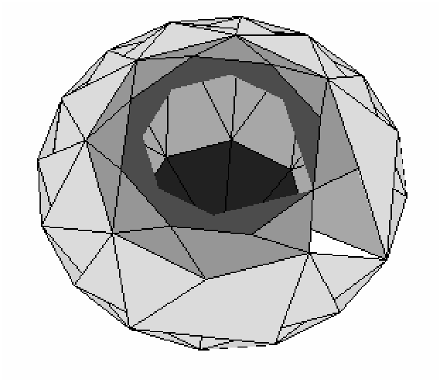




An axonometric picture of a diamond created by eight rotations of its eighth part (left). DfW5 gives a nice picture in its 3D-Plot Window (left below).

The DfW5-Diamond

Next example shows a space tube created by a helix:



We use the SPACE_TUBE() from GRAPHICS.MTH together with my auxiliary function FIG():

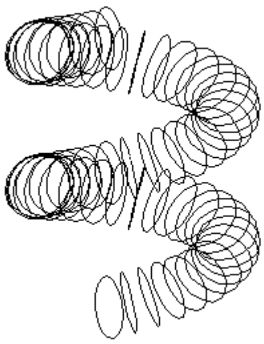
`SPACE_TUBE([2·COS(t),3·SIN(t),0.7·t],t,1,s)`

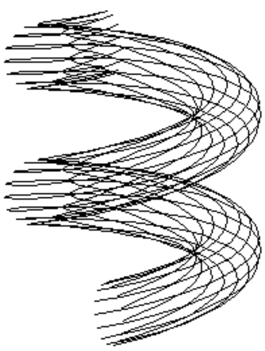
FIG(param_form,par1,par2,par1_start,par1_end,steps1,par2_start,par2_end,steps2) returns lists of points which form the families of parameter curves.

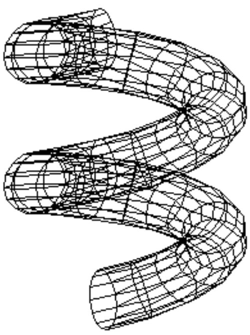
`FIG(SPACE_TUBE([2·COS(t),3·SIN(t),0.7·t],t,1,θ),t,θ,−2·π,2·π,60,0,2·π,15)`

In DfW5 we will be able to produce a 3D-plot from the parameter form and from the point list as well.

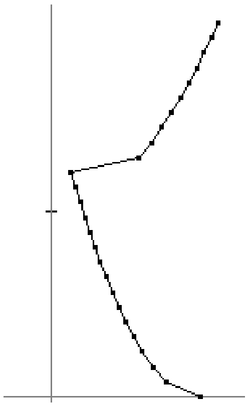
In times of DfW4 or even earlier – in DfD – we produce images of the parameter curves:





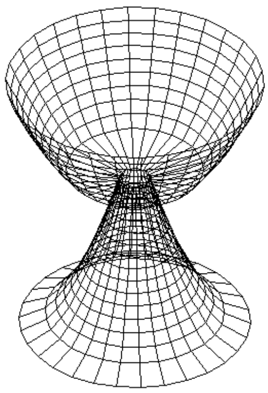


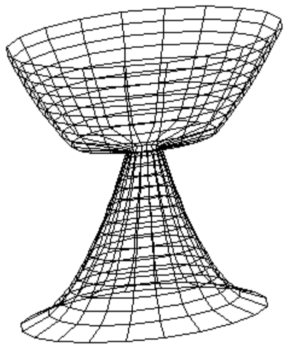
`OBL(FIG(SPACE_TUBE([2COS(t),3SIN(t),0.7t],t,1,θ),t,θ,−2·π,2·π,60,0,2π,15),0.5,40°)`
`COPROJECTION(OBL(.........,40°))` and `COMPL(OBL(.......,40·°))`
(from left to right)

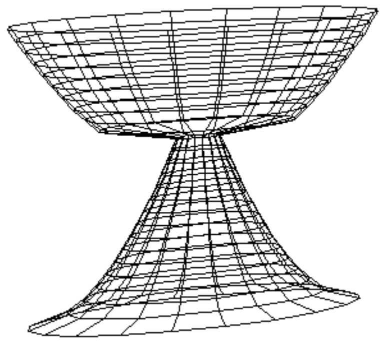


mer is a list of points which describes half of the cross section of a solid of revolution created by rotating this "curve" around the vertical axis.

P1(mer,20) and P2(mer,20) result in the two families of parameter curves, dividing the full rotation into 20 parts. COMPL (P1(mer,20)) combines both families and applying any projection we will see a wire frame representation of the wine glass. Cheers to you all -
- and a Happy New Year 2000.







ISO(COMPL(P1(mer, 30)))   KAV(COMPL(P1(mer, 20)))   Try CENTR1(..., [.....])

It was last year, winter time, when I met my friend Otto Wurnig from Graz, Styria. He invited me to give a lecture on *Graphics with DERIVE* for students and teachers. And he added a challenge. He said to me: 'I saw a *MATHEMATICA* presentation showing solids of revolution with shadings and hidden lines – I know that in the moment we cannot do that with DERIVE and it is not so necessary from my point of view – and with *MATHEMATICA* we added the discs to have an imagination of a volume integral. You are familiar with graphics in DERIVE, try to prepare that for your workshop." That was it. Now, look how I tried to satisfy Otto:

Many of you will agree that students often have problems to imagine the solids created by revolving a curve or a segment of a curve around the *x*- or the *y*-axis. I start with a piece of a parabola. With the tools which you have been introduced until now it is not too difficult to produce the point lists of the two possible surfaces: rotate around $y = 0$ for $-2 \le x \le 3$ and rotate around $x = 0$ for $1.5 \le y \le 6$. To produce the disks it is helpful to transform the generating curve into a step function, which then being rotated will produce the disks.

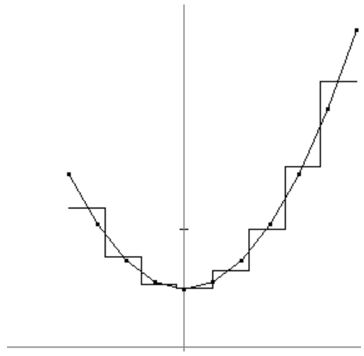
F1:=x^2/2+1 defines the function

F_X(expr,variable,start,end,stepnumber) parameterizes the graph

F_X(f1,x,-2,3,10)

TREP_X(expr,variable,start,end,stepnumber) gives the step function over function expr.

TREP_X(f1,x,-2,3,8)



ROTF_X(expr,variable,start,end,n1,n2) returns the point list of the surface with n1 and n2 parameter curves (parallel circles and cross sections).

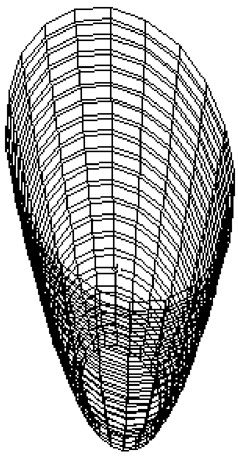
```
ROTF_X(f1,x,-2,3,10,20)
```

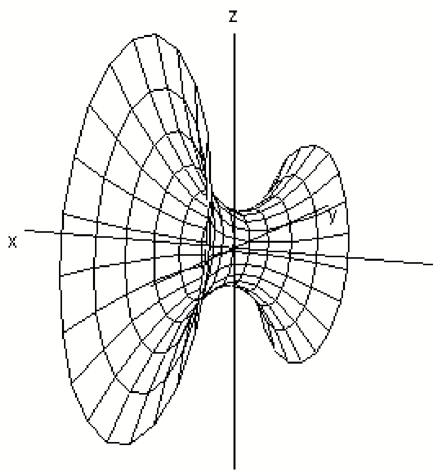
ROTAPP_X(expr,variable,start,end,n1,n2) returns the point list of n1 disks, and the circles approximated by 20 segments.

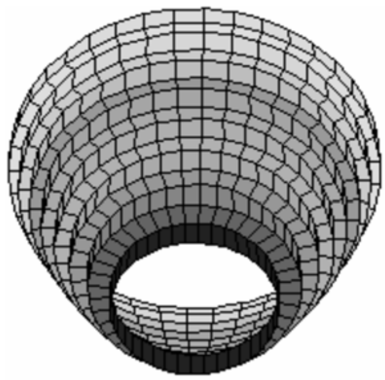
```
ROTAPP_X(f1,x,-2,3,10,20)
```

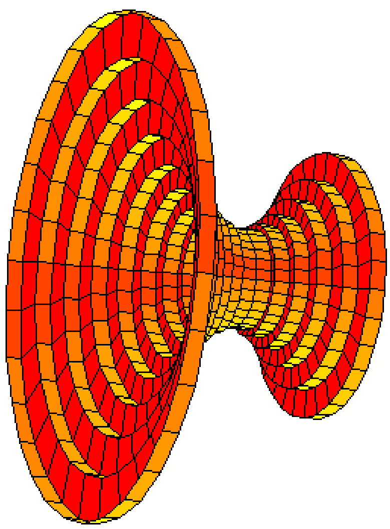
`g1:=√(2y-2)` is used for rotation around the vertical axis. Functions F_Y, TREP_Y, ROTF_Y and ROTAPP_Y are working in the respective way. See the demofile PROJDEMO.DMO.

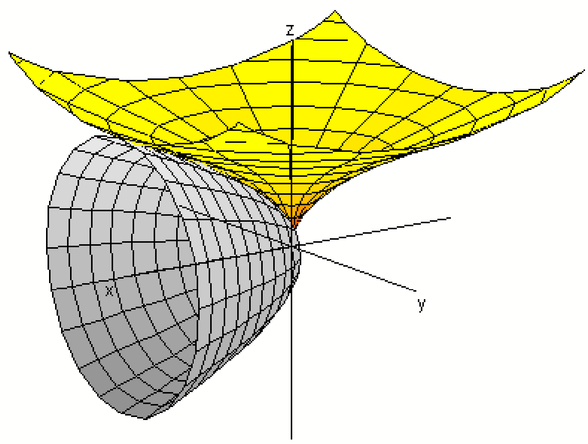
Here are two images (axonometric and central perpective) from DfW4 and DfD and three DfW5-(α-Version) creations.





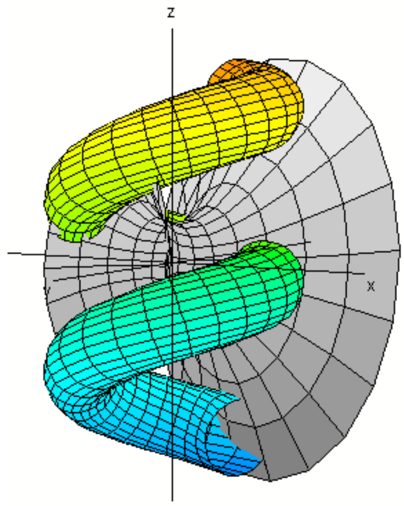






The right plot shows the solids generated by rotations of a parabola around the *x*- and *y*-axes (DERIVE 6.10)

DERIVE for WINDOWS 5 (and DERIVE 6, of course) makes it possible to create such monsters - see below: this is a cross-breed of a helix and a solid of revolution. Imagine it in colours and rotating it on your PC-screen, then you have an idea what DfW5 can do – among many other features.



Intersection Pyramid – Plane, Calculate and Draw!!

(Preload PROJECT.MTH, then load PYRAMID.MTH)

Given is a pyramid with base ABCD[A(3,0,0), B(5,5,0), C(0,7,0), D(0,0,0)] and a plane E given by its intersection points with the axes: E[(XE(12,0,0), YE(0,12,0), ZE(0,0,12)]

```
[o:=[0,0,0],xe:=[12,0,0],ye:=[0,12,0],ze:=[0,0,12]]
axes:=[xe,o,ye,o,ze]
[s:=[3,3,8],a:=[3,0,0],b:=[5,5,0],c:=[0,7,0],d:=[0,0,0]]
"The pyramid, given as a space polygon"
pyr:=[a,b,c,d,a,s,b,c,s,d]
```

$$\mathrm{AXOPIC}\left([pyr,\ axes],\ \frac{1}{2},\ \frac{3}{4},\ 1,\ 45\cdot°,\ 30\cdot°\right)$$

```
[px := [10, 0, 0], py := [0, 12, 0], pz := [0, 0, 8]]
G(u, v) := u + t·(v - u)
E(u, v, w) := u + α·(v - u) + β·(w - u)
"Intersection segment SA ∩ plane E --> vertex A1:"
SOLVE(G(s, a) = E(px, py, pz), [t, α, β])
```

$$\left[\ t = \frac{11}{25}\quad \alpha = \frac{7}{50}\quad \beta = \frac{14}{25}\ \right]$$

$$\lim_{t\to 11/25} G(s,\ a) = \left[3,\ \frac{42}{25},\ \frac{112}{25}\right]$$

$$a1 := \left[3,\ \frac{42}{25},\ \frac{112}{25}\right]$$

In the same way one obtains points B1, C1 and D1 giving the quadrilateral, which forms the intersection figure (see the file on diskette).

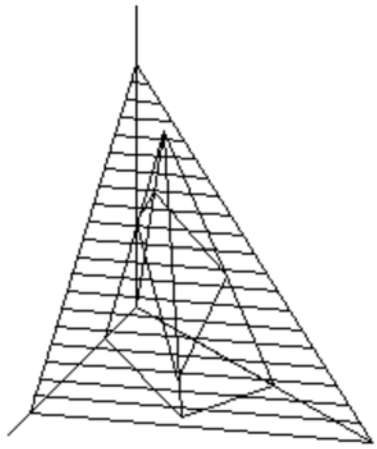
```
traces := [px, py, pz, px]
```

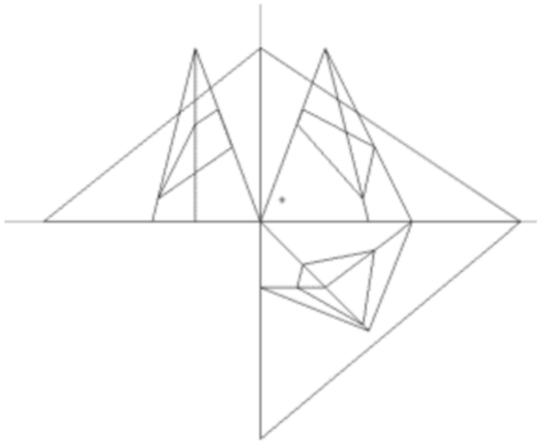
are the intersection lines of plane E with the projection planes XY, XZ and YZ.

I want to shade the intersecting plane E by horizontal segments. A nice exercise for working with the parameter form of a line in R^3.

```
[p1 := x + t1·(z – x) : p2 := y + t1·(z – y)}
pl := p1 + t2·(p2 – p1)
shade := VECTOR(VECTOR(pl, t2, 0, 1), t1, 0, 1, 0.05)
```

You can see an axonometric picture of the whole construction. Please imagine different colours.
On the diskette you will find an additional shading of the intersection figure.

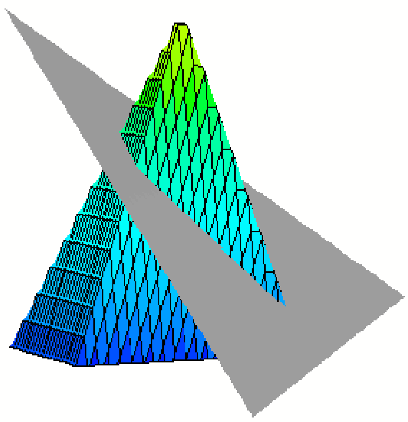


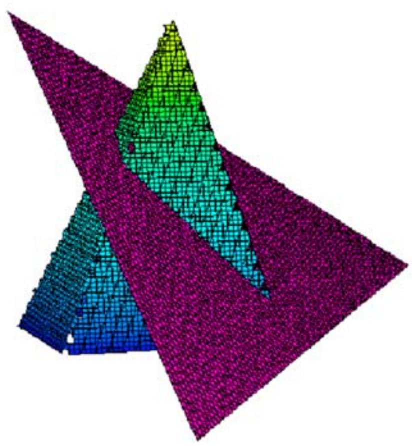


After all you can use FRONTV, TOPV and SIDEV to create a nice sketch of the results.

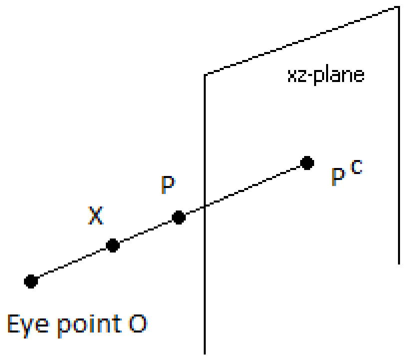
Working with MIN and MAX you can produce a 3D-Plot in DfW and DfD. DfW5 and DERIVE 6 show a pretty presentation of the pyramid together with the intersecting plane. (left under)
See also a presentation using one of my favourite pieces of software DPGraph2000. I had to convert the equations of all the planes from parameter form into implicit form and then express the figure using Boolean operations:

```
graph3d((20*x-8*y+3*z-60<=0 & 8*x+20*y+7*z-140<=
0&3*z-8*x<=0&3*z-8*y<=0,              x/10+y/12+z/8<=1))
```





How to find the Central Projection:



Central Projection of point P from eye point O onto *xz*-plane gives picture point P^c.

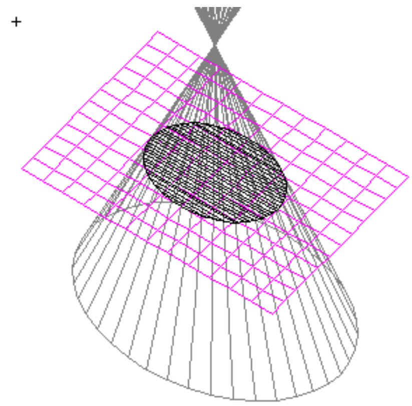
We find the intersection point of line OP with the projection plane:

$$\vec{x} = \vec{o} + t(\vec{p} - \vec{o}) \cap x_2 = 0 \rightarrow o_2 + t(p_2 - o_2) = 0$$

$$\rightarrow t = \frac{-o_2}{p_2 - o_2}.$$

Substituting for t we obtain the coordinates of Pc, written in matrix form:

$$\vec{p}^{\,c} = \frac{1}{p_2 - o_2} \begin{pmatrix} -o_2 & o_1 & 0 \\ 0 & 0 & 0 \\ 0 & o_3 & -o_2 \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$



Further Examples:

Use DERIVE to calculate the various conics intersecting a (double cone) with planes.

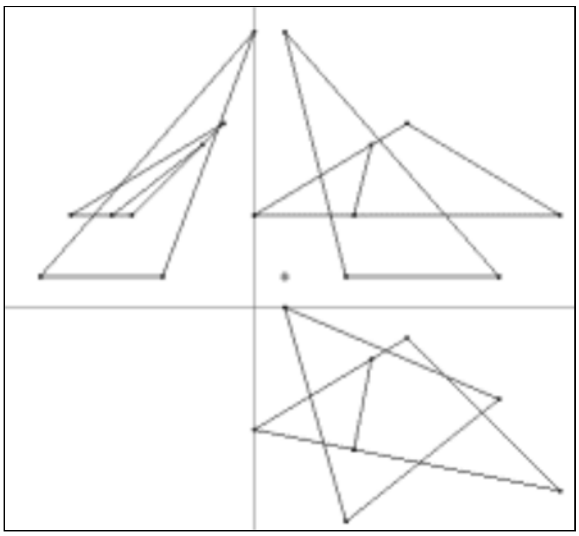
Represent the figures (cone, plane, intersection curve) in an appropriate way.

You can find a proposed solution on the diskette CONIC.MTH

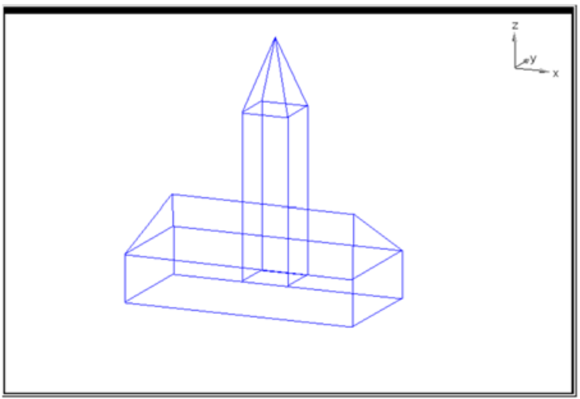
Maybe a nice example for the classroom (Calculation & Presentation as well):

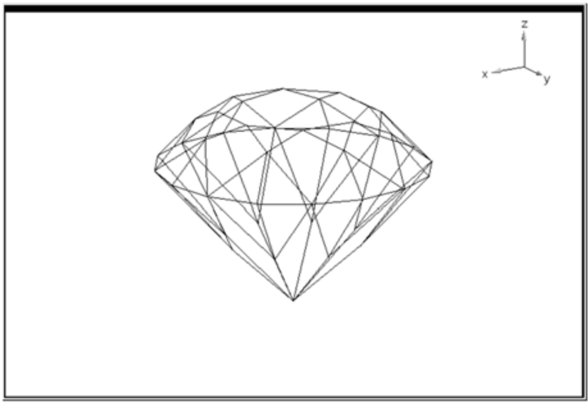
Calculate and present the intersection of two triangles ΔABC [A(0,1,9), B(7,3,1), C(3,8,1) and ΔUVW [U(4,0,3), V(6,10,3), W(1,5,6)].

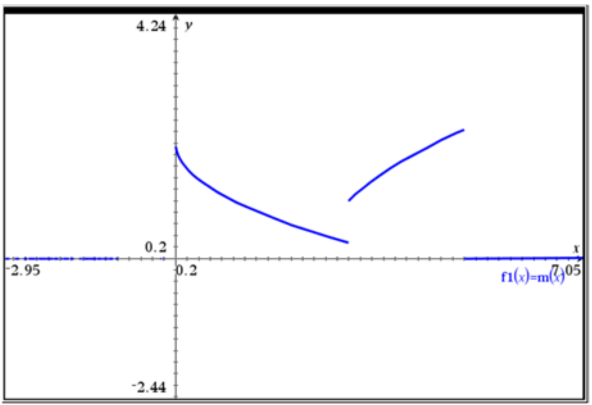
(TRIASECT.MTH)

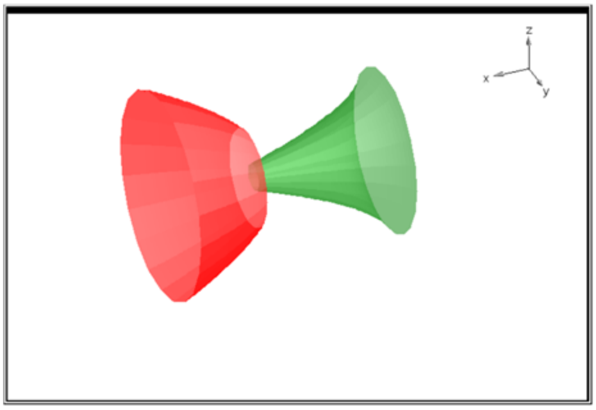


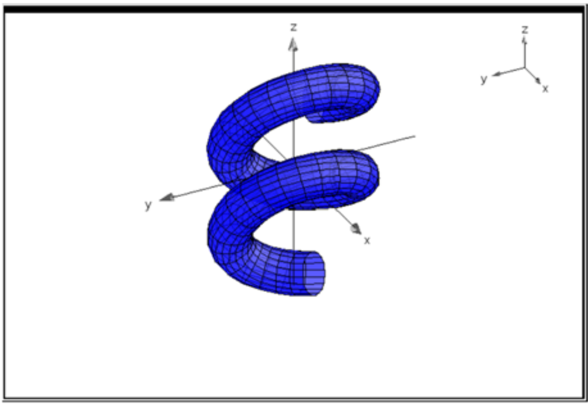
See some objects from above generated with TI-NspireCAS using Geneviève Savard's great library Geo3D.tns which will be presented in DNL#103

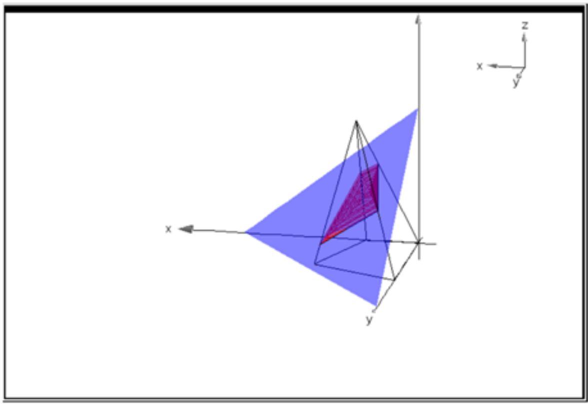


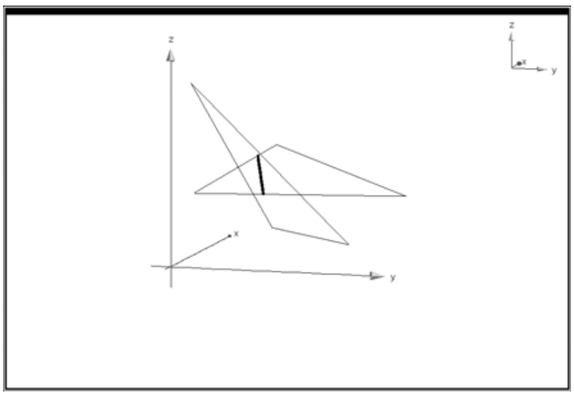


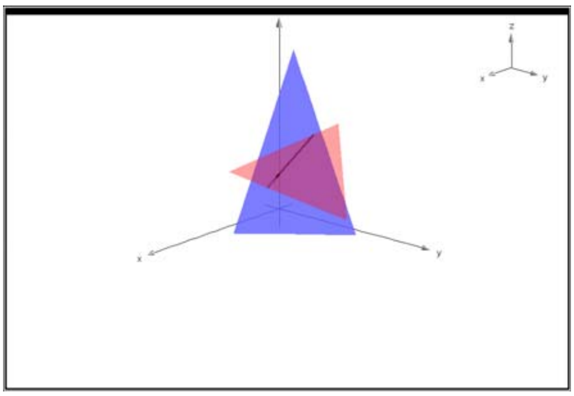












Just in time to celebrate the millenium Alfonso contributed his Calendar Problems:

CALENDAR PROBLEMS

Alfonso J. Población Sáez
E.U. Politécnica, Valladolid, España

In our daily life, there exist a lot of familiar things which, by usual, we give them little importance, although many are quite singular and interesting, even sometimes surprising. One of them, I think, is our calendar, a system that pretends to measure time and varies a lot from a culture to another. New modifications are proposed continuously trying to fix the mistakes made in the past and in present, looking for the imposible: to racionalize the irrational. So, today I will consider some aspects related with our calendar for which it is needed to know some astronomical terms and historical events, but don't panic, I will try to be brief. You will find some calls to drill exercises that are proposed at the end. I hope these comments could help in order to propose questions or practices for secondary level students.

Our current calendar is based in the *Tropical Year* because this concept rules the seasons. The Tropical Year is the interval of the elapsed time between two consecutive crossings of the *Vernal Equinox* (the first day of Spring) by the Sun. The Tropical Year is equal to 365 days 5 hours 48 minutes 46 seconds = 365.242199, but this is an average value; if we measure a concrete year, its length will not coincide exactly with this value. From the values of the last 2000 years, it is accepted an average length of 365.2422 days.

And now a bit of historical background. Egyptians used a 365-days calendar. As time passed, they noticed that the Vernal Equinox came earlier and earlier, so they thought in a reform that never was put in practice (1). In 46 B.C., Julius Caesar stated a new calendar, consisted in a solar year of 365 days with an extra day every fourth year, trying to correct the decimal excess 0.242199. Its implementation had a lot of incidents (2).

But the defect of the effect on the date of the seasons still remained. Besides this, the calendar became increasingly inaccurate with respect to the regulation of Easter and the ecclesiastical calendar, because, for instance by the 16th century, Easter was on the way to slipping into summer. Pope Gregory XIII introduced a reform: ten days were omitted (October 5, 1582 Thursday was followed by October 15, Friday), and the rule for leap years was changed. Now a year would be a leap year if it was divisible by 400, so 1600 and 2000 are leap years, but 1700, 1800 and 1900 are not (3). The errors with respect to the solar year were surprisingly reduced with this system (4).

However the Julian date is not completely forgotten. In astronomy, for many purposes, a continuous count of days is convenient. Finally, I enclose a short DERIVE file I made that converts the Julian date into calendar date and viceversa. Remember that the Julian one measures the number of elapsed days since noon on January 1, 4713 B.C.

```
#1:   " Calendar Date ----> Julian Date "
#2:   [PrecisionDigits:=20,Notation:=Decimal]
#3:   K(y,m,d):=y+m/100+d/0000
#4:   A(y):=FLOOR(y/100)
#5:   B(y,m,d):=IF(K(y,m,d)<1582.1015,0,2-A(y)+FLOOR(A(y)/4))
#6:   JD(y,m,d):=FLOOR(365.25*y)+FLOOR(30.6001*(m+1))+1720994.5+d+B(y,m,d)
#7:   JULIAN(y,m,d):=IF(m>2,JD(y,m,d),JD(y-1,m+12,d))
#8:   " If we also desire to count hours and minutes "
#9:   JD_2(y,m,d,h,mn):=JD(y,m,d)+(h+mn/60)/24
#10:  JULIAN_2(y,m,d,h,mn):=IF(m>2,JD_2(y,m,d,h,mn),JD_2(y-1,m+12,d,h,mn))
#11:  JULIAN(1987,2,15)
#12:  2446841.5                                             ;Simp(#12)
#13:  JULIAN_2(1987,2,15,12,0)
#14:  2446842                                               ;Simp(#14)
#15:  JULIAN_2(1990,2,16,15,12)
#16:  2447939.1333333333333                                 ;Simp(#16)
#17:  JULIAN(-4713,1,1)
#18:  -365.5                                                ;Simp(#18)
#19:  JULIAN_2(-4713,1,1,12,0)
#21:  -365                                                  ;Simp(#20)
```

In this first part, we introduce the year (y), month (m), day (d), hour (h) and minutes (mn). If the month is January or February, we put m as 13 or 14 and y-1. If the date is before October 15, 1582 (Gregorian Reform), then B=0. B calculates the leap years and the value 1720994.5 represents the days before year 1.

```
#22:  " Julian Day ----> Calendar Date "
#23:  Z(j_d):=FLOOR(j_d+0.5)
#24:  F(j_d):=j_d+0.5-Z(j_d)
#25:  AA(j_d):=FLOOR((Z(j_d)-1867216.25)/(36524.25))
#26:  A_(j_d):=IF(Z(j_d)<2299161,Z(j_d),Z(j_d)+1+AA(j_d)-FLOOR(AA(j_d)/4))
#27:  B_(j_d):=A_(j_d)+1524
#28:  C_(j_d):=FLOOR((B_(j_d)-122.1)/(365.25))
#29:  D_(j_d):=FLOOR(365.25*C_(j_d))
#30:  E_(j_d):=FLOOR((B_(j_d)-D_(j_d))/(30.6001))
#31:  G_(j_d):=B_(j_d)-D_(j_d)+F(j_d)-FLOOR(30.6001*E_(j_d))
#32:  M_(j_d):=IF(E_(j_d)<13.5,E_(j_d)-1,E_(j_d)-13)
#33:  Y_(j_d):=IF(M_(j_d)>2.5,C_(j_d)-4716,C_(j_d)-4715)
#34:  DATE(j_d):=[Y_(j_d),M_(j_d),G_(j_d)]
#35:  DATE(2437401)
#36:  [1961,4,11.5]                                         ;Simp(#35)
#37:  DATE(2437402)
#38:  [1961,4,12.5]                                         ;Simp(#37)
#39:  DATE(2437401.5)
#40:  [1961,4,12]                                           ;Simp(#39)
#41:  DATE(2371629.5)
#42:  [1781,3,14]                                           ;Simp(#41)
```

Questions:

1.- Using the Egyptian calendar, what date should be the Vernel Equinox after one hundred years?

2.- What is the error caused by the Julian calendar? When will the Vernel Equinox be after one hundred years?

3.- Until October 5, 1582, what was the cumulative gap?

4.- What is the error made by the Gregorian calendar? Could you explain mathematically this good aproximation to reality?

5.- Try to understand the DERIVE file listed above.

References:

[1] Beskin, N.M., *Fracciones maravillosas*, Editorial Mir, Moscú, 1987

[2] Vorontsov-Veliaminov, *Problemas y ejercicios prácticos de Astronomía*, Ed. Mir, Moscú, 1979

ANSWERS

1.- As the error is 5h 48m 46s per year □ 0.242199 days, after 100 years, we have 24.2199 days which will lead us to April 15.

2.- $365\frac{1}{4}$ = 365d 6h, that exceeds 11min 14sec the real time. This error is 0.0078009 days per year. After 100 years this gives us 0.78009 and hence after 400 years the error is about 3.12036 days.

3.- From the year 1 to the year 1582 the calendar drifted off the mean solar year by 1581* 0.0078009 = 12.333222 days. Why Pope Gregory XIII removed only 10 days? It has to do with the First Council of Nicea in 325. There it was determined the date of Easter. From 325 to 1582 the calendar diverged by 1257*0.0078009 = 9.80 (10.05 if we take 0.008), so aproximately ten days were needed to be removed since the Council of Nicea.

4.- First, we will try to explain how to solve the alternance of the leap years. We will represent the length of a year like a continued fraction, i.e.

1 year = 365d 5h 48m 46s = [365; 4, 7, 1, 3, 5, 64] days

Note that this fraction is obtained from an empirical calculation of the length of the year, so it is nonsense to say anything about its rationality or irrationality. It's an accepted quantity, and by this reason it will be expressed like a finite fraction:

$$\frac{5h\,48m\,46s}{1\ day} = \frac{20926\ \ seconds}{86400\ \ seconds} = \frac{10463}{43200}$$

Making use of the file explained to solve diophantine equations in this section in past newsletters, we have that

n	1	2	3	4	5	6	7
w_n	0	4	7	1	3	5	64
p_n	0	1	7	8	31	163	10463
q_n	1	4	29	33	128	673	43200

The column $p_n/q_n = 1/4$ gives us the Julian approximation, a leap year every four. The next column leads us a more precise approximation, 7 leap years every 29, which is $365\frac{7}{29}$ days per year.. It's more adjusted but also more complicated. We can build the following table:

Alternance of leap years		Length of year	Error
Number of leap years	Period		
1	4	365 days 6 hours	- 11 min 14 sec
7	29	365d 5h 47m 35s	+1min 11 sec
8	33	365d 5h 49m 5s	- 19 sec
31	128	365d 5h 48m 45s	+1 sec

The third case was proposed by the great poet and mathematician Omar Khayyam (1050-1123). The fourth version is highly exact. In 1864, the Russian astronomer Medler proposed its introduction in his country when the 20th century began. It was only necessary to omit a leap year (considering it ordinary, 365d) every 128 years, because in our current calendar we have 32 leap years every 128. However this modification was not admitted.

Now let's solve the Gregory XIII question. If we begin by the ratio 31:128, we can substitute the period of 128 by another more convenient, for example 400. So, $31/128 = x/400$ gives us $x = 96.875 \approx 97$. This is precisely the Gregorian calendar: 97 leap years every 400 years. Did he make this calculation?

Keeping an eye into the history of Science, we can conclude that was not possible. First, they did not know the length of year so exactly as we do. Gregory XIII's committee used the astronomical tables composed by the Council of Toledo by order of king Alfonso X (1221-1284). In those tables, a year = 365d 5h 49m 16s, so it exceeded the real in 27 seconds. But Gregory XIII thought his calculation was less than the real in 4 seconds. Besides this, the continued fractions were not known by that time. The way they probably thought could be like this: By Toledo's tables, the Julian year exceeds the real in 10 min 44 seconds, so

$$\frac{24\ hours}{10\ \text{min}\ 44\ \text{sec}} = \frac{86400}{644} \approx 134$$

So to correct the Julian calendar, it was necessary to omit a leap year every 134 years. But this is a complicated operation because the year 134 could not be leap, so that as 400/3 ≈ 134, in 400 years we only have to omit three times a leap year. And this is the Gregorian calendar.

Experiments Using CBL/CBR and the TI-92 – New Conceptions for Teaching Science

Heinz-Dieter Hinkelmann, Korneuburg, Austria

1. Introduction

With this title a new series should be started to enlarge the possibilities of application of the TI-92 under the aspect of natural science. For that means experiments based on CBL™ (Computer Based Laboratory) and CBR™ (Computer Based Ranger), both products of *Texas Instruments*, are described as well as the measuring sensors of CBL-basic equipment and the sensors and probes of the *Vernier* company. My friend *Josef Böhm* had the idea of this project. He told me about that on a hiking-tour in the mountains. And now I try to turn this idea into reality.

It is planned to show one or two experiments for the natural science lessons and their mathematical analysis. If anybody has more ideas, suggestions or questions, just please contact me by using the following adress:

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The measuring instruments will be shortly described for those, who don´t know them. Today´s edition describes CBL and the following editions of DNL will explain CBR and the sensors and probes.

2. How Does the CBL Work?

The CBL is a portable, handheld, battery-operated, data collection device, which can be used in math and science lessons. The data can be collected by a large amount of sensors. For that purpose the CBL must be connected with TI-92 (TI-82, TI-83, TI-85, TI-86 or TI-89).

Programs on the calculator control data collection with the CBL. The data are stored as lists in the memory and can be analyzed and evaluated.

The CBL unit is an intelligent device with its own microprocessor and memory for collecting data and temporarily storing collected data. The CBL can also be used as a stand-alone device. But the system becomes very effective when it is used interactively with a calculator.

The CBL has six connections that are called channels to connect probes: three analog channels (CH1, CH2, and CH3), one ultrasonic motion detector channel (SONIC), one digital input channel (DIG IN), and one digital output channel (DIG OUT). The analog channels accept the three sensors included with the CBL and a wide range of *Vernier* sensors. You can collect data at rates of up to 10,000 points per second for up to 512 points per channel. Additionally, you can collect data on up to five channels simultaneously.

The CBL system includes the CBL interface, the "CBL System Guidebook", the "CBL System Experiment Workbook", batteries, carrying

case, and three probes:

- Temperature probe: uses a thermistor to measure temperature
- Light probe: uses a phototransistor to measure irradiance
- Voltage probe: a generic probe to read any voltage between ±10 Volts

The mode of sensor working (*Texas Instrument* or *Vernier*) together with the CBL is rather similar: Data of an electric size (voltage, current, or resistance) are measured and transferred to CBL. The so called „AutoIDENT“-feature allows the CBL unit to automatically identify specific probes when you connect them to the CBL channels. It determines what kind of data is going to be measured, and loads an equation for converting the data into the appropriate measurement unit.

The light probe´s output is a voltage that is linearly proportional to the amount of irradiance sensed. The AutoIDENT resistor contained in the probe causes the the CBL software to automatically convert the measured voltage to mW/cm^2 units. The range of light over which the probe is sensitive is 10 $\mu W/cm^2$ to 1 mW/cm^2. The light probe can be used in the visible and near infrared light range. It is designed to work in air only; it is not waterproof.

The temperature probe measures the resistance of a thermistor. A thermistor is a variable resistor whose resistance decreases nonlinearly with increasing temperature. The best-fit approximation to this nonlinear characteristic is the Steinhart-Hart equation. The AutoIDENT resistor contained in the probe causes the CBL software to automatically convert the measured resistance to °C by the aid of the equation. The temperature probe is sensitive between –20°C to +125°C. It is water resistant (maximal for 24 hours). You can use it with some chemicals which are listed in the handbook.

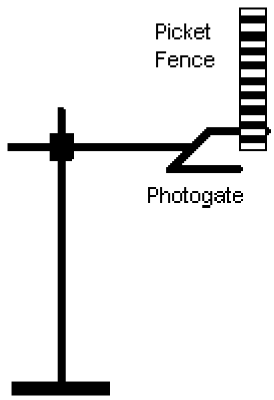
Of course the voltage probe does not need any transformation equations. The AutoIDENT resistor contained in the probe causes the CBL software to automatically measure voltage. The black hook should be connected to ground and the red hook to the signal voltage.

3. A Preview of Selected Classical Experiments

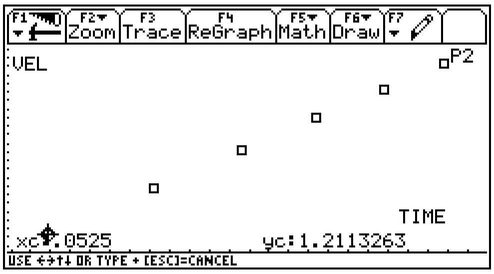
We have worked out some results of experiments without any prescription of their progress (detailed prescriptions are given in further editions of DNL) to show you some possibilities of this system.

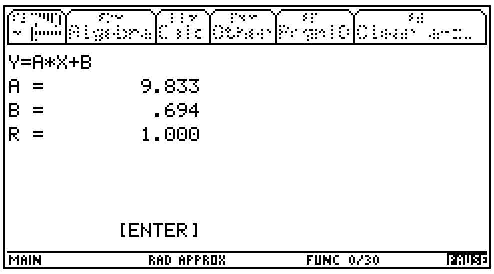
3.1 Picket Fence Free Fall / Measurement of Acceleration of Earth

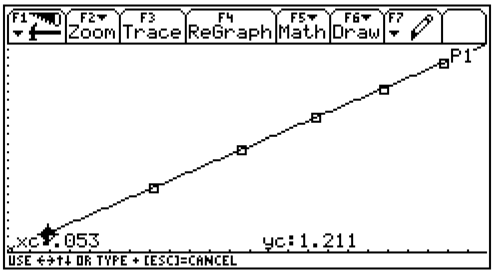
If objects are dropped from a certain height in the atmosphere down onto the Earth, air resistance and friction are at work. A heavier object, that has less air resistance, will move towards the ground with almost constant acceleration (acceleration of Earth g). The Vernier Picket Fence (a piece of clear plastic with evenly spaced black bars on it) produces nearly no air resistance when it drops. When the Picket Fence passes through a Photogate, the CBL will measure the time from the leading edge of one bar blocking the beam until the leading edge of the next bar blocks the beam.



The result could be like that: The velocity-time graph, the result of the regression calculation (A=9,83 m/s^2) and the graph together with the regression line are represented:





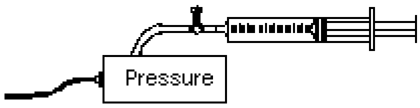


3.2 Boyle-Mariotte´s Law

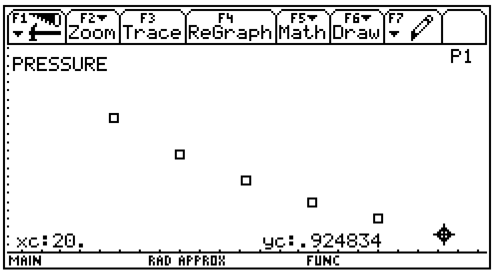
Gases tend to expand or contract so as to fill its container. Consequently, increasing or decreasing the volume of a fixed mass of gas has an effect on its pressure. The relationship is summarized in Boyle-Mariotte´s law: The volume, V, of a gas varies inversely with its pressure, p, when the temperature of the gas remains constant. Stated mathematically:

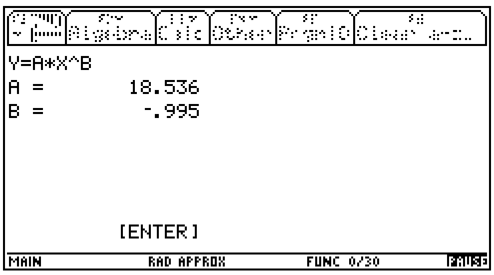
$$p.\ V = const.$$

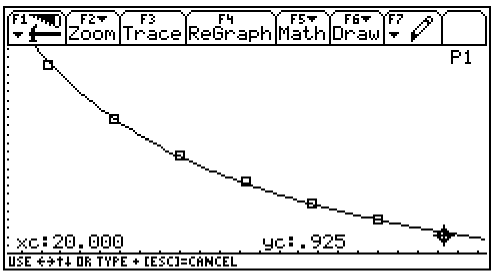
This statement can be easily controlled by adding a large hypodermic syringe with a short piece of tubing.



The result could look like the following: The pressure-volume graph, the result of the regression calculation and the graph together with the regression line are shown.



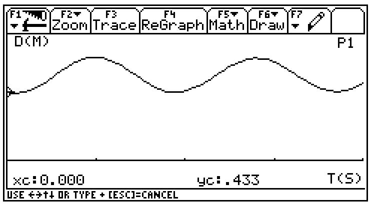


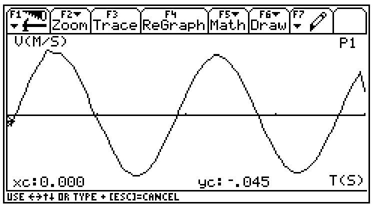


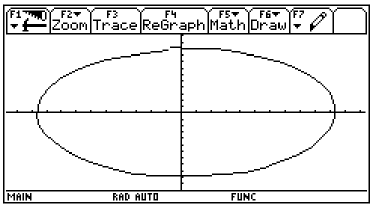
3.3 The Simple Pendulum, Measurements by the CBR

Harmonic vibrations can be produced using a simple pendulum. A yo-yo or a mass attached to the end of a piece of string can act as a simple pendulum. Using the CBR we measure the distance and the velocity of a simple pendulum versus time. The aim is to plot the relationship between the horizontal displacement of the pendulum from ist position when stationary and ist horizontal velocity.

The result could be like that: The distance-time graph, the velocity-time graph and the velocity-distance graph (of a complete oszillation) are shown.







Letter of the Editor

Dear Members of the DUG-family,

This is a short letter of the Editor, because I wanted to present some photographs from the ACDCA Summer Academy 1999 on page 1. I hope that the print will not debase the quality of the scanned pictures. Once more many thanks to Richard Schorn, the senior delegate, for sending a pile of pictures. Try the website:

`http://www.acdca.ac.at`

to find (nearly) all lectures of this DERIVE and TI-92 event for downloading. Many thanks to the delegates for sending the papers in electronic form and to Walter Wegscheider, ACDA´s webmaster.

Very soon we will have 2000, but don´t forget that the true end of this millenium is on 31 December 2000 together with the end of DUG´s 1st decade. Nevertheless I found some appropriate contributions for this DNL to join the common 2K hysteria (ACDC9, Casus Nulli Anni, Mixed History).

H-D. Hinkelmann starts a new series to introduce CBL and CBR together with its various probes. In Austria we - the maths teachers - are launching a new offensive for "subject overlapping" teaching and Physics, Chemistry, Biology are our first partners. I am sure that also "pure" DERIVIANS will be interested how the natural sciences can be brougth into classroom without any bulky and heavy equipment.

You might miss Johann Wiesenbauer´s TITBITS in this issue. Johann intended to present some introductory examples of DfW5´s programming features. But because we had no chance to ask for SWHH´s permission to publish unreleased DfW5 code and because Johann was very busy with new lectures on University we left this important and undoubtely interesting intention for the next DNL.

At the end of my short letter I want to thank you all for your cooperation, which we all experience as friendship at so many occasions, reaching over all borders and continents. We hope that we can continue our work in the years lying in front, they all beginning with the challenging "2". There are several possibilities to meet next year and I am sure that we will see old - and make new friends.

Noor and Josef - your DUG - team sends the best seasonal greetings, many peaceful days together with your family and a happy, healthful and prosperous New Year 2000.

Some months ago I received a large envelope containing some sheets of paper with a lot of formulae and some interesting plots of surfaces and families of curves. Mr Fritsch contacted me and told that he – as a non-prof mathematician and astronomer – has found out very interesting facts concerning the so called "Three Body Probem". He used DERIVE, Acrospin and later CYCLON89. I was also interested in that subject matter and happy enough to meet anyone who likes to do maths as his hobby I visited Mr Fritsch in the hospital and we together improved the graphics with the notebook on his bed. After our talk I "consulted" the Internet to inform myself. I found some very informative internet addresses, which migth be also of interest for you:

http://www.mathsoft.com/asolve/constant/lpc/lpc.html Laplace Limit Constant

http://www.treasure-troves.com/physics/LagrangePoints.html

http://www.treasure-troves.com/physics/Newtonian2-BodyProblem.html

http://www.treasure-troves.com/physics/Newtonian3-BodyProblem.html

http://www.treasure-troves.com/physics/JacobiIntegral.html

(By the way, the treasure .... website is a Must for everybody interested in science)

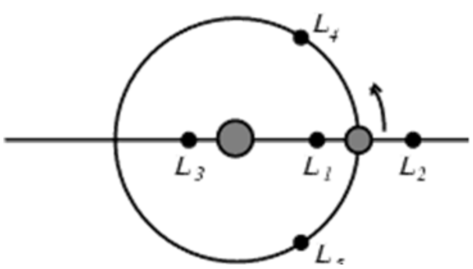
As Mr Fritsch hopes that there are some readers who might share his interest in celestial mechanics he asked me to add his postal address at the end of his contribution.

Mapping of the JACOBI-Limit Surfaces together with the Contour curves

Ludwig Fritsch, Vienna, Austria

I used the wonderful features of DERIVE together with CYCLONE98 to produce representations of three dimensioned mathematical functions. It is necessary to describe the mathematical astronomical problem – "the Restricted 3-Body Problem".

In Celestial Mechanics only the "Restricted 2-Body Problem" is solveabel analytically – it describes a motion of a celestial body with its mass concentrated on one point and the motion of a mass around a central body. If there are three or more masses the mutual attraction forces influence the paths of the bodies. Mathematical treatment of this problem is very complicated and only possible in numerical way for a limited space of time. In astronomical sense this problem is named "More Body Problem", which can be treated numerically with High-End-Computers.



For two large co-orbiting bodies with nearly circular orbits of masses m_1 and m_2 (with $m_1 > m_2$), and a third small body having the same revolution period n as the other two, there are five points where the gravitational forces of the two large bodies exactly balance the Centrifugal Force felt by the small body. The points are called Lagrange points, and an object placed at one of these points would remain in the same position with respect to the other two. Points L_4 and L_5 are located at the vertices of Equilateral Triangles with base along the m_1m_2 line, and are truly stable. Points L_1, L_2, and L_3 are quasi-stable. There are, however, orbits around them in the planes perpendicular to the axis connecting the two major bodies which are almost stable, requiring only small occasional corrections. These periodic orbits are approximated by
(from http://www.treasure-troves.com/physics/LagrangePoints.html)

In this paper we want to present the isocurves of the so called JACOBI Integral in a 3D-presentation.

Even "problème restreint" (Poincaré) needs some assumptions: Three "point masses" of spatial extension zero are located in such a way, that the third mass (m ≈ 0 in comparison with both other masses) moves on a coplanar path with excentricity 0 (= circle) around the central mass, e.g sun or earth. The center mass should be much more than the others. In our solar system we can find these conditions (nearly) three times realized and there they are of importance:

- in the system Sun – Jupiter – the 14 Trojans (Group of Asteroids with relatively few mass which are moving around Jupiter´s Lagrange Points L_4 and L_5).
- very important in the system earth – moon – space craft
- and the system Earth – Moon – Dustclouds. The Polish astronomer K. Kordylewski assured that he could see these dustclouds in the very clear nights of Polish mountain regions on Lagrange locations L_4 and L_5. [1]

The references give the following equation for the limit surfaces, with one real side for a given velocity and an imaginary other side.

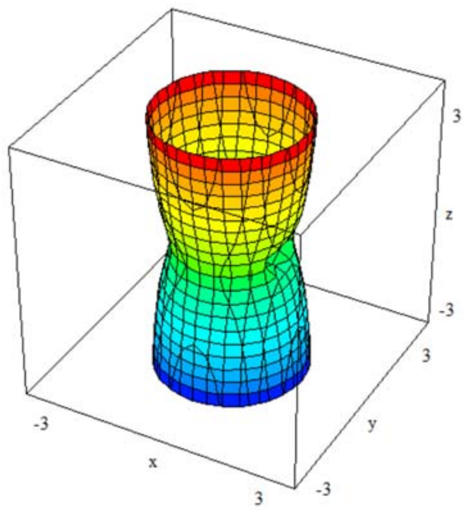
$$(1-\mu)\left(\rho_1^{\;2}+\frac{2}{\rho_1}\right)+\mu\left(\rho_2^{\;2}+\frac{2}{\rho_2}\right)=C+\mu(1-\mu)+z^2$$

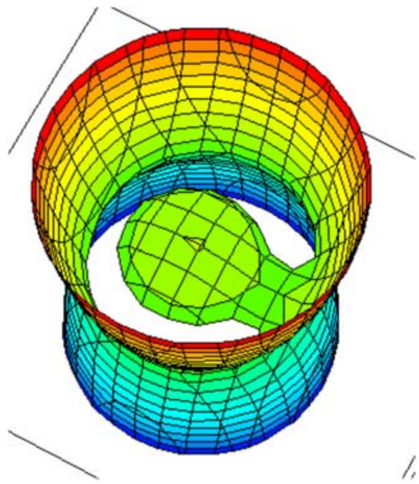
$$\text{with: } \rho_1^{\;2}=(\mu+x)^2+y^2+z^2 \quad \text{and} \quad \rho_2^{\;2}=(1-\mu-x)^2+y^2+z^2$$

$$C = C_{\min} = 3.000 \quad \text{und} \quad \mu = \text{mass ratio earth – moon} = 1{:}81.3003.$$

I chose the fictitious value 0.08 for μ and 3.15 for *C*. I used *DERIVE* to develop and to edit the formula and transferred it with cut and paste to CYCLONE98. (I used "bipolar coordinates" because that makes plotting faster and more accurate).

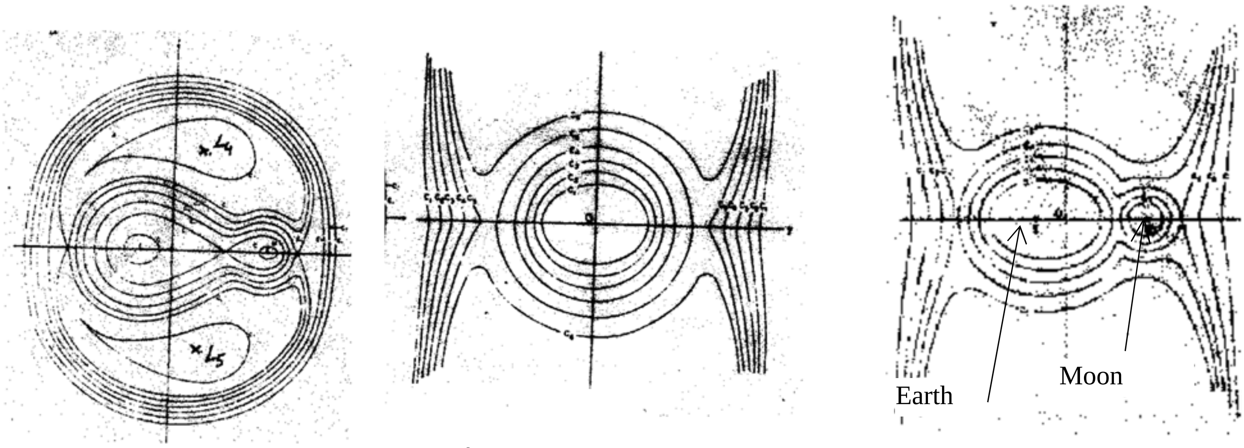
```
graph3d(0.92*((((0.08 + x)^2 + y^2 + z^2)^(1/2))^2 + 2/((0.08 + x)^2
+ y^2 + z^2)^(1/2)) + 0.08*((((0.92 - x)^2 + y^2 + z^2)^(1/2))^2 +
2/((1 - 0.08 - x)^2 + y^2 + z^2)^(1/2)) - 0.08*0.92 - z^2 = 3.15)
```





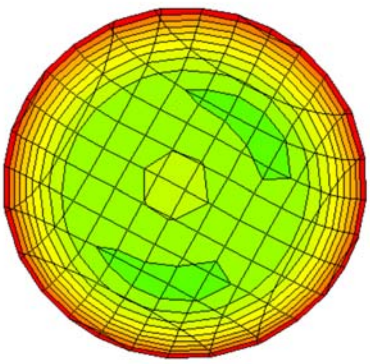
Immediately we see the surface in colours (from a deep blue to a wonderful red). The cursor keys let the figure rotate in all directions and PgUp/PgDn let it increase and decrease. So we get interesting views into the "inner life" of the figure. Additionaly we can create "sliders" and observe the various intersection planes parallel to xy-, xz- and yz-plane.

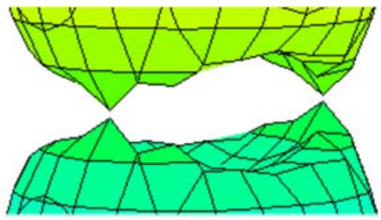
1914 MOULTON [1] produced contour lines. He draw them by hand only schematically – unfortunately he had no *DERIVE* available – because there are equations of order 16 to be solved. Here you can find the scanned MOULTON – Contours.



Curves of zero relative velocity in (x,y) plane, (y,z)plane,..(x,z) plane

In the next CYCLONE98 picture with a slightly changed value for C one can observe two kidney formed contour lines which surround the Lagrange Points L_4 and L_5.

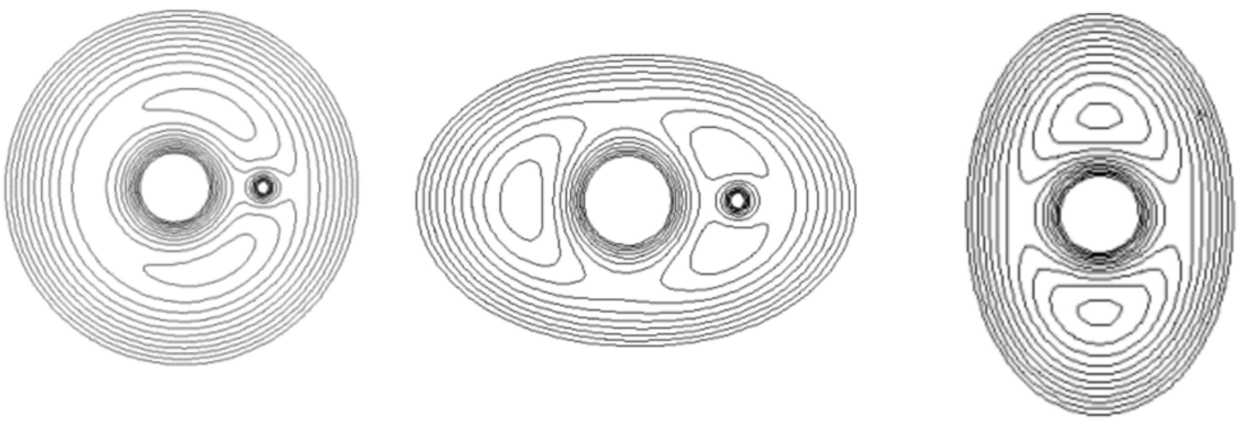




From another point of view one can imagine the Lagrange Points L_4 and L_5.

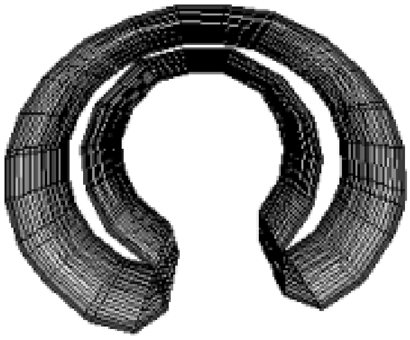
This "slice" gives an impression of the Earth – Moon – system (with a space ship as third body!). It is a slider in y-direction.

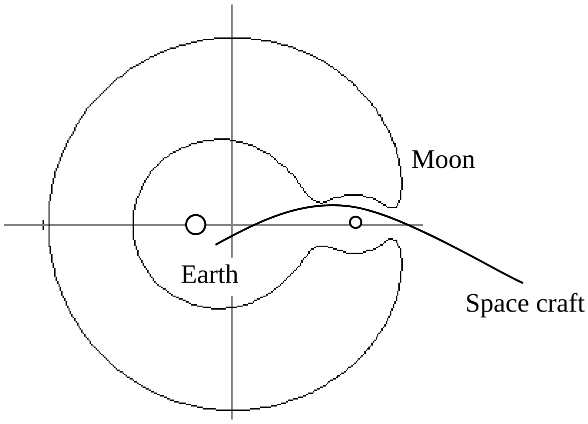
Finally I tried to reproduce MOULTON´s contour curves using the implicit plotting facilities of *DERIVE* for Windows 4 and I was really surprised about the outcomes. Compare the plots below with MOULTON´s original drawings:



You can easily obtain them in DERIVE, first defining a function `F(x,y,z,c)`, for the various contour levels *C*. The VECTOR-command varies the *C*-values and setting one of the three variables = 0 gives the family of curves. You can interpret the two circles als locations for earth and moon.

Simplifying and plotting `VECTOR(F(x,y,0,c),c,3,5,0.2)` returns the left picture on page 36.





The last two figures show the JACOBI limit surfaces - but with a not completely closed contour line around M_2 - which is the moon in this case. The figure shows that the rocket doesn´t need additional energy to overcome the C 3.18-line, and this fact helped the constructors of the APOLLO-project rockets for more suitable weight ratios.

References:

[1] Moulton, F.R., *An Introduction to Celestial Mechanics*, DOVER - publ. 1914, repr. 1960
[2] Guthmann, A., *Einführung in die Himmelsmechanik und Ephemeridenrechung*, B.I. Wissenschaftsverlag 1994

Address of the Author: Ludwig Fritsch, Anton Boschgasse 22, 1210 Wien, Austria

Steady State Material Balances on a Separation Train

(from `http://www.polymath-software.com/ASEE/`)

This is a nice example of applied mathematics in the field of chemistry. It is an application of linear simultaneous equations.

Xylene, styrene, toluene and benzene are to be separated with the array of distillation columns that is shown where F, D, B, D1, B1, D2 and B2 are the molar flow rates in mol/min.

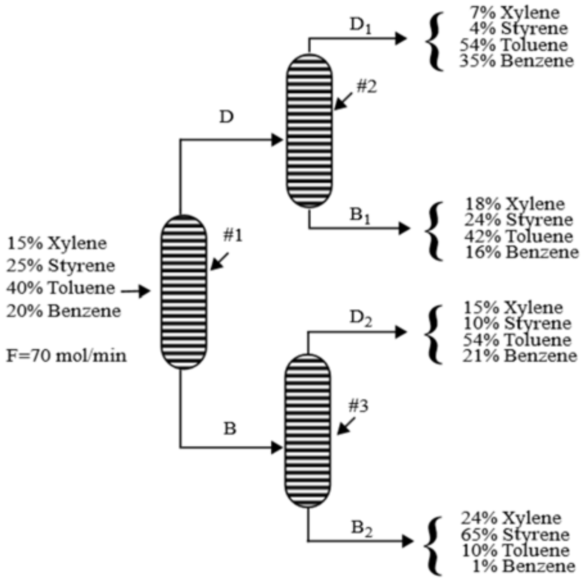
Material balances on individual components on the overall separation train yield the equation set

Xyl: $0.07D_1+0.18B_1+0.15D_2+0.24B_2 = 0.15\times70$

Sty: $0.04D_1+0.24B_1+0.10D_2+0.65B_2 = 0.25\times70$

Tol: $0.54D_1+0.42B_1+0.54D_2+0.10B_2 = 0.40\times70$

Ben: $0.35D_1+0.16B_1+0.21D_2+0.01B_2 = 0.20\times70$



Overall balances and individual component balances on column #2 can be used to determine the molar flow rate and mole fractions from the equation of stream D from

Molar Flow Rates: $D = D_1 + B_1$

Xylene: $X_{Dx}D = 0.07D_1 + 0.18B_1$
Styrene: $X_{Ds}D = 0.04D_1 + 0.24B_1$
Toluene: $X_{Dt}D = 0.54D_1 + 0.42B_1$
Benzene: $X_{Db}D = 0.35D_1 + 0.16B_1$

where X_{Dx} = mole fraction of Xylene, X_{Ds} = mole fraction of Styrene, X_{Dt} = mole fraction of Toluene, and X_{Db} = mole fraction of Benzene.

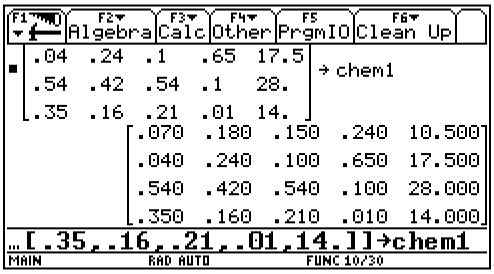
Similarly, overall balances and individual component balances on column #3 can be used to determine the molar flow rate and mole fractions of stream B from the equation set

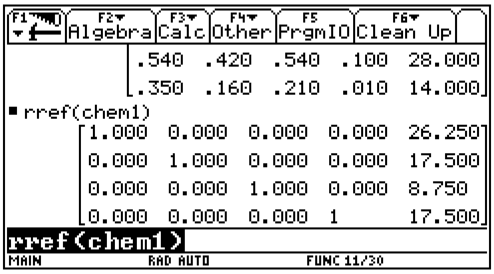
Molar Flow Rates: $B = D_2 + B_2$

Xylene: $X_{Bx}B = 0.15D_2 + 0.24B_2$
Styrene: $X_{Bs}B = 0.10D_2 + 0.65B_2$
Toluene: $X_{Bt}B = 0.54D_2 + 0.10B_2$
Benzene: $X_{Bb}B = 0.21D_2 + 0.01B_2$

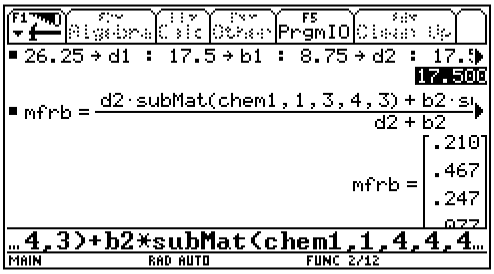
a) Calculate the molar flow rates of streams D_1, D_2, B_1, B_2.
b) Determine the molar flow rates and composites of streams B and D.

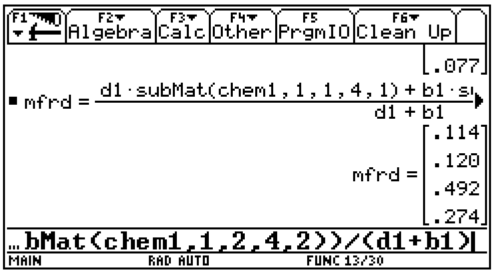
We define a matrix chem1 and then apply rref(chem1) to obtain the solutions for D_1, B_1, D_2 and B_2.





$$\begin{pmatrix} D_1 = 26.25 \\ B_1 = 17.50 \\ D_2 = 8.75 \\ B_2 = 17.50 \end{pmatrix}$$





Composition of B

Composition of D

subMat(chem1,1,3,4,3) gives the third column of matrix chem1. Now it is easy to perform a vector calculation to find the vector of B´s mole rates. Similarly, one can find the solution vector for the composition of D.

This is the way to do it with *DERIVE*: First define matrix chem1 and right hand side vector b:

#4: $b := [0.15\cdot 70,\ 0.25\cdot 70,\ 0.4\cdot 70,\ 0.2\cdot 70]$

#5: $chem1^{-1}\cdot b = [26.25,\ 17.5,\ 8.75,\ 17.5]$

#6: $[d1 := 26.25,\ b1 := 17.5,\ d2 := 8.75,\ b2 := 17.5]$

#7: $\dfrac{d2\cdot chem1'_3 + b2\cdot chem1'_4}{d2 + b2} = [0.21,\ 0.4666,\ 0.2466,\ 0.07666]$

#8: $\dfrac{d1\cdot chem1'_1 + b1\cdot chem1'_2}{d1 + b1} = [0.114,\ 0.12,\ 0.492,\ 0.274]$

At the ACDCA Summer Academy in Gösing Sergey presented an animated graph of a Tsunami. At this occasion he was asked about plotting implicit functions in 3 variables. He claimed that he has submitted an article on this subject, but Josef refuses to publish it. I didn´t refuse, but waited for an appropriate occasion. I think that now - wainting for DfW5 and trying DPGraph2000 - is the right moment tp present Sergey´s

Implicit Multivalue Bivariate Function in 3D

Email: svb@rpl.mpgu.msk.su, svb@scan.msk.su

This demo uses IMP_SURF.MTH utility. (See IMP_SURF.BAT – all on diskette.)

ACROSPIN, J.Boehm`s filter ACD & S.Chaney's TSR macro ENCORE is optional but strongly recommended as it support fast CLOSED SURFACES ANIMATION

MAIN IDEA: Split Plotting rectangular box into small rectangular subboxes
(cells:=cubes) or pyramids (cells:=pyramids) and find function
surface intersections (if any) with all subbox's sides.

```
IMP_SURF_(f,x,c,l,g) = a vector of 3D lines = [ [[x1,y1,z1],[x2,y2,z2]]
                                                ,[x3,..
                       f= implicit function f(x). f or f1=f2
                       x=[x_variable_name, y_..,z_..],
                       c=[center_x,center_y,..],
                       l=[length_x,..],
                       g=[grids_x,..]

IMP_SURF(f,m) = a vector of 3D lines = [ [[x1,y1,z1],[x2,y2,z2]],[x3,..
                       f=f or f1=f2 or [f1,f2,..] or [[f1],[f2],..]
                       m=[[x,center_x,length_x,grids_x],[y,cen-
ter_y,..],[z,..]]

PROJECT(r,a) = rotates a vector of 3D lines by angles from a=[x_angle,y_..,z_.]
               & projects the result on the yz-plane.
```

ROTATE(a) = a matrix that rotates 3D points by 3 angles from vector a=[x,y,z].
It is equivalent to successive application of ROTATE_X(x),
ROTATE_Y(y) & ROTATE_Z(z) from GRAPHICS.MTH DERIVE utility
(3*3*3 matrices product).

Do NOT assign the following global names ANY VALUE:
LINES, LIN, TRI, TRI_H, CUBE, CUBES, CUBES_, SQRX, SQRY, SQRZ,
IMP_SURF_, IMP_SURF, ROT, ROTATE, PROJECT, PYRAMIDS

USAGE:
1. Select the Plot box (center, size, number of subboxes) for the function f
2. approX IMP_SURF(f,box)
3. goto 4 or 6
4. apply J.Boehm's filter ACD.EXE (option2) to convert the result to
   ACROSPIN format & rotate the image i.e.
   in DERIVE PC/XM:
   - Press Ctrl-P & wait several seconds until ACROSPIN grid appear
   - Rotate the image with Arrow keys
   in DfW:
   - Switch to 3D window (rightmost DERIVE menu button)
   - Press Alt-P to Plot the surface in 3D window
   - Make 6 keystrokes:

Alt-File__WritetoAcrospin__H__Enter__Overwrite__Yes
to save data in the file H.ACD and display it with ACROSPIN
- Rotate the image with Arrow keys
6. Press Esc to return from 3D surface rotation to DERIVE
5. rotate & project the result on yz-plane i.e.

approX PROJECT(result,angles) & plot the result in 2D window.

WARNING!
Compute time greatly increase with number of grids (cubes). Be patient!
Only 4*4*4 and less grids can be used in DERIVE PC. Use DERIVE XM
for more detailed images.

NOTE: The utility do not display points that are in the corners of subcubes.
If it happened - shift the center of the Plot Box a little bit to see
all points & lines.

IMP_SURF.BAT demonstrates some 3D Plots. Try it, please!

Some examples:

$$box2 := \begin{bmatrix} x & 0 & 5 & 4 \\ y & 0 & 5 & 4 \\ z & 0 & 5 & 4 \end{bmatrix}$$

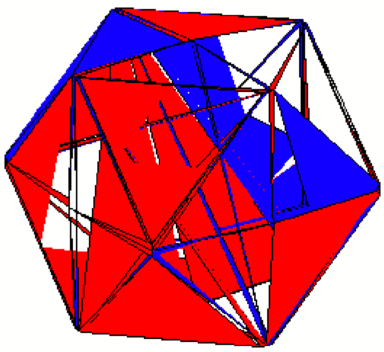
$$f2 := IMP\_SURF(x^4 + y^4 + z^4 = 5,\ box2)$$

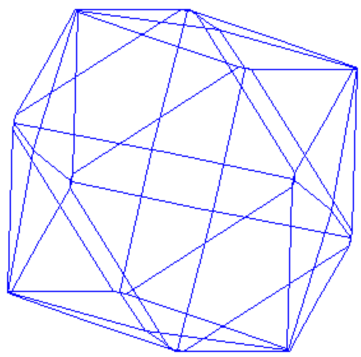
PROJECT(f2, [0.2, 0.2, 0.2])

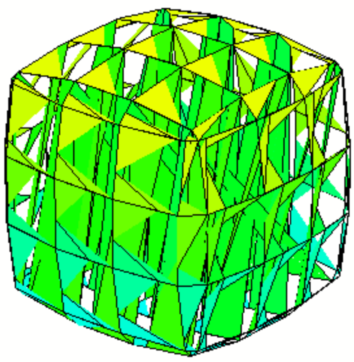
$$box3 := \begin{bmatrix} x & 0 & 5 & 7 \\ y & 0 & 5 & 7 \\ z & 0 & 5 & 7 \end{bmatrix}$$

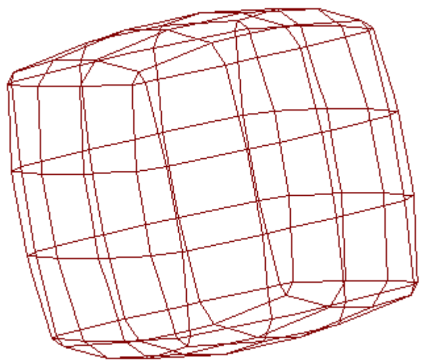
$$f3 := IMP\_SURF(x^4 + y^4 + z^4 = 5,\ box3)$$

PROJECT(f3, [-0.2, 0, -0.5])



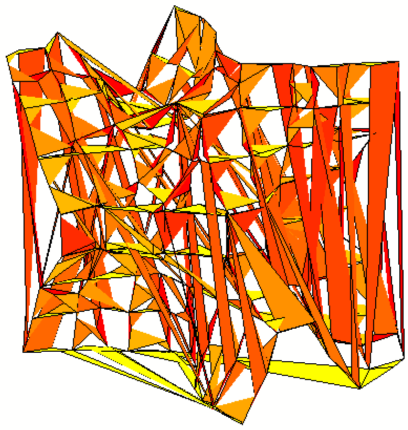


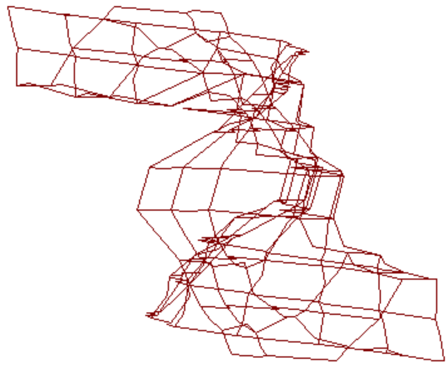


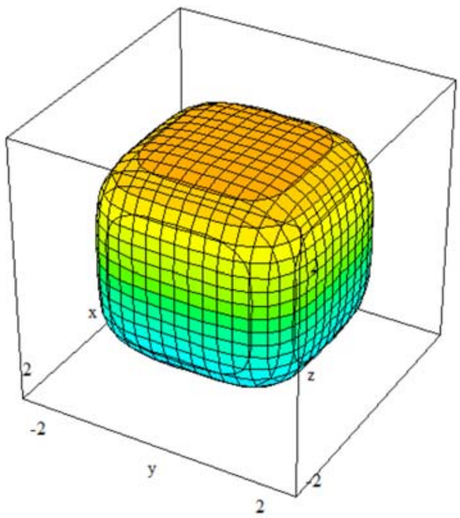


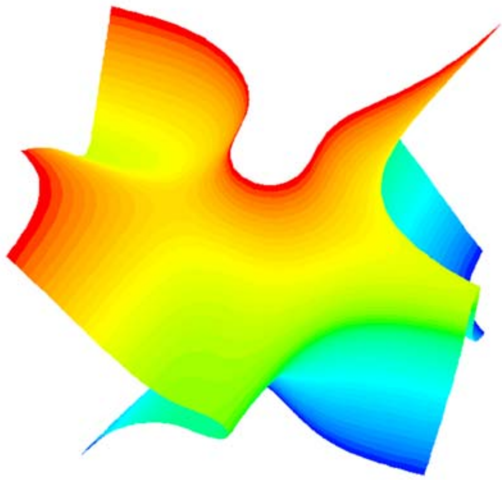
$$f4 := \mathrm{IMP\_SURF}(2\cdot x^3\cdot z^2 + 5\cdot y^3\cdot x^2 + y^2\cdot z^3 = 1,\ \mathrm{box3})$$

$$\mathrm{PROJECT}(f4,\ [0.2,\ 0,\ -0.5])$$



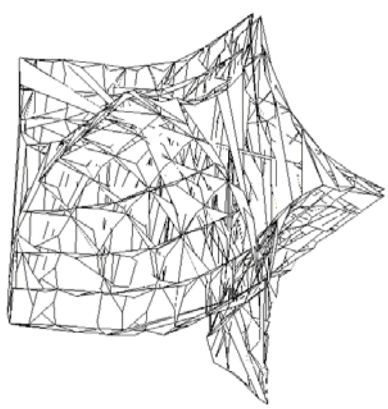


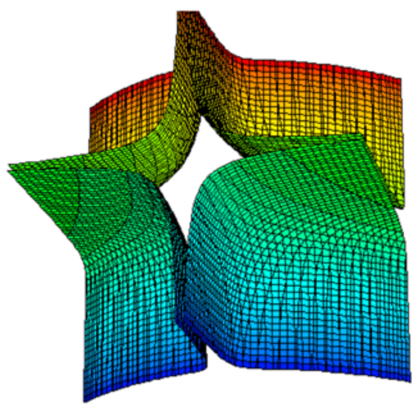




$$\mathrm{box4} := \begin{bmatrix} x & 0 & 6 & 8 \\ y & 0 & 6 & 8 \\ z & 0 & 6 & 8 \end{bmatrix}$$

$$\mathrm{IMP\_SURF}\left(\frac{x^4\cdot z^2}{2} - 5\cdot z^2\cdot x^2\cdot y + \frac{x\cdot y\cdot z}{4} = 10,\ \mathrm{box4}\right)$$





Today's mathematics and technology applied to Peaucellier's inversor.

Renée Gossez, Athénée Royal d'Uccle 1, Bruxelles, Belgium.
Jacqueline Sengier, Université Libre de Bruxelles, Bruxelles, Belgium.

The problem of constructing a linkage that produces a linear movement has remained unsolved for many years in the early 19th century. The conjecture was even made that it was impossible. James Watt had found a solution, but it was only approximate. In 1864, Charles Peaucellier, a French naval officer, finally invented a very simple linkage performing the task and called since then Peaucellier's inversor.

In Belgium, an important part of the curriculum in the last year of secondary school, consists of analytic geometry, transformations of the plane and complex numbers. The curriculum also contains explicit recommendation to use technology.

Studying Peaucellier's inversor gives the possibility to apply all these mathematical tools and to use the technology to perform calculations and graphs.

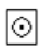
Contents.

1. Linkages.
2. Historical context and purpose of the Peaucellier's inversor.
3. *What* is the inversor made of and *how* does it work ?
4. Lets try with the TI 92 (geometry application).
5. *Why* does it work ?
   5.1. proof using analytic geometry and the TI 92 to perform the calculations;
   5.2. proof using the inversion in the plane and the geometry application of the TI 92;
   5.3. proof using complex numbers and the Y= and Graph applications of theTI 92.

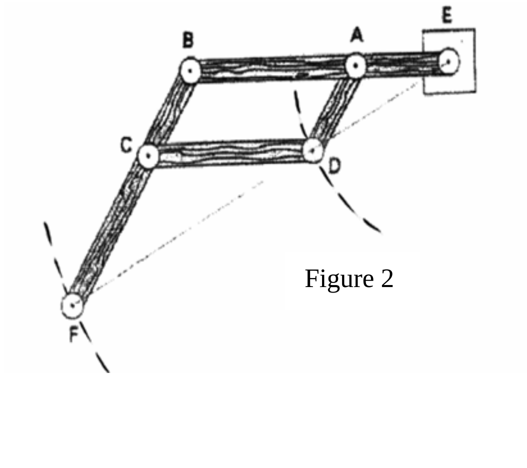
1. Linkages.

A linkage consists of a system of interconnected rigid rods in the plane, some of them being attached to fixed points about which they are free to turn, in such a way that the whole system has just enough freedom to allow one point on it to describe a certain curve.

Examples:

 represents a fixed point

COMPASS Figure 1



SCHEINER & LANGLOIS-Pantograph

Figure 2

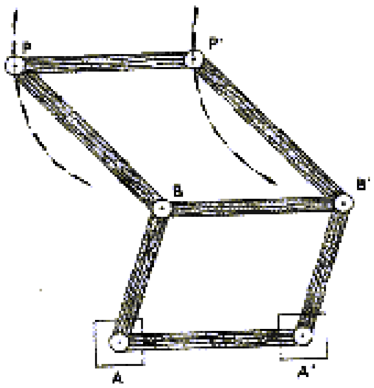


Figure 3
KEMPE Translator

Two facts have stimulated the interest of mathematicians and engineers for linkages in the late 18th and in the 19th centuries :

- the development of machine construction
- the work of Alfred Kempe (England, 19th century).

Kempe proved that any curve in the plane whose equation is polynomial can be constructed with the help of a linkage.

2. Historical context.
Purpose of the Peaucellier's inversor.

Linkages have long been used in machine construction.

One famous example, the "Watt parallelogram" (figure 4) was invented and used by James Watt (1764) to guide the piston P of a steam engine along a segment of straight line.

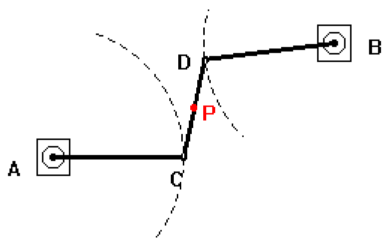
Though satisfactory in the case of Watt's steam engine, the result was only *approximative.* (Point P of figure 5 is actually moving on a curve with a 8 shape).

So, a new problem was raised, namely the construction of a linkage able to produce a *precise* linear movement.

> *This problem remained unsolved for a long time and the conjecture was even made that it was impossible !*

Nevertheless, in 1864, surprising all the specialists, a French naval officer, Charles Peaucellier, invented a very simple linkage called since then Peaucellier's inversor *which realizes this objective.*

Unfortunately, Peaucellier's invention came too late : at that time, efficient lubricants had been introduced and other mechanical solutions had been found so that the technical problem for steam engines had lost its significance.
It is said that Peaucellier's inversor was used only once, but in a prestigious place : it was used in the ventilation system of Westminster Palace !



|AC| = |BD| = a |CD| = b

Figure 4

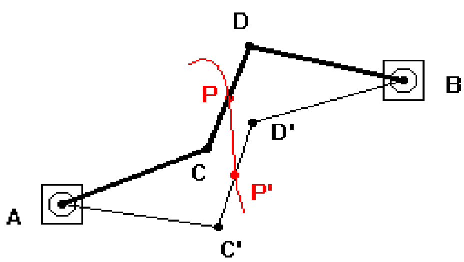
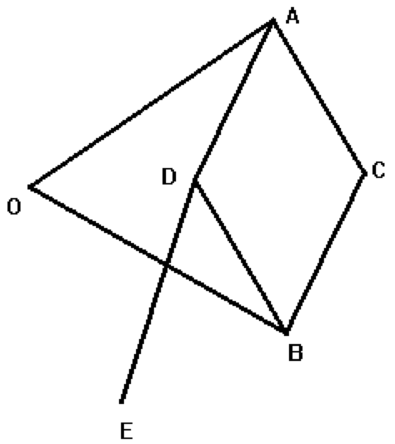


Figure 5

3. What is Peaucellier's inversor made of and how does it work ?

It consists of seven rigid rods, two of length a, four of length b, and a seventh of arbitrary length.
O and E are two fixed points such that |EO| = |ED|.
The entire apparatus is free to move subject to the given conditions.

As |EO| = |ED|, D necessarily describes an arc of a circle about E.
When D moves on this arc, C describes a segment of a straight line.



|OA|=|OB|=a |DA|=|AC|=|CB|=|BD|=b

Figure 6

4. Let's try with the TI 92.

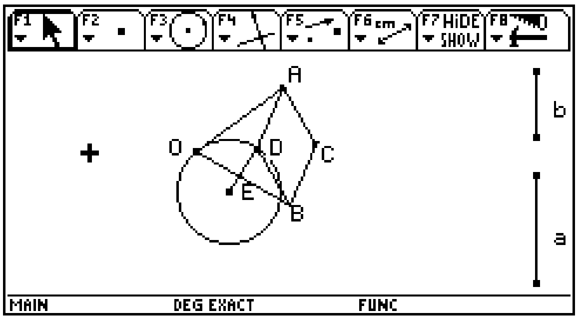


Figure 7

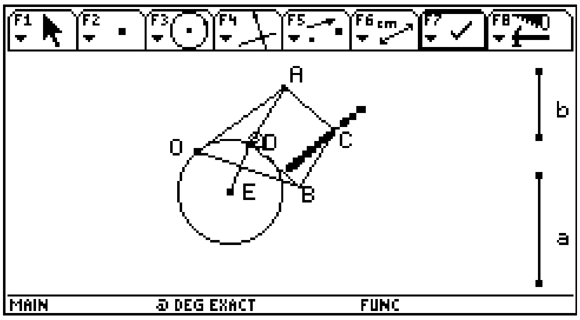


Figure 8

5. Why does it work ?

5.1. Proof using analytic geometry.

Let *x* and *y* be the coordinate axes and E the point (1, 0).

We want to find the equation of the locus of C when D moves on the circle Γ (center E, ray |EO|) and prove that this locus is a line.

Let (α, β) be the coordinate of D. As D is moving on circle Γ we have $(\alpha-1)^2+\beta^2=1$.
(in fact D only moves on an arc of Γ but we do not take this supplementary condition into account here).

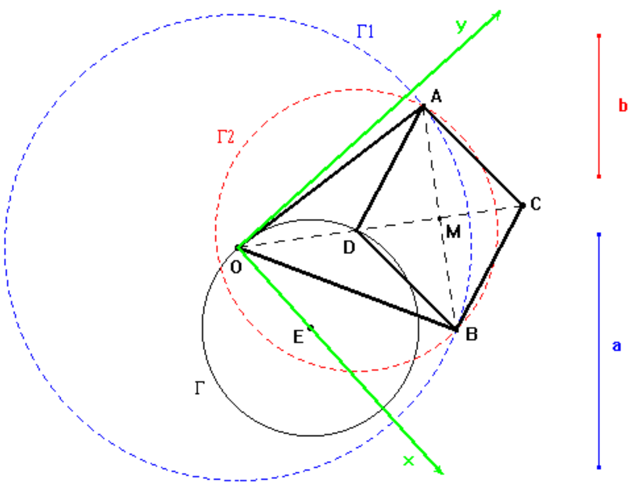


Figure 9

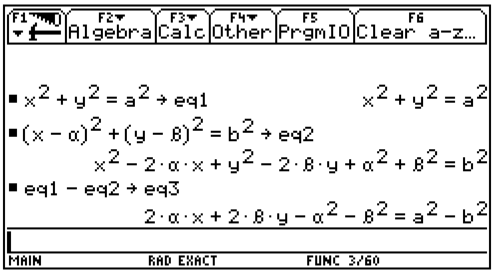


Figure 10

A and B both belong to circle Γ_1 (center O, ray a) and to circle Γ_2 (center D, ray b).
$\Gamma_1 \equiv eq1$, $\Gamma_2 \equiv eq2$ and hence

$AB \equiv eq1 - eq2$ (eq3)

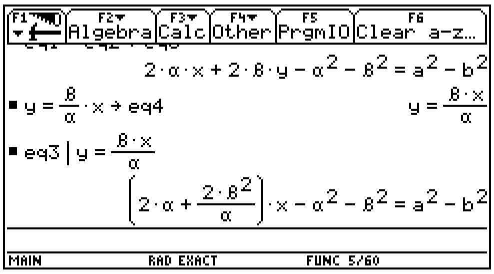


Figure 11

$OD \equiv y = \dfrac{\beta}{\alpha}x$ (eq4)

$M = AB \cap OD$. Let (mx, my) be the coordinate of M.

(mx, my) is the solution of system $\begin{cases} eq3 \\ eq4 \end{cases}$.

In figures 11 to 15, we solve this system step by step

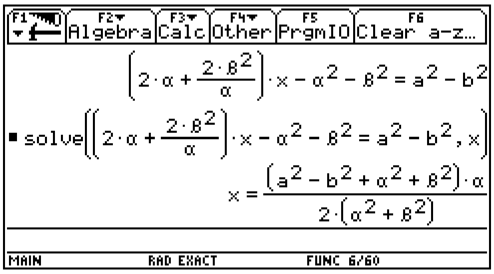


Figure 12

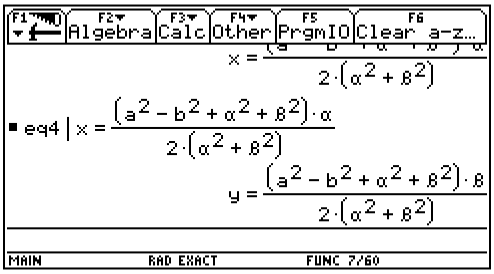


Figure 13

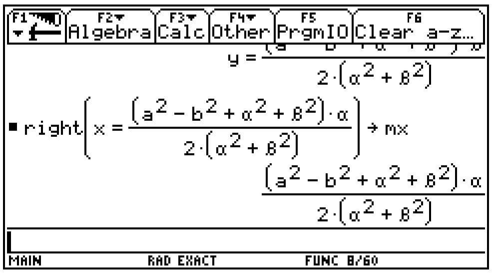


Figure 14

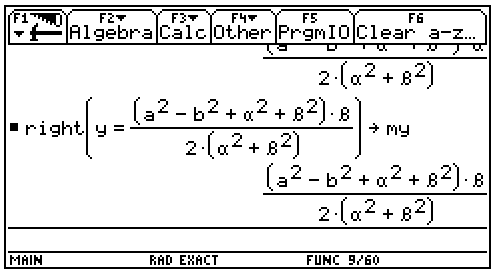


Figure 15

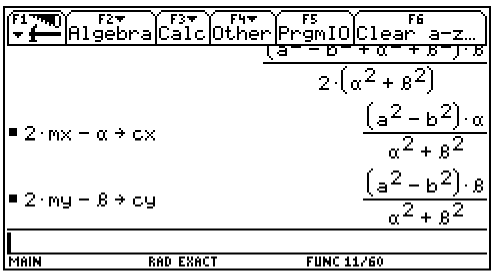


Figure 16

C(cx, cy) is the symmetric of D with respect to M i. e. that

$$\begin{cases} \dfrac{cx + \alpha}{2} = mx \\ \dfrac{cy + \beta}{2} = my \end{cases} \Rightarrow \begin{cases} cx = 2mx - \alpha \\ cy = 2my - \beta \end{cases}$$

Finally, the parametric equations of the locus of C are given by : $\begin{cases} x = \dfrac{(a^2 - b^2)\alpha}{\alpha^2 + \beta^2} \\ y = \dfrac{(a^2 - b^2)\beta}{\alpha^2 + \beta^2} \end{cases}$ (figure 16)

where α and β verify the condition $(\alpha - 1)^2 + \beta^2 = 1 \Leftrightarrow \alpha^2 + \beta^2 = 2\alpha$.

As a consequence the cartesian equation of the locus of C is $x = \dfrac{a^2 - b^2}{2}$ which is the equation of a line.

5.2. Proof using the inversion in the plane and the geometry application of the TI 92.

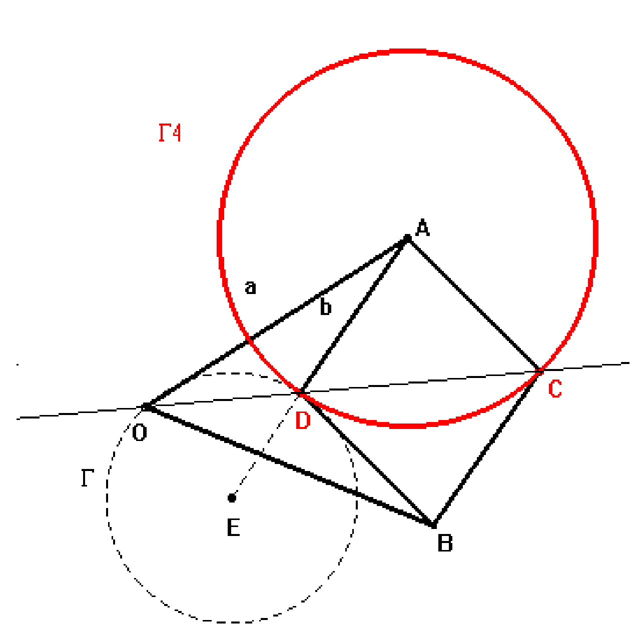


Figure 17

The inversion with center O and power k^2 is the transformation of the plane applying the point P on the point P' such that $\overrightarrow{OP}.\overrightarrow{OP'} = k^2$.

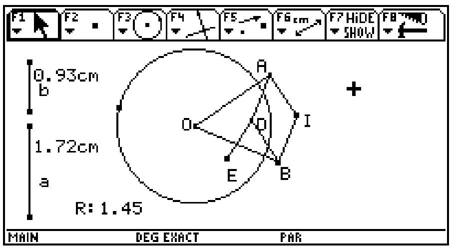
Property : the inverse of a circle Λ containing the inversion center O, is a line. This line is perpendicular to the diameter of Λ through O.

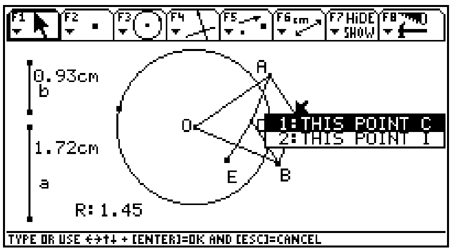
Let Γ4 be the circle with center A, ray b.
The power of point O with respect to the circle Γ4 is given by
$\overrightarrow{OD}.\overrightarrow{OC} = a^2 - b^2$ which is a constant.

As a consequence :
Point C is the inverse of D by the inversion φ with center O, power $a^2 - b^2$.
As D is moving on the circle Γ, C moves on a line. This line is perpendicular to OE.

Let's check this with the TI 92 :

We construct the inversion circle with center O and radius $\sqrt{a^2 - b^2}$.
We then construct φ(D) and call this point I. Point I clearly coïncides with C. (Figures 18, 19)





5.3. Proof using complex numbers and the Y= and Graph applications of the TI 92.

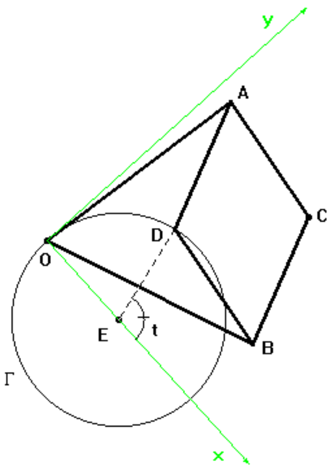


Figure 20

It is easy to show that in the complex plane, $z'=\dfrac{a^2-b^2}{\overline{z}}$ (1) is the equation of the inversion with center O(0,0) and power a^2-b^2.

From equation (1) and from the parametric equations of the locus of

$$D:\begin{cases} x = 1+\cos(t) \\ y = \sin(t) \end{cases}$$

we easily obtain that the locus of C has equation $x=\dfrac{a^2-b^2}{2}$:

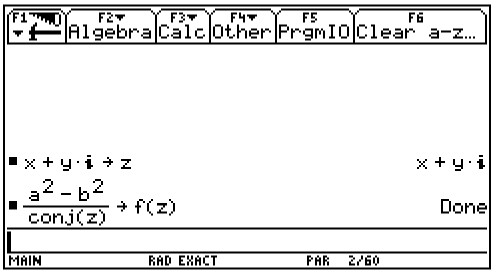


Figure 21

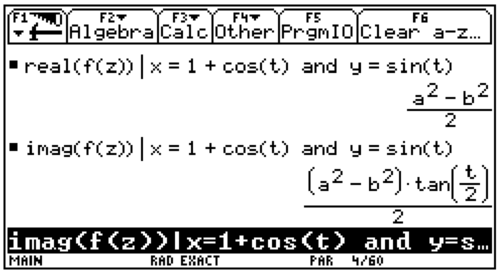


Figure 22

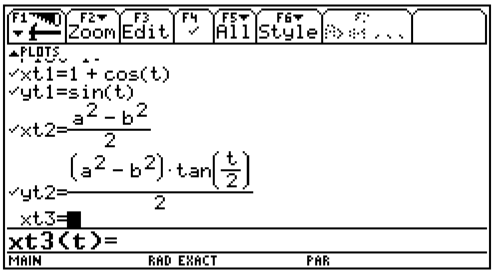


Figure 23

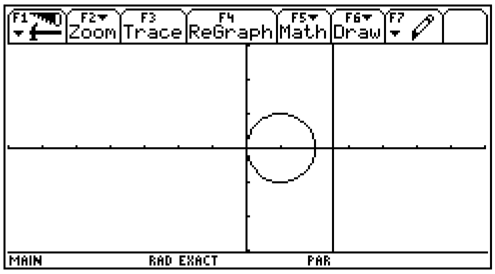


Figure 24

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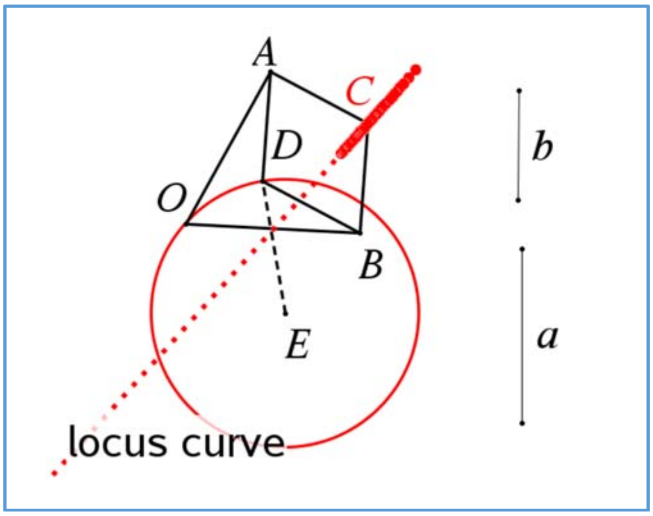
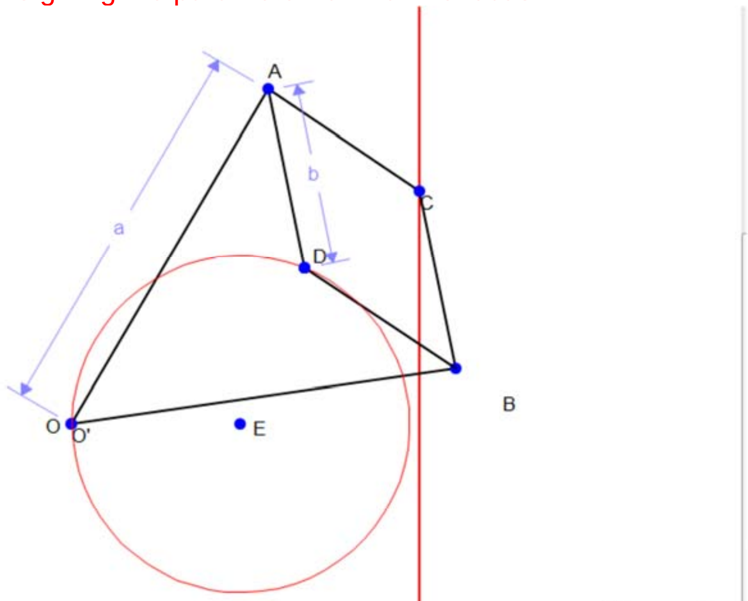
- Géométrie synthétique moderne, Paul Rossier, Vuibert, Paris, 1961.
- What is mathematics, Courant, Robbins, 14th edition Oxford University Press.
- Faire de la géométrie en jouant avec CABRI-Géomètre, Tome 2, Roger Cuppens, Publication n° 105 de l'APMEP.
- Les systèmes articulés, François Rideau, Pour la Science n° 136, février 1989, p. 94.
- Géométrie élémentaire et réalité. Introduction à la pensée géométrique, Erich Ch. Wittmann, Didier Hatier, 1999.

See the TI-NspireCAS construction below (Trace and locus of point C when D is moving on the circle).



Screen shot of Geometry Expressions giving the parameter form of the locus.



$$z_0 \Rightarrow \text{implicitize}\left(\frac{18+a^2-b^2+18\cdot\cos(t)+\left|18-a^2+b^2+18\cdot\cos(t)\right|}{12}, \frac{(3+3\cdot\cos(t))\cdot\left(18+a^2-b^2+18\cdot\cos(t)+\left|18-a^2+b^2+18\cdot\cos(t)\right|\right)\cdot\sin(t)}{36\cdot(1+\cos(t))^2}, t\right)=0$$

I copied the expression to DERIVE and after some manipulation I received the parameter form of the locus in a much more comfortable form.

This is exact the vertical line ploted as locus in GE.

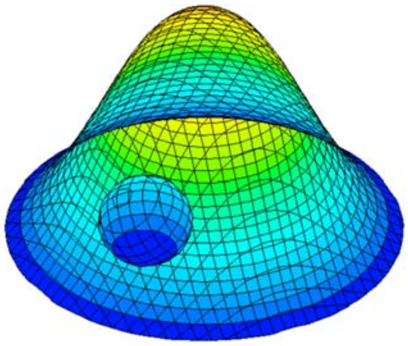
$$\#13: \quad x = \frac{a^2 - b^2}{6}$$

$$\#14: \quad y = \frac{(a^2 - b^2)\cdot \text{TAN}\left(\frac{t}{2}\right)}{6}$$

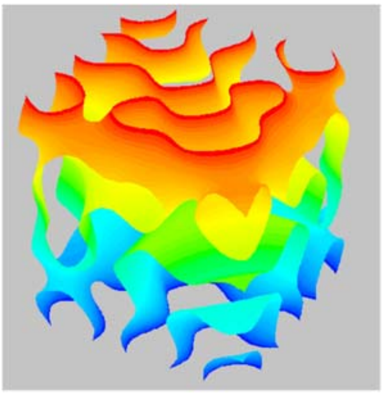
David Halprin from Australia sent a nice collection entitled "Vermischte Geschichte" (Mixed History) full of ´Epi-taphs/grams for Mathematicians of Disparate Fame´:

Some of the mathematicians, cited in some papers, have great worth, yet their importance may occasionally be overlooked, resulting in the potential loss of some ´gold below the surface´. David Halprin

Don´t laugh, this is infinitely serious! Eureka, I don´t have to take a bath any more, in order to have a revelation.
Archimedes (c 287 – 212 BC), Greece



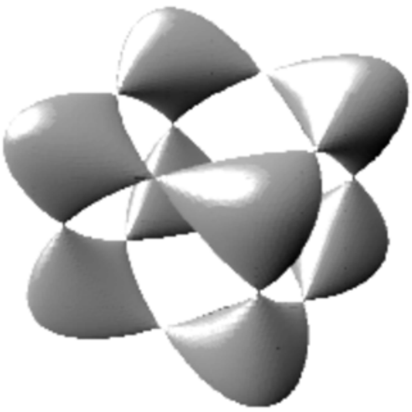
I have transcended to a
´π in the Sky´.
C.L.F. von Lindemann
(1852-1939), Germany

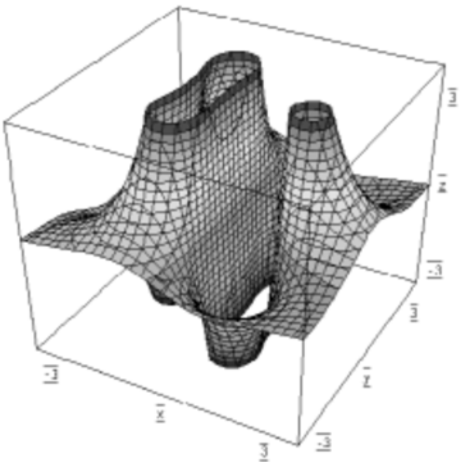


Ernesto Cesaro has thrown his last curve.
The ultimate condition for immobility of a straight guy.
In summary I have diverged to infinity; add that up!
Ernesto Cesaro (1859 – 1906), Italy

Now, at last, I am counting my infinities.
Georg Cantor (1845-1918), Germany

A cognitive Dissonance;
My ashes are in a non-Poisson distribution.
S.D. Poisson (1781-1840),France





I opine that I am no longer living;
falsify that if you can!
Karl Popper (1902 – 1996), Austria/England

Curtis-Wayne Kutzler, a nth degree cousin, drew my attention to the following text:

The Y0K Problem

(Translated from a Latin Scroll dated 2BC):

Dear Cassius,

Are you still working on the Y Zero K problem? This change from BC to AD is giving us a lot of headaches and we haven't much time left.

I don't know how people will cope with working the wrong way around.

Having been working happily downwards forever, now we have to start thinking upwards.

You think someone would have thought of it earlier and not left it to us to sort it all out at this minute.

I spoke to Ceasar the other evening. He was livid that Julius hadn't done something about it when he was sorting out the calendar. He said he could see why Brutus turned nasty. We called Consultus, but he simply said that continuing downwards using minus BC won't work and as usual charged a forturne for doing nothing useful. Surely, we will not have to throw out all our hardware and start over again? I suppose the Macrohardus Consortium will make another fortune out of this.

Money lenders are paranoid of course! They have been told that all usury rates will invert and they will have to pay their clients to take out loans. It is an ill wind...

As for myself, I just can't see the sand in an hourglass flowing upwards. I have heard that there are plans to stable all horses at midnight at the turn of the year as there are fears that they will stop and try to run backwards, causing immense damage to chariots and possible loss of life. Some say the world will cease to exist at the moment of transition. Anyway, we are still continuing to work on this blasted Y Zero K problem. I will send a parchment to you if anything further develops.

If you have any ideas please let me know,

Plutonius