THE DERIVE - NEWSLETTER #45

THE BULLETIN OF THE DERIVE USER GROUP + TI 92

Contents:

March 2002

Our friend Leon Magiera from Wroclaw University of Technology, Poland, proudly announces his book:

General Physics Problem Solving with *CAS* Derive
2001, ISBN 1-59033-057-9, $89.

Symbolic calculations are one of the computer's most significant applications, because these operations allow the user more time to spend on analyzing results or addressing different problems.Several programs perform symbolic calculations and computer algebra, but *DERIVE* is particularly noteworthy because of its low price and many features. This book focuses on solving general problems. These problems are practical, as opposed to the more theoretical questions dealt with in other works.

Contents:

References for *Some Statistics-Tools for DERIVE and the TI-89/92*, pp 35.

[1] **Applied Statistics, A First Course in Inference**, Graybill a.o., Prentice Hall, 1998

[2] **Statistics for Business and Economics**, McClave a.o., Prentice Hall, 2001

[3] **Elementary Statistics**, Larson & Farber, Prentice Hall, 2000

[4] **Investigating Statistics with the TI-92**, Kelly, Brendan Kelly Publ. Inc, 1997

[5] **A Concise Course in A-Level Statistics**, Crawshaw a.o., Stanley Thornes, 1990

[6] **Statistics, Concepts and Controversies**, Moore, Freeman & Co, 1996

All the books presented above are not CAS-related books, but I can recommend each of them as a rich source of applications together with clear representations of the concepts of descriptive and inferential statistics as well. Josef

A list of *DERIVE* and TI-92 related book is following. Unfortunately I didn´t come across any non-german publication during the last months with one exception, Leon´s book. If you know about any publications, then please send a short note.

[1] **Analysis 2, Ein Arbeitsbuch mit *DERIVE*, Rüdeger Baumann,**
296 S., Klett, 2002, ISBN 3-12-739514-0bk-teachware SR-23

Endlich liegt die Fortsetzung der erfolgreichen **Analysis 1** vor. Die Kapitelüberschriften lauten: Fortsetzung der Differenzialrechung, Integrierbare Funktionen, Funktionen mehrerer Veränderlicher, Kurven in Ebene und Raum, Modelle mit Differenzengleichungen.

Dazu gibt es eine auch Begleitdiskette unter ISBN 3-12-739515-9

[2] **Animationen mit *DERIVE* und DPGraph in Mathematik und Physik, R Wonisch,**
Klassische Themen der Schulmathematik und Physik werden durch beeindruckende Animationen ergänzt. 58 S. (+ Diskette), bk-teachware SR-28

[3] **CAS-Pilotaufgaben, Landesinstitut für Erziehung und Unterricht. Stuttgart**
Sammlung von 12 Abitur Pilotaufgaben mit *DERIVE*, TI-89/92/92+ (teilw. Lösungen), 40 Seiten

[4] **CAS-Pilotaufgaben für BOS, Landesinstitut für Erziehung und Unterricht. Stuttgart**
Sammlung von 10 Abitur Pilotaufgaben mit *DERIVE*, TI-89/92/92+ (samt Lösungen), 40 Seiten

Alle genannten Bücher können auch über **bk-teachware** bezogen werden.
Weitere aktuelle Informationen auf http://shop.bk-teachware.com
There is also a list of English books available in Bernhard´s shop.

Dear DUG members,

Let us start together the 12th DUG year. It is wonderful to have you all back in our community. And it will be wonderful to meet many of you in Vienna at VISIT-ME 2002. We received plenty of submissions for both parts of our Conference, totally more than 130. On this page you find all offered and accepted 90 min workshops (+ the country of the presentators) and on the last page I gave a list of the *DERIVE* & TI-92 Conference lectures. The lectures entitled in German will be held in German only.There is another bundle of interesting lectures, sub-mitted for the ACDCA Summer Academy. As a registered participant you can choose among a rich variety of lectures and workshops. In the next future we will set the abstracts on our website. Don´t forget to register in time.

Peter Lüke-Rosendahl wrote a mail asking me to update my picture in my Letter. I agree with him, but in this letter I take another picture. Some of you might remember that I once as proud grandfather resented our first grand-daughter Kimberly. Now I can offer two grand-daughters, Kim and Yvonne working on grandpa´s table, using his spectacles and *TI*-89. Next time I´ll present our grandson Dominic, maybe accompanied by a very young sister or brother.

Concerning Peter, you find two contributions dealing with Peter´s Knife. Rüdiger found a connection to an earlier *DNL* contribution.

David Halprin´s article on "Dunes" is not connected with Frank Herbert´s Phantasy Saga "Dune", but gives an excellent example for Cross Curriculum treating a topic.

Unfortunately I don´t have so much space as usual for my words and I have to close hoping to meet many of you in Vienna – VISIT ME!!

Josef

Workshops submitted and accepted (so far) for VISIT-ME 2002 (in alphabetical order)

A picture tells a story of well over 1,000 words (GER)
Absolute Value: Geometrically, Algebraically and Technologically (USA)
Computer Algebra within a Spreadsheet-Style Environment (UK)
Drawing Julia Fractals on the TI-92 (BEL)
Dynamic Geometry in Classroom (AUT)
Exploring the Dynamic Geometry of Calculus with Geometer´s Sketchpad 4.0 (USA)
Geometric Proof in Upper Middle School (MEX)
Maple in the Process of Teaching Higher Mathematics (UKR)
Primality Testing and Factoring Large Numbers with DERIVE (AUT)
Programming in DERIVE – Some introductory examples (AUT)
Programming with TI-Interactive (USA)
Some general-purpose toold for 2D- and 3D-linear trans. geometry with DERIVE (UK)
Technology as a pedagogical amplifier in developing student´s geometric intuiton ....(USA)
The digital camera and mathematics – what´s the connection? (USA)
The Use of Prolog Programming Language as a CAS (UKR)
TI-83+ Calculator Workshop (SWI)
Use the program editor of the TI-89/92 to write a program to solve cubic equations (USA)

Download all *DERIVE*- and TI-89/92 files from

http://www.acdca.ac.at/t3/dergroup/index.htm

http://www.bk-teachware.com/main.asp?session=375059

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE & TI-92 User Group*. It is published at least four times a year with a contents of 44 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE* and the *TI-92/89* as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

As many of the *DERIVE* Users are also using the *TI-92/89* the *DNL* tries to combine the applications of these modern technologies.

Editor: Mag. Josef Böhm
A-3042 Würmla
D´Lust 1
Austria
Phone/FAX: 43-(0)2275/8207
e-mail: nojo.boehm@pgv.at

Contributions:

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE & TI-92 Newsletter* will be.

Next issue: June 2002
Deadline 15 May 2002

Preview: Contributions waiting to be published

A Utility file for complex dynamic systems, Lechner, AUT
Tanz der Wallace-Geraden / Dance of Wallace-Lines, Baumann, GER
Various Training Programs
Type checking, Finite continued fractions, Welke, GER
Kaprekar´s "Self numbers", Schorn, GER
Examples for Statistics, Roeloffs, NL
Some simulations of Random Experiments, Böhm, AUT
Wonderful World of Pedal Curves, Böhm, AUT
Another Task for End Examination, Lechner., AUT
Cooperation DP-Graph and *DERIVE*, Wonisch, GER
Applications for Random Permutations, Schlöglhofer, AUT
Tools for 3D-Problems, Lüke-Rosendahl, GER
Penrose Inverse of a Matrix, Karsten Schmidt, GER
Differential Equations in Secondary School, Günter Schödl, AUT
Four Calender Functions, Richard Schorn, GER
Putzer´s Method for the Calculation of e^{At}, Francisco Fernández, ARG

and
Setif, FRA; Vermeylen, BEL; Leinbach, USA; Aue, GER; Koller, AUT, Keunecke, GER, ...

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Herstellung: Selbstverlag

Theresa Shelby, Hawaii tshelby@ti,com

Hello all,

Several of you have requested that the plot windows have a feature for annotations similar to the "Font of New Text Objects" menu item in the Algebra Window Options > Display menu.

We think this is a very good idea and will look at incorporating it a future point release of Derive 5. At the very least, we could store the last annotation font selected and not revert back to the DfW5 Printer font for each new annotation.

Thank you for your continued suggestions to improve Derive!

Aloha,
Theresa

Root Hunting

Jim FitzSimons cherry@GETNET.NET

How can *DERIVE* do this simplification?

$$v2 = \frac{v0\cdot(2\cdot r + 25)}{4\cdot(r\cdot(\sqrt{3} + 2) + 25\cdot\sqrt{3} + 50)}$$

$$v2 = \frac{v0\cdot(2 - \sqrt{3})\cdot(2\cdot r + 25)}{4\cdot(r + 25)}$$

Ignacio Larrosa Cañestro, A Coruña (España) ignacio.larrosa@eresmas.net

Multiplying numerator and denominator by

#3: $r\cdot(2 - \sqrt{3}) - 25\cdot\sqrt{3} + 50$

#4: $$\frac{v0\cdot(2\cdot r + 25)\cdot(r\cdot(2 - \sqrt{3}) - 25\cdot\sqrt{3} + 50)}{4\cdot(r\cdot(\sqrt{3} + 2) + 25\cdot\sqrt{3} + 50)\cdot(r\cdot(2 - \sqrt{3}) - 25\cdot\sqrt{3} + 50)}$$

expanding both separated and factoring (Radical), I get:

#5: $$\frac{r^2\cdot v0\cdot(4 - 2\cdot\sqrt{3}) + r\cdot v0\cdot(150 - 75\cdot\sqrt{3}) + v0\cdot(1250 - 625\cdot\sqrt{3})}{4\cdot(r\cdot(\sqrt{3} + 2) + 25\cdot\sqrt{3} + 50)\cdot(r\cdot(2 - \sqrt{3}) - 25\cdot\sqrt{3} + 50)}$$

#6: $$\frac{r^2\cdot v0\cdot(4 - 2\cdot\sqrt{3}) + r\cdot v0\cdot(150 - 75\cdot\sqrt{3}) + v0\cdot(1250 - 625\cdot\sqrt{3})}{4\cdot r^2 + 200\cdot r + 2500}$$

#7: $$\frac{v0\cdot(2 - \sqrt{3})\cdot(2\cdot r + 25)}{4\cdot(r + 25)}$$

Saludos

Josef Böhm, Würmla

Hi, root hunters,

working with roots is often a challenge.
Ignacio used an old rule to eliminate the root from the denominator. I tried also a very old rule and rewrote the denominator by first outfactoring 25 and then the expression $\left(\sqrt{3}+2\right)$. The expression resulting is now recognized by *DERIVE* and one immediately obtains the desired result.

Using some oldfashioned techniques by hand firstly:

$$\#9: \quad \frac{v0\cdot(2\cdot r + 25)}{4\cdot(r\cdot(\sqrt{3} + 2) + 25\cdot\sqrt{3} + 50)} = \frac{v0\cdot(2\cdot r + 25)}{4\cdot(r + 25)\cdot(\sqrt{3} + 2)}$$

Just Simplify!

$$\#10: \quad \frac{v0\cdot(2\cdot r + 25)}{4\cdot(r\cdot(\sqrt{3} + 2) + 25\cdot\sqrt{3} + 50)} = \frac{v0\cdot(2 - \sqrt{3})\cdot(2\cdot r + 25)}{4\cdot(r + 25)}$$

Jim FitzSimons

How can I get DERIVE to simplify this?

$$\#11: \quad v2 - v3 =$$

$$\frac{r\cdot v0\cdot(r\cdot(338994333\cdot\sqrt{3} + 674007334) + 8349875000\cdot\sqrt{3} + 16633550000)}{2\cdot(r\cdot(666\cdot\sqrt{3} + 1001) + 16650\cdot\sqrt{3} + 25025)\cdot(r\cdot(674666\cdot\sqrt{3} + 1015999) + 16650000\cdot\sqrt{3} + 25025000)}$$

$$\#12: \quad v2 - v3 = \frac{r\cdot v0\cdot(2 - \sqrt{3})}{2\cdot(r + 25)}$$

From the original equation #11 an intermediate result may be obtained by expanding in r:

$$\#13: \quad v2 - v3 = v0\cdot\left(1 - \frac{\sqrt{3}}{2}\right) - \frac{25\cdot v0\cdot(331\cdot\sqrt{3} + 4)}{2\cdot(r\cdot(666\cdot\sqrt{3} + 1001) + 16650\cdot\sqrt{3} + 25025)}$$

Simplification of this then yields:

$$\#14: \quad v2 - v3 = \frac{r\cdot v0\cdot\left(\frac{331\cdot\sqrt{3}}{2} + 2\right)}{r\cdot(666\cdot\sqrt{3} + 1001) + 16650\cdot\sqrt{3} + 25025}$$

Simplify again and the new result is:

$$\#15: \quad v2 - v3 = \frac{r\cdot v0\cdot(331\cdot\sqrt{3} + 4)}{2\cdot(r\cdot(666\cdot\sqrt{3} + 1001) + 16650\cdot\sqrt{3} + 25025)}$$

Obviously this last result is not very different from the preceding one.
However, it is curious that *DERIVE* will cycle repeatedly between the two with successive simplifications.

If $\sqrt{3}$ is replaced by a variable, say x, in the last form of the equation. and then simplifying this expression the result is as follows:

$$\#16:\quad v2 - v3 = \frac{r\cdot v0\cdot(331\cdot x + 4)}{2\cdot(r\cdot(666\cdot x + 1001) + 16650\cdot x + 25025)}$$

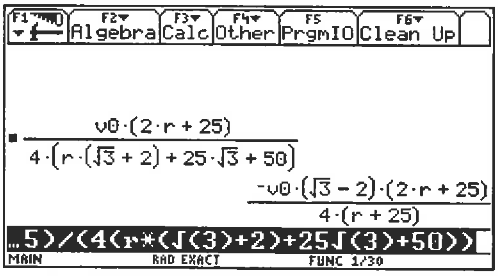
$$\#17:\quad v2 - v3 = \frac{r\cdot v0\cdot(331\cdot x + 4)}{2\cdot(r + 25)\cdot(666\cdot x + 1001)}$$

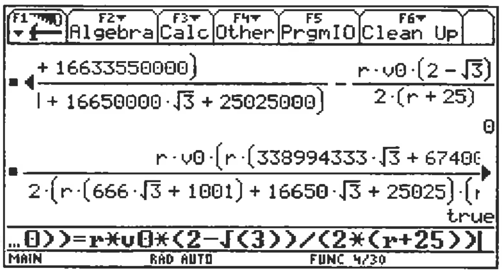
Replacing x by SQRT(3) and simplifying twice will yield the final result:

$$\#18:\quad v2 - v3 = \frac{r\cdot v0\cdot\left(1 - \frac{\sqrt{3}}{2}\right)}{r + 25}$$

$$\#19:\quad v2 - v3 = \frac{r\cdot v0\cdot(2 - \sqrt{3})}{2\cdot(r + 25)}$$

As you can see, the TI-92 is able to perform the first task – but it is unable to do the second simplification.





But the TI does recognize the equivalence of the two expressions!

Josef

Valeri Anisiu anisiu@math.ubbcluj.ro

Hello to all,

I tried to automatize the simplification using SUBST. Unfortunately, *DERIVE* refuses to substitute some subexpressions (=operands of a certain level).

For example,

```
SUBST(r·y + 2·r + 25·√3 + 50, √3, x) = r·y + 2·r + 25·√3 + 50
```

Leaving the SQRT(3) at it is, even if 25*SQRT(3) is an operand of the first expression and SQRT(3) is (or should be) an operand for 25*SQRT(3) (as the function STRING confirms).

Note however that the substitution works using the menu Simplify/Subexpression! and also for expressions like 25+SQRT(3). Similar situations arise when trying to substitute for example #i with i.

Let us hope that the next release of DERIVE will fix this behavior.

Enric Puig, Spain enricpuig@TELELINE.ES

Hello,

Are people aware that after November 2000, there is no further input in the Archives of Derive-news? You can check this at: http://www.mailbase.ac.uk/lists/derive-news/archive.html.

Theresa Shelby,Hawaii tshelby@ti.com

Hi Enric,

The mailing list Derive-News was moved and archives after November 2000 are available at

http://www.jiscmail.ac.uk/lists/derive-news.html

Regards,
Theresa Shelby

Next is part of an answer from Albert Rich to Peter Scofield. You can learn about Procedural Programming, Josef.
By the way, have a look to ARROWS.DFW. Peter will give a lecture and a workshop at the VISITME, 10 – 13 July in Vienna.

Hi Peter,

I will include your improved version of `Arrows.dfw` in the Users directory for version 5.05 of Derive. Thank you.

I liked your idea for a function that appended the columns of any number of matrices so much that I generalized the existing APPEND_COLUMNS function in VECTOR.MTH to do it. It takes advantage of the ability in Derive 5 to define functions that can accept any number of arguments (see "Procedural Programming" in the on-line help for details). Also if given a single argument that is a list of matrices, APPEND_COLUMNS also appends those matrices, just like your APPEND_COLIST does.

```
#1:   MATRIX?(a, r_) := VECTOR?(a) ∧ EVERY(VECTOR?(r_), r_, a)

      APPEND_COLUMNS Args :=
        If DIM(Args) = 1 ∧ VECTOR?(Args↓1) ∧ EVERY(MATRIX?(a_), a_, Args↓1)
#2:        APPEND(VECTOR(a_`, a_, Args↓1))`
           APPEND(VECTOR(a_`, a_, Args))`
```

For example:

$$\#3:\quad \text{APPEND\_COLUMNS}\left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c & d \\ e & f \end{bmatrix}, \begin{bmatrix} g \\ h \end{bmatrix}\right) = \begin{bmatrix} a & c & d & g \\ b & e & f & h \end{bmatrix}$$

$$\#4:\quad \text{APPEND\_COLUMNS}\left[\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c & d \\ e & f \end{bmatrix}, \begin{bmatrix} g \\ h \end{bmatrix}\right] = \begin{bmatrix} a & c & d & g \\ b & e & f & h \end{bmatrix}$$

Both simplfy to the same matrix!

Aloha, Albert.

Stefan Welke, Germany
spwelke@aol.com

Concerning Trig Equations, DNL#44, pp 23

It is clear that there are two periodic functions, hence each solution x provokes another solution $x + 2\pi$. By inspecting the plot one convinces himself, that within an interval of length 2π two solutions can be found, if the interval starting point is not a solution, and that there are three solutions if it is. Then the endpoint of the interval is also a solution. It is more elegant to work with half open intervals: All intervals of form [a,a+2π] contain two solutions.

```
SOLUTIONS(5·COS(t) = 6·SIN(t) - 1, t)
```

$$\left[-\text{ATAN}\left(\frac{4\cdot\sqrt{15}+11}{119}\right)-\frac{3\cdot\pi}{4},\ \text{ATAN}\left(\frac{4\cdot\sqrt{15}-11}{119}\right)+\frac{\pi}{4},\ \text{ATAN}\left(\frac{4\cdot\sqrt{15}-11}{119}\right)-\frac{7\cdot\pi}{4}\right]$$

We see, that *DERIVE* indeed offers three exact solutions, say x_1, x_2 and x_3. Immediately we can see that $x_3 + 2\pi = x_2$. x_2 and x_3 are endpoints of a closed interval of length 2π. So we can conclude that each set of solutions is of the following form

$$\mathbb{L} = \{ x_3 + 2k\pi \mid k \in \mathbb{N} \} \cup \{ x_1 + 2k\pi \mid k \in \mathbb{N} \}$$

What could be improved in the *DERIVE* solution is the order of appearance of the solutions.

Because of its periodicity this equation is easy to solve in comparison with

$$3/2 * x * \cos(x) - 1/2 *\sin(x) = 0 \iff 3*x = \tan(x)$$

One can see that the straight line $y = 3x$ intersects the infinite number of branches of the tangens line, but because of the slope of the line one cannot refer on periodicity and we cannot expect exact solutions. I don´t believe that there is an exact solution. So also CAS has found its borders. But it is remarkable enough that *DERIVE* found the exact solutions for the first problem.
Best regards,
Stefan Welke

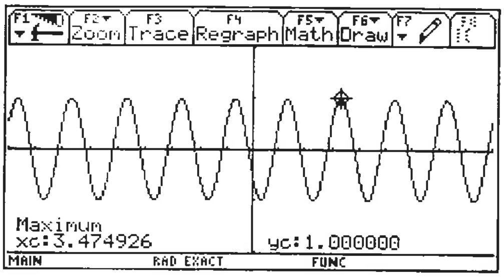
Thomas Laumann, Hamburg

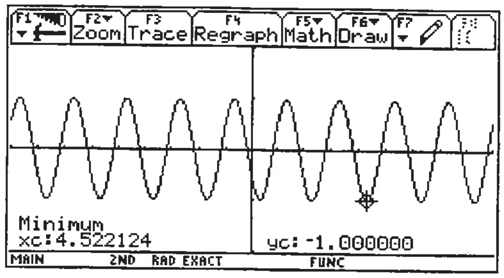
Trig Problems again

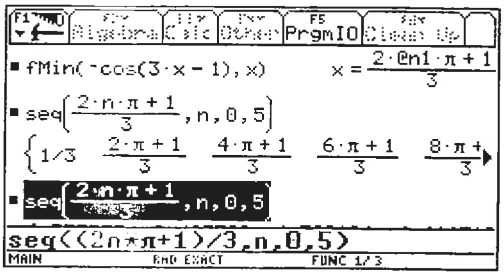
How can I solve problems like the following on the *TI*-92?

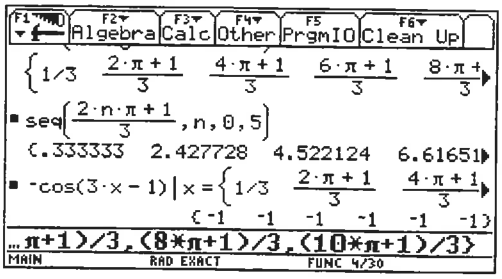
Task 1: Give the the positions of maximum and minimum values of $y = -\cos(3x - 1)$

Proposed Solution:









@n1 is an integer parameter.

(to becontinued page 27)

Sand Dunes

David Halprin, North Balwyn, Australia, davidlaz@net2000.com.au

In broad overview, dunes are rounded, elliptical, parabolic or crescentic mounds of loose sand, which have been piled up by wind action in the areas of lakes, (lacustrine dunes), rivers, sea coasts and deserts.

There are 5 or 6 supposed basic dune-types, as well as a few other variations , so herin is an approximate mathematical treatment for the simpler ones, since exactness is a pointless goal with this type of natural phenomenon, that evolves over long periods of time, and can be looked at either as a flowing object or as a stationary one, depending on one´s approach.

Dunes are all composite structures, that can be deconstructed mathematically. Their aerial views/profiles and lateral profiles will be examined as well as the rippled surfaces of some.

These on left hand side, are sand dunes in irregular patches and assorted shapes.

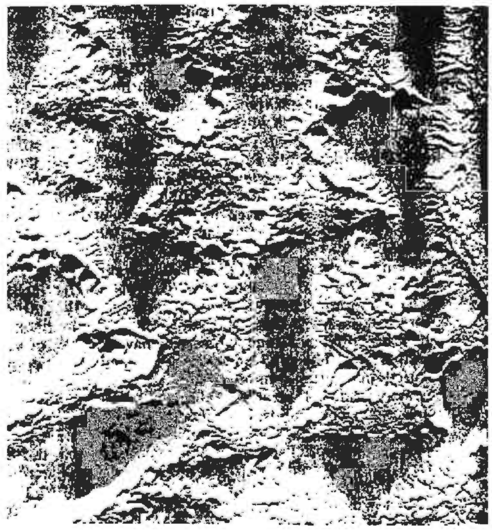


Figure 1: Irregular Dune

Sand Dunes and Sand Sheets result from wind-blown sand and are gently undulating surfaces with low relief and in mounds. Their creation, growth, migration and patterns depend mainly on the variety of sand(s), the winds, the local vegetation (flora), fauna and topology. They frequently overlap with ohe other., forming a network of domea and basins.

The smalles dunes are about a metre high and cover about ten square metres. The largest may be over 400 metres high and cover several square kilometres. The grains of dune sands vary in diameter from 0.1 – 0.7 mm.

The winds have three sources of variation, (direction, speed and turbulence.) The wind is retarded near the dune surface by friction. There is a certain threshold velocity before sand will be picked up, depending on grain size, then once initiated, can be maintained by a lower wind speed.

Desert winds produce deposition of sediment whenver their velocities are checked by opposing winds, a hill or mountain, or some other cause of wind shadows. Sedimentary deposits of wind-blown dust (loess [1]) are exemplified by bluffs as thick as 100 metres and/or widespread blankets of loess.

Sea winds blow sand off to the rear part of the beach, where it forms small hummocks. They join up to form fore-dunes, and in many cases, several rows of dunes are formed. In some cases the dunes migrate to adjacent low-lying planes. In other cases, parabolic dunes are formed with their summits coastward. These summits, in time, may be broken through by the winds, forming a series of ridges parallel to the prevailing winds. These could be the ´Seif Dunes´.

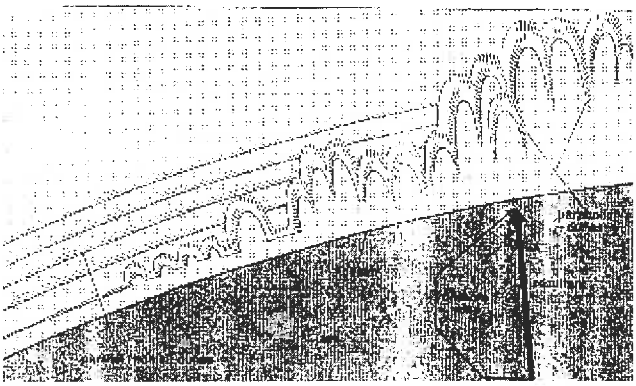


Figure 2: Shore Parabolic Dune

This is the outline of frontal and parabolic shore dunes behind the beach, and the onshore wind resultant.

So this writer's approach for the aeolian process of the Sand Dune is to consider a particle (grain) of saltating (jumping) or creeping sand being transported by the wind, where its fate depends on local phenomena and conditions. If there is some solid interface, (such as the nether end of a pre-existing Barchan dune, a tree, a shrub, or other obstruction), then it may become part of a climbing dune at that spot, depositing as accretion sand. However, if it encounters a foetal form of a gestating (potential) dune, then this 'foetus' will act as seed dune, (nucleus), and there are eight main 'canonic' possibilities:-

Firstly, re the 'seed-dunes' or 'fore-dunes'.

The seed-dune may be a patch of sand, perhaps resulting from windblown sand having been deposited in a ground hollow previously, now attracting further sand, because of the drag causing the wind to change its velocity gradient as it ancounters the patch. This often builds into a 'Barchan', and it moves in the direction of the wind by the firm áccretion sand' being eroded from the windward side and deposited, (called "traction and creep"), on the lee side, (slip face), as loose 'avalanche sand', with a steep face of about 30 – 33 degrees maximum, since the coefficient of friction will not support a steeper angle of repose, and if it builds to 34 degrees, there is a scarring avalanche to re-establish 30 – 33 degrees.

The seed dune may be easily blown along the plain and therefore offer little, if any, resistance to any oncoming grains of sand, so if it takes any recognisable shape it could be that of a 'Barchan' with mainly accretion sand at the windward side and as a veneer over most the main mass, (which is avalanche sand, which type is mostly on the lee side also). This may be a down-wind migrating dune. Since wind directions are frequently variable throughout the year causing ' dune-stress', the dunes may migrate in different directions. Over a long period of time, the overall net wind component is the determining factor for such migration. The net component, in some case may be zero, in which case dunes may migrate to and fro, but will tend to remain in a fixed average position through time.

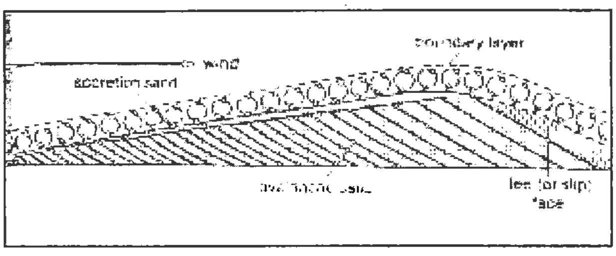


Figure 3: Dune Formation and Migration

This shows the distribution of sand on, and within, a dune due to wind action. Sand is shown raining down onto the slip (lee) face, leading to migration.

1) Parabolic Dunes

The seed dune may be fairly firmly planted, perhaps with a vegetative overgrowth, and therefore offer a substantial resistance to the impulsive force of the flying sand, and a 'Parabolic Dune' may start up.

It appears as a U-shaped scoop of sand, with points tapering to windward and a rounded nose pointing downwind. Only the nose of the dune is active, in that the arms are usually anchored by vegetation and can be 'amputated' as the dune moves downwind and the former arms may cease to appear to be dunes.

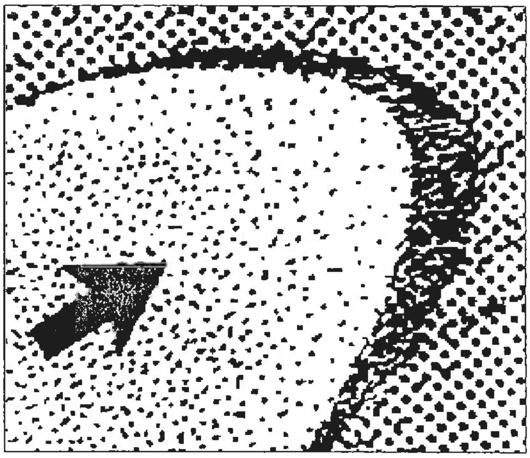
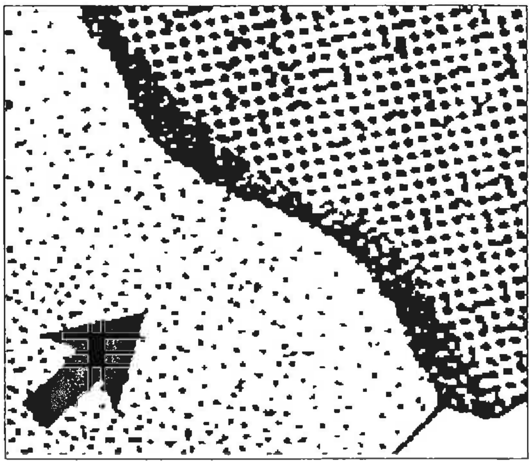


Figure 4: Parabolic Dune

2) Transverse Dunes



When the wind moves large quantities of sand, it buries all vegetation and forms high transverse dunes. The crests (ridges) roll along in nearly straight lines for hundreds of feet, orthogonal to the wind's dominant direction. They are unaffected by vegetation.

Figure 5: Transverse Dune

3) Barchan Dunes

It is a special type of transverse dune, which embodies the classic image of a sand dune, a crescent shape with tips jutting forward (leeward) in elegant curves as it moves across the landscape. It usually forms as a solitary dune, where winds have limited sands available for shaping. But as more sand accumulates, the resulting 'compound barchan' eventually merges as a transverse dune.

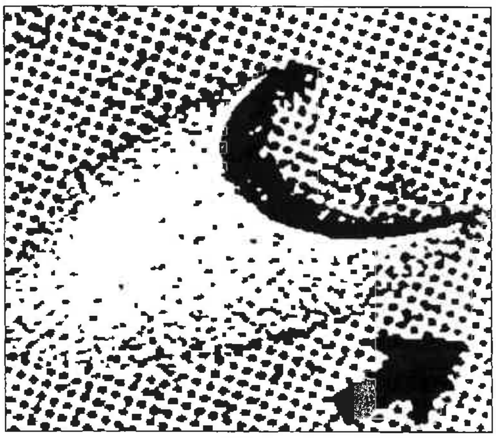


Figure 6: Barchan Dune (above)

Figure 7: Mega-Barchan Dune (left)

4) Reversing Dunes

Where reversing winds have brought migrating transverses dunes to a standstill, this results in a subset of transverse dunes, called 'Reversing Dunes'. During their formation the slip faces first in one direction, then in another. Sometimes, with reversal of the wind, newly-arrived sand has nowhere to go but up to the top of the transverse dunes. In a special case, the excess sand, which accumulates at the far end of the transverse dunes, piles against the steep flanks of the mountains in 'climbing dunes'.

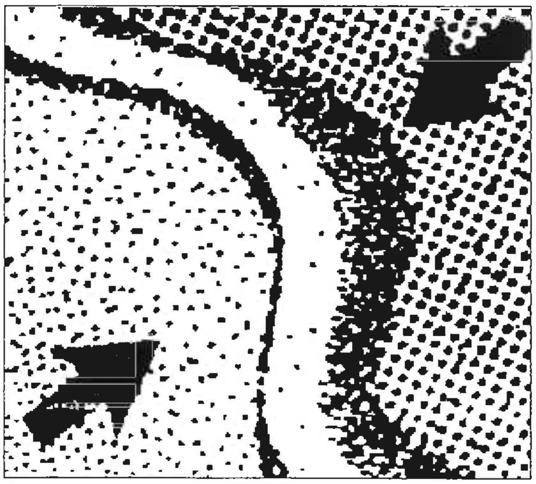


Figure 8: Reversing Dune

5) Climbing Dunes

When turbulent winds rise over obstacles, they pile sand in loose masses called climbing dunes. With little distict shape, and less grace, they often form from sand blown past other dunes, but then slowed by a cliff or ridge..

6) Seif Dunes

A Seif dune is a linear sand dune of height up to 90 metres, and length up to 100 km, and its width is about six times its height, (uop to 550 metres). It is an elongated ridge, generally orieted with its long direction of two or more winds, blowing at acute angles to each other. The dune crestconsists of a series of peaks and gaps, and the steep (slip) face may change sides of the dune according to changes in wind direction. Mostly, it rests on a coarse sand sheet. It grows in height and width by the action of cross winds, and in length by the action of winds parallel to its longitudinal axis.

The linear Seif dune depends on a seasonal cycle of wind variation, having steep slopes on each side and a sharp sinuos summit and it is subject to cross winds that cause the ridges to be aligned with the resultant of these cross winds.

The Seif dunes can have very puzzing paterns, such as two crossing directions and/or meanders.

7) Whaleback Dunes

A whaleback dune is a very large elongated dune.

8) Star-Shaped Dunes

They are formed by irregular winds.

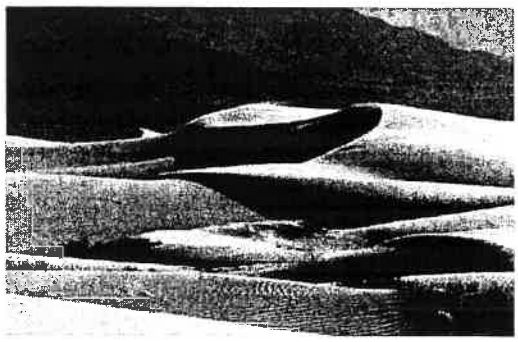


Figure 9: Landscape with Dunes

The upper surfaces of many dunes may be approximated mathematically as inclined planes, many of which have small asymetrical parallel ripples, orthogonal to the wind, with wavelengths of 7-18cm, which are subject to traction and impact creep and may disappear with high velocity winds.

$$y = 0.324919x + 0.40081x^2 - 0.187354x^3 + 0.0185603x^4$$

This quartic approximates a ripple profile or a dune profile, with a 32 degree leeward tangent ad a gentle windward slope starting with a tangent of 18 degrees and building its steepness.

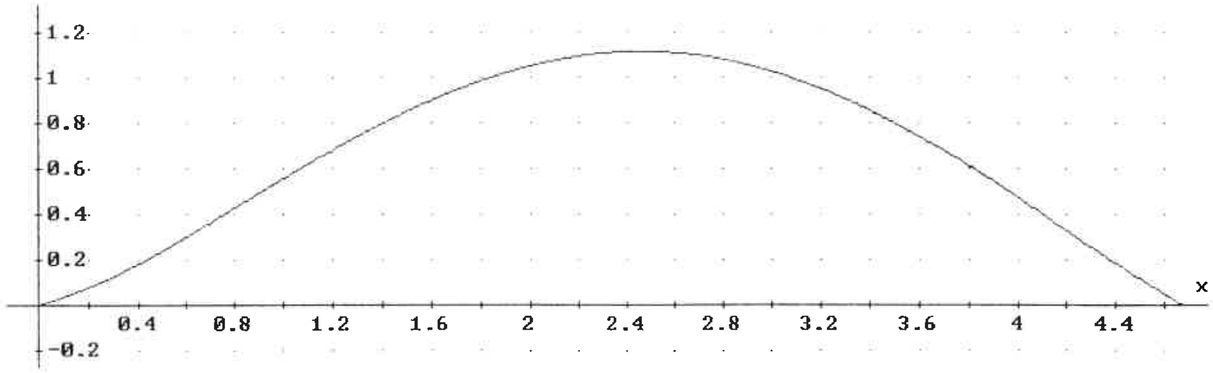


Figure 10: Ripple or Dune Profile

By the way of preliminary, one can make a mathematical statistics analogy for the dune analysis with the methodology to derive a catenary.

The Catenary

It can be looked upon as a hanging chain, comprising a system of particles, (an infinite amount of links), each of which is a point on the curve. One can consider it either ´in toto´ or just consider one link and the forces acting on it and its adjacent neighbours, being resolved vertically and horizontally to evole two simultaneous equations in intrinsic coordinates, which reduce to a Whewell Intrinsic Equation in ´s´ and φ to give the locus of the general point of the curve.

Comment of the Editor:
Whewell Equation is an Intrinsic Equation which expresses a curve in terms of its arc length *s* and tangential angle φ. An Intrinsic Equation is an equation which specifies a curve in terms of intrinsic properties such as arc length, radius of curvature, and tangential angle instead of with reference to artificial coordinate axes. Intrinsic equations are also called Natural Equations. The Tangential Angle φ(*t*) - in German: Drehwinkel - for a plane curve *r*(*t*) is given by

$$\frac{r'(t)}{|r'(t)|} = \begin{pmatrix} \cos(\varphi(t)) \\ \sin(\varphi(t)) \end{pmatrix} = e^{i\varphi(t)}$$

References: Eric W. Weisstein, CRC Concise Ecyclopedia of Mathematics
Alfred Gray, Differentalgeometrie

Let T be the tension at the general point and T_0 the tension at the vertex. Let w be the weight per unit length (each link, say) and the tangential angle is the intrinsic coordinate φ.

Resolving horizontally, the vertically we have:-

$$T_0 = T \cdot \cos\varphi$$
$$w \cdot s = T \cdot \sin\varphi$$

$s = k \cdot \tan\varphi$ where $k = \dfrac{T_0}{w}$ Whewell Intrinsic Equation Type-1

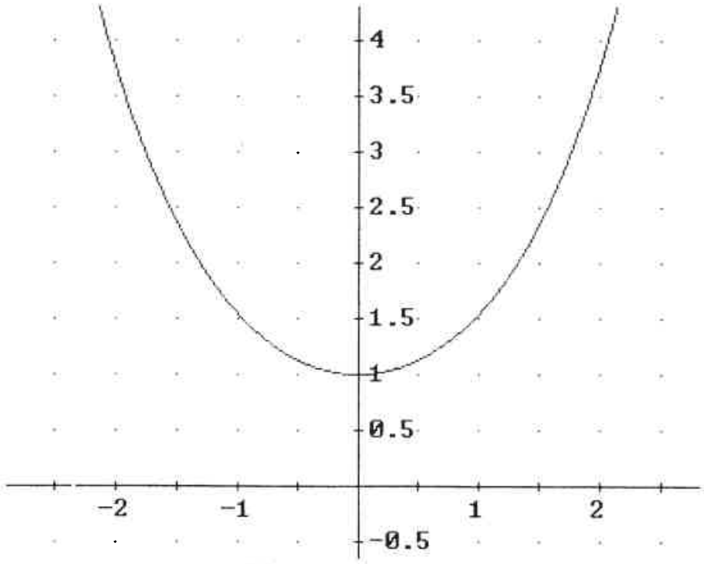


Figure 11: Catenary

Derivation

The mathematical methodology hereunder has made use of the ′Two Williams′ approach, using the philosophy of William of Ockham and the intrinsic geometry of William Whewell, (as well as Leonhard Euler and Ernesto Cesaro).

Let the horizontal force of the wind per unit length be W, therefore the force on a section of the arc of the dune curve from 0 to s is $W \cdot s$. N is the horizontal reactive force orthogonal to the curve of the dune and the tangential angle is the intrinsic coordinate φ.

Then we resolve the forces in the wind direction. The resolution of the forces orthogonal to the dune merely reaffirm the coefficient of friction to be as expected.

$$s = V_\varphi$$
$$W \cdot s = N \cdot \cos\varphi + \mu \cdot N \cdot \sin\varphi$$
$$\mu \cdot N \cdot \cos\varphi = N \cdot \sin\varphi \quad \therefore \mu = \tan\varphi$$

$$\therefore W \cdot s = N(\cos\varphi + \sin\varphi \tan\varphi) = N\frac{1}{\cos\varphi} = N\sec\varphi$$ Whewell Intr. Equation Type-1

$$\therefore \rho = \frac{ds}{d\varphi} = \frac{N}{W} \cdot \sec\varphi \cdot \tan\varphi = k \cdot \sec\varphi \cdot \tan\varphi$$ Euler Intrinsic Equation Type-1

Initially, $x = y = \varphi = 0$ hence:-

$$\frac{dx}{d\varphi} = \rho \cdot \cos\varphi = k \cdot \tan\varphi \quad \therefore \quad x = \pm k \cdot \ln(\sec\varphi)$$

$$\frac{dy}{d\varphi} = \rho \cdot \sin\varphi = k \cdot \tan^2\varphi \quad \therefore \quad y = k \cdot \left|\tan\varphi - \varphi\right|$$

Vector for plotting Barchan or Parabolic Dune

$$\begin{bmatrix} k \cdot \ln(\sec\varphi) & k \cdot \left|\tan\varphi - \varphi\right| \\ -k \cdot \ln(\sec\varphi) & k \cdot \left|\tan\varphi - \varphi\right| \end{bmatrix}$$

(You can find all the equations on the respective *DERIVE* file.)

Next is the vector for plotting dunes rotated thru α radians:

$$\begin{bmatrix} k \cdot \ln(\sec\varphi) \cdot \cos\alpha + k \cdot \left|\tan\varphi - \varphi\right| \cdot \sin\alpha & -k \cdot \ln(\sec\varphi) \cdot \sin\alpha + k \cdot \left|\tan\varphi - \varphi\right| \cdot \cos\alpha \\ -k \cdot \ln(\sec\varphi) \cdot \cos\alpha + k \cdot \left|\tan\varphi - \varphi\right| \cdot \sin\alpha & k \cdot \ln(\sec\varphi) \cdot \sin\alpha + k \cdot \left|\tan\varphi - \varphi\right| \cdot \cos\alpha \end{bmatrix}$$

$$y = k \cdot \left(\sqrt{e^{\left|\frac{2x}{k}\right|} - 1} - arc\sec\left(e^{\frac{x}{k}} \right) \right)$$

$$\frac{dy}{dx} = \pm\sqrt{e^{\left|\frac{2x}{k}\right|} - 1}$$

So we now have the pair of parametric Cartesian euqations for ease of plotting, which is much more suitable than the Cartesian equation $y = y(x)$.

These equations yield a curve resembling a Catenary or Parabola, and it is representative of a horizontal planar cross section of the sand dune profile.

Obviously the height of such a section will yield a similar-shaped curve, larger close to the base and smaller towards the summit. In other words we can deem the various horizontal planar cross sections as being similar curves, a nested set of parallel curves, (approximately), if they were all projected onto the base plane. These can be easily represented in intrinsic coordinates:-

$\rho = k \cdot \sec\varphi \cdot \cos\varphi$ (Ground level tracing of dune)

Vector for plotting nth-level nested curve

$$\begin{bmatrix} k \cdot \ln(\sec\varphi) & k \cdot \left|\tan\varphi - \varphi\right| - n \\ -k \cdot \ln(\sec\varphi) & k \cdot \left|\tan\varphi - \varphi\right| - n \end{bmatrix}$$

$$\text{VECTOR}\left(\begin{bmatrix} \frac{1}{2} \cdot \text{LN(SEC}(\varphi)) & \frac{1}{2} \cdot \left|\text{TAN}(\varphi) - \varphi\right| - n \\ -\frac{1}{2} \cdot \text{LN(SEC}(\varphi)) & \frac{1}{2} \cdot \left|\text{TAN}(\varphi) - \varphi\right| - n \end{bmatrix}, n, -2, 0, 0.5 \right)$$

with $0 \leq \varphi \leq \pi$.

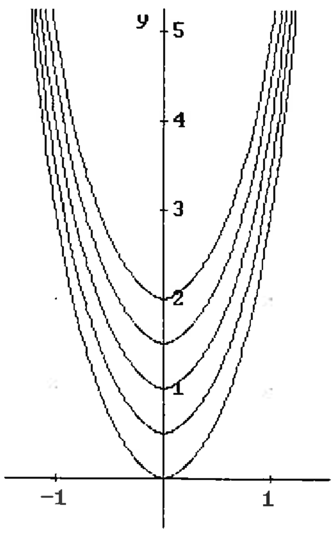


Figure 12: Nested Horizontal Sections of Dune

Now to five diagrams, the first of which has a *k*-factor of 0.9 and depends on a unidirectional wind.

The next four are with a *k*-factor of 1/2. The second is with the wind direction at π/24 (radians) to the unidirectional wind (Figure 15) and the third with -π/24 to that same wind (Figure 16), while the fourth is the overlay of the two of them. There was no point to include the dune caused by the undiverted wind stream since it is subsumed (embodied) by the other two diagrams.

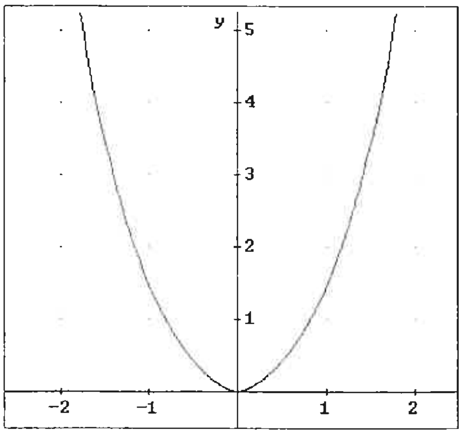


Figure 13: Barchan Dune with $k = 9/10$

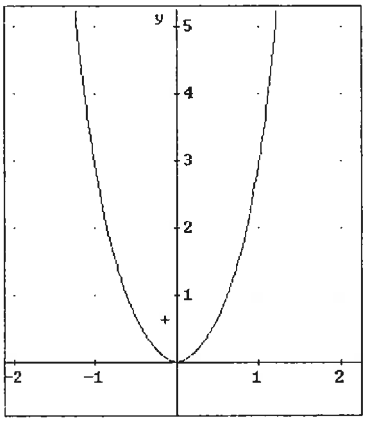


Fig 14: Barchan Dune with k=1/2

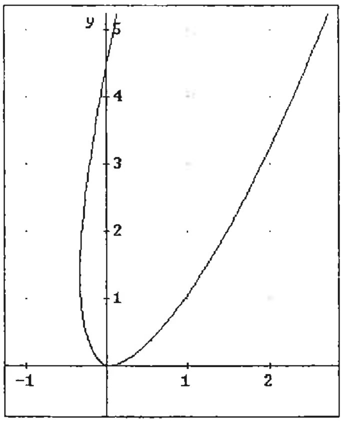


Fig 15: B. D. with k=1/2, α=π/12

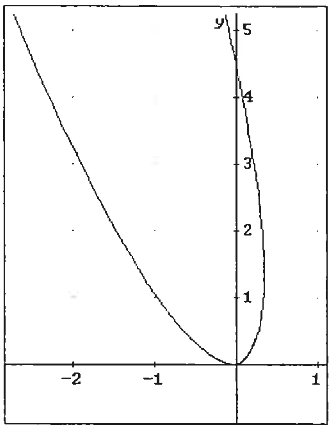


Fig 16: B. D. with k=1/2, α=-π/12

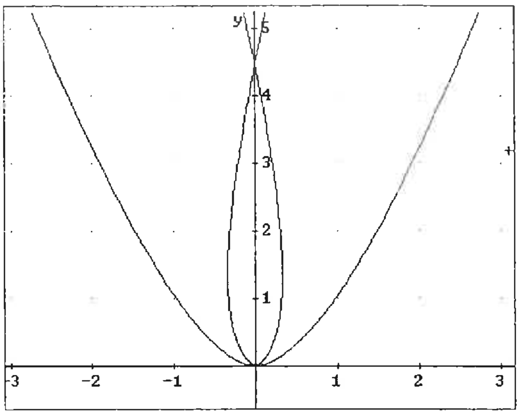


Figure 17: Composite Barchan

Since the turbulence, Froude number etc. are different on the lee side from the windward side of a dune, a barchan can be drawn accordingly, with, for example, *k*-values of 1/2 and 1/3, as below:

$$\begin{bmatrix} \frac{1}{2}\cdot \mathrm{LN}(\mathrm{SEC}(\varphi)) & \frac{1}{2}\cdot |\mathrm{TAN}(\varphi) - \varphi| \\ -\frac{1}{2}\cdot \mathrm{LN}(\mathrm{SEC}(\varphi)) & \frac{1}{2}\cdot |\mathrm{TAN}(\varphi) - \varphi| \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3}\cdot \mathrm{LN}(\mathrm{SEC}(\varphi)) & 2 + \frac{2}{3}\cdot |\mathrm{TAN}(\varphi) - \varphi| \\ -\frac{2}{3}\cdot \mathrm{LN}(\mathrm{SEC}(\varphi)) & 2 + \frac{2}{3}\cdot |\mathrm{TAN}(\varphi) - \varphi| \end{bmatrix}$$

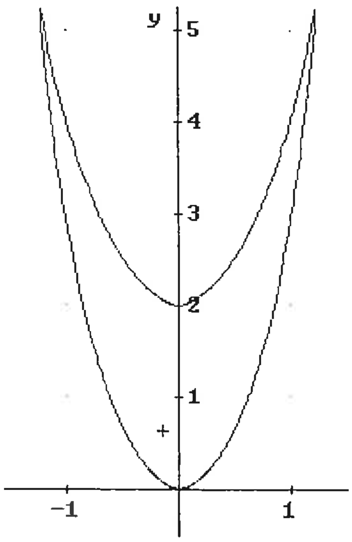


Figure 18: Barchan with $k = 1/2$ and $k = 2/3$

In the first case, there is virtually no force of reaction when the oncoming particle meets up with the dune under a saltating curtain and therefore the k-value in the equation tends to zero in one instance, giving a very narrow U-curve (Barchan), OR there is a negative value for k thereby giving an inverted U-curve, (Parabolic Dune).

However, the usual value for k would be just below unity in the case of the U-Curve dunes (Barchans), where there is a force of reation almost equal to the force of the impact of the ssand particle.

Although there was no intention to allow for the boundary between laminar and turbulent flow conditions, (by calculating the **Froude** number and the **Reynolds** number), neverthless Froude seems to have inserted his post mortal influence.

The Froude number, $F = k + 1$ seems to be an approximate correlation, except the above equation cannot represent a standig wave, therefore k cannot be zero, since that is a singular example for the transition phase. (See Appendix.)

Of course, this argument assumes no turbulence, and both constant wind direction and speed, (prevailing direction and mean velocity), and the collision is a roughly horizontal impact, but minor variation of the wind direction through a valley, say would merrely ′round off the edges′ and shape it into a composite curve, as though one overlays the base curve with a slightly rotated version of itself, see diagrams above.

Retrospectively, the main consideration for this approximation is:-

Is the shape, thus derived, in accordance with the current theory/theories and or paradigm(s), or are we precluded from a correct prediction due to some unforeseen factor)s)?

Granted, it does not take into consideration of adhesion, the chaotic states that arise due to charge, (built up by friction against air molecules as well as other sand particles), thermal conveection and buoyancy in the wind, and shear instabilities. But these are more in the quantitative sphere of influence. This paper is a qualitative analysis of the morphology, in the main. If the sand sticks, it adds to the dune mass and if it flies off, we forget it. It is just a mound of windblown sand, that moves in the direction of the wind by the sand from the windward and being transported over the mound to be laid down on the leeward slope, in addition to it growing by more sand being transported with the same wind.

APPENDIX

The **Froude number** is a dimensionless parameter, used to idicate the influence of gravity in fluid mechanics, (for inviscid and incompressible fluids), including movement of dust by winds in dune formations etc.. It is the ratio of an interial force to a gravitational force:-

$F = \frac{v}{\sqrt{g\,d}}$ where F is the Froude number, v is the velocity of a small surface wave, g is the gravitational acceleration and d is the depth of flow.

When $F < 1$, small surface waves can move upstream/upwind, (tranquil flow)
When $F > 1$, they will be carried downstream/downwind, (shooting flow)
When $F = 1$, the critical Froude number, the velocity of flow is equal to the velocity of the surface waves, hence we have a standing wave, representing a transition.

The Froude number together with the **Reynolds number**, in fluid mechanics, is a dimensionless number representing the ratio of intertial force to viscous force, (fluid intertia to sheer strress). It is used as a criterion to determine whether fluid (liquid or gas) flow is absolutely steady, [smooth, (streamkined or laminar)] or, ´on-the-average-steady´ with small turbulent fluctuations. The transition between laminar and turbulent flows occurs in the range of values beginning from 1000 to 2000 and extending to between 3000 and 5000.

POSTLUDE

Although this canonic form of a dune curve is not as accurate as one might prefer, there is a scope for improvement thuswise:-

Von Mises wrote a wonderful book on aeroplane wing profiles many years ago. His basic curve, with which to work, (his canonic form), was the *Joukowski* aerofoil profile. This had most of the desireable features, yet was deficient from the engineering perspective, since it would not have flown very successfully, as determined by wind-tunnel tests using similar Reanolds numbers for the model and the actual intended wing construction.

By some ingenious mathematical artifice, he amdended the profile to be aerodynamically sound, (by some rounding at the cusp). This writer no longer has access to that book, so cannot implement the methodology of von Mises to adjust the dune curve canonical form in like manner to give it an aerodynamically-acceptable form. Perhaps a reader may have some suggestions?

BIBLIOGRAPHY

Encyclopaedia Brittanica 15th Edition 1773
Great Sand Dunes, Stephen A. Trimble, Paragon Press 1978

Comment of the Editor:
You will find two DERIVE-files belonging to this article. Dune2002 *refers directly to it and* DUNEPLOT *is from a former version, but shows some examples describing the "Intrinsic Equation" resulting from s = V(φ).*

[1] It might be of interrest for you that we also have in German the word "Löss". The Löss regions are very fertile and some of Austria´s best vines ripe on Löss grounds. Very famous are the cellars of city Retz. The people there dug 5 stores deep in the Löss ground.

From Benno Grabinger: I hope that you and the DERIVE community like the plot:

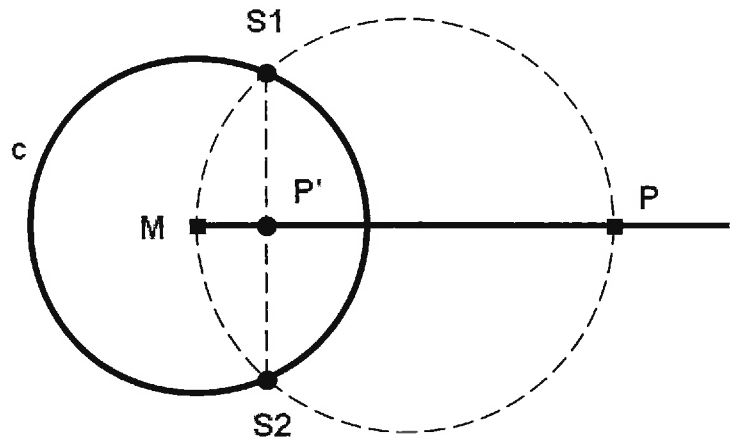
```
       v(i) :=
         If i = 1
            ["*"]
#1:         If i = 11
               ["III"]
               APPEND(["*"], v(i - 1))

#2:    Simplify #3

#3:    VECTOR([v(i)], i, 1, 11)
```

Some remarks on Shoemaker's Knife and Steiner-Chains

Stefan Welke, Bonn/Germany, Spwelke@aol.com

Abstract: P. Lüke-Rosendahl considers in [1] a special case of the Appolonius-problem, i.e. to construct a circle which touches three given circles. This contribution presents a different approach. The use of inversion with respect to a circle allows a generalisation of the given problem: The circle, which touches the three given semicircles forming Shoemaker's Knife, is the first one of a sequence of inscribed circles called a Steiner-Chain.

1 Inversion with respect to a circle



Definition
Consider a circle c with midpoint M and radius r. The inversion of a point P with respect to the circle c is a point P', which lies on the ray starting at M that contains P, such that the product of the distances of P and P' from M equals r^2, i.e.: $\overline{MP} \cdot \overline{MP'} = r^2$.

The figure above indicates how to construct P' with the circle of Thales if P is given.

Inversion with respect to a circle is an anticonformal one-to-one map of the extended plane, i.e. the plane with the point at infinity, onto itself. The center M is mapped at the point ∞ and vice versa. The most important mapping properties for our purposes are listed in the following

Theorem Inversion with respect to circle maps

(1) straight lines through the center M of the circle onto themselves

(2) circles through the center M onto straight lines not passing M

(3) circles not passing M onto circles.

If we consider straight lines as circles with infinite diameter, then inversion maps circles onto circles.

Straight lines, circles, and other algebraic curves are the sets of zeros of polynomial equations $p(x,y)=0$. Consider the circle c_1 with center $(5\,|\,0)$ and radius $r_1 = 6$ as example. The corresponding polynomial is $p(x,y)=(x-5)^2+y^2-36$. We shall invert this circle with respect to the circle i with center $(-1\,|\,0)$ and radius 1. Since $(-1\,|\,0)$ is a point on c_1, this circle will be mapped onto a straight line. We let *DERIVE* perform the necessary calculations. The following *DERIVE*-function transforms the polynomial $p(x,y)$ of an algebraic curve γ into the polynomial $p'(x,y)$ of the image curve γ' under inversion with respect to a circle with center m and radius r:

#1: $$\mathrm{inv}(p,\ m,\ r) := \mathrm{NUMERATOR}\left(\mathrm{FACTOR}\left(\mathrm{SUBST}\left(p,\ [x,\ y],\ m + \frac{r^2\cdot([x,\ y] - m)}{([x,\ y] - m)\cdot([x,\ y] - m)}\right)\right)\right)$$

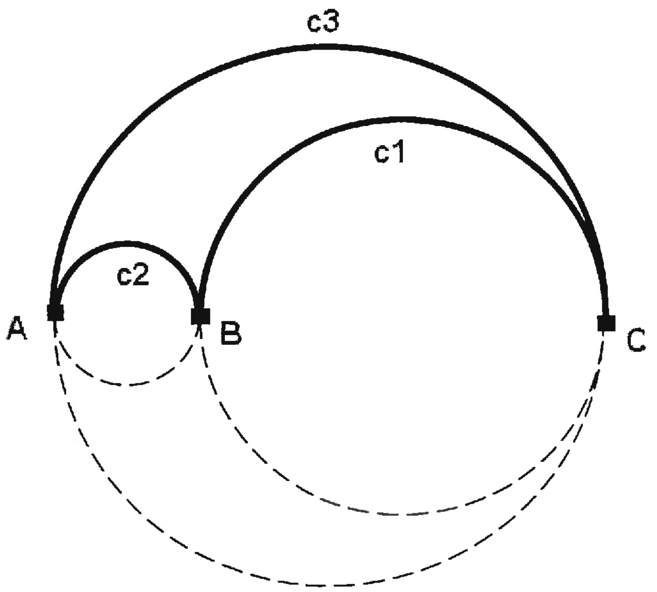
#2: $$\mathrm{circle1} := (x - 5)^2 + y^2 - 36$$

#3: $$\mathrm{inv}(\mathrm{circle1},\ [-1,\ 0],\ 1)$$

#4: $$-\ 12\cdot x - 11$$

We can see that the image is a straight line, perpendicular to the $x-$axis, and its equation is $-12x - 11 = 0$.

2 Finding the incircle of Shoemaker's Knife geometrically



An arbelos or Shoemaker's Knife is a triangle bounded by half-circles such that the vertices A, B and C are collinear. Another formulation of the problem solved in [1] is: Find the incircle of the triangle ABC ! This is a particular case of the Appolonius-problem as mentioned above. The solution, which was presented in [1], was purely algebraic: At first the coordinates of the center of the incircle were calculated as the coordinates of an intersection point of two ellipses, then came the radius, and at last the equation of the incircle was formed.

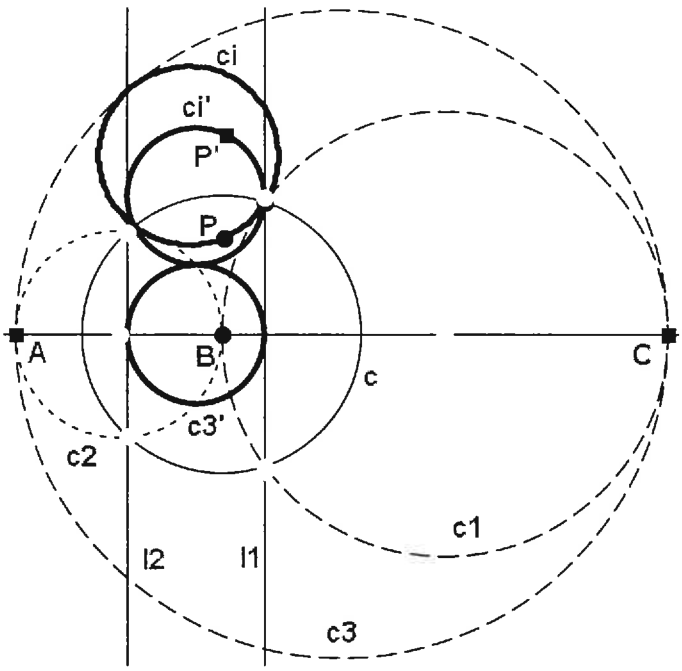
We will present a geometric solution of this problem.

This allows a realisation with a DGS and a direct computation of the polynomial that defines the incircle.

The half-circles which form the arbelos are parts of the three circles c_1, c_2 and c_3. Let us choose a fourth circle c with center B as the circle of inversion. Then the images of c_1 and c_2 under inversion with respect to c are two parallel straight lines $l_1 := c_1'$ and $l_2 := c_2'$ which are perpendicular to the straight line s through A, B and C. The image of c_3 under inversion is a circle c_3' with center on s, and the parallel lines l_1 and l_2 are tangents to that circle.

Now assume that the incircle c_i is given. Then the image of c_i under inversion is a circle c_i' subject to the following conditions: (1) c_i' touches c_3' from above, (2) l_1 and l_2 are tangents to c_i'.

This circle c_i' is constructed easily and thus we only need to invert that circle with respect to c to obtain the incircle. The last result is due to the fact that inversion is an involutionary or self-inverse mapping: The image of the image is the original itself. The next figure is a screen shot of a construction with the DGS DynaGeo [2]. This program has the same advantage as Cabri II that inversion with respect to a circle is a built-in feature.



The incircle c_i is the locus of a point P' which is the image of a point P on the circle c_i'. This is a dynamic construction.

A purely Euclidean construction with ruler and compass is possible too: One can construct c_i with the help of the points where the tangents l_1 and l_2 touch c_i'. The images of these points under inversion are the points where the incircle touches c_1 and c_2. The third point, where the incircle touches c_3, is the image of the point where c_i' touches c_3'. Now we have three points of the incircle and construction of c_i is possible.

3 The equation of the incircle

Now let c_1 be the circle with radius $r_1 = 6$ and center $M_1 = (5 \mid 0)$ which is `circle1` in our example above. Let c_2 have center $M_2 = (-3 \mid 0)$ and radius $r_2 = 2$, and finally let c_3 have center $M_3 = (3 \mid 0)$ and radius $r_3 = 8$. Note that this is just the example that is treated in [1].

We perform the inversion with respect to the circle c with center $B = (-1 \mid 0)$ and radius 1. l_1 intersects the $x-$axis at $x_1 = -1 + \frac{1}{12} = -\frac{11}{12}$, and l_2 intersects this axis at $x_2 = -1 - \frac{1}{4} = -\frac{5}{4}$. Thus the center of c_i' is $M = (\frac{x_1 + x_2}{2} \mid x_1 - x_2) = (-\frac{13}{12} \mid \frac{1}{3})$ and the radius is $r = \frac{x_1 - x_2}{2} = \frac{1}{6}$

The following calculations with *DERIVE* are self-explanatory:

```
                          2     2
#5:    circle2 := (x + 3)  + y  - 4

                          2     2
#6:    circle3 := (x - 3)  + y  - 64

                       1
#7:    x1 := -1 + ————
                      12

                       1
#8:    x2 := -1 - ————
                       4
```

#9: $$m := \left[\frac{x1 + x2}{2},\ x1 - x2\right]$$

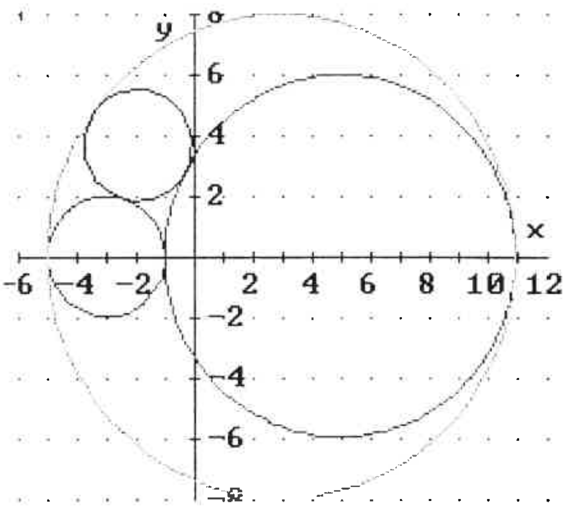
#10: $$r := \frac{x1 - x2}{2}$$

#11: $$cii := ([x,\ y] - m)\cdot([x,\ y] - m) - r^2$$

#12: $$\frac{144\cdot x^2 + 312\cdot x + 144\cdot y^2 - 96\cdot y + 181}{144}$$

#13: inv(cii, [-1, 0], 1)

#14: $$13\cdot x^2 + 50\cdot x + 13\cdot y^2 - 96\cdot y + 181$$



This is exactly the same result as in [1]. The polynomial in #14 is up to a factor of 13 equivalent to the equation presented there.

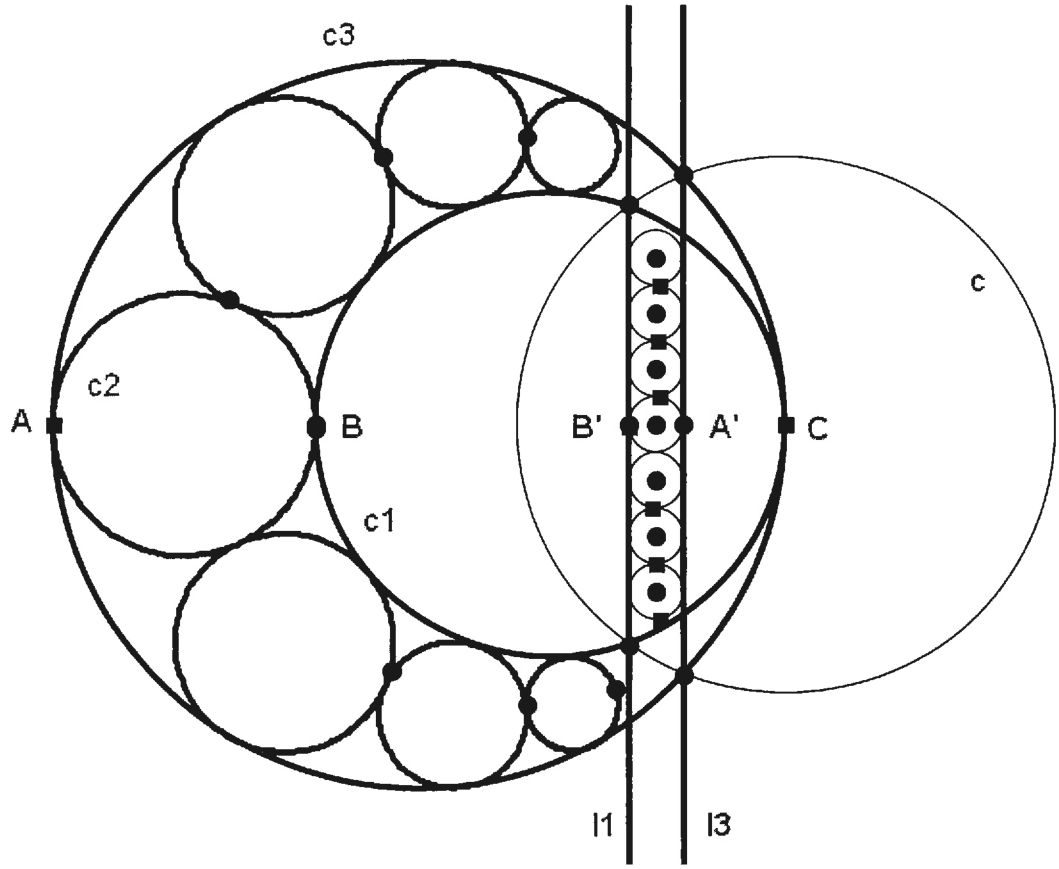
A mathematician's mind never rests. Now that we have found the incircle we can easily see that the incircle is the second circle of an infinite sequence of circles $\{\gamma_0 := c_2, \gamma_1 := c_i, \gamma_3, \ldots\}$ that lie between c_1 and c_3, touching both circles, and each circle γ_n touches its predecessor as well as its successor. Such a sequence of circles is called a Steiner-Chain.

4 Geometric construction of Steiner-Chains

Originally a Steiner-Chain, as studied by the swiss mathematician Jakob Steiner, is a chain of circles filling the space between two circles, one of which lies completely in the interior of the other. In our situation the circles c_1 and c_3 have a common point $C = (11 \mid 0)$. We now choose this point C as the center of a circle c. Then we perform an inversion with respect to this circle c. The images of the circles c_1 and c_3 are two parallel straight lines l_1 and l_2 that are tangents to a twofold infinite chain $\Gamma := \{\ldots, \gamma_{-2}, \gamma_{-1}, \gamma_0, \gamma_1, \gamma_2, \ldots\}$ of circles between these two lines. The first one, γ_0 with its center on the $x-$axis, is the image of c_2.

The image $\Gamma' := \{\ldots, \gamma'_{-2}, \gamma'_{-1}, \gamma'_0, \gamma'_1, \gamma'_2, \ldots\}$ of this chain under inversion is just the desired continuation (in two directions) of the sequence starting with c_2 and the incircle.

The following figure is another screen shot of a dynamic construction with DynaGeo. Again a purely Euclidean construction of every circle is possible.



5 The equations of the Steiner-Chain

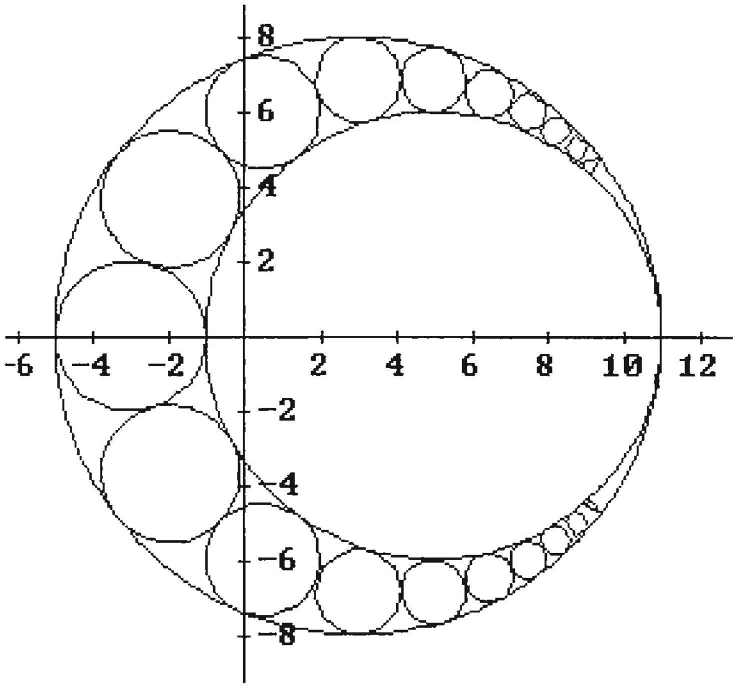
We perform the inversion with respect to the circle c with center $C=(11\,|\,0)$ and radius 1. l_1 intersects the $x-$axis at the point B' with $x_1=11-\frac{1}{12}=\frac{131}{12}$ and l_3 intersects this axis at A' with $x_3=11-\frac{1}{16}=\frac{175}{16}$. Thus the center of γ_k is $M_k=(\frac{x_1+x_2}{2}\,|\,k(x_2-x_1))=(\frac{1049}{96}\,|\,\frac{1}{48})$ and the radius is $r=\frac{x_2-x_1}{2}=\frac{1}{96}$.

The following calculations with *DERIVE* are self-explanatory again:

```
#16:  [x1 := 11 - 1/12, x2 := 11 - 1/16]

#17:  m(k) := [(x1 + x2)/2, k·(x2 - x1)]

#18:  r := (x2 - x1)/2

#19:  γ(k) := ([x, y] - m(k))·([x, y] - m(k)) - r^2

#20:  inv(γ(k), [11, 0], 1)

#21:  x^2·(k^2 + 12) + 2·x·(36 - 11·k^2) + y^2·(k^2 + 12) - 96·k·y + 121·k^2 + 60
```

#22: VECTOR($x^2 \cdot (k^2 + 12) + 2 \cdot x \cdot (36 - 11 \cdot k^2) + y^2 \cdot (k^2 + 12) - 96 \cdot k \cdot y + 121 \cdot k^2 + 60 = 0$, k, -9, 9)

#23: $[93 \cdot x^2 - 1710 \cdot x + 93 \cdot y^2 + 864 \cdot y + 9861 = 0,\ 76 \cdot x^2 - 1336 \cdot x + 76 \cdot y^2 + 768 \cdot y + 7804 = 0, \ldots\ldots,\ 93 \cdot x^2 - 1710 \cdot x + 93 \cdot y^2 - 864 \cdot y + 9861 = 0]$



6 Conclusions & remarks

CAS and DGS have their merits in different areas. In geometry a combination of both is superior to the use of CAS or DGS alone. Using a DGS lets you see what you are thinking about. Then you can translate your thoughts into the algebraic language with the aid of a CAS.

You can construct only a few links of a Steiner-Chain with a DGS, but once you know how to proceed, you can produce hundreds of equations of these links with a CAS.

A first reference for inversions, the Appolonius-problem and Steiner-Chains is [3]. The behaviour of simple algebraic curves under inversion with respect a circle is demonstrated in some detail in [4].

References

[1] P. Lüke-Rosendahl, "Shoemaker's Knife from another Point of View", DNL #44, December 2001

[2] R. Mechling, DynaGeo-Homepage: http://www.dynageo.de/

[3] C. S. Ogilvy, "Excursions in Geometry", Oxford University Press, NY, 1969

[4] S. Welke, "Inversion of Elementary Algebraic Curves with Respect to a Circle", Proceedings of the International Derive and TI-92 Conference, Schloß Birlinghoven, July 1996, Ed.: Bärbel Barzel

Peter´s Shoemaker´s Knife and Renée-Jaqueline´s Peaucellier Inversor

Rüdiger Baumann, Celle, Germany, baumann-celle@t-online.de

Hallo Josef,
gerade ist *DNL* Nr.44 angekommen. Es ist immer ein erfreuliches Ereignis, wenn der rostrote Umschlag im Briefkasten steckt. Anregende Lektüre mit vielen wertvollen Tipps und Tricks sowie Neuigkeiten aus der *DERIVE*-Familie.

Die Geschichte mit dem „Schustermesser" war ja richtig spannend – bis zum schlußendlichen Erfolg. Sollte das „ ....*from another Point of View*" im Titel andeuten, dass es noch andere, eventuell einfachere, Lösungen gibt?

Vielleicht vermag der Peaucellier-Inversor (siehe Gossez & Sengier in *DNL* Nr.36, Seite 37) hier weiterzuhelfen? Er kann das Schustermesser geradebiegen, dann werden die Kreise hineinpraktiziert und schließlich das Messer – samt Kreisen – wieder krummgebogen. In *DERIVE* sieht das so aus:

DNL#44 just arrived. It is always enjoyable finding the rust-colored envelop in the mail box....

The story treating the "Shoemaker´s Knife" was really exciting – until the final success. Can it be that the "... *from another Point of View*" in the title should indicate that there are other, maybe simpler solutions?

Maybe that the Peaucellier-Inversor (Gossez & Sengier, *DNL#36*, p 37) might help. It can bend the Knife to straight lines, then the circles will be put between and finally the lines together with the lines are bent back. In *DERIVE* it looks as follows.

(You can download also an English version, Josef)

#1: $\text{Kreis}(M, r) := M + r\cdot[\text{COS}(t), \text{SIN}(t)]$

#2: $k1 := \text{Kreis}([1, 0], 1)$

#3: $k2 := \text{Kreis}\left(\left[\frac{3}{4}, 0\right], \frac{3}{4}\right)$

#4: $k3 := \text{Kreis}\left(\left[\frac{7}{4}, 0\right], \frac{1}{4}\right)$

#5: $\text{Bild1} := [k1, k2, k3]$

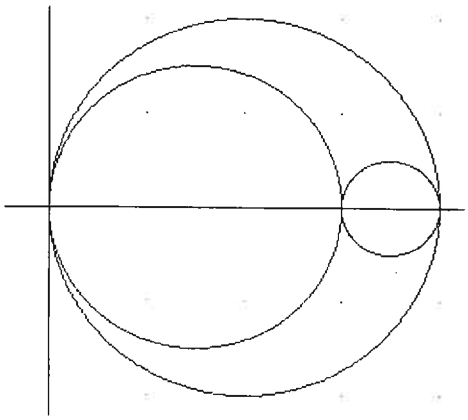


Bild 1

Nun tritt der Inversor in Aktion; er wird (frei nach Gossez & Sengier) so definiert:
Now the Inversor enters the stage. It is defined - according Gossez & Sengier - as follows:

#6: $f(P) := \left[\frac{P_1}{P_1^2 + P_2^2}, \frac{P_2}{P_1^2 + P_2^2}\right]$

#7: $\text{Bild2} := [f(k1), f(k2), f(k3)]$

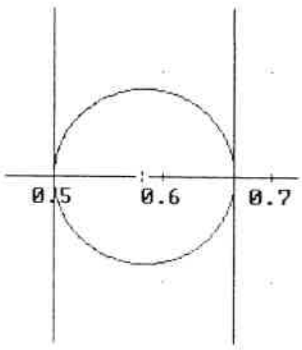


Bild 2

Die beiden Kreise durch den Nullpunkt (den Mittelpunkt des Inversionskreises) sind zu Geraden geworden (Bild 2)

The two circles passing the origin (center of the inversion circle) have been transformed to lines.

```
#8:   Kreiskette1(n) := VECTOR(Kreis([[7/12, k/6], 1/12], k, 1, n)
#9:   [Kreiskette1(10), bild2]
```

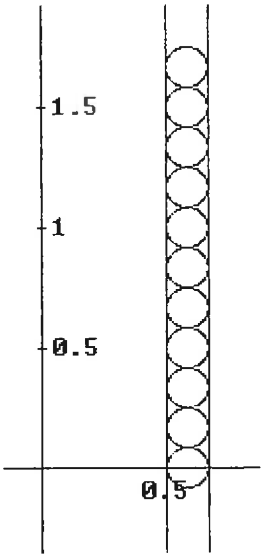


Bild 3

In Bild 2 wird eine gerade Kreiskette eingefügt und der Aufruf `Kreiskette1(10)` liefert Bild 3:

In Bild2 a straight chain of circles is inserted and `Kreiskette(10)` (= `Circlechain(10)`) *results in Bild 3:*

Die gerade Kreiskette wird mit f zurücktransformiert (da f gleich der Umkehrabbildung) ist und der Aufruf `Kreiskette2(10)` liefert Bild 4.

The straight chain of circles is transformed back by applying f again (f is equal to its inverse mapping) and plotting `circlechain(10)` *leads to Bild 4.*

```
#10:  Kreiskette2(n) := VECTOR(f(k), k, Kreiskette1(n))
#11:  [Kreiskette2(10), Bild1]
```

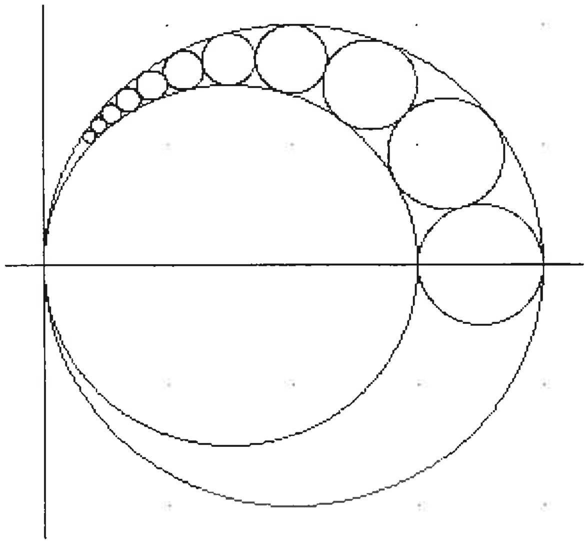


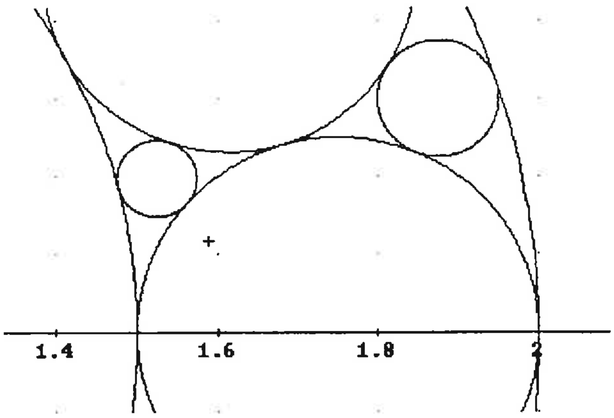
Bild 4

Auf der nächsten Seite findet Ihr noch zwei Aufgaben zur Anwendung der Inversion.

On the next page you can find two more problems for applying inversion.

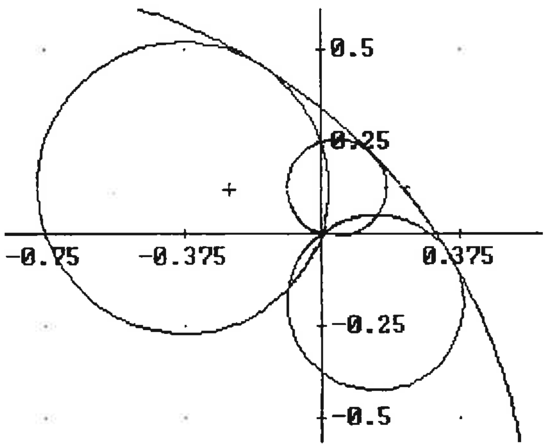
1. Kann man in die Zwickel weitere Kreisketten einfügen – und zwar nach allen Richtungen? (Apollonian Gasket, siehe Mandelbrot: Fractals, 1977, Seite 187)

1. Is it possible to insert additional chains of circles into the gussets – and in all directions? (Apollonian Gasket, see Mandelbrot; Fractals, 1977, p 187).

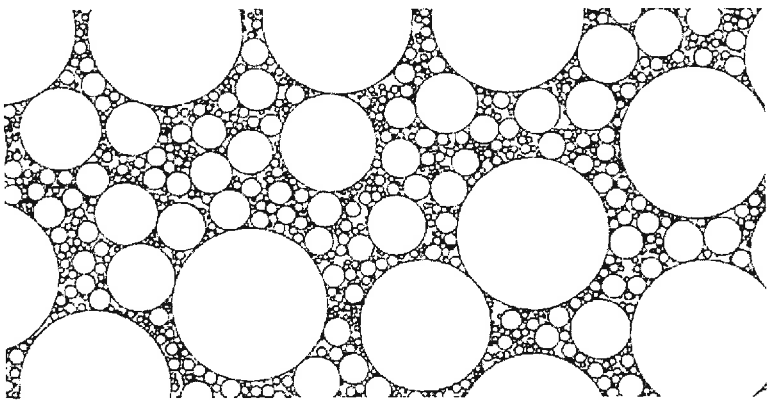


2. Zu drei Kreisen, die sich paarweise schneiden und einen Punkt gemeinsam haben, soll ein Kreis konstruiert werden, der die drei Kreise von außen berührt.

2. Given are three circles which are intersecting in pairs and have one point in common. Find a fourth circle touching the given ones from outside.

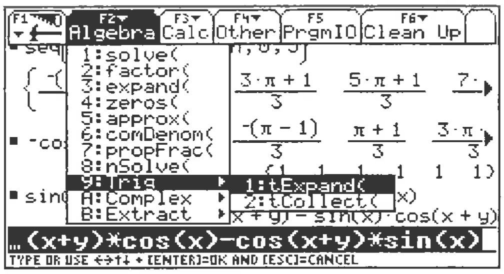


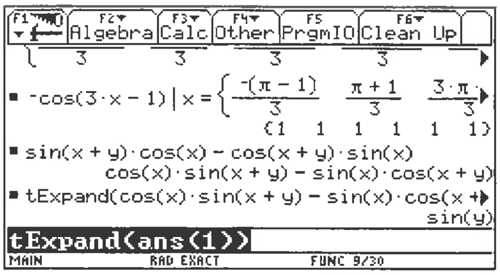
I found a nice "Osculatory packing": each successively placed circle on the plane needs only to be tangent to at least one previous circle. (Pickover, Computers, Patterns, Chaos and Beauty, Dover Publ., 2001).
A very recommendable book and - a challenge? Josef.



Task 2 Simplify the expression: $\sin(x+y) * \cos x - \cos(x+y) * \sin x$

Proposed Solution:

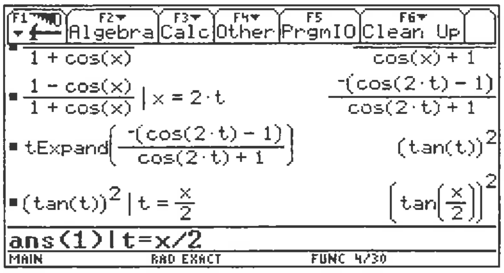




It is always recommended to apply `Trig / tExpand` or `tCollect`, very similar doing it by hands – but then you have to apply that $\sin^2 x + \cos^2 x = 1$.

Task 3 Simplify the function by transforming it in a function of $x/2$: $y = (1 - \cos x)/(1 + \cos x)$

Proposed Solution:

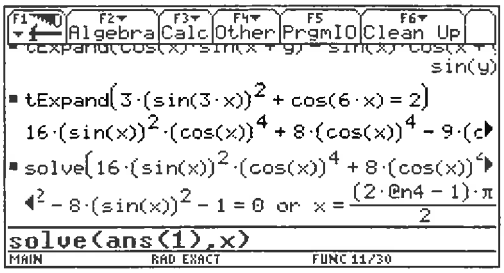


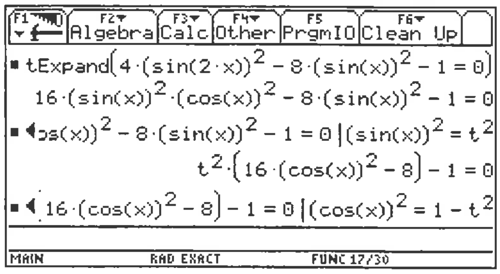
That's very easy, but you have to know the trick, how to change to the half angle.

One can reproduce the transformation by applying `tExpand(cos(2t))`, too.

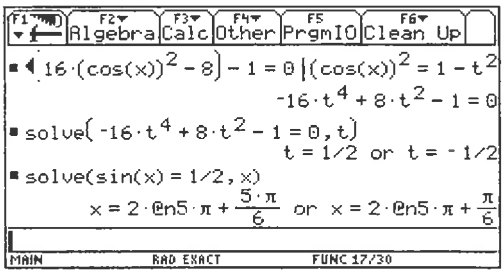
Task 4 Find all solutions of $3\sin^2(3x) + \cos(6x) = 2$ for $0 \le x \le 2\pi$.

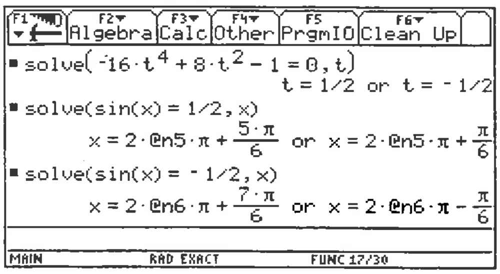
Proposed Solution:
First again use `tExpand` and try to `Solve` the equation. A product appears and one family of solutions can be read off immediately:





Replacing the parameter by 1 and 2 gives the first two sulutions, pi/2 and 3pi/2 ; the remaining equation must be treated separately – it reminds me on a quadratic, I substitute t^2 for $\sin^2 x$ and $1 - t^2$ for $\cos^2 x$ which results in a biquadratic equation for t.





Resubstituting leads to the other solutions: 5pi/6, pi/6, and. 7pi/6, pi/6+2pi = 11pi/6 respectively.

Problems with transmitting DATA-tables from a *TI*-92+ to a *TI*-92:

The following problem was reported by Tania Koller, St.Pölten, Austria

F1 | F2 Plot Setup | F3 Cell | F4 Header | F5 Calc | F6 Util | F7 Stat

DATA	Zeit	Weg			
	c1	c2	c3	c4	c5
1	0	0			
2	10	675			
3	20	1300			
4	30	1875			
5	40	2400			
6	50	2875			
7	60	3300			

c1=seq(n,n,0,140,10)

MAIN RAD AUTO FUNC

In some classes we have a "mixed population" of *TI*-92 and *TI*-92+.

At any occasion we wanted to transfer a DATA-file from a PLUS to an ordinary *TI*-92 and we came across a strange incompatibilty.

The first attempt ended with an error message on the PLUS. Despite the fact that the message remains on the screen it is not able to transfer it via *GraphLink* to the PC.(*Maybe that the PLUS is ashamed about its misbehaviour, Josef*)

Tania and her students successfully tried to overcome this nasty problem. See their solution:

ERROR
ice
Data type
ESC=CANCEL

F1 | F2 Plot Setup | F3 Cell | F4 Header | F5 Calc | F6 Util | F7 Stat

DATA	Zeit	Weg			
	c1	c2	c3	c4	c5
1	0	0			
2	10	675			
3	20	1300			
4	30	1875			
5	40	2400			
6	50	2875			
7	60	3300			

c1=

MAIN RAD AUTO SEQ

The problem is caused by the `seq`-command! So do the following: Work as usually with the `seq`-command, but then go back to cell `c1` and delete in `c1=` the `seq()`-command.

You will see, that the values in column1 will remain the same. Now you can transfer the DATA-file without any problems.

Grandsons´ Spring Coil

In begin of January I received a letter from Paul Higgins:

I am trying to get *DERIVE*5 to draw a picture of the enclosed photograph. It is like my grandsons toy which was called Slinky. A coiled spring on top of the stairs and it falls down by gravity from one stair to another. If it is laid on its side and joined round on itself it is again like the enclosed picture.
Many many thanks,
Paul Higgins

Let me start with the general form of a torus knot line (a winded curve around an elliptic torus):

```
torusknot(a, b, c, p, q) := [(a + b·COS(p·t))·COS(q·t), (a + b·COS(p·t))·SIN(q·t),
  c·SIN(p·t)]

torusknot(4, 3, 3, 50, 1)
```

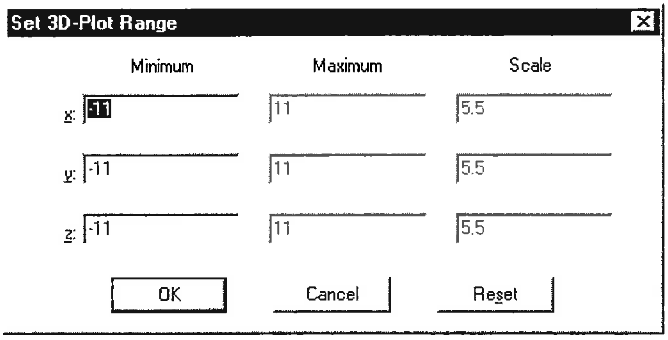
Plot Parameters

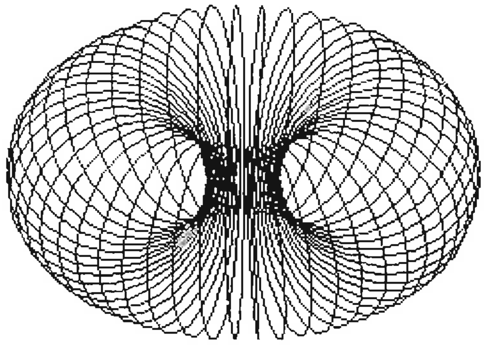
	Minimum	Maximum	Number of Panels
s:	0	1	1
t:	-PI	PI	1000

Switch to the 3D-Plot Window, issue Insert Plot and set the parameters.

Only *t* is important, because it is a space curve and not a surface. But you need to enter Min and Max for s, too.

Then set the Plot Range, to be sure to see the figure.



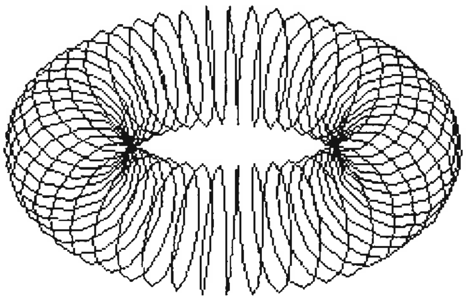


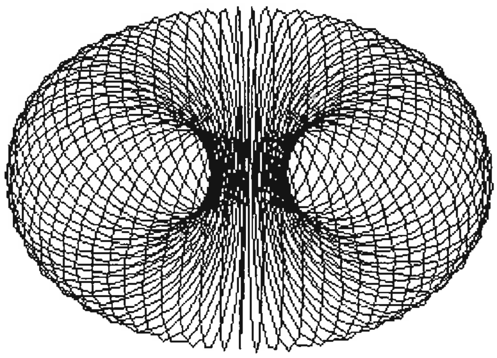
Our spring coil is a special form of the torus knot line, because it is not an elliptic torus, so b = c, and we have only one kind of windings, so q = 1. This helps to produce a special springcoil-function.

```
springcoil(r1, r2, p) := [(r1 + r2·COS(p·t))·COS(t), (r1 + r2·COS(p·t))·SIN(t),
  r2·SIN(p·t)]
```

`springcoil(4, 3, 50)` shows the same figure as above.

`springcoil(8, 3, 50)` and `springcoil(4, 3, 80)`



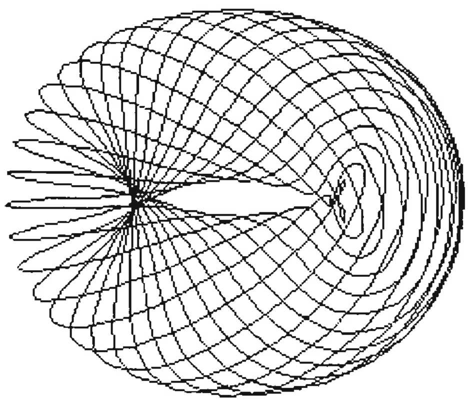


Finally see here a true torus knot line with both kinds of windings:

`torusknot(8, 3, 5, 27, 16)`

try also

`torusknot(8, 3, 5, 16, 27)`



One important additional hint: Take care that in the Declare > Simplification Settings Menu Trigonometry is set on Auto. Otherwise you can get strange graphs!!

Wythoff's Nim

Richard Schorn, Kaufbeuren, Germany
richard.schorn@t-online.de

The Game

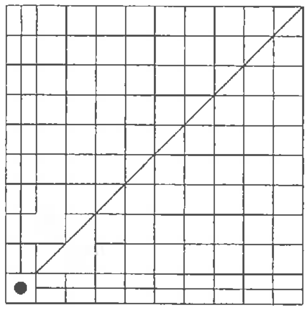


Figure A

In chapter 8 of his book "Penrose Tiles to Trapdoor Ciphers" Martin Gardner describes two simple two-person games. The first one is played on a chessboard with a single queen. The second one is played with two piles of counters resembling the classical game of Nim.

In the chessboard game which has no name (according to Gardner) the first player puts the queen on any cell in the top row or in the column farthest to the right. The queen moves in the usual way but only to the west, southwest or south. The player alternate moves. The player who moves the queen to the lower left corner is the winner.

Now for the winning strategy:

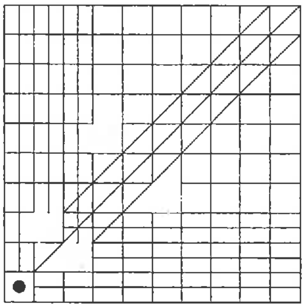


Figure B

If a player reaches a cell in the left utmost column, in the bottom row or on the main diagonal he is a sure loser, because the other player can get the queen in the next to the cell in the lower left corner (Figure A). If a player gets his queen to one of the gray cells he is the winner, because the opponent must make a move so that the queen gets to the goal in the next move. The two gray cells are "safe" for the player who can get to them. To find safe cells for this player one has to block all "ways" leading to the gray cells in Figure A. Obviously the new gray cells in Figure B are safe for this player. Continuing the process of blocking and searching for safe cells one arrives at Figure C. Eventually one finds the safe cells for the whole chessboard (not only 8x8 cells!).

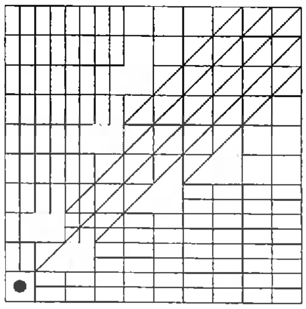


Figure C

The other game, "Wythoffs Nim" is said to be an old Chines game, which in 1907 the Dutch mathematician W.A.Wythoff reinvented and analysed. The play starts with two piles of counters. A move consists in taking any number of of counters from either pile. A player may remove an entire pile. There is a second kind of move: A player may take the same number of counters from both piles (not allowed in standard Nim). The player who takes the last counter wins.

Surprisingly this game and the unnamed game on the chessboard are isomorphic! To see this one has to assign coordinates (x|y) for the cells. The cell at the lower left corner (the goal) bears the coordinates (0|0).A westward move of the queen diminishes x, a southward move diminishes y and a move along the diagonal makes x and y smaller by the same amount. In Wythoff´s Nim x and y stand for the numbers of counters in each pile.

Example: Using a standard chessboard (8x8) it is clever for the first player to put the queen on cell (4|7) (or (7|4)). The second player has no chance to get the queen on a safe cell. The first player will eventually win the game, because after the first move of his opponent he can move the queen to a safe cell.

Safe Cells

To find the safe cells one can use the geometrical procedure outlined above. The resulting sequences have rather unusual properties and are interesting in themselves. Wythoff discovered that the members in sequence **<x>** are simply multiples of the golden ratio $\Phi = \frac{1+\sqrt{5}}{2}$ = 1,61803398... rounded down to the next integer, $\mathbf{x_i = [i\ \Phi]}$. The sequence **<y>** is constructed in a similar way: $\mathbf{y_i = [i\ \Phi^2]}$. Three of the strange properties can be seen by inspection of the first members.

I	x_i	y_i
1	1	2
2	3	5
3	4	7
4	6	10
5	8	13
6	9	15
7	11	18
8	12	20
9	14	23
10	16	26

i	x_i	y_i
11	17	28
12	19	31
13	21	34
14	22	36
15	24	39
16	25	41
17	27	44
18	29	47
19	30	49
20	32	52

i	x_i	y_i
21	33	54
22	35	57
23	37	60
24	38	62
25	40	65
26	42	68
27	43	70
28	45	73
29	46	75
30	48	78

1. $x_i + i = y_i$
2. No member of one sequence is member of the other.
3. Merging the two sequences results in the set of the natural numbers.

Program

The generation of the two sequences relies on some of the mentioned properties:
Start with 1 as the x-value of the first safe pair. Add this to its position number to obtain 2 as the y-value. The x-value of the next pair is the smallest positive integer not previously used, it is 3. The y-value is 5, the sum of 3 and the position number. The next x-value is 4, the smallest integer not yet used.

My program for DERIVE is based on this algorithm in Gardners book.

```
         w(z, W := [[1, 1, 2]], x := 3, i := 2, M := {1, 2}) :=
           Prog
             Loop
               If i - 1 = z
                  RETURN W
               W := APPEND(W, [[i, x, i + x]])
#1:            M := M ∪ {x, i + x}
               i := i + 1
               Loop
                 If MEMBER?(x, M)
                    x := x + 1
                    exit

#2:      w(30)
```

Bibliograpy

Much of this article was taken more or less literally from Gardners book. A lot of further information was disregarded, e.g. Beatty sequences, misère Nim, save cells for other pieces (king, rook, bishop and "mixtures" as King-knight). Gardner published a very large bibliography till 1987. Somewhat newer literature can be found in Sloanes Encyclopedia of Integer Sequences.

[1] *Gardner, Martin*: Penrose Tiles to Trapdoor Ciphers, W.H.Freeman and Co, , 1989, ISBN 0-7167-1987-8

[2] *Sloane, N.J.A. / Plouffe,Simon*: Encyclopedia of integer sequences, Academic Press Inc, 1995, ISBN 0-12-558630-2

Some Statistics – Tools for *DERIVE* and the *TI*-89/92 Part 1

Josef Böhm, Würmla, Austria

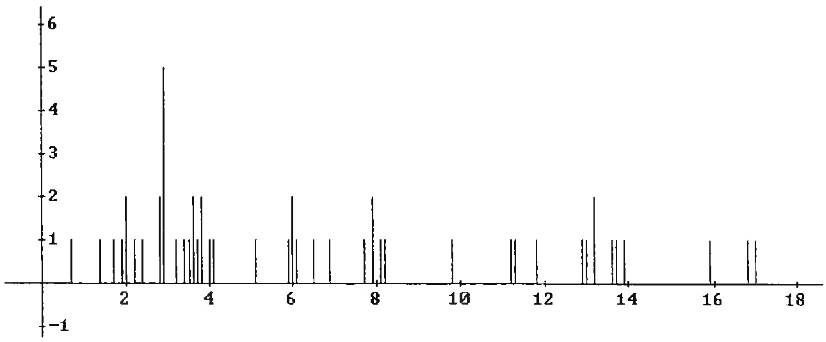
In this paper I'd like to present some tools for statistics using *DERIVE*-functions and programs. The *TI*-92 offers a lot of statistics representations like histograms, boxplots, regression lines, ... But this list can be also accomplished by a pie chart, by a stem & leaf diagram and others.

What we do need first are data. I use a list of Radon-Gas Concentrations [pc/l][1] of buildings belonging to one company

```
radon := [13.6, 2.8, 2.9, 3.8, 15.9, 1.7, 3.4, 13.7, 6.1, 16.8, 7.9, 3.5,
    2.2, 4.1, 3.2, 2.9, 3.7, 2.9, 2, 2.9, 11.2, 1.9, 2, 6, 2.9, 7.7, 5.1,
    13.2, 3.8, 13.9, 2.4, 7.9, 1.4, 5.9, 6.5, 11.8, 13.2, 2.8, 6.9, 0.7,
    12.9, 3.6, 3.6, 8.1, 17, 8.2, 9.8, 13, 11.3, 4, 6]
```

The most common representation is a frequency diagram which is delivered by plotting **freqdiag(list)**.

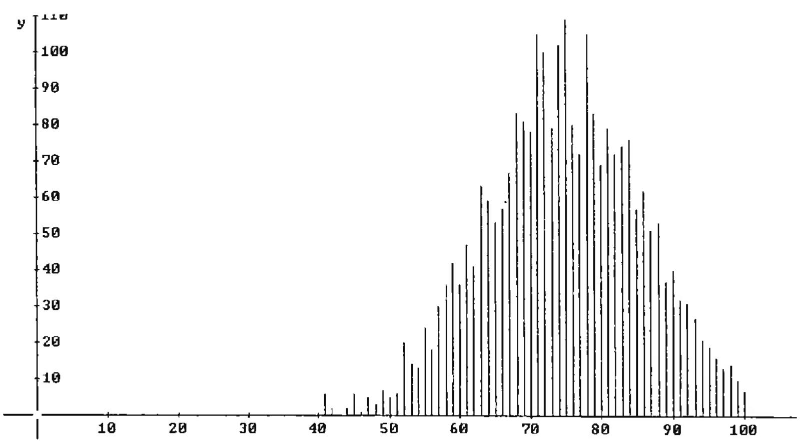
FREQDIAG(radon)



FREQDIAG(scores)

scores is a list of 2600 SAT scores, which I imported from a spreadsheet file (see *DNL*#41).

(All functions and programs used in this contribution can be downloaded. See Letter-of-the-Editor-page!).



The non-graphic form is the frequency table, which can be achieved by **FREQTAB(list)** und **FRETAB(list)**. The first gives the absolute frequencies in a horizontal table and the latter in a vertical one.

FREQTAB(radon) =

0.7	1.4	1.7	1.9	2	2.2	2.4	2.8	2.9	3.2	3.4	3.5	3.6
1	1	1	1	2	1	1	2	5	1	1	1	2

3.7	3.8	4	4.1	5.1	5.9	6	6.1	6.5	6.9	7.7	7.9	8.1	8.2	9.8	11.2	11.3
1	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1	1

11.8	12.9	13	13.2	13.6	13.7	13.9	15.9	16.8	17
1	1	1	2	1	1	1	1	1	1

FREPRG(radon)

0.7	1
1.4	1
1.7	1
1.9	1
2	2
2.2	1
2.4	1

FREQPRG(list) and **FREPRG(list)** do the same, but are based on a program, but very much faster.

Relative frequency tables **FREQPRGREL(list)** and **FREPRGREL(list)** are easy to realize by a slight modification of the two functions from above.

Let´s assume that we have to set up three classes houses, Class 1 showing a concentration up to 4.0 pc/l, Class 2 with a concentration exceeding 4.0 but less than or equal 10 16.0 pc/l and the dangerous Class3 with a value above 16.0 pc/l.

CLASSINT(classes,datalist) and **CLASSINTPR(classes,datalist)** return the absolute and relative frequencies respectively of the data given in datalist.
classes = $[x_1, x_2, x_3, x_4, \ldots, x_{n+1}]$ are the boundaries of the *n* classes, which are defined as being right open ($x_1 \le x < x_2, \ldots, x_n \le x < x_{n+1}$).

```
classint([0, 4.1, 16.1, 50], radon) = [24, 25, 2]
classintpr([0, 4.1, 16.1, 50], radon) = [47.0588, 49.0196, 3.92156]
```

CLASSPRP(props,data,variable) result in the same – now collected – frequency lists, but you can declare the properties of the classes in the list props:

prp := $[x \le 4,\ 4 < x \le 16,\ x > 16]$

CLASSPRP(prp, radon, x) =

$x \le 4$	24	47.0588
$4 < x \le 16$	25	49.0196
$x > 16$	2	3.92156

Let´s have two sidesteps now, the first of them showing the respective *DERIVE*-function:

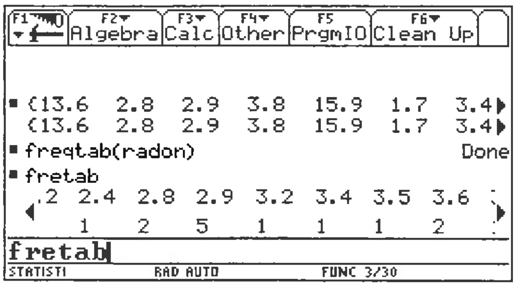
$$\text{CLASSPRP(props, data, x\_)} := \text{VECTOR}\left(\left[\text{props}_i,\ \text{DIMENSION(SELECT(props}_i\text{, x\_, data))},\ \frac{\text{DIMENSION(SELECT(props}_i\text{, x\_, data))}\cdot 100}{\text{DIMENSION(data)}}\right],\ \text{i, 1, DIMENSION(props)}\right)$$

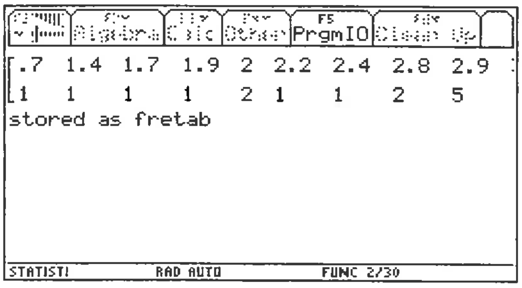
The second sidestep will lead us to the *TI*-92, to perform the same classification of a datalist:

For pupils – young and old as well– it is not so funny to have different names for functions working on different platforms. So I tried to stick to the same function and program names together with the same parameter list as far as possible. One difference is that you should enter lists between braces { } on the *TI*. So one can easily switch from the PC to the Handheld and vice versa. Let me show, how it works.

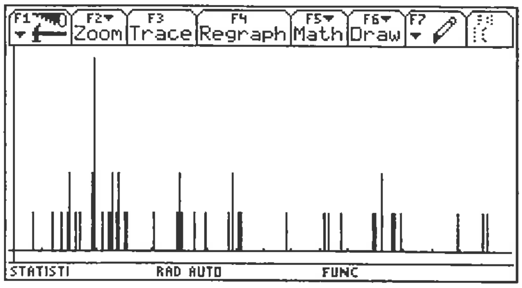
I don´t want to bore you with program listings, you can download and study them on your own. And I am sure that many of you will.

Edit the data as a list and call the program `freqtab(listname)`.



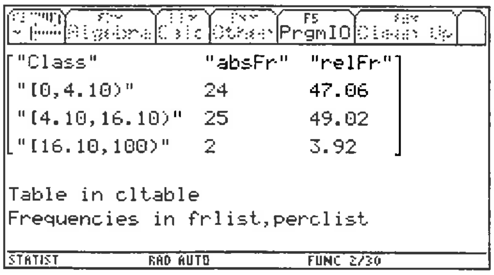


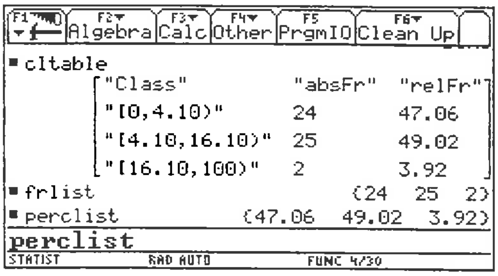
Applying `freqdiag(listname)` returns a frequency diagram of the absolute frequencies in the Graph-window.



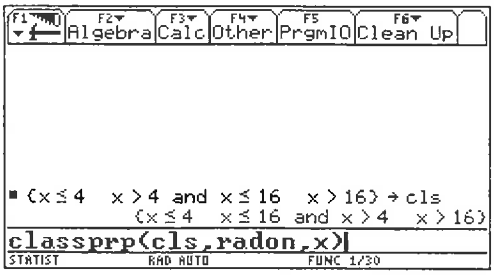
For working with classes use functions like in *DERIVE*:

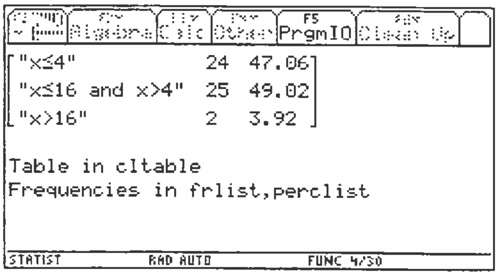
`classint({0,4.1,16.1,100},radon)`





`classprp({x <= 4.1, x>4.1 and x <= 16.1, x>16.1},radon,x)`





At several occasions you have classified data at your disposal and you want to give them a graphic representation form. And additionally in many cases you might find that the classes have various widths – which makes interpretation and graphic representation more difficult. One example follows (it was part of end examination 2001 on my school – but without using *DERIVE*. The students worked with the *TI* in the usual way – not using any additional functions like those presented in this contribution).

`smoke` is the number of female smokers of a population who are smoking a certain number of cigarettes/day The widths of the classes (buckets) are given by their boundaries in list `smoke_b`. Be careful, the buckets have different widths.

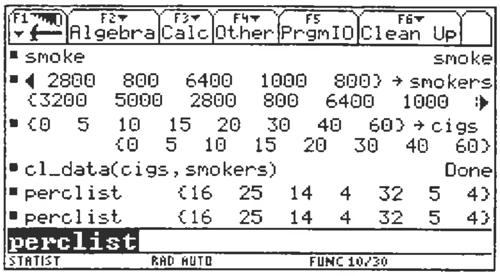
Use my *DERIVE*-function to treat classified data: **`cl_data(bounds,data)`**.

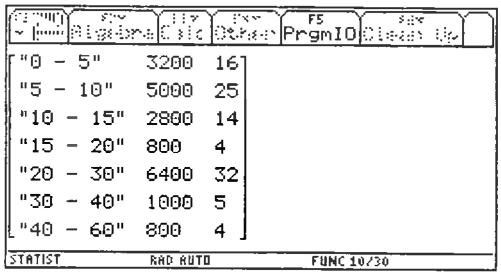
```
smoke := [3200, 5000, 2800, 800, 6400, 1000, 800]
smoke_b := [0, 5, 10, 15, 20, 30, 40, 60]
```

cl_data(smoke_b, smoke) =

0 - 5	3200	16
5 - 10	5000	25
10 - 15	2800	14
15 - 20	800	4
20 - 30	6400	32
30 - 40	1000	5
40 - 60	800	4

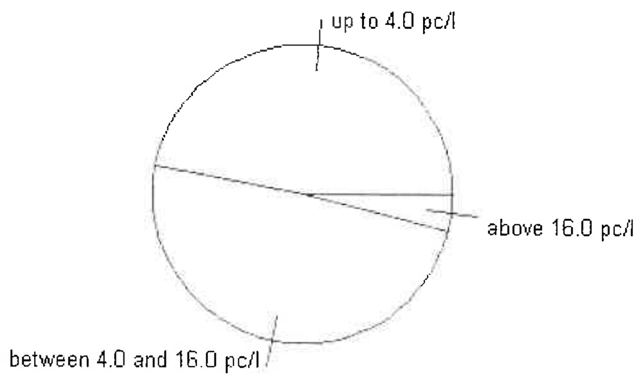
This is the way, how the TI is working. We will need this `perclist` for the next performance



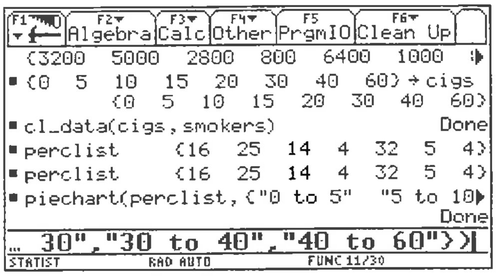


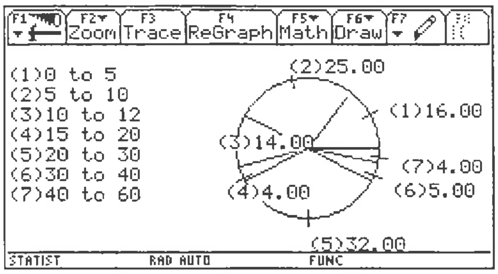
The most popular the many data presentation forms is the pie chart, but we miss it in *DERIVE* and on the TI-family as well. Make your own pie diagram, using **`piechart(freqlist)`**:

```
PIECHART(classint([0, 4.1, 16.1, 50], radon))
```



In *DERIVE* we have to add the annotations, but on the *TI* we can add the annotations in a list and using the graphing features, we can produce a diagram together with the labels.





(I did not include the settings for the Graph window in my program. So please set ZoomDec, Axes and Grid off, in order to obtain a nice diagram).

Next task: have a list of data and you want to collect the data in classes of equal width for later plotting a histogram. The *TI* is able to produce a histogram of ungrouped data. *DERIVE* is not, but only for the next lines, then *DERIVE* will plot wonderful histograms.

GROUP_FREQTAB(list,start,end,number_of_buckets) returns a table of – again right open – classes of equal widths together with the class frequencies and finally

HISTO(list,start,end,number_of_buckets) produces a plot of the respective histogram.

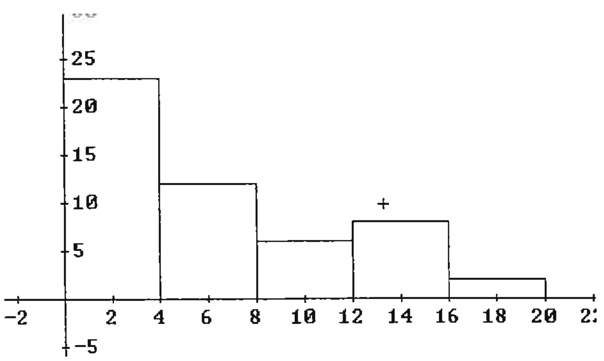
To overcome problems if the last element of the list coincides with the right boundary of the last class I adapted my function, so that the last class is formed by a closed intervall $[x_n,x_{n+1}]$. You may be interested in inspecting the functions in the *DERIVE* file.

This additional property is important for automatically generated grouped frequency tables.

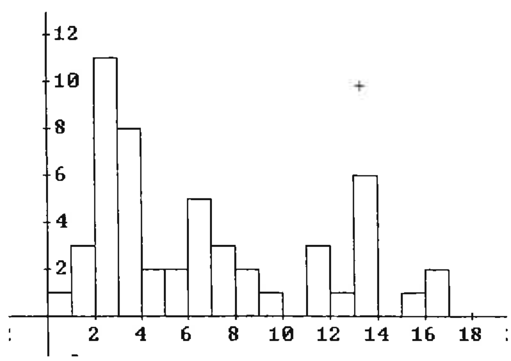
GROUP_FREQTAB(radon, 0, 20, 5) =

0 – 4	23	45.0980
4 – 8	12	23.5294
8 – 12	6	11.7647
12 – 16	8	15.6862
16 – 20	2	3.92156

#47: HISTO(radon, 0, 20, 5)

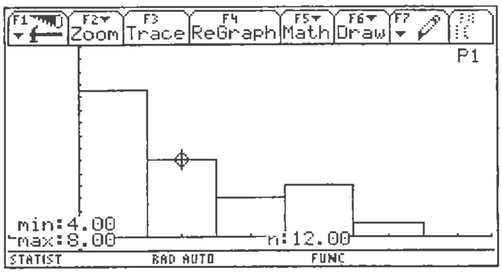


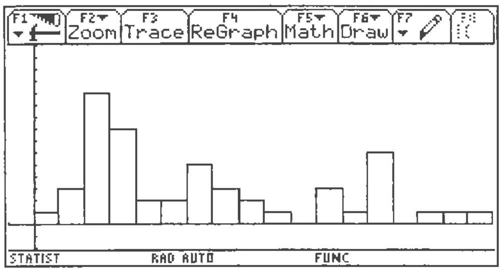
#48: HISTO(radon, 0, 17, 17)



Don't blame me for distributing 51 elements into 17 classes. I know the rule of thumb to have equal or less than $\sqrt{n}$ classes for n sample values. And I'll consider that in special functions. But I need this example violating this rule seriously to demonstrate an inconsistency between the *TI*-generated histogram and other histograms first.

I fill one column in a newly opened *TI*-DATA-sheet and produce the equivalent histograms on the *TI*.





The left plots don´t show any differences, but the right ones do. So we need to add some changes to make the functions for the two systems consistent – and also make them consistent to most of the textbooks. What is the reason for the different forms? Which one is right?

I will continue that discussion in the next *DNL* issue.

GROUP_FREQTAB(radon, MIN(radon), MAX(radon), 5)

0.7 – 3.96	23	45.0980
3.96 – 7.22	9	17.6470
7.22 – 10.48	6	11.7647
10.48 – 13.74	9	17.6470
13.74 – 17	4	7.84313

This is another table using the minimum- and maximum element as outermost boundaries. The price we pay are uncomfortable class intervals.

We meet the same fact using automatic grouping applying the above mentioned rule of thumb:

```
ncls(list) :=
  If DIM(list) ≤ 400
     FLOOR(√(DIM(list)))
     20
```

`ncls` is the number of classes (and blocks in the histogram).

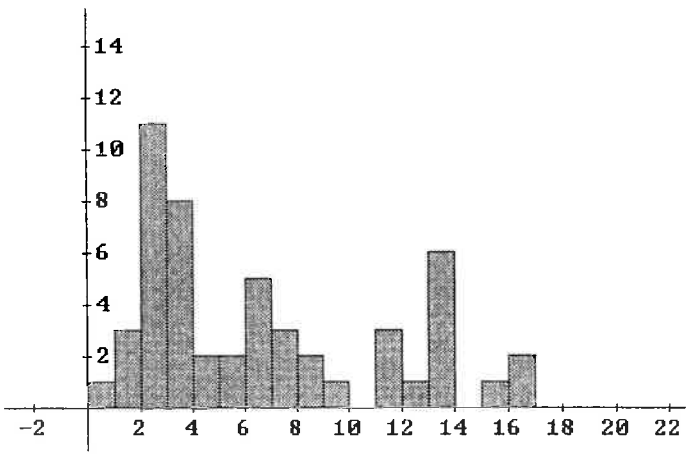
GROUP_FREQTAB_AUT(radon) =

0.7 – 3.02857	15	29.4117
3.02857 – 5.35714	11	21.5686
5.35714 – 7.68571	6	11.7647
7.68571 – 10.0142	6	11.7647
10.0142 – 12.3428	3	5.88235
12.3428 – 14.6714	7	13.7254
14.6714 – 17	3	5.88235

Here and now I´ll add a "shaded histogram".

`histosh(list,start,end,number_of_bucktes)` returns nice shaded histograms.

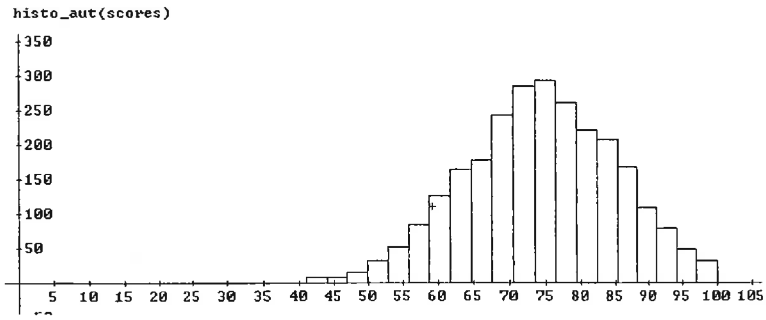
histosh(radon, 0, 17, 17)



What´s about the 2600 SAT-results?

See the top of the table together with the histogram:

(It takes *DERIVE* some time!)



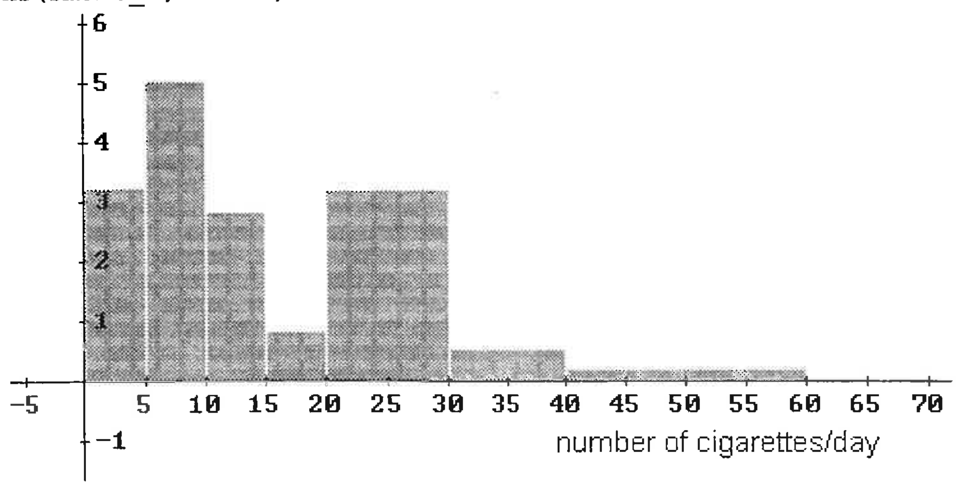
GROUP_FREQTAB_AUT(scores)

41 – 43.95	8	0.3076
43.95 – 46.9	9	0.3461
46.9 – 49.85	15	0.5769
49.85 – 52.8	31	1.192
52.8 – 55.75	51	1.961
55.75 – 58.7	84	3.230
58.7 – 61.65	125	4.807
61.65 – 64.6	163	6.269
64.6 – 67.55	177	6.807

The data of the smokers from above show buckets of various widths. In such cases it is dangerous to produce a histogram, because it often leads to biased representations, comparing always the areas of the bars. In order to avoid that, one has to adapt the **heights** of the bars that their areas are proportional to the frequencies, which should be represented. So automatically we are led to the *density function*, if the total area of all bars is fixed as 1 = 100%. In this case we are working with relative frequencies only. (For plotting the histogram you need not simplify the heights!)

```
heights(smoke_b, smoke) = [3.2, 5, 2.8, 0.8, 3.2, 0.5, 0.2]
```

```
plot_dens(smoke_b, smoke)
```



We asked our students: "*How would you label the y-axis?*" - What is your answer?

I showed all histograms – except the last one – with absolute frequencies. In the file `statut1.mth` you can find **histrel(...)** and **histoshrel(...)** to create the respective plots.

In the next *DNL* I'll continue with Boxplot and Modified Boxplot for *DERIVE*, Stem & Leaf Diagram for *DERIVE* and the *TI*, One-Variable-Statistics for *DERIVE* and comfortable use of various regression forms.

For references see the Book Shelf please.

Titbits from Algebra and Number Theory (21)

by Johann Wiesenbauer, Vienna

Before turning to the main topic of this column let me ask you the following question which has come up in a recent discussion via emails: By what features of his programs can you tell a proficient DERIVE-programmer most likely? I know this question is far from being easy and in fact there may be many different answers. As for me I would give the following advice: Simply check all auxiliary variables in his programs to see if they are really necessary! The same goes for SUB's , if there are any. In my experience, most programmers are far too "generous", when it comes to the introduction of new auxiliary variables and whenever lists and matrices are involved their programs usually abound with unnecessary SUB's, i.e. references to indices. In many cases those SUB's can be simply replaced by calls of the far more efficient functions FIRST( ) and REST( ).

To see what I mean, let's for example compute the greatest common divisor d of two positive integers a and b using their prime factorizations. Of course, this is sheer madness from an algorithmic point of view, since factoring of large numbers is known to be a very hard problem, whereas Euclid's algorithm (cf. DNL #38)) for the computation of the gcd(a,b) is very fast even for numbers with thousands of digits. All the same - just as a nice programming challenge following a suggestion by Enric Puig - let's do it this way. Take a look at my following program that does the trick without any auxiliary variable or occurrence of SUB's ! (The letter p in the name of function should remind of the fact that we use the prime factorizations of the arguments.)

```
gcdp(a, b) :=
  Prog
    a := SORT(APPEND(FACTORS(a), FACTORS(b)))
    b := 1
    Loop
      If DIM(a) < 2
         RETURN b
      If FIRST(FIRST(a)) = FIRST(FIRST(REST(a)))
         Prog
           b :* FIRST(FIRST(a))^FIRST(REST(FIRST(a)))
           a := REST(a)
      a := REST(a)
```

Of course, using gcd(a,b)=gcd($|a|,|b|$) as well as gcd(a,b)=$|a|+|b|$, whenever a=0 or b=0, we could have easily extended the domain of a and b to the set of integers, but this program isn't intended for real use anyway.

Speaking of gcd(a,b), currently (ver. 5.05) it is not quite clear from the description in the online help that my function EXTENDED_GCD(a,b) in the file number.mth also works for Gaussian integers a and b. For example, you could also compute

$$\text{EXTENDED\_GCD}(4 + 7\cdot\hat{\imath}, -6 + 7\cdot\hat{\imath}) = [2 + \hat{\imath}, [2 - 2\cdot\hat{\imath}, 1 + 2\cdot\hat{\imath}]]$$

which means that 2+i is a greatest common divisor of a = 4 + 7i and b = – 6 + 7i in the Ring **Z**[i] and

$$[4 + 7\cdot\hat{\imath}, -6 + 7\cdot\hat{\imath}]\cdot[2 - 2\cdot\hat{\imath}, 1 + 2\cdot\hat{\imath}] = 2 + \hat{\imath}$$

(Note that in general there are four gcd's in **Z**[i] and the others you get from the output one by multiplying it with all powers of i.)

Considering the special case of coprime integers a and b the output of extended_gcd(a,b) will be of the form [1,[x,y]], where x and y are integers such that 1= xa+yb. As a consequence, every integer c can be expressed by a integer linear combination of a and b. (Simply multiply the last equation by c !) In contrast, assuming that a and b are positive coprime integers and $c \in \mathbf{N}$ (**N** denotes here and in the following the set of natural numbers {0,1,2,..}), the Diophantine equation

$$c = xa+yb$$

hasn't always got solutions x,y∈**N**. For example, as you can easily check if a = 3 and b = 5 there is no such solution exactly for c= 1,2,4,7. As in this example, there are always only finitely many exceptional values of c though. (Try to prove this!) The problem of finding these exceptional values of c's (in particular, the greatest one!) and of solving the equation above for all other c's is often called the "stamp problem" in the literatur. (Here a and b are the values of two postage stamps and c is any value that can be achieved by using these two stamps only!)

In order to solve it let's consider the auxiliary set

$$S=\{xa+yb \mid 0 \le x < b, 0 \le y < a\}$$

Obviously, for all c∈S the equation c=xa+yb is solvable in **N**. More generally, for all c of the form c = s+ k (ab) with k∈**N** and s∈S there is a solution. (Simply increase either x by kb or y by ka in the solution x,y∈**N** for c=s !). On the other hand, it is not too difficult to see that the converse is also true: Any c such the equation above has a solution x,y∈**N** must be representable in this form. (Try to prove this!) Is your head near splitting? Well, let's again consider our example, where a=3 and b=5. Then S is given by

```
S := SORT(VECTOR(3·v  + 5·v , v, {0, ..., 4}·{0, ..., 2}))
                    1      2

          [0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 19, 22]
```

Now it is very instructive, if we rearrange the elements in the following way:

```
[VECTOR(IF(MEMBER?(x, S), x, *), x, 0, 14), VECTOR(IF(MEMBER?(x, S), x, *), x, 15, 29)]
```

0	*	*	3	*	5	6	*	8	9	10	11	12	13	14
*	16	17	*	19	*	*	22	*	*	*	*	*	*	*

Thus two facts become visible, which are also true in the general case:

1. S is a complete set of representatives mod ab. (For the cognoscenti this is actually an easy consequence of the so-called Chinese Remainder Theorem.)

2. There is a unique correspondence between the 4 numbers in S exceeding ab, namely 16, 17, 19 and 22 and the 4 gaps in the first row indicated by stars. But those four gaps correspond in turn to the numbers c=1,2,4,7 from above which cannot be represented in the form c=xa+ya with x,y∈**N** !

What is therefore the greatest number c∈ **N** not representable in this form? In our example it is 7 = $=22-15$, where 22 ist the biggest number in S, which we got for x=4, y=2. In general, this number is given by $a(b-1)+b(a-1)-ab = ab-a-b$.

If you feel like it, you could also try to give a formula for the number of c's for which the Diophantine requation is not solvable in **N**. (A few weeks ago someone on the Mathematics forum of Compuserve tried to do exactly this, but failed. You can look up my solution there in DERIVE-notation.) Moreover you could try to solve the analogous problem for 3 or even more stamps, but be warned: It is much harder! Needless to say that you will also find a lot of stuff on the "stamp problem" on internet, including programs to compute the quantities of stamps needed for a given c.

Let's turn to a completely different topic now. In my last column a introduced a number of useful routines for computations in symmetric groups. This time I will go a step further and set up some tools for computations in the more general class of symmetric semigroups.

Please note that you should use the currently latest version of DfW, i.e. 5.05, for this, since we are going to use "genuine" global variables in our programs. (As you may know, in former versions 5.xx global variables could only be be used but not permanently created within programs. In order to create them an explicit assignment outside of a program had been necessary! In my eyes, this has been clearly the most spectacular change for a long time and I welcome it wholeheartedly!)

In the following H_n denotes the symmetric semigroup over {1,2,..,n} for some fixed n. The elements of H_n, i.e. the mappings from {1,2,..,n} into itself, will be written in the form $[a_1, a_2, ..., a_n]$, meaning that $1 \mapsto a_1, 2 \mapsto a_2, \dots, n \mapsto a_n$. We call this the vector representation of the mapping in the following. As a rule, we prefer to refer to mappings by their symbolic name though, if one had been defined (see the function DEF(var,u,v) below).

And here are all setting and routines.

```
InputMode := Word

DisplayFormat:=Compressed

init:=PROG(defs:=[],ok)
```

defs is the global variable I was talking about. If you call the routine init, defs will be set to the empty list [ ]. (Of course, in this case the direct assignment defs:=[ ] would have been much shorter, but you could modify it according to your needs and I like the idea of having a routine that marks the beginning of a "session" and initializes all global variables.) defs contains all definitions of variables in terms of elements of H_n. We will see examples before long.

```
op(u,v):=
  Prog
    u:=FIRST(REST(FIRST(SELECT(FIRST(1_)=u,1_,defs))))
    v:=FIRST(REST(FIRST(SELECT(FIRST(1_)=v,1_,defs))))
    u:=[VECTOR(u↓k_,k_,v)]
    FIRST(FIRST(APPEND(SELECT(REST(1_)=u,1_,defs),[u])))
```

op(u,v) defines the composition of u and v, which are the symbolic names of some elements of H_n. The output is an element of H_n or rather its symbolic name, if one is available.

```
def(var,u,v):=
  Prog
    If ¬ VECTOR?(u)
       u:=op(u,v)
    v:=SELECT(FIRST(u_)=u,u_,defs)
    If v≠[]
       RETURN APPEND(STRING(FIRST(FIRST(v))),"!!!")
    v:=SELECT(REST(u_)=[u],u_,defs)
    If v≠[]
       RETURN APPEND(STRING(FIRST(FIRST(v))),"!!!")
    defs:=APPEND(defs,[[var,u]])
    "ok"
```

By means of def(var,u,v) a new variable var can be introduced. Here either u and v are the symbols of some elements of H_n, in which case var is defined to be their product, or u is the definition of a mapping and v is missing, in which case u gets the symbolic name given by var. In particular, the new definition of a variable will be appended to the list defs. Note that def(var,u,v) will not allow "coincidences", where a mapping gets two different names. In case of such a coincidence the older name will be shown as a result (followed by 3 exclamation marks!) and nothing else happens.

```
curtable:=VECTOR(VECTOR(op(u,v),v,defs COL 1),u,defs COL 1)
```

table yields the current operation table for all elements defined so far. Again, whenever it is possible, symbolic names are used. Hence, note that the occurrence of mappings in vector representation in this table clearly indicates that the operation for the set of current variables is not yet "closed".

```
vars:=MAP_LIST((FIRST(defs`))   ,k_,{1,...,DIM(defs)})
                             k_
```

Vars gives the currents set of symbolic names of all variables defined so far. We prefer the set notation to the list notation here, because often cartesian products of several copies of vars are needed in order to check certain conditions.

Phew! You can't make head or tail out of it so far? Don't worry! Everything will become perfectly clear if we compute some examples, which is just what we are going to do now.

Let's begin we computations in S_4 which looks familiar due to the computations in my last two "Titbits". In the following I defined the identity element e and the permutation a=[2,3,4,1] as well as some of its powers. When computing a^4 and trying to assign to the output, namely e, a new name, this coincidence was detected by the program as ahown below.

```
init=ok
def(e,[1,2,3,4])=ok
def(a,[2,3,4,1])=ok
def(a2,a,a)=ok
def(a3,a2,a)=ok
def(a4,a3,a)=e!!!
```

The current operation table looks this.

curtable

e	a	a2	a3
a	a2	a3	e
a2	a3	e	a
a3	e	a	a2

In particular, it is closed, that is $\{e, a, a^2, a^3\}$ form a subgroup of S_4. Now let's add b=[3,2,1,4] and see what happens!

```
def(b,[3,2,1,4])=ok
```

curtable

e	a	a2	a3	b
a	a2	a3	e	[4,3,2,1]
a2	a3	e	a	[1,4,3,2]
a3	e	a	a2	[2,1,4,3]
b	[2,1,4,3]	[1,4,3,2]	[4,3,2,1]	e

```
[def(ab,a,b),def(a2b,a2,b),def(a3b,a3,b)]=[ok,ok,ok]
```

curtable

e	a	a2	a3	b	ab	a2b	a3b
a	a2	a3	e	ab	a2b	a3b	b
a2	a3	e	a	a2b	a3b	b	ab
a3	e	a	a2	a3b	b	ab	a2b
b	a3b	a2b	ab	e	a3	a2	a
ab	b	a3b	a2b	a	e	a3	a2
a2b	ab	b	a3b	a2	a	e	a3
a3b	a2b	ab	b	a3	a2	a	e

What we have finally got here is the dihedral group D_4, i.e. the 8-element group of symmetries of a square, which we used in the last "Titbits" to get all solutions of the n-queens problems "modulo symmetries". It goes without saying that this operation table is far more instructive (and less space consuming!) as the the one in vector representation.

Are you ready for a more difficult example? After all, I wanted to show you that my routines also work in symmetric semigroups.

Suppose you want to construct the free algebra A generated by {a,b} that obeys the laws

$$(xy)z = x(yz), \quad x^3 = x, \quad xyx = x^2y$$

for all $x,y,z \in A$. If you from products starting with a and b you will sooner or later arrive at the following 12 products:

$$a, a^2, b, b^2, ab, a^2b, ab^2, a^2b^2, ba, b^2a, ba^2, b^2a^2$$

It is easy to see that are no more products, because if you multiply any of these products by a from the left, you will get no new products, and the same goes for b. For example, you get

$$a \cdot ba = aba = a^2b, \quad a \cdot b^2a = ab^2a = a^2b^2, \quad a \cdot ba^2 = (aba)a = a^2ba = a(aba) = a(a^2b) = a^3b = ab$$

The crucial question is: Are those 12 products really different or is there any trick we just don't see that would show for example the coincidence of a^2b^2 and b^2a^2?

Here is where our tool comes into play. Let's number the products above with 1,2,...,12. Then the left multiplication with a is given by the mapping [2,1,5,7,6,5,8,7,6,8,5,7] and the left multiplication by b is similarly given by the mapping [9,11,4,3,10,12,9,11,10,9,12,11]. (Check this!) Hence, all we have to do is to input all products above and wait for our program to report a coincidence! But there is none!

```
init=ok
def(a, [2,1,5,7,6,5,8,7,6,8,5,7])=ok
def(a2,a,a)=ok
def(b, [9,11,4,3,10,12,9,11,10,9,12,11])=ok
[def(b2,b,b),def(ab,a,b),def(a2b,a2,b),def(ab2,a,b2),def(a2b2,a2,b2)]=[ok,ok,ok,ok,ok]
[def(ba,b,a),def(b2a,b2,a),def(ba2,b,a2),def(b2a2,b2,a2)]=[ok,ok,ok,ok]
```

And here comes the operation table of the free algebra we are looking for in all its glory!

*	a	a2	b	b2	ab	a2b	ab2	a2b2	ba	b2a	ba2	b2a2
a	a2	a	ab	ab2	a2b	ab	a2b2	ab2	a2b	a2b2	ab	ab2
a2	a	a2	a2b	a2b2	ab	a2b	ab2	a2b2	ab	ab2	a2b	a2b2
b	ba	ba2	b2	b	b2a	b2a2	ba	ba2	b2a	ba	b2a2	ba2
b2	b2a	b2a2	b	b2	ba	ba2	b2a	b2a2	ba	b2a	ba2	b2a2
ab	a2b	ab	ab2	ab	a2b2	ab2	a2b	ab	a2b2	a2b	ab2	ab
a2b	ab	a2b	a2b2	a2b	ab2	a2b2	ab	a2b	ab2	ab	a2b2	a2b
ab2	a2b2	ab2	ab	ab2	a2b	ab	a2b2	ab2	a2b	a2b2	ab	ab2
a2b2	ab2	a2b2	a2b	a2b2	ab	a2b	ab2	a2b2	ab	ab2	a2b	a2b2
ba	ba2	ba	b2a	ba	b2a2	b2a	ba2	ba	b2a2	ba2	b2a	ba
b2a	b2a2	b2a	ba	b2a	ba2	ba	b2a2	b2a	ba2	b2a2	ba	b2a
ba2	ba	ba2	b2a2	ba2	b2a	b2a2	ba	ba2	b2a	ba	b2a2	ba2
b2a2	b2a	b2a2	ba2	b2a2	ba	ba2	b2a	b2a2	ba	b2a	ba2	b2a2

Oh no, no space left and there is so much more I should have told you! Well, maybe the next time!

(j.wiesenbauer@tuwien.ac.at)

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